Helical black holes with higher derivative corrections

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Work in progress with Daniel Brattan, Abhiram Kidambi and Amos Yarom

Motivation

- 5D RN black holes in AdS unstable to helical phase if CS-term in

$$S = \int d^5x\sqrt{-g}[(R+12) - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}] - \frac{2}{3}\alpha\int F\wedge F\wedge A$$

large enough, i.e. $\alpha > \alpha_c \approx 0.2896$ [Nakamura, Ooguri, Park]

- 5D minimal gauged $\mathcal{N}=2$ SUGRA:

$$\alpha_s = \frac{1}{2\sqrt{3}} \approx 0.2887 < \alpha_c$$

- But $\frac{\alpha_c - \alpha_s}{\alpha_c} \approx 0.003$

- Could higher derivative corrections to α_s and α_c lead to $\alpha_s > \alpha_c$, i.e. an instability?
- New BH solutions in string theory with helical structure of horizon?
- Potential applications in AdS/CFT?

[Related work: Takeuchi; Ooguri, Park]

- Minimal 5D SUGRA relevant:

For any supersymmetric solution of $D = 10$ or $D = 11$ supergravity that consists of a warped product of $d+1$ dimensional anti-de-Sitter space with a Riemannian manifold M, $AdS_{d+1} \times_w M$, there is a consistent Kaluza-Klein truncation on M to a gauged supergravity theory in $d+1$-dimensions for which the fields are dual to those in the superconformal current multiplet of the d-dimensional dual SCFT. [Gauntlett, Varela]

Overview

- Review instability without higher derivative corrections
  [Nakamura, Ooguri, Park; Donos, Gauntlett]

- Review higher derivative corrections to 5D minimal $\mathcal{N} = 2$ gauged SUGRA and RN black hole
  [Myers, Paulos, Sinha]

- Instability with higher derivative corrections

- Outlook

Instability without higher derivative corrections

- Instability most likely to occur at $T = 0$

  $\Rightarrow$ consider extremal RN

- Near horizon geometry $AdS_2 \times \mathbb{R}^3$

  $\Rightarrow$ look for modes with $m^2_{AdS_2} < m^2_{BF} = -\frac{1}{4r_2^2}$

r_2 : AdS_2-radius

- Ansatz: [Donos, Gauntlett; cf. also Nakamura, Ooguri, Park]

$$ds^2 = \frac{-dt^2 + dr^2}{12r^2} + d\vec{x}^2 + Q^2 dt^2 + 2Q\omega_2 dt$$

with

$$\omega_2 = \cos(kx_1)dx_2 - \sin(kx_1)dx_3$$

- Killing vectors: $\partial_{x_2},\ \partial_{x_3}$

$$\partial_{x_1} - k(x_2\partial_{x_3} - x_3\partial_{x_2})$$

- $A = \frac{E}{12r}dt + b\omega_2$ with $E = 2\sqrt{6}$ near horizon electrical field

- To linear order in b, Q (enough to analyze onset of instability):

$$(\Box_{AdS_2} - k^2)\psi + E\,\Box_{AdS_2} b = 0$$

$$(\Box_{AdS_2} - k^2)b - 4\alpha E k b + E\psi = 0$$

with $\psi = -12 r^2 Q'$

- Strategy: (1) Determine effective mass $m^2(k,\alpha)$

  (2) Determine $k_0(\alpha)$ minimizing $m^2(k,\alpha)$ for fixed $\alpha \Rightarrow m^2_{\min}(\alpha) = m^2(k_0(\alpha),\alpha)$

  (3) Find α_c for which $m^2_{\min}(\alpha) < m^2_{BF}$ for $\alpha > \alpha_c$

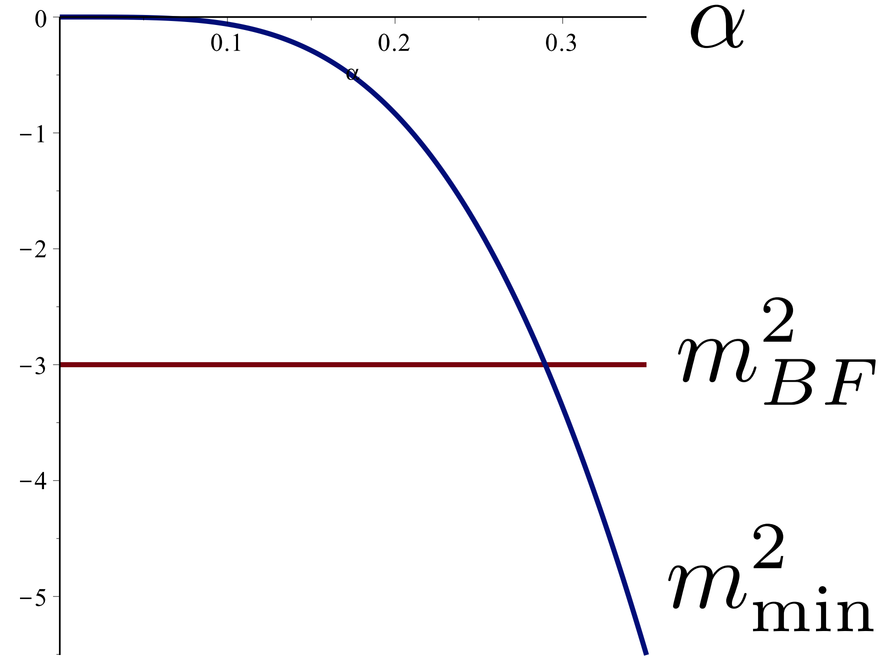
Concretely:

(1) $$\det \begin{pmatrix} m^2 - k^2 & Em^2 \\ E & m^2 - k^2 - 4\alpha E k \end{pmatrix} = 0$$

$$\Rightarrow m^2 = \frac{1}{2}\left(2k^2 + E^2 + 4\alpha E k - E\sqrt{E^2 + 8\alpha E k + 4k^2(1+4\alpha^2)}\right)$$

(2) $$k_0 = E\frac{2\alpha + 4\alpha^3 + \alpha\sqrt{1 + 4\alpha^2 + 16\alpha^4}}{1 + 4\alpha^2}$$

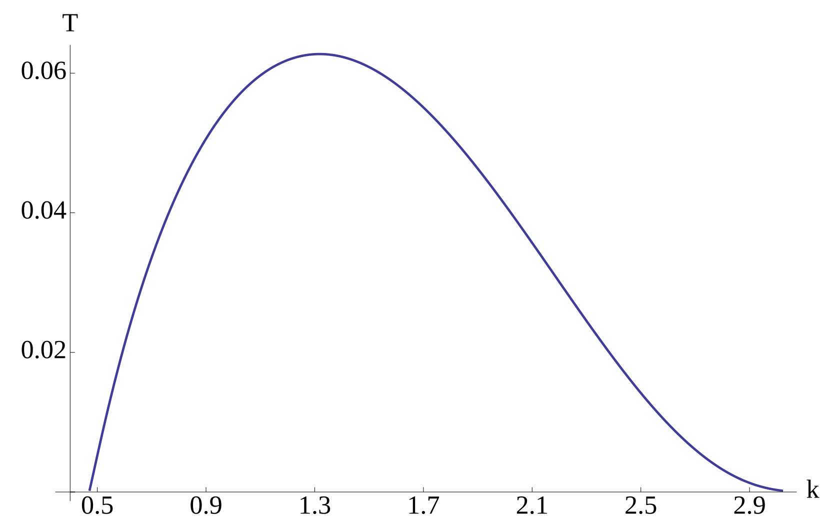
(3) $\alpha_c = 0.2896$



- α_c coincides with value obtained by looking for normalizable fluctuations in full geometry

[Nakamura, Ooguri, Park]

- For $\alpha > \alpha_c$ instability appears for range of k
  e.g. $\frac{\alpha}{\alpha_c} \approx 1.47$



[Taken from: Donos, Gauntlett]

- Solution with particular $k(T)$ minimizes free energy at fixed T

[Donos, Gauntlett]

Higher derivative terms

- Most general form up to 4 derivatives (modulo field redefinitions and partial integration) [Myers, Paulos, Sinha]

$$S = S_0 + \int d^5x\sqrt{-g}\,[c_1 R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma} + c_2 R_{\mu\nu\rho\sigma}F^{\mu\nu}F^{\rho\sigma} + c_3(F^2)^2 + c_4 F^4 + c_5\epsilon^{\mu\nu\rho\sigma\tau}A_\mu R_{\nu\rho\alpha\beta}R_{\sigma\tau}{}^{\alpha\beta}]$$

- $c_1 = \frac{1}{8}\frac{c-a}{c}$

$$T^a_a = \frac{c}{16\pi^2}W_{abcd}W^{abcd} - \frac{a}{16\pi^2}(R_{abcd}R^{abcd} - 4R_{ab}R^{ab} + R^2)$$

Weyl-tensor

4D Euler density

- If A dual to $\mathcal{N}=1$ R-symmetry current:

$$c_2 = -\frac{c_1}{2}\,, \quad c_3 = \frac{c_1}{24}\,, \quad c_4 = -\frac{5c_1}{24}\,, \quad c_5 = \frac{c_1}{2\sqrt{3}}\,, \quad \alpha_s = \frac{1-288c_1}{2\sqrt{3}}$$

- Leave c_i arbitrary for moment, but for sensible derivative expansion need $\forall_i : c_i \ll 1$

$\Rightarrow$ supersymmetric case: $c \sim a \gg 1$

- Following above strategy, need to take into account

[Myers, Paulos, Sinha]

★ Correction to condition for extremality:

$$\frac{q^2}{r_0^6} = 2[1-48(c_1-2(2c_3+c_4))]$$

★ Correction to AdS_2-radius: $r_2^2 = \frac{1}{12} + (4c_2 + 16c_3 + 8c_4)$

$$\Rightarrow m_{BF}^2 = -\frac{1}{4r_2^2} = -(3-144c_2-576c_3-288c_4)$$

Ansatz:

$$ds^2 = \frac{-dt^2 + dr^2}{(12 - 576c_2 - 2304c_3 - 1152c_4)r^2} + d\vec{x}^2 + Q^2dt^2 + 2Qdt\omega_2$$

$$A = \left(\frac{2\sqrt{6}}{12} - 4\sqrt{6}(c_1 + 2c_2 + 4c_3 + 2c_4) \right) r^{-1}dt + b\omega_2$$

Plug this into Einstein & Maxwell eqs.

$\Rightarrow$ 4th order eqs. for b, Q

- Toy example: $Q'' + AQ' + BQ + cQ''' = 0$

Define: $\psi = Q + cQ'$

$$\Rightarrow \psi'' + A\psi' + B\psi = -\cancel{cQ'''} + \cancel{cQ'''} + AcQ'' + BcQ'$$

$$\Rightarrow (1 - cA)\psi'' + (A - cB)\psi' + B\psi = 0(+\mathcal{O}(c^2))$$

- Toy example: $Q'' + AQ' + BQ + cQ''' = 0$

  Define: $\psi = Q + cQ'$

$$\Rightarrow \psi'' + A\psi' + B\psi = -c\cancel{Q'''} + c\cancel{Q'''} + AcQ'' + BcQ'$$

$$\Rightarrow (1 - cA)\psi'' + (A - cB)\psi' + B\psi = 0(+\mathcal{O}(c^2))$$

- In our case, define

$$\psi = -12r^2Q' - c_1 2304 r^3 Q'' - c_1 576 r^4 Q''' - c_2 96\sqrt{6} r^2 b''$$

$$\Rightarrow (1 + a_1)\Box_{AdS_2}\psi + (-k^2 + a_2)\psi + (2\sqrt{6} + a_3)\Box_{AdS_2} b + a_4 b = 0$$

$$(1 + a_5)\Box_{AdS_2} b + (-k^2 - 8\sqrt{6}k\alpha + a_6)b + a_7\Box_{AdS_2}\psi + (2\sqrt{6} + a_8)\psi = 0$$

$\forall_i : a_i(k, \alpha, c_j)$ constants linear in c_j

Result:

$$\alpha_c = \alpha_c^{(0)} + 11.82c_1 + 37.06c_2 + 183.67c_3 + 55.00c_4 - 12.61c_5$$

i.e. in supersymmetric case:

$$\alpha_c = \alpha_c^{(0)} - 14.16c_1$$

$$\frac{1}{8}\frac{c-a}{c}$$

Result:

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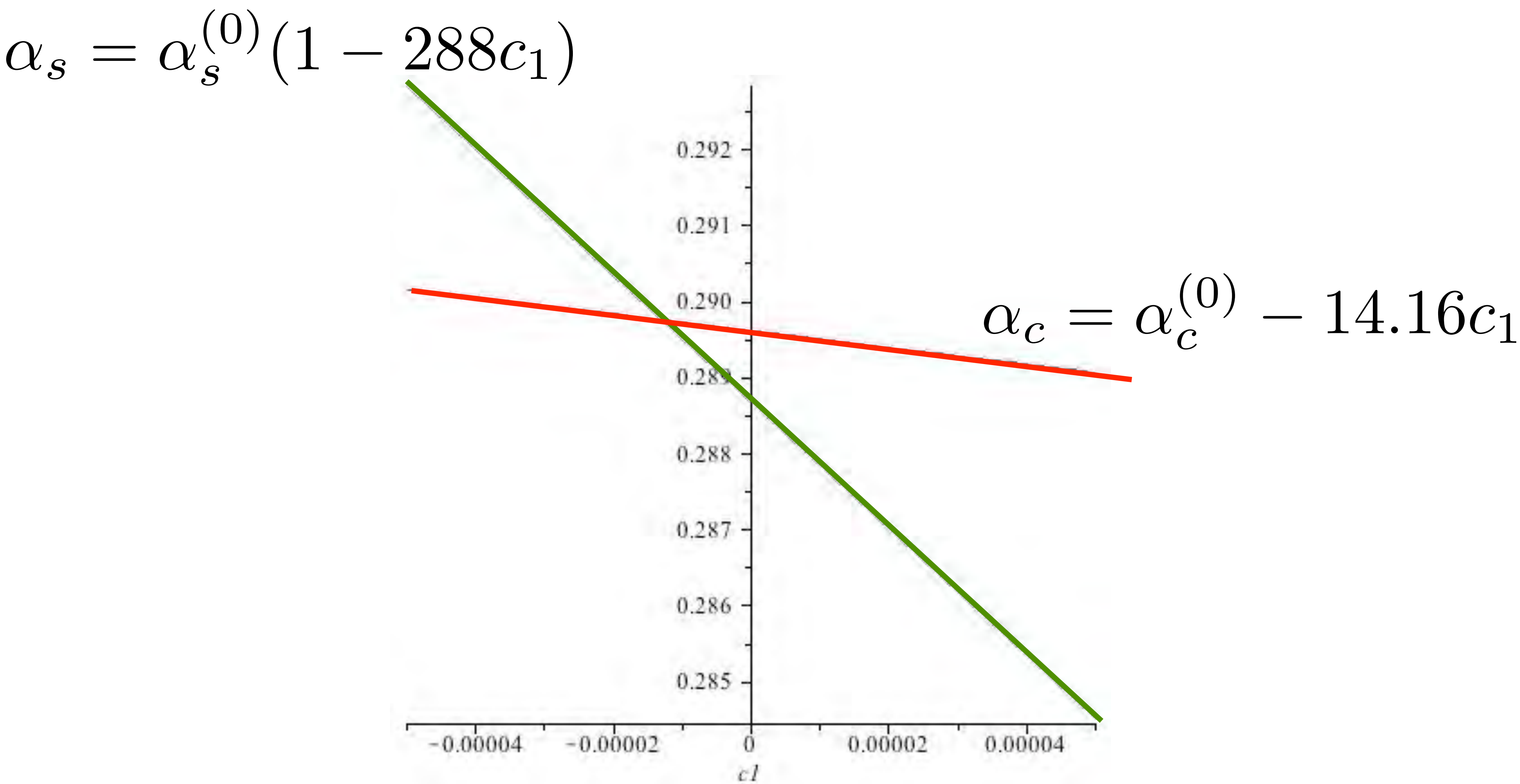
i.e. in supersymmetric case:

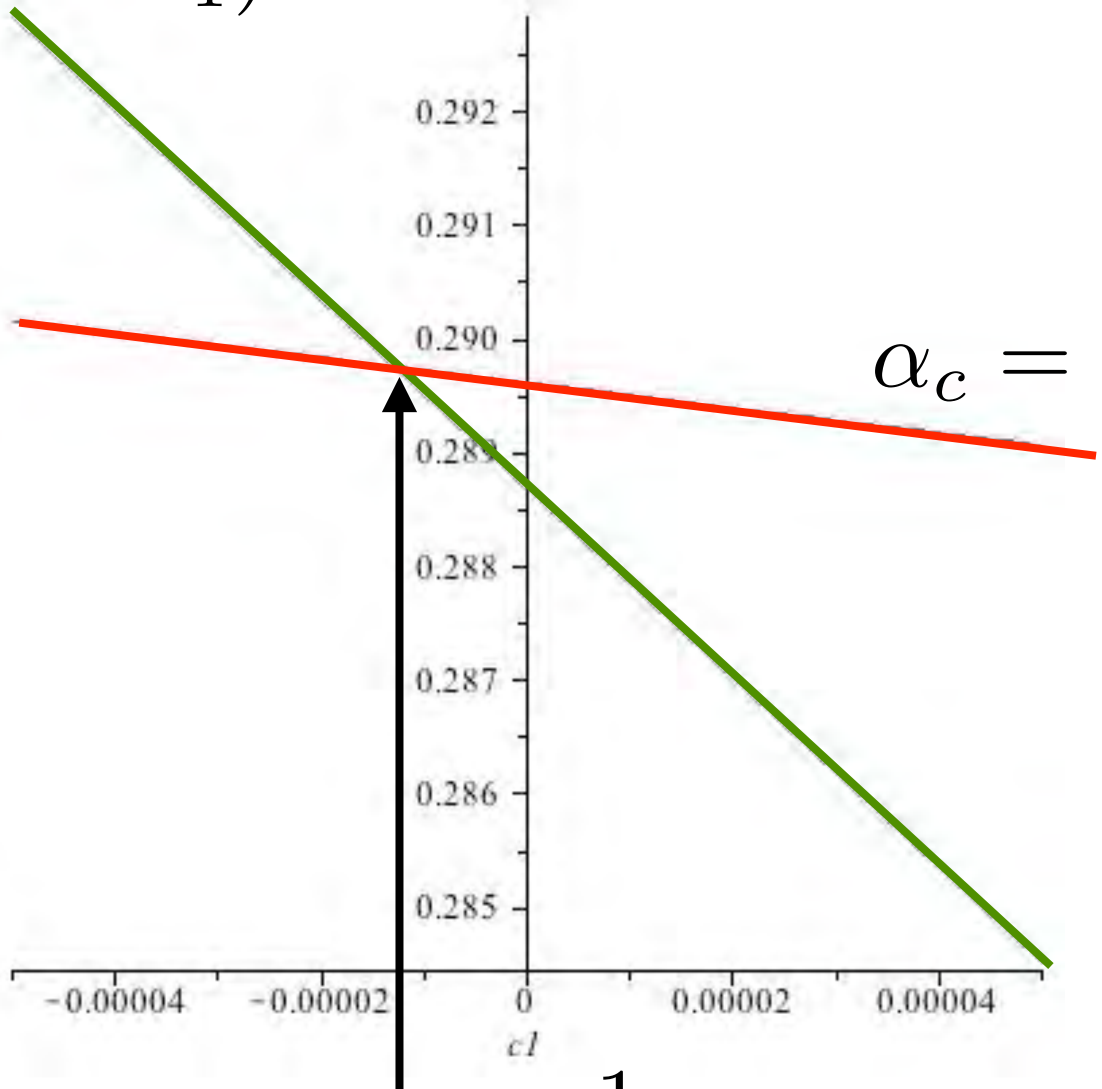
$$\alpha_c = \alpha_c^{(0)} - 14.16 c_1$$

$$\frac{1}{8}\frac{c-a}{c}$$

But α_s also decreases with positive c_1


$\alpha_s = \alpha_s^{(0)}(1 - 288c_1)$
$\alpha_c = \alpha_c^{(0)} - 14.16c_1$
0.292
0.291
0.290
0.289
0.288
0.287
0.286
0.285
-0.00004
-0.00002
0
0.00002
0.00004
c1

$$\alpha_s = \alpha_s^{(0)}(1 - 288c_1)$$

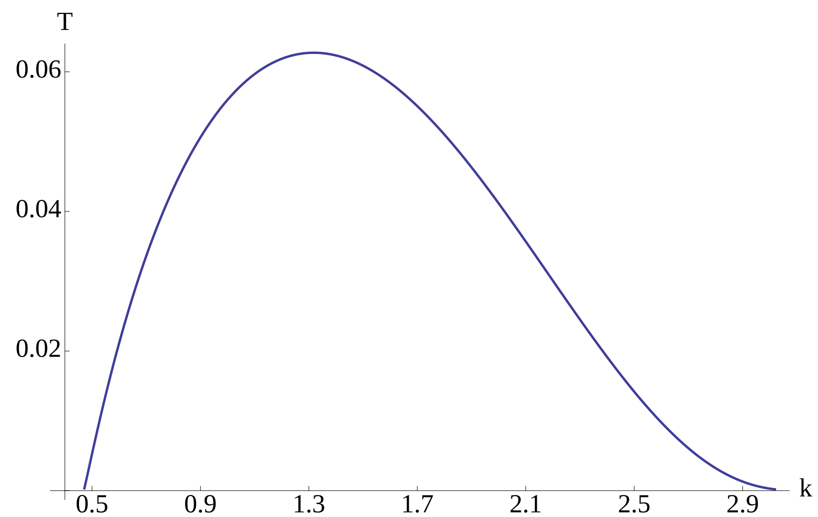


$$\alpha_c = \alpha_c^{(0)} - 14.16c_1$$

$$c_1 = \frac{1}{8}\frac{c-a}{c} = -1.30027 \times 10^{-5}$$

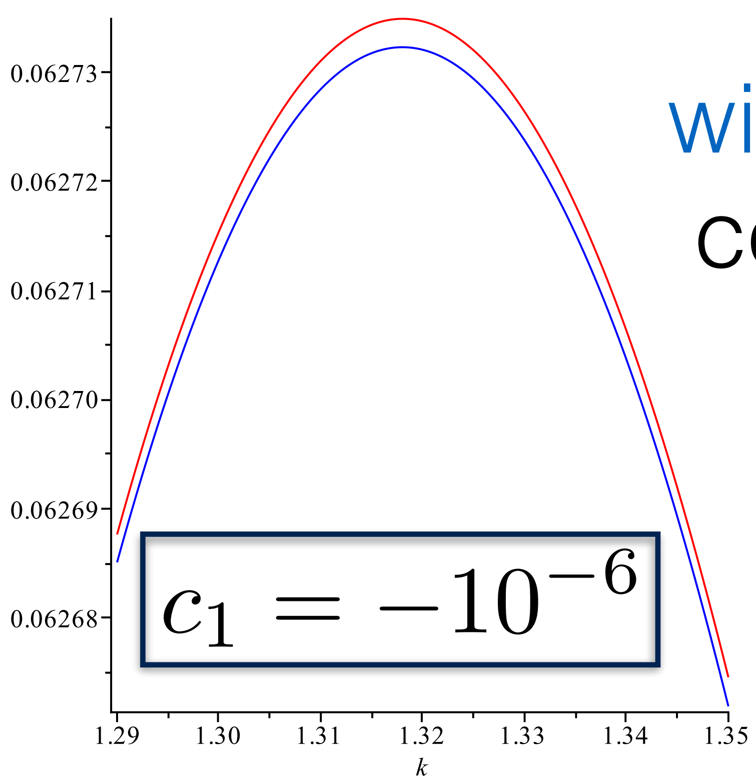
Analysis in full background

- Reminder: $\left(\frac{\alpha}{\alpha_c} \approx 1.47\right)$

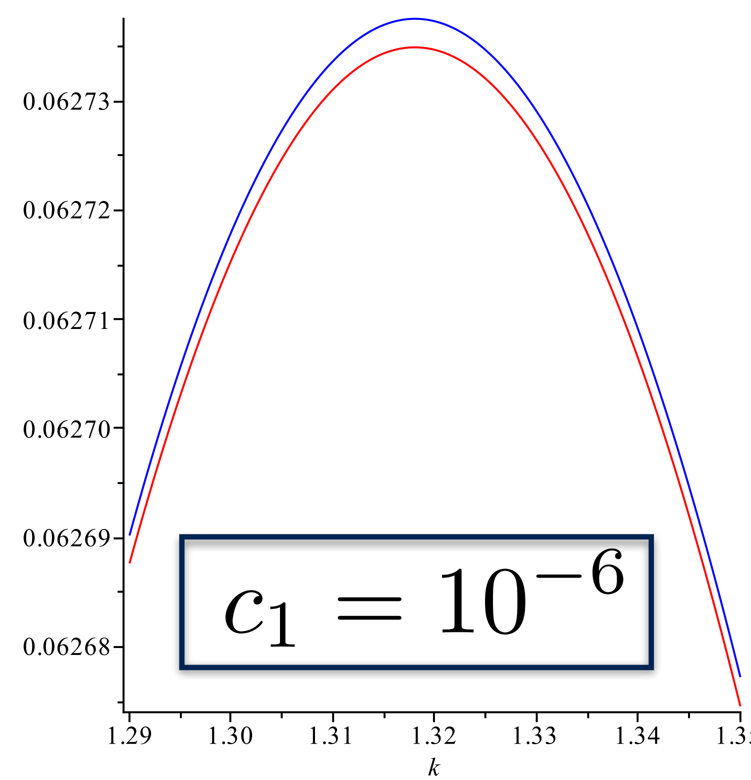


[Taken from: Donos, Gauntlett]

- Higher derivative corrections (still $\alpha \approx 1.47\alpha_c$):



with & without corrections



$c - a$

- Violations of the KSS bound: $\frac{\eta}{s} = \frac{1}{4\pi}\left(1 - \frac{c-a}{c} + \ldots\right)$

  [Brigante, Liu, Myers, Shenker, Yaida; Buchel, Myers, Sinha]

- Mixed current-gravitational anomaly:

  [Anselmi, Freedman, Grisaru, Johansen]

$$D_a J^a = \frac{c-a}{24\pi^2} R_{abcd}\tilde{R}^{abcd}$$

- Superconformal indices at high $T = \beta^{-1}$

$$\sum (-1)^F e^{-\beta(\Delta + \frac{1}{2}R)} \approx e^{-\frac{16\pi^2}{3\beta}(a-c)}$$

[Di Pietro, Komargodski]

- Single-trace higher spin gap in large N SCFT

[Camanho, Edelstein, Maldacena, Zhiboedov]

$$\left|\frac{a-c}{c}\right| \lesssim \frac{1}{\Delta_{\mathrm{gap}}^2}$$

dim. of lightest higher spin single-trace operator

- In CFTs with $a > c$, universal term in entanglement entropy can become negative for certain higher genus entangling surfaces

[Perlmutter, Rangamani, Rota]

Results on $c - a$

- In $\mathcal{N} = 1$ SCFT: $\frac{1}{2} \leq \frac{a}{c} \leq \frac{3}{2}$ [Hofmann, Maldacena]

  with $\frac{a}{c} = \frac{3}{2}$ for free theory with only vector multiplets.

  However, $\frac{a - c}{c} = \frac{1}{2} \not\ll 1$

- „Normal“ large N_c CFTs (with $SU(N_c), SO(N_c), Sp(N_c)$ gauge group) have $c > a$ [Buchel, Myers, Sinha]

- Non-Lagrangian theories arising as IR limit of world-volume theories of N M5-branes wrapping Riemann surface with $g > 1$ can have $a - c > 0$

  [Gaiotto; Gaiotto, Maldacena]

Conclusion

- Higher derivative corrections to minimal 5D gauged $\mathcal{N}=2$ SUGRA could make R-charged RN black holes unstable if dual $\mathcal{N}=1$ CFT has $a > c$

- Outlook:

  - ★ Confirm near horizon result by fluctuations in full geometry

  - ★ $\alpha \to \infty$ limit should be tractable analytically

[Ovdat, Yarom]

ευχαριστώ πολύ!