The correspondence between free fermionic models and orbifolds

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Motivation

- String theory provides the most promising framework for a fundamental theory of physics.
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The landscape problem

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- We would like to understand how many models exist that closely resemble the SM (MSSM) and ultimately find a dynamical way to select among them...
- There have been extensive computer scans towards that goal, both in the (bosonic) orbifold and in the free fermionic formulation.
  Fischer, Ratz, Torrado, Vaudrevange 2013, Faraggi, Rizos, Sonmez 2014, ...

  And of course there are other approaches as well...
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- –Which formalism is better?
  –"Ours!"

Motivation

It would be very useful to have a dictionary between

- the orbifold formalism (OF)
  Dixon, Harvey, Vafa, Witten '87
- and the free fermionic formalism (FFF)
  Antoniadis, Bachas, Kounnas '87, Kawai, Lewellen, Tye '87

that would allow us to compare the previous results.

Previous work

Various aspects of the correspondence have been discussed in the past
Kiritsis, Kounnas '97, Gregori, Kounnas, Rizos '99, Donagi, Faraggi '04, Donagi, Wendland '08

However, a complete model builder's dictionary that would allow for a computational comparison is still missing.

Motivation

Bonus:

Equivalent formulations of particular models allow us to use tools from one formalism to solve difficult problems in the other. For example:

- It is much easier to construct asymmetric orbifold actions in the FFF than in the OF.
- It is much easier to move in the Narain moduli space in the OF but not in the FFF.
- and more...

Orbifold models

We are interested in **toroidal orbifolds**. Such models are specified by:

1) A **Narain lattice** on which the internal 6 dimensions (and the gauge degrees of freedom) are compactified.
2) An **orbifold action** compatible with the lattice.
3) A choice of the relative phases when we have more than one action (**discrete torsion**).

Free fermionic models

In these models all the degrees of freedom needed to cancel the conformal anomaly are implemented as worldsheet fermions.

Free fermionic models are specified by:

1) A set of **basis vectors** that describe the boundary conditions of the worldsheet fermions around the cycles of the worldsheet torus.

2) A choice of the relative phases between different basis vectors (**discrete torsion**).

Converting from one to the other

To convert a free fermionic model to an orbifold we must know how to implement the following steps:

1) Choose how to bosonize, *ie.* which fermions to combine.
2) Extract the **Narain lattice** from the basis vectors.
3) Extract the **orbifold action** from the basis vectors.
4) Extract the orbifold phases (**discrete torsion**) from the free fermionic phases.

1) Choosing which fermions to combine

In FFF the supercurent is non-lineraly realized in terms of the worlsheet fermions λ^i:

$$T_F = \psi_\mu \partial X^\mu + f_{IJK}\lambda^I\lambda^J\lambda^K.$$

The requirement of $N = 1$ spacetime SUSY restricts the constants f_{IJK} to be the stucture constants of $SU(2)^6$. This groups the fermions into 6 sets with three fermions each, call them χ_I, y_I, w_I, $I = 1, \dots 6$.

We choose to group y_I and w_I together and convert them into X_I.

2) Extracting the Narain lattice

In a 2d CFT bosons and fermions are equivalent and we can convert from one to the other using

$$y + iw = :e^{iX}:$$

which is know as the **bosonization/fermionization formula**.

The relation above assumes that the bosons are compactified on a circle with a specific radius (or on a specific lattice in the general case). This is known as the **fermionic point** in the moduli space of lattice compactifications.

2) Extracting the Narain lattice

The geometric data of the orbifold model can be read from the untwisted part of the partition functions in the two formalisms, *i.e.*

$$\mathcal{Z}_{\text{FFF}} = \underbrace{\sum_{\alpha,\beta} C\left[{\alpha \atop \beta}\right] Z\left[{\alpha \atop \beta}\right]}$$

$$\downarrow$$

$$\mathcal{Z}_{\text{orbi}} = \mathcal{Z}_{\text{untwisted}}(G, B, A, g) + \mathcal{Z}_{\text{twisted}}$$

$$\|$$

$$\sum_{p_L, p_R} q^{\frac{1}{2} p_L(G,B,A,g)^2}\, \bar{q}^{\frac{1}{2} p_R(G,B,A,g)^2}$$

2) Extracting the Narain lattice

As an example, the free fermionic model with basis vectors $\{\mathbb{1}, \mathbf{S}\}$ corresponds to a bosonic model with:

$$G = \mathbb{1}_6$$

$$B = \begin{pmatrix} 0 & -1 & \cdots & -1 \\ 1 & \ddots & \ddots & \vdots \\ \vdots & & \ddots & -1 \\ 1 & \cdots & 1 & 0 \end{pmatrix}_{6\times 6} \quad , \quad A = -\begin{pmatrix} 1 & \cdots & 1 \\ 2 & \cdots & 2 \\ \vdots & \vdots & \vdots \\ 13 & \cdots & 13 \\ 13/2 & \cdots & 13/2 \\ 15/2 & \cdots & 15/2 \\ 2 & \cdots & 2 \end{pmatrix}_{16\times 6}$$

2) Extracting the Narain lattice

$$g = \begin{pmatrix}
2 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
-1 & 2 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & -1 & 2 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & -1 & 2 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & -1 & 2 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & -1 & 2 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & -1 & 2 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & -1 & 2 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
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0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 2 & -1 & 0 & 0 & 0 & 0 \\
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0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 2 & 1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 4
\end{pmatrix}$$

3) orbifold action from the basis vectors

Using

$$y + iw = \,:e^{iX}:$$

we see that:

- When
$$y + iw \to -(y + iw) \quad \Rightarrow \quad X \to X + \pi$$
(shift action)

- When
$$y + iw \to y - iw \quad \Rightarrow \quad X \to -X$$
(twist action)

- When
$$y + iw \to -y + iw \quad \Rightarrow \quad X \to -X + \pi$$
(roto-translational action)

4) Extracting the discrete torsion

$$\mathcal{Z}_{\mathsf{FFF}} = \sum_{\alpha,\beta} \quad C\big[{}^{\alpha}_{\beta}\big] \quad Z\big[{}^{\alpha}_{\beta}\big]$$

$$\updownarrow$$

$$\mathcal{Z}_{\mathsf{orbi}} = \sum_{h,h':[h,h']=0} \quad C\big[{}^{h}_{h'}\big] \quad Z\big[{}^{h}_{h'}\big]$$

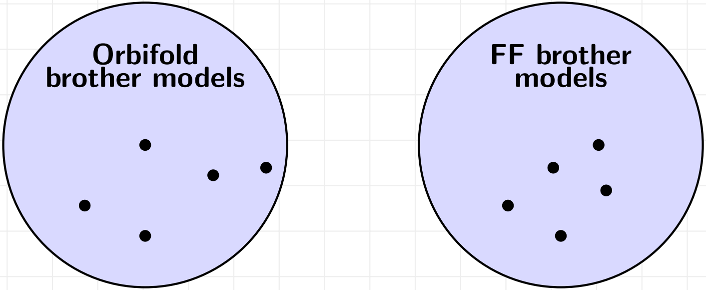
This should be a straightforward task...

4) Extracting the discrete torsion

Points to consider:

- There are phases that in one formalism are included in the C part and in the other in the Z part!
- The identification of phases depends on the exact algorithm for the identification of the orbifold action (step 3).

  We can see **mirage torsion** on both sides: Ploger, Ramos-Sanchez, Ratz, Vaudrevange '07



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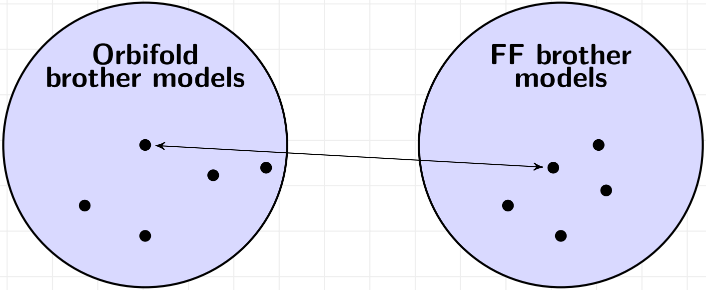

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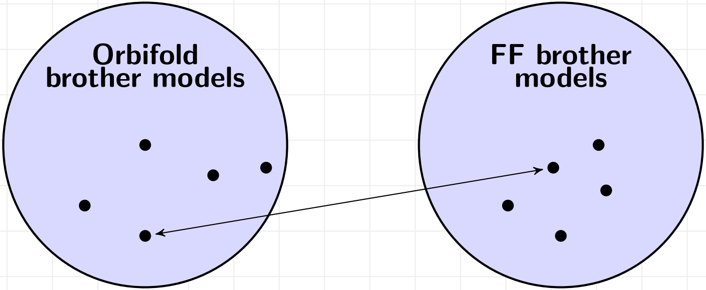

Summary and outlook

1. The heterotic string provides a nice framework to construct (semi-)realistic models. Understanding the **moduli space** of heterotic models is of great importance.
2. Free fermionic and orbifold models are related and we can translate from one to the other.
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Thank you very much!