Exact correlation functions in 4d $\mathcal{N} = 2$ SCFTs

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$d = 4,\ \mathcal{N} = 2$

Continuous families of $\mathcal{N} = 2$ SCFTs

$\mathcal{N} = 2$ superconformal manifold

➡ space parametrized by $\mathcal{N} = 2$ exactly marginal couplings

spectrum, correlation functions ... vary continuously across this space

exact coupling constant dependence?

Sector of interest:

correlation functions of local 1/2-BPS operators

- non-trivial
- rich geometric structure
- largely computable...

$\mathcal{N} = 2$ chiral rings

- R-symmetry of d=4 $\mathcal{N} = 2$ SCFTs : $SU(2)_R \times U(1)_R$

- $\mathcal{N} = 2$ chiral primary operators ϕ_I **(+ anti-chiral)**

$SU(2)_R$ - neutral, 1/2-BPS

$$\overline{Q}^i_{\dot{\alpha}} \cdot \phi_I = 0 \ , \quad \dot{\alpha} = \pm \ , \quad i = 1, 2$$

In short multiplets saturating the bound Dolan-Osborn, ‘02

$$\Delta \geq \frac{|R|}{2}$$

scaling dimension R-charge

Chiral ring data

- Chiral primaries form a ring (under OPE)

$$\phi_I(x)\,\phi_J(0) = C_{IJ}^K\,\phi_K(0) + \dots$$

- 2-point functions

$$\langle \phi_I(x)\,\overline{\phi}_J(0)\rangle = \frac{g_{I\bar{J}}}{|x|^{2\Delta}}$$

- 3-point functions

$$C_{IJ\bar{K}} = C_{IJ}^L\,g_{L\bar{K}}$$

$$\langle \phi_I(x)\phi_J(y)\bar{\phi}_K(z)\rangle = \frac{C_{IJ\bar{K}}}{|x-y|^{\Delta_I+\Delta_J-\Delta_K}|x-z|^{\Delta_I+\Delta_K-\Delta_J}|y-z|^{\Delta_J+\Delta_K-\Delta_I}}$$

0

On a conformal manifold the 2- & 3-point function coefficients

$$g_{I\bar{J}} \ , \quad C_{IJ\bar{K}}$$

are non-trivial functions of the marginal coupling constants.

☞ $\mathcal{N} = 4$ is special: non-renormalization theorems

Lee-Minwalla-Rangamani-Seiberg ’98, ...,
Baggio-de Boer-Papadodimas ’12

☞ Access to all extremal *N*-point functions

$$\langle \phi_{I_1}(x_1) \cdots \phi_{I_n}(x_n) \bar{\phi}_J(y) \rangle$$

$$R_J + \sum_k R_{I_k} = 0$$

Baggio-VN-Papadodimas, ’14

Geometry I

An infinitesimal deformation

$$\delta S = \frac{\delta\lambda^i}{4\pi^2}\int d^4x\, \mathcal{O}_i(x) + \frac{\delta\bar{\lambda}^i}{4\pi^2}\int d^4x\, \overline{\mathcal{O}}_i(x)$$

preserves the $\mathcal{N} = 2$ superconformal invariance iff it is the descendant of a (anti)-chiral primary with $\Delta = 2\ , \quad R = \pm 4$

$$\mathcal{O}_i = Q^4 \cdot \phi_i\ , \quad \overline{\mathcal{O}}_i = \bar{Q}^4 \cdot \bar{\phi}_i$$

! indices $i, j, ...$ for $R = 4$ chiral primaries

Zamolodchikov metric

The coefficient of the 2-point function

$$\langle \mathcal{O}_i(x) \overline{\mathcal{O}}_j(0) \rangle = \frac{G_{i\bar{j}}}{|x|^8}$$

Zamolodchikov '86

defines a metric on the conformal manifold **M**.

$\mathcal{N} = 2$: with this metric **M** is a complex Kaehler manifold

$$G_{i\bar{j}} = \partial_i \partial_{\bar{j}} \mathcal{K}$$

Kaehler potential and localization

Gerchkovitz-Gomis-Komargodski ’14

also Gomis-Ishtiaque ’14

$$\mathcal{K} = 192 \log Z_{S^4}$$

Pestun ’07

Sphere PF Z_{S^4} can be computed <u>exactly</u> with localization

➠ determines 2-point functions of the chiral primaries ϕ_i

$$g_{i\bar{j}} = \frac{G_{i\bar{j}}}{192} = \partial_i \partial_{\bar{j}} \log Z_{S^4}$$

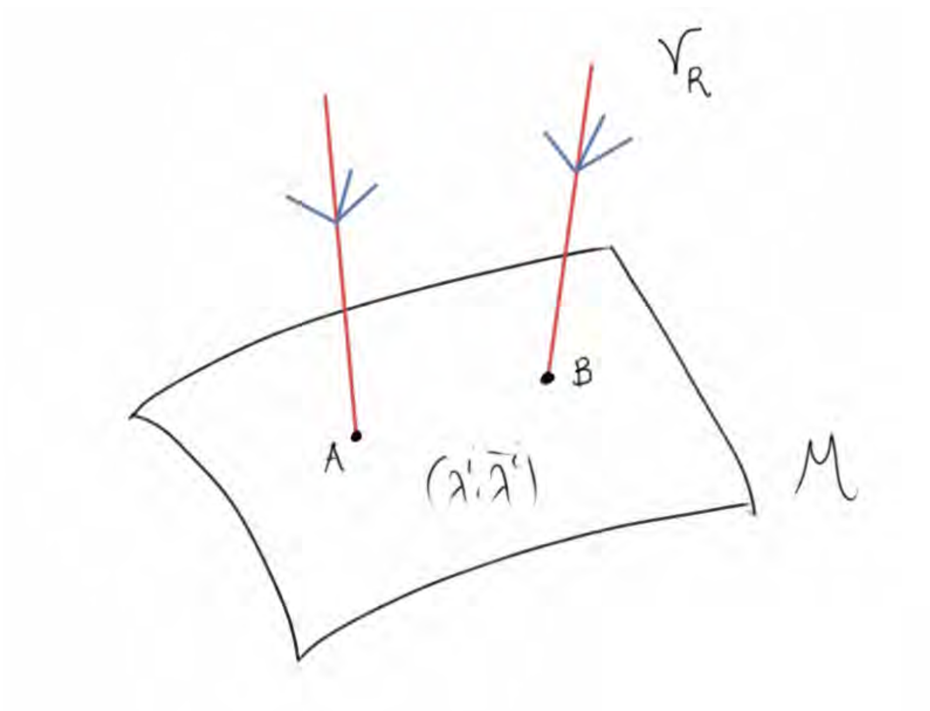
part of $g_{I\bar{J}}$ data

Geometry II

Operator mixing and quantum renormalization

➠ chiral primaries as sections of vector bundles $\mathcal{V}_R$

with non-trivial connection



$$(\nabla_\mu)^L_K = \delta^L_K \partial_\mu + (A_\mu)^L_K$$

$$C_{I\,K}^{L} \;:\; \mathcal{V}_{R_I} \otimes \mathcal{V}_{R_K} \;\to\; \mathcal{V}_{R_L}$$

Superconformal Ward identities imply

tt* equations

holomorphic vector bundles

$$(F_{ij})_K^{\ L} = (F_{\bar{i}\bar{j}})_K^{\ L} = 0$$

$$(F_{i\bar{j}})_K^{\ L} = -\left[C_i, \bar{C}_j\right]_K^{\ L} + g_{i\bar{j}}\delta_K^L \left(1 + \frac{R}{4c}\right)$$

2d: Cecotti-Vafa 1991

4d: Papadodimas ’09

topological-anti-topological fusion

Holomorphic gauge

- Practical to select a particular scheme

  converts *tt** equations to PDEs for 2- and 3-point functions

- Holomorphic vector bundles ➟ holomorphic gauge $(A_{\bar{j}})_K^{\ L} = 0$

$$\frac{\partial}{\partial \bar{\lambda}^j}\left(g^{\bar{M}L}\frac{\partial}{\partial \lambda^i} g_{K\bar{M}}\right) = C_{iK}^P g_{P\bar{Q}} C^{*\bar{Q}}_{\bar{j}\bar{R}} g^{\bar{R}L} - g_{K\bar{N}} C^{*\bar{N}}_{\bar{j}\bar{U}} g^{\bar{U}V} C_{iV}^L - g_{i\bar{j}}\delta_K^L$$

$$\frac{\partial}{\partial \bar{\lambda}^j} C_{IJ}^K = 0$$

$$\frac{\partial C_{jK}^L}{\partial \lambda^i} - \frac{\partial C_{iK}^L}{\partial \lambda^j} = g^{\bar{Q}L}\partial_i g_{P\bar{Q}} C_{jK}^P - C_{jP}^L g^{\bar{Q}P}\partial_i g_{K\bar{Q}} - (i \leftrightarrow j)$$

- **d=2 N=(2,2): very restrictive set of equations**

  solution almost unique Cecotti-Vafa 1991

- **d=4: how restrictive?**

  a recognizable (integrable) structure in these equations?

  a complete solution from a `few' data?

Example: $\mathcal{N} = 2$ superconformal QCD

$\mathcal{N} = 2$ SYM, gauge group *SU(N)* $\oplus$ 2*N* hypermultiplets

$\mathcal{N} = 2$ chiral ring generators φ *complex scalar in vector multiplet*

$$\phi_\ell \propto \mathrm{Tr}\left[\varphi^\ell\right] \quad , \quad \ell = 2, 3, \ldots, N$$

Complex 1-dimensional conformal manifold

$$\mathcal{O}_\tau = Q^4 \cdot \phi_2$$

complexified gauge coupling $\quad \tau = \dfrac{\theta}{2\pi} + \dfrac{4\pi i}{g_{YM}^2}$

SU(2)

Baggio-VN-Papadodimas '14

- 1 chiral ring generator
- No degeneracies
- The chiral primary operators are $\phi_{2n} \propto \left(\mathrm{Tr}\left[\varphi^2\right]\right)^n$
- We normalize $\phi_{2n}\ ,\quad n > 1$ so that

$$\phi_2(x)\phi_{2n}(0) = \phi_{2n+2}(0) + \dots$$

or

$$C_{2n\ 2m}^{2(n+m)} = 1$$

consistent with holomorphic gauge

Solve for the 2-point function coefficients

`trivial' in *N=4*
(group theory)

$$\langle \phi_{2n}(x)\bar{\phi}_{2n}(0)\rangle = \frac{g_{2n}(\tau,\bar{\tau})}{|x|^{4n}}$$

highly non-trivial
in *N=2*

NOTE: equivalently, in basis of orthonormal 2-point functions we study the exact 3-point functions

*tt** equations

Recursive

$$\partial_\tau \partial_{\bar\tau} \log g_{2n} = \frac{g_{2n+2}}{g_{2n}} - \frac{g_{2n}}{g_{2n-2}} - g_2$$

$$g_0 = 1 \ , \quad n = 1, 2, \ldots$$

<u>semi-infinite</u> Toda chain

$$\partial_\tau \partial_{\bar\tau} q_n = e^{q_{n+1} - q_n} - e^{q_n - q_{n-1}} \ , \quad n = 2, \ldots$$

$$g_{2n} = \exp\left(q_n - \log Z_{S^4}\right)$$

☛ **one datum, e.g.** g_2 **from localization, determines all !!!**

Predictions for perturbation theory

0-instanton sector

$$g_2^{(0)} = \frac{3}{8}\frac{1}{(\mathrm{Im}\tau)^2} - \frac{135\,\zeta(3)}{32\,\pi^2}\frac{1}{(\mathrm{Im}\tau)^4} + \frac{1575\,\zeta(5)}{64\,\pi^3}\frac{1}{(\mathrm{Im}\tau)^5} + \dots\ ,$$

$$g_4^{(0)} = \frac{15}{32}\frac{1}{(\mathrm{Im}\tau)^4} - \frac{945\,\zeta(3)}{64\,\pi^2}\frac{1}{(\mathrm{Im}\tau)^6} + \frac{7875\,\zeta(5)}{64\,\pi^3}\frac{1}{(\mathrm{Im}\tau)^7} + \dots\ ,$$

$$g_6^{(0)} = \frac{315}{256}\frac{1}{(\mathrm{Im}\tau)^6} - \frac{76545\,\zeta(3)}{1024\,\pi^2}\frac{1}{(\mathrm{Im}\tau)^8} + \frac{1677375\,\zeta(5)}{2048\,\pi^3}\frac{1}{(\mathrm{Im}\tau)^9} + \dots\ ,$$

$$g_8^{(0)} = \frac{2835}{512}\frac{1}{(\mathrm{Im}\tau)^8} - \frac{280665\,\zeta(3)}{512\,\pi^2}\frac{1}{(\mathrm{Im}\tau)^{10}} + \frac{1913625\,\zeta(5)}{256\,\pi^3}\frac{1}{(\mathrm{Im}\tau)^{11}} + \dots\ ,$$

…

SU(N)

Baggio-VN-Papadodimas '15
to appear

- More chiral ring generators: $\phi_\ell \propto \mathrm{Tr}\left[\varphi^\ell\right] \ , \quad \ell = 2, 3, \ldots, N$

  non-trivial degeneracies...

- In conventions where $C_{K\,L}^{K+L} = 1$ the *tt** equations become

$$\partial_{\bar{\tau}}\left(g^{\bar{M}_\Delta L_\Delta}\partial_\tau g_{K_\Delta \bar{M}_\Delta}\right) = g_{K_\Delta+2,\bar{R}_\Delta+\bar{2}}\,g^{\bar{R}_\Delta L_\Delta} - g_{K_\Delta \bar{R}_\Delta}\,g^{\bar{R}_\Delta-\bar{2},L_\Delta-2} - g_2\,\delta^{L_\Delta}_{K_\Delta}$$

Preliminary observations

Assume there is a **constant** linear transformation

$$\phi'_K = \mathcal{M}_K{}^L \, \phi_L$$

that 1) diagonalizes $g_{K\bar{L}}$

and 2) retains the OPE

$$\phi'_2 \, \phi'_K = \phi'_{K+2} + \dots$$

☞ *tt* eqs reduce to a decoupled sequence of Toda chains*

Such a transformation requires highly non-trivial properties

- $g_{K\bar{L}}$ need to obey specific relations *`horizontal relations'*

- gauge connection will be reducible

if chiral primaries at scaling dimension Δ have degeneracy D

the holonomy is not U(D) but $U(1)^D$

(in primed basis no quantum mixing)

- OPE $\phi_2\,\phi'_K = \phi'_{K+2} + \ldots$ requires group-theoretical identities at tree-level

`vertical relations'

Examples. Assume $(\mathrm{Tr}[\phi^2])^n \longrightarrow_{\mathcal{M}} (\mathrm{Tr}[\phi^2])^n$. We need:

(1) the ratios

$$R_{2n,\bar{K}} = \frac{\langle(\mathrm{Tr}[\varphi^2])^n(x)\ \bar{\phi}_K(0)\rangle}{\langle(\mathrm{Tr}[\varphi^2])^n(x)(\mathrm{Tr}[\bar{\varphi}^2])^n(0)\rangle}$$

do not renormalize (horizontal relations)

(2) ratios at different levels are related (vertical relations)

$$R_{2n,\bar{K}} = R_{2n+2,\bar{K}+\bar{2}} = \frac{\langle(\mathrm{Tr}[\varphi^2])^{n+1}(x)\ (\bar{\phi}_K\mathrm{Tr}[\bar{\varphi}^2])(0)\rangle}{\langle(\mathrm{Tr}[\varphi^2])^{n+1}(x)(\mathrm{Tr}[\bar{\varphi}^2])^{n+1}(0)\rangle}$$

Verified by **explicit 3-loop computations !!!**

Example: *SU(4),* Δ = 6

$$(Tr[\varphi^2])^3\ ,\quad Tr[\varphi^2]Tr[\varphi^4]\ ,\quad (Tr[\varphi^3])^2$$

$$\frac{g^{12}}{(16\pi)^6}\begin{pmatrix} 232560 - \frac{8241345\,\zeta(3)\,g^4}{4\pi^4} & 99180 - \frac{14058765\,\zeta(3)\,g^4}{16\pi^4} & 6480 - \frac{229635\,\zeta(3)\,g^4}{4\pi^4} \\ 99180 - \frac{14058765\,\zeta(3)\,g^4}{16\pi^4} & 55935 - \frac{30324105\,\zeta(3)\,g^4}{64\pi^4} & 8100 - \frac{1012095\,\zeta(3)\,g^4}{16\pi^4} \\ 6480 - \frac{229635\,\zeta(3)\,g^4}{4\pi^4} & 8100 - \frac{1012095\,\zeta(3)\,g^4}{16\pi^4} & 58320 - \frac{1454355\,\zeta(3)\,g^4}{4\pi^4} \end{pmatrix}$$

Many more checks.

Also preliminary evidence of full decoupling.

Consequence: in general *SU(N)* theory

$$\langle \phi_{2n}(x)\bar{\phi}_{2n}(0)\rangle = \frac{g_{2n}(\tau,\bar{\tau})}{|x|^{4n}}$$

continues to obey

$$\partial_\tau \partial_{\bar{\tau}} \log g_{2n} = \frac{g_{2n+2}}{g_{2n}} - \frac{g_{2n}}{g_{2n-2}} - g_2$$

➡ ***solution from SU(N) S^4 partition function***

Horizontal & vertical relations fix many more mixed correlators

Outlook

- The above ansatz solves the SU(N) *tt** equations

  - Is this the choice of the gauge theory?

  (horizontal and vertical relations do not appear to come from Ward identities)

  - Specific external data are needed to solve the Toda chains. How are these computed exactly?

- Fruitful approach to an unexplored class of non-perturbative dynamics in 4d QFTs

  Surprising lessons (non-renormalization theorems in $\mathcal{N} = 2$?)

- Many more directions...

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