Black holes in $\mathcal{N} = 8$ supergravity

Eighth Crete Regional Meeting in String Theory, Nafplion

David Chow

University of Crete

9 July 2015

Introduction

4-dimensional $\mathcal{N} = 8$ (maximal) supergravity:

- Low energy regime of M-theory compactified on T^7
- In perturbation theory, UV finite up to 4 loops, ...?
- Setup for addressing black hole microscopics

Aim: Find general (stationary, non-extremal, asymptotically flat) black hole of $\mathcal{N} = 8$ supergravity

STU supergravity: truncation of $\mathcal{N} = 8$ with 4 $\mathrm{U}(1)$ gauge fields

Classical U-duality (global symmetries) (Cvetič Hull 96) $\Longrightarrow$
General STU solution is generating solution for general $\mathcal{N} = 8$ solution, mixing matter fields, but same 4d metric

Equivalent aim: Find general black hole of STU supergravity

- 10 parameters: mass, rotation, 4 electric, 4 magnetic
  (5 electromagnetic charges suffice, but 8 is more symmetric)

Based mainly on 1404.2602 (with Geoffrey Compère)

Outline of solution generating technique

STU supergravity: 4d metric + 4 vectors + 6 scalars

reduce on $t \to$ 3d metric + 5 vectors + 11 scalars

Hodge dualize $\to$ 3d metric + 16 scalars

3d scalars parameterize coset (Breitenlohner Maison Gibbons 88)

$$G/K = \mathrm{SO}(4,4)/\mathrm{SL}(2,\mathbb{R})^4$$

Lagrangian: 3d euclidean gravity + scalars (in $\mathrm{SO}(4,4)$ matrix $\mathcal{M}$)

$$\mathcal{L}_3 = R \star_3 1 - \tfrac{1}{8}\mathrm{Tr}[\star_3(\mathcal{M}^{-1}\,\mathrm{d}\mathcal{M}) \wedge (\mathcal{M}^{-1}\,\mathrm{d}\mathcal{M})]$$

Efficient 1-step generating technique:

$$\mathrm{d}s_3^2 = \mathrm{d}s_3^2, \qquad \boxed{\mathcal{M} = g^\sharp \mathcal{M}_0 g}$$

$\mathcal{M}_0$: seed solution g: $\mathrm{SO}(4,4)$ group element

$\mathcal{M}$: new solution $\sharp$: $\mathrm{SO}(4,4)$ generalized transpose

(Chong Cvetič Lü Pope 04, Gal'tsov Scherbluk 08, Bossard Michel Pioline 09)

STU supergravity

4-dimensional $\mathcal{N} = 2$ supergravity coupled to 3 vector multiplets

Bosonic fields:

- Metric: g_{ab}
- 4 $\mathrm{U}(1)$ gauge fields: A^I, $I = 1, 2, 3, 4$ (or dual fields $\widetilde{A}_I$)
- 6 real scalars: φ_i "dilatons", χ_i "axions", $i = 1, 2, 3$
  (3 complex scalars $z_i = \chi_i + \mathrm{i}e^{-\varphi_i}$, sometimes called S, T, U)

Bosonic Lagrangian of form

$$\mathcal{L}_4 = R \star 1 - \tfrac{1}{2}\textstyle\sum_i(\star\mathrm{d}\varphi_i \wedge \mathrm{d}\varphi_i + \mathrm{e}^{2\varphi_i} \star \mathrm{d}\chi_i \wedge \mathrm{d}\chi_i)$$
$$- \tfrac{1}{2}\textstyle\sum_{I,J}[F_{IJ}(\boldsymbol{\varphi}, \boldsymbol{\chi}) \star F^I \wedge F^J + G_{IJ}(\boldsymbol{\varphi}, \boldsymbol{\chi}) F^I \wedge F^J]$$

Scalars parameterize coset $\overline{G}/\overline{K} = (\mathrm{SL}(2, \mathbb{R})/\mathrm{U}(1))^3$

Special case: Einstein–Maxwell theory (A^I all equal, $\varphi_i = \chi_i = 0$)

Field equations invariant under $\mathrm{SL}(2, \mathbb{R})^3$ and triality (permuting $\mathrm{SL}(2, \mathbb{R})$'s)

STU supergravity continued

Bosonic Lagrangian

$$\begin{aligned}\mathcal{L}_4 = R\star 1 &- \tfrac{1}{2}\textstyle\sum_{i=1}^{3}(\star \mathrm{d}\varphi_i \wedge \mathrm{d}\varphi_i + \mathrm{e}^{2\varphi_i}\star \mathrm{d}\chi_i \wedge \mathrm{d}\chi_i)\\ &- \tfrac{1}{2}\textstyle\sum_{i=1}^{3} \mathrm{e}^{2\varphi_i-\varphi_1-\varphi_2-\varphi_3}\star(\widetilde{F}_i - \chi_i F^4)\wedge(\widetilde{F}_i - \chi_i F^4)\\ &- \tfrac{1}{2}\mathrm{e}^{-\varphi_1-\varphi_2-\varphi_3}\star F^4 \wedge F^4 + \chi_1\chi_2\chi_3 F^4\wedge F^4\\ &+ \tfrac{1}{2}\textstyle\sum_{\{i,j,k\}=\{1,2,3\}}(\chi_i \widetilde{F}_j \wedge \widetilde{F}_k - \chi_j\chi_k \widetilde{F}_i \wedge F^4)\end{aligned}$$

$(\widetilde{A}_1, \widetilde{A}_2, \widetilde{A}_3, A^4)$ duality frame: triality invariance manifest, and prepotential exists

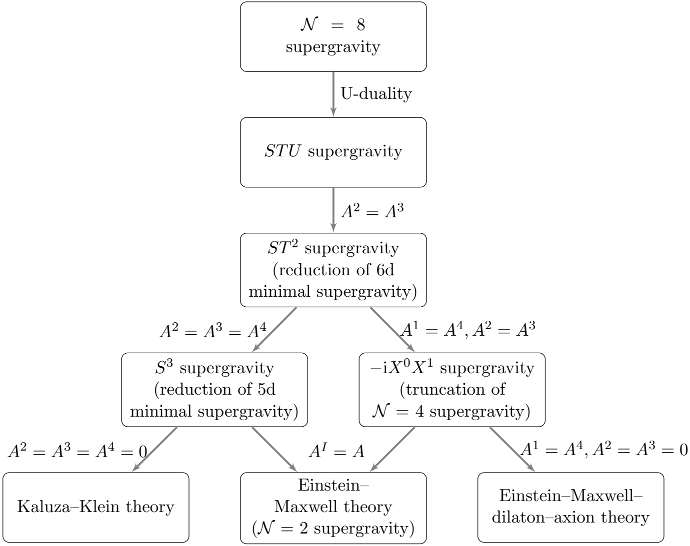
$\mathrm{SL}(2,\mathbb{R})_1 \times \mathrm{SL}(2,\mathbb{R})_2 \times \mathrm{SL}(2,\mathbb{R})_3$ global symmetry:

$$\begin{pmatrix}\widetilde{F}_1\\ F^4\end{pmatrix} \to \Lambda \begin{pmatrix}\widetilde{F}_1\\ F^4\end{pmatrix}, \qquad \begin{pmatrix}\widetilde{F}_2\\ F^3\end{pmatrix} \to (\Lambda^{\mathsf{T}})^{-1} \begin{pmatrix}\widetilde{F}_2\\ F^3\end{pmatrix}$$

$$\begin{pmatrix}\widetilde{F}_4\\ F^1\end{pmatrix} \to \Lambda \begin{pmatrix}\widetilde{F}_4\\ F^1\end{pmatrix}, \qquad \begin{pmatrix}\widetilde{F}_3\\ F^2\end{pmatrix} \to (\Lambda^{\mathsf{T}})^{-1} \begin{pmatrix}\widetilde{F}_3\\ F^2\end{pmatrix}$$

$$\Lambda = \begin{pmatrix}a & b\\ c & d\end{pmatrix} \in \mathrm{SL}(2,\mathbb{R})_1, \qquad z_1 \to \frac{az_1 + b}{cz_1 + d}$$

Truncations of $\mathcal{N} = 8$ supergravity



Basic example: 4d gravity $\to$ 3d

Restrict to stationary solutions $\implies$ Kaluza–Klein reduce on t

Reduction ansatz (timelike):

$$\mathrm{d}s^2 = -\mathrm{e}^{2U}(\mathrm{d}t + \omega_3)^2 + \mathrm{e}^{-2U}\,\mathrm{d}s_3^2$$

Hodge dualize ω_3 to 3d scalar σ:

$$\mathrm{d}\omega_3 = -\tfrac{1}{2}\mathrm{e}^{-4U}\star_3\mathrm{d}\sigma$$

$\implies$ 3d euclidean gravity + scalars:

$$\mathcal{L}_3 = R\star_3 1 - 2\star_3\mathrm{d}U\wedge\mathrm{d}U - \tfrac{1}{8}\mathrm{e}^{-4U}\star_3\mathrm{d}\sigma\wedge\mathrm{d}\sigma$$

Hidden $\mathrm{SL}(2,\mathbb{R})$ "Ehlers symmetry" revealed in 3d:
Scalars (U,σ) parameterize $\mathrm{SL}(2,\mathbb{R})/\mathrm{U}(1)$

Basic example: 4d gravity → 3d: $\mathrm{SL}(2,\mathbb{R})/\mathrm{U}(1)$ coset

$\mathfrak{sl}(2,\mathbb{R})$ generators:

Cartan	positive-root	negative-root
$H = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$,	$E = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}$,	$F = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}$

Scalars (U, σ) parameterize $\mathrm{SL}(2,\mathbb{R})/\mathrm{U}(1)$ coset: $\mathcal{V} = \mathrm{e}^{-UH}\mathrm{e}^{-\sigma E/2}$

Define $\mathcal{M} \equiv \mathcal{V}^{\mathsf{T}}\mathcal{V} \in \mathrm{SL}(2,\mathbb{R})$:

- $\mathcal{M}$ is "gauge-invariant", i.e. $\mathcal{V} \sim k\mathcal{V}$ for $k \in \mathrm{SO}(2) = \mathrm{U}(1)$
- 3d scalars (U, σ) can be extracted from $\mathcal{M}$

3d theory as manifest $\mathrm{SL}(2,\mathbb{R})$ coset:

$$\begin{aligned}\mathcal{L}_3 &= R \star_3 1 - 2 \star_3 \mathrm{d}U \wedge \mathrm{d}U - \tfrac{1}{8}\mathrm{e}^{-4U} \star_3 \mathrm{d}\sigma \wedge \mathrm{d}\sigma \\ &= R \star_3 1 - \tfrac{1}{8}\mathrm{Tr}[\star_3(\mathcal{M}^{-1}\,\mathrm{d}\mathcal{M}) \wedge (\mathcal{M}^{-1}\,\mathrm{d}\mathcal{M})]\end{aligned}$$

Basic application: generate Taub–NUT from Schwarzschild

STU supergravity $\to$ 3d

Reduction ansatz:

$$\mathrm{d}s^2 = -\mathrm{e}^{2U}(\mathrm{d}t + \omega_3)^2 + \mathrm{e}^{-2U}\,\mathrm{d}s_3^2,$$

$$A^I = \zeta^I(\mathrm{d}t + \omega_3) + A^I_{(3\mathrm{d})}, \qquad \widetilde{A}_I = \widetilde{\zeta}_I(\mathrm{d}t + \omega_3) + \widetilde{A}_{I(3\mathrm{d})}$$

16 3d scalars after Hodge dualization:

- U, σ (dualizing ω_3)
- 8 electromagnetic scalars ζ^I, $\widetilde{\zeta}_I$ (dualizing $\widetilde{A}_{I(3\mathrm{d})}$, $A^I_{(3\mathrm{d})}$)
- 3 dilatons φ_i, 3 axions χ_i

3d scalars parameterize $\mathrm{SO}(4,4)/\mathrm{SL}(2,\mathbb{R})^4$

$\mathrm{SO}(4,4)$ generalized transpose $\sharp$:

$$g^\sharp = \eta g^{\mathsf{T}} \eta^{-1}, \qquad \eta = \mathrm{diag}(-1,-1,1,1,-1,-1,1,1)$$

$\mathrm{SO}(4,4)$ matrices satisfy $\mathcal{M}^\sharp = \mathcal{M}$

STU supergravity $\to$ 3d: $\mathrm{SO}(4,4)/\mathrm{SL}(2,\mathbb{R})^4$ coset

28 $\mathfrak{so}(4,4)$ generators:

Cartan (4)	Positive-root (12)	Negative-root (12)
H_Λ	E_Λ, E^{Q_I}, E^{P^I}	F_Λ, F^{Q_I}, F^{P^I}

$\Lambda = 0$: Ehlers $\mathrm{SL}(2,\mathbb{R})$ $\Lambda = 1,2,3$: scalar fields $\mathrm{SL}(2,\mathbb{R})^3$
$I = 1,2,3,4$: gauge fields

16 scalars parameterize coset in Iwasawa gauge $g = \not{g_K}\, g_H\, g_E$:

$$\mathcal{V} = \exp(-U H_0)\exp(\tfrac{1}{2}\textstyle\sum_i \varphi_i H_i)\exp(-\sum_i \chi_i E_i)$$
$$\exp[-\textstyle\sum_I(\zeta^I E^{Q_I} + \widetilde{\zeta}_I E^{P^I})]\exp(-\tfrac{1}{2}\sigma E_0)$$

3d Lagrangian with manifest $\mathrm{SO}(4,4)$ symmetry:

$$\mathcal{L}_3 = R \star_3 1 - \tfrac{1}{8}\mathrm{Tr}[\star_3(\mathcal{M}^{-1}\,\mathrm{d}\mathcal{M}) \wedge (\mathcal{M}^{-1}\,\mathrm{d}\mathcal{M})], \quad \mathcal{M} \equiv \mathcal{V}^\sharp \mathcal{V}$$

Useful to define 1-form $\mathcal{N} \in \mathfrak{so}(4,4)$ (Breitenlohner Maison 00):

$$\mathrm{d}\mathcal{N} = \mathcal{M}^{-1}\star_3 \mathrm{d}\mathcal{M} = \mathrm{d}\omega_3\, F_0 + \textstyle\sum_I(\mathrm{d}A^I_{(3\mathrm{d})}\, F^{P^I} - \mathrm{d}\widetilde{A}_{I(3\mathrm{d})}\, F^{Q_I}) + \ldots$$

Generating the charged solution

Method generalizes "Harrison transformation" of Einstein–Maxwell theory, which uses $\mathrm{SU}(2,1)/\mathrm{S}(\mathrm{U}(1,1)\times\mathrm{U}(1))$ coset

1. Initial seed solution: Kerr–Taub–NUT (3 parameters m, n, a)
2. Act with group element (adds 8 charge parameters δ_I, γ_I)

$$g = \exp\left(-\sum_I \gamma_I (E^{P^I} - F^{P^I})\right) \exp\left(-\sum_I \delta_I (E^{Q_I} - F^{Q_I})\right)$$

3. Final 11-parameter solution: 3d scalars from $\mathcal{M} \to g^\sharp \mathcal{M} g$ ($\mathcal{N} \to g^{-1}\mathcal{N}g$ gives directly 3d 1-forms $\omega_3, A^I_{(3\mathrm{d})}, \widetilde{A}_{I(3\mathrm{d})}$)

Seed Kerr solution often generates NUT charge
$\implies$ start with more general Kerr–Taub–NUT seed instead

Choice of g motivated by:

- Symmetry in gauge fields
- $g^\sharp g = \mathbb{I}$: preserves asymptotic flatness at spatial infinity

Summary of general charged solution

11 physical charges	11 parameters
Mass M, NUT charge N	m, n
Angular momentum J	a
Electric charges Q_I	δ_I
Magnetic charrges P^I	γ_I

Set $N = 0$ (linear constraint on seed m, n)
$\rightarrow$ 10-parameter asymptotically flat family

Unifies various previous solutions (Demiański Newman 66, Rasheed 95, Cvetič Youm 96, Matos Mora 96, Larsen 99, Lozano-Tellecha Ortín 99, Chong Cvetič Lü Pope 04, Giusto Saxena Ross 07, Compère de Buyl Stotyn Virmani 10)

Thermodynamics

Thermodynamic quantities satisfy usual relations:

- First law: $\mathrm{d}E = T\,\mathrm{d}S + \Omega\,\mathrm{d}J + \sum_I (\Phi^I\,\mathrm{d}\overline{Q}_I + \Psi_I\,\mathrm{d}\overline{P}^I)$
- Smarr relation: $E = 2TS + 2\Omega J + \sum_I (\Phi^I \overline{Q}_I + \Psi_I \overline{P}^I)$

Product of inner and outer horzion areas (Larsen 97, Cvetič Larsen 97, Cvetič Gibbons Pope 10):

$$S_+ S_- = 4\pi^2 \left(J^2 + \Delta(Q_I, P^I)\right) \in \pi^2\mathbb{Z}$$

Cayley hyperdeterminant:

$$\Delta = \tfrac{1}{16}[4(\textstyle\prod_I Q_I + \prod_I P^I) + 2\sum_{I<J} Q_I Q_J P^I P^J \\ - \textstyle\sum_I (Q_I)^2 (P^I)^2]$$

- $\mathrm{SL}(2,\mathbb{R})^3$-invariant and triality-invariant
- Special case of $\mathrm{E}_{7(7)}$ quartic invariant $\diamondsuit$ (Kallosh Kol 96)

Non-extremal entropy formula:

$$S_+ = 2\pi\left(\sqrt{\Delta + F} + \sqrt{-J^2 + F}\right)$$

$S_+, J, \diamondsuit$ $\mathrm{E}_{7(7)}$-invariant $\implies$ $F(M, Q_I, P^I, z^i_\infty)$ also invariant

Killing tensors and separability

Kerr metric admits Killing–Stäckel tensor:

$$K_{ab} = K_{(ab)}, \quad \nabla_{(a}K_{bc)} = 0, \quad K^{ab}P_a P_b \text{ conserved on geodesics}$$

General STU supergravity charged black holes admit Killing–Stäckel tensor $\widetilde{K}_{ab}$ in "string frame" $\mathrm{d}\widetilde{s}^2 = \Omega^2\,\mathrm{d}s^2$
$\implies$ Conformal Killing–Stäckel tensor

$$Q_{ab} = Q_{(ab)}, \quad \nabla_{(a}Q_{bc)} = q_{(a}g_{bc)}, \quad (q_a = \tfrac{1}{6}(\nabla_a Q^b{}_b + 2\nabla_b Q^b{}_a))$$

in Einstein frame, components $Q^{ab} = \widetilde{K}^{ab}$

- $Q^{ab}P_a P_b$ conserved on null geodesics

Separation of:

- Massive/massless geodesics in string/Einstein frame
- Massless Klein–Gordon equation in Einstein frame

$\mathrm{U}(1)^4$ gauged supergravity and AdS black holes

Maximal $\mathcal{N}=8$, $\mathrm{SO}(8)$-gauged supergravity truncates to $\mathcal{N}=2$, $\mathrm{U}(1)^4$-gauged supergravity (Cvetič *et al.* 99):

$$\mathcal{L}_{\text{gauged}} = \mathcal{L}_{\text{ungauged}} + g^2 \textstyle\sum_i (2\cosh\varphi_i + \chi_i^2 \mathrm{e}^{2\varphi_i}) \star 1$$

g: gauge-coupling constant/inverse AdS radius

Potential breaks 4d global symmetry $\mathrm{SL}(2,\mathbb{R})^3 \to \mathrm{U}(1)^3$
$\implies$ previous techniques fail, so guess instead

Two classes of asymptotically AdS generalizations:

- Static solutions, 10 parameters: mass, AdS radius, 4 electric charges, 4 magnetic charges, (S^2, $\mathbb{R}^2$, H^2 horizon)
- Rotating solutions, 8 parameters: mass, NUT charge, rotation, AdS radius, 2 electric charges, 2 magnetic charges ($\mathrm{U}(1)^2$ truncation of $\mathcal{N}=4$, $\mathrm{SO}(4)$-gauged supergravity)

Includes known subcases (Duff Liu 99, Chong Cvetič Lü Pope 05, Lü Pang Pope 13, Lü 13)

Summary

$\mathrm{SO}(4,4)$ solution generating technique for STU supergravity

Solutions:

- General rotating black holes of STU supergravity with 11 parameters: mass, NUT, rotation, 4 electric, 4 magnetic
  $\implies$ General black holes in $\mathcal{N}=8$ supergravity with 59 parameters: mass, NUT, rotation, 28 electric, 28 magnetic
- Also some AdS black holes of $\mathrm{U}(1)^4$-gauged supergravity

Some properties:

- $\mathrm{E}_{7(7)}$-invariant non-extremal entropy formula:

$$S = 2\pi(\sqrt{\lozenge + F} + \sqrt{-J^2 + F})$$

- Killing tensors, separability

Open problems:

- Beyond coset models: inverse scattering, more general $\mathcal{N}=2$ theories
- Complete proof of uniqueness

This research has been co-financed by the European Union (European Social Fund, ESF) and Greek national funds through the Operational Program "Education and Lifelong Learning" of the National Strategic Reference Framework (NSRF), under the grant schemes "Funding of proposals that have received a positive evaluation in the 3rd and 4th Call of ERC Grant Schemes" and the program "Thales".

Summary of general charged solution: metric

Coordinates: (t, r, ϕ, u), $u = n + a\cos\theta$

Metric:

$$\mathrm{d}s^2 = -\frac{R-U}{W}(\mathrm{d}t + \omega_3)^2 + W\left(\frac{\mathrm{d}r^2}{R} + \frac{\mathrm{d}u^2}{U} + \frac{RU\,\mathrm{d}\phi^2}{a^2(R-U)}\right)$$

$$R = r^2 + a^2 - n^2 - 2mr, \qquad U = a^2 - (u-n)^2$$

$W^2(r, u)$ quartic

Summary of general charged solution: matter

Gauge fields:

$$A^I = -W \frac{\partial}{\partial \delta_I}\left(\frac{\mathrm{d}t + \omega_3}{W}\right)$$

Scalar fields:

$$\chi_i = \frac{f_i}{r^2 + u^2 + g_i}, \qquad \mathrm{e}^{-\varphi_i} = \frac{W}{r^2 + u^2 + g_i},$$

$$f_i(r,u) = 2(mr + nu)\xi_{i1} + 2(mu - nr)\xi_{i2} + 4(m^2 + n^2)\xi_{i3},$$

$$g_i(r,u) = 2(mr + nu)\eta_{i1} + 2(mu - nr)\eta_{i2} + 4(m^2 + n^2)\eta_{i3}$$