Semi-holography for QCD

Ayan Mukhopadhyay
CCTP and CCQCN, University of Crete

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Based on

E. Iancu and AM, "*A semi-holographic model for heavy ion collisions*", arXiv:1410.6448 (published in JHEP)

Ongoing projects with E. Iancu (CEA-Saclay, France), A.Rebhan, S. Stricker F. Preis (IFT, Vienna Institute of Technology) and Di-Lun Yang (UoC)

Part 0: Introduction and Plan

The proposal for semi-holography

Wilsonian RG leads to effective QFTs with small number of parameters near fixed points.

Needed: a new framework that describes physics at intermediate scales with a small number of effective parameters

Semi-holography: a general framework combining perturbative and holographic approaches consistently at intermediate scales

EXAMPLE: QCD

Asymptotic freedom

Semi-holographic framework with *small number* of effective parameters — justifying both ends?

“Chiral Lagrangian effective” QFT — Sakai-Sugimoto-Witten holographic model

Plan

- **Part 1: The general formulation of semi-holography**
- **Part 2: QGP**
- **Part 3: Hadrons and nuclei**
- **Part 4: Outlook**

Part 1:General Formulation

Vlasov equation

$$\left(\frac{\partial}{\partial t} + \mathbf{v} \cdot \nabla_{\mathbf{x}} - e\left(\mathbf{E} + \mathbf{v} \times \mathbf{B}\right) \cdot \nabla_{\mathbf{p}}\right) f\left(\mathbf{x}, \mathbf{p}, t\right) = 0.$$

$$\nabla \cdot \mathbf{E} = 4\pi\rho, \quad \nabla \times \mathbf{B} = 4\pi\mathbf{j} + \frac{\partial \mathbf{E}}{\partial t}$$

$$\nabla \cdot \mathbf{B} = 0, \quad \nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$

Electromagnetic fields do not have independent dynamics

$$\rho = -e \int \mathrm{d}^3 p\, f\left(\mathbf{x}, \mathbf{p}, t\right), \quad \mathbf{j} = -e \int \mathrm{d}^3 p\, f\left(\mathbf{x}, \mathbf{p}, t\right) \frac{\mathbf{p}}{p^0(\mathbf{p})}$$

Semi-holography

(i) Strongly coupled soft fields are *operators of a holographic theory* coupling to perturbative hard fields

(ii) These mean-fields are solved by (d+1)-gravitational theory *self-consistently*

(iii) The full data for initial + boundary conditions are given by perturbative hard fields *alone*

Note it is now a single theory not two coupled theories!

Why effective?

(i) Dimensions of all couplings (hard-soft, hard-hard) and gravitational parameters are set by an intermediate scale

(ii) It should be shown that only a few minimalist couplings matters in an appropriate range of energy scales (or temperatures)

Non-Fermi liquids

AM and G.Policastro, PRL **111**, no.22, 221602 (2013) [arXiv: 1306.3941[hep-th]]

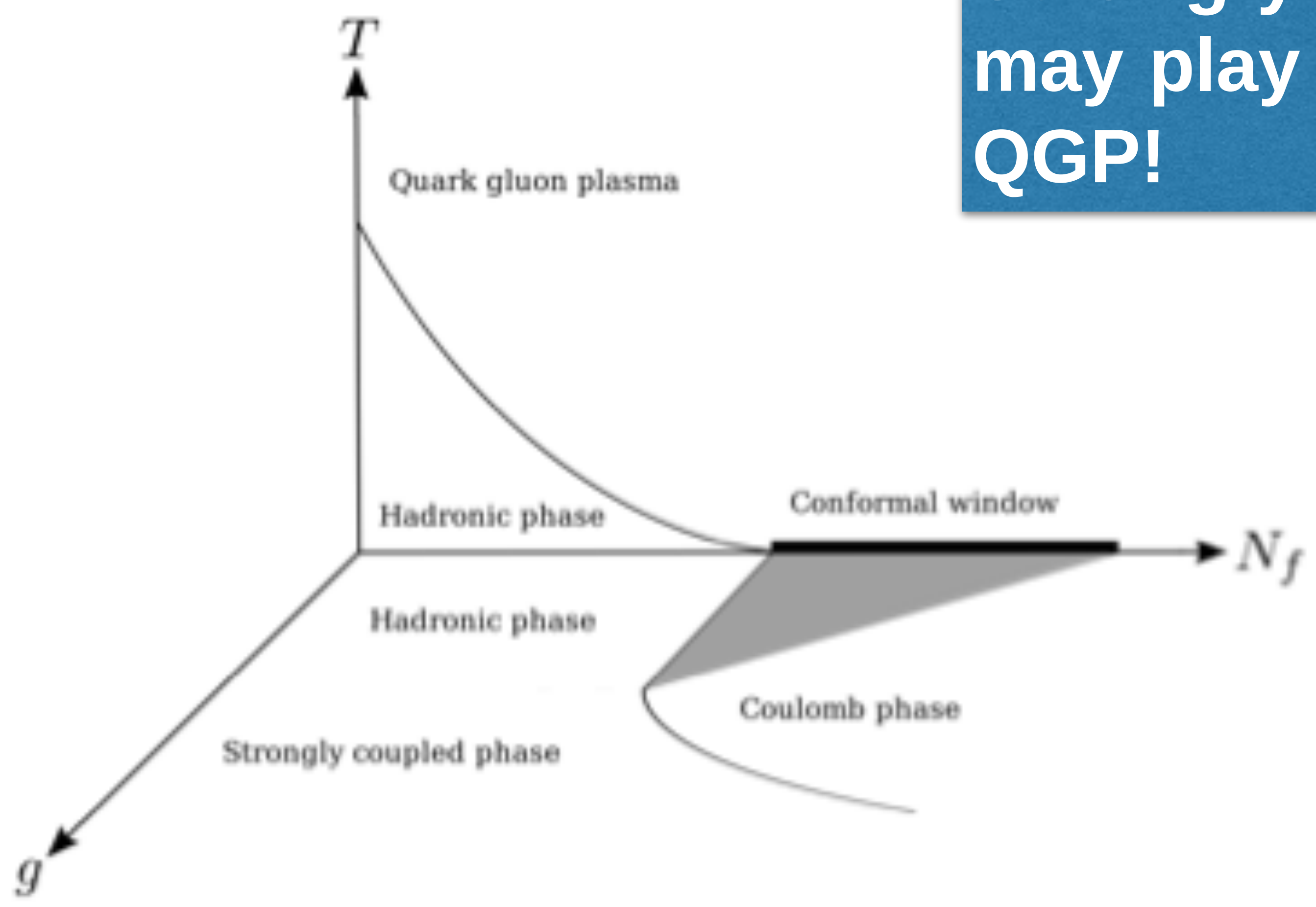
First proposed by myself and G. Policastro for non-Fermi liquids in certain limits (2013).

It was shown that the minimalist semi-holographic model of Polchinski and Faulkner (2010) was stable towards non-minimalist deformations.

Also a generalisation of Landau's Fermi liquid theory followed.

The case for QGP

Strongly coupled IR-CFT may play a dominant role in QGP!

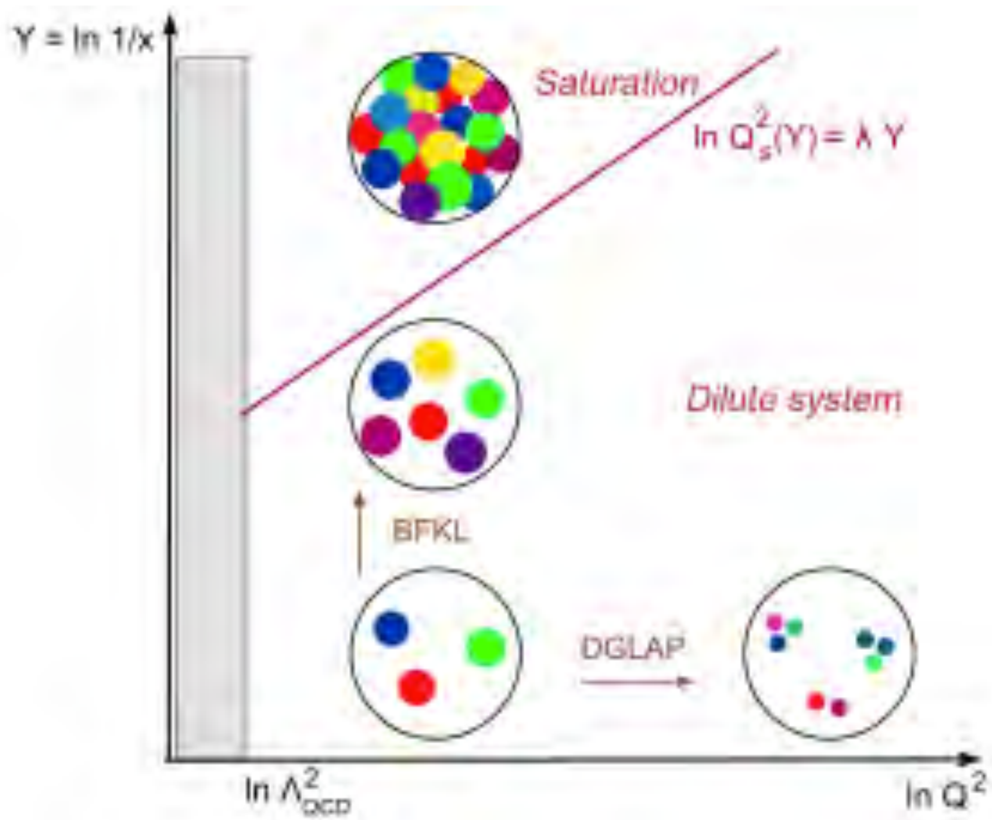


Reproduced from arXiv:1410.20136 by MP Lombardo et.al.

Part 2:The quark-gluon plasma

Why combine weak and strong coupling approaches

Qualitative similarity to strongly coupled N=4 SYM at finite temperature explains many tantalising properties of QGP



The initial gluons however are abundantly at the saturation scale (CGC theory)

$$G(x, Q^2 = Q_s^2(x)) \sim \mathcal{O}\left(\frac{1}{\alpha_s^2}\right)$$

$$Q_s^2(x) = Q_0^2 \left(\frac{x}{x_0}\right)^{\lambda}$$

$$Q_0 = 1\text{Gev}, \quad \lambda = 0.288, \quad \text{x}_0 = 0.0003$$

Minimalist approach

Assume only two IR-CFT operators of the holographic theory with dimension 4

$$\mathcal{H}, \quad \mathcal{T}^{\mu\nu}$$

The minimalist semi-holographic model for QGP is:

$$S = -\int \mathrm{d}^4x \, \frac{1}{4N_c} \mathrm{Tr}(F_{\alpha\beta}F^{\alpha\beta}) - \frac{\beta}{Q_s^4} \int \mathrm{d}^4x \, h \, \mathcal{H} - \frac{\gamma}{Q_s^4} \int \mathrm{d}^4x \, t_{\mu\nu} \mathcal{T}^{\mu\nu}$$

$$h(x) = \frac{1}{4N_c} \mathrm{Tr}(F_{\alpha\beta}F^{\alpha\beta}) \qquad t_{\mu\nu}(x) = \frac{1}{N_c} \mathrm{Tr}\Big(F_{\mu\alpha}F_{\nu}{}^{\alpha} - \frac{1}{4}\eta_{\mu\nu}F_{\alpha\beta}F^{\alpha\beta}\Big)$$

Finally:

$$S^{\mathrm{on-shell}}_{(d+1)-grav} = -\frac{\beta}{Q_s^4}\int \mathrm{d}^4x\, h\,\mathcal{H} - \frac{\gamma}{Q_s^4}\int \mathrm{d}^4x\, t_{\mu\nu}\mathcal{T}^{\mu\nu}$$

Then in the holographic gravity problem:

non-normalizable mode for bulk dilation: $\frac{\beta}{Q_s^4}h$ — **Input**

normalizable mode for bulk dilation: $\mathcal{H}$ — **Output (via regularity)**

boundary metric is: $g^{(\mathrm{b})}_{\mu\nu} = \eta_{\mu\nu} + \frac{\gamma}{Q_s^4}t_{\mu\nu}$ — **Input**

normalizable mode of bulk graviton is: $\mathcal{T}_{\mu\nu}$

$$\mathcal{T}^{\mu\nu} = g^{(b)\mu\rho}\mathcal{T}_{\rho\sigma}g^{(b)\sigma\nu}$$

Output (via regularity)

The 5D minimalist gravity theory dual to the (deformed) IR theory:

$$\begin{aligned} R_{MN} - \frac{1}{2} R G_{MN} - \frac{6}{l^2} G_{MN} &= \nabla_M \Phi \nabla_N \Phi - \frac{1}{2} G_{MN} \nabla_P \Phi \nabla^P \Phi, \\ G^{MN} \nabla_M \nabla_N \Phi &= 0 \end{aligned}$$

The boundary conditions for the gravity theory is supplied by "glasma" equations

$$D_\mu F^{\mu\nu} = -\frac{\beta}{Q_s^4} D_\mu (F^{\mu\nu} \mathcal{H}) + \frac{\gamma}{Q_s^4} D_\mu (F^{\mu\nu} \mathcal{T}^{\alpha\beta} \eta_{\alpha\beta}) - \frac{2\gamma}{Q_s^4} D_\mu (\mathcal{T}^{\mu\alpha} F_\alpha{}^\nu + F^\mu{}_\alpha \mathcal{T}^{\alpha\nu})$$

Both are to be solved *iteratively*.

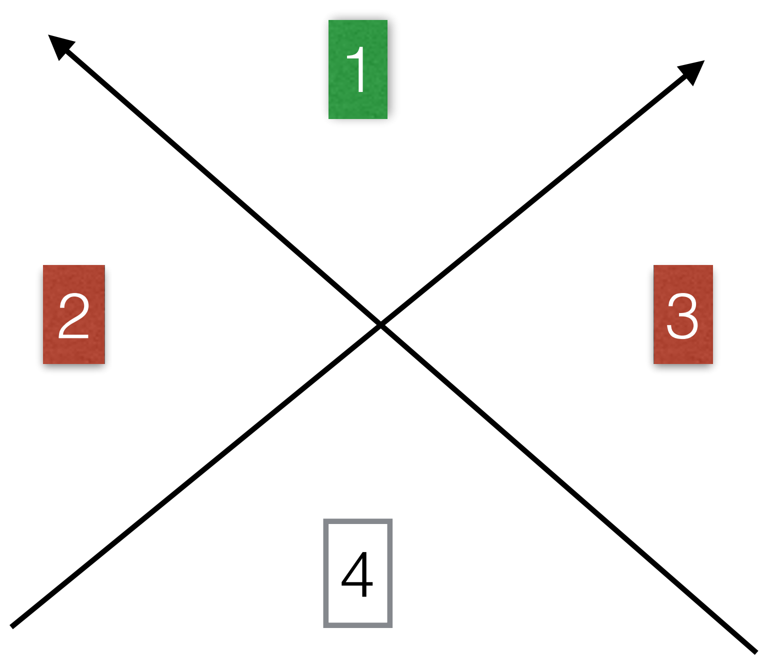
Note there are two minimalist versions which differ by non-minimalist terms.

Initial Conditions

In the CGC picture, the colliding nuclei can be thought of as shock waves. In region 2 and 3, the gauge fields are pure gauge

$$A^+ = A^- = 0\,,$$

$$A^i(x) = \theta(-x^+)\theta(x^-)A^i_{(1)}(\mathbf{x}_\perp) + \theta(-x^-)\theta(x^+)A^i_{(2)}(\mathbf{x}_\perp)$$



$$A^i_{(1,2)}(\mathbf{x}_\perp) = \frac{\mathrm{i}}{g} U_{(1,2)}(\mathbf{x}_\perp)\partial_i U^\dagger_{(1,2)}(\mathbf{x}_\perp),$$

$$U_{(1,2)}(\mathbf{x}_\perp) = \mathrm{P}\,\exp\left(-\,\mathrm{ig}\int \mathrm{dx}^{\mp}\frac{1}{\nabla^2_\perp}\rho_{(1,2)}(\mathrm{x}^{\mp},\mathbf{x}_\perp)\right)$$

$$\tau = \sqrt{t^2 - (x^3)^2} = \sqrt{2x^+x^-}\,, \qquad \eta = \frac{1}{2}\ln\frac{x^+}{x^-}$$

Continuity and regularity of the em-tensor imply (Kovner, McLerran, Weigert '95)

At $\tau = O^+$

$$A^i = A^i_{(1)} + A^1_{(2)}, \qquad A^\eta = \frac{\mathrm{ig}}{2}\left[A^i_{(1)}, A^i_{(2)}\right], \qquad \partial_\tau A^i = \partial_\tau A^\eta = 0$$

The initial conditions remain unchanged for the semi-holographic glasma.

Thus initial conditions are determined completely by the stochastic nuclear charges

$$\langle \rho^a_{(m)}(\mathbf{x}_\perp)\rho^b_{(n)}(\mathbf{y}_\perp)\rangle = g^2\mu^2\,\delta_{mn}\,\delta^{ab}\,\delta^{(2)}(\mathbf{x}_\perp - \mathbf{y}_\perp) \qquad g^2\mu^2 = Q_s^2$$

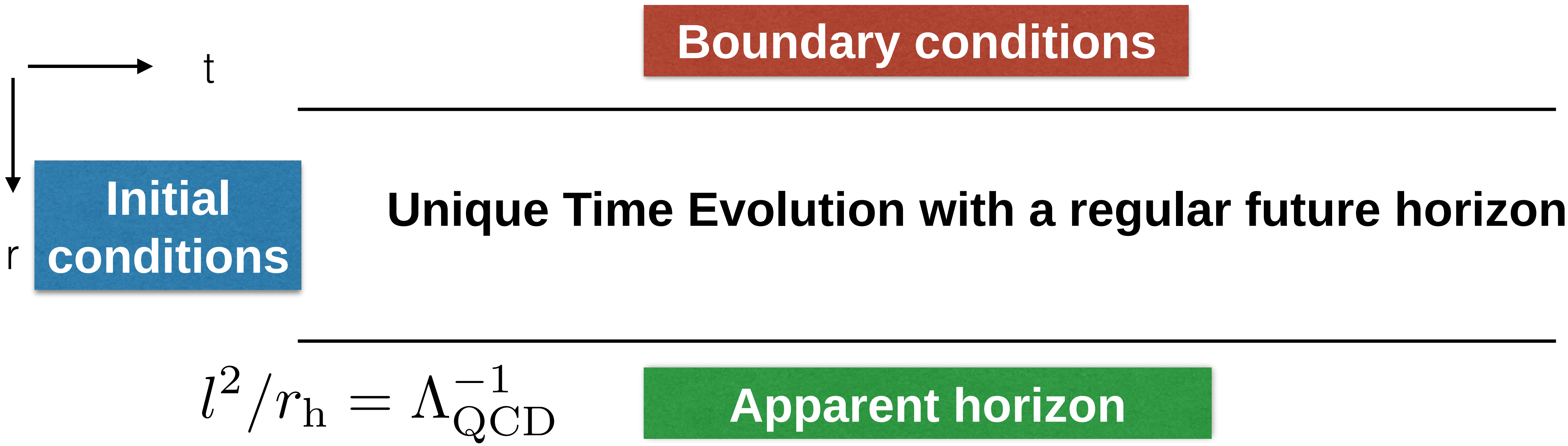
Let the bulk metric be written as:

$$\mathrm{d}s^2 = G_{mn}(X)\mathrm{d}x^m\mathrm{d}x^n + 2\mathrm{d}\tau\left(\mathrm{d}r - A(X)\mathrm{d}\tau - F_m(X)\mathrm{d}x^m\right)$$

For the gravitational problem to have a unique solution, we need initial conditions for:

$$\hat{G}_{mn} = G_{mn}/(\det G), \quad \mathcal{T}_{\tau\tau}, \quad \mathcal{T}_{\tau m}, \quad \Phi, \quad \partial_\tau \Phi$$

Chesler-Yaffe algorithm: put an apparent horizon at a fixed value of r



Uncertainty principle tells us that minimum time to produce soft gluons is

$$\tau \sim 1/p_\perp \geq 1/Q_s$$

In order to simulate this effect we make the hard-soft couplings time-dependent

$$\beta = \beta_0 \tanh\big(\alpha_0 \tau Q_s\big), \qquad \gamma = \gamma_0 \tanh\big(\alpha_0 \tau Q_s\big)$$

As the soft modes are in the vacuum initially:

$$\begin{aligned} G_{\eta\eta}(r,\eta,x^i) = \frac{r^2}{l^2}\tau^2,\ G_{ij}(r,\eta,x^i) = \frac{r^2}{l^2}\delta_{ij},\ G_{\eta i}(r,\eta,x^i) &= 0, \\ \mathcal{T}_{\tau\tau}(\eta,x^i) = \mathcal{T}_{\tau\eta}(\eta,x^i) = \mathcal{T}_{\tau i}(\eta,x^i) &= 0, \\ \Phi(r,\eta,x^i) = \partial_\tau \Phi(r,\eta,x^i) \ &= \ 0 \end{aligned}$$

Iterative algorithm

(i) Solve the glasma equations first disregarding soft fields

(ii) Substitute the boundary conditions in the gravitational problem

(iii) Obtain the expectation values of the IR-CFT operators from gravity and re-solve the glasma equations

(iv) Obtain the new boundary conditions in gravity from the new glasma solution

(v) Continue until there is convergence

At each step of the iteration keep the same initial conditions for both the glasma and the gravitational problem

Conceptual issues

It seems that UV and IR live in different metric apparently.

Momentum constraints of Einstein's equations

$$\nabla_\mu \mathcal{T}^{\mu\nu}(x) = \frac{\beta}{Q_s^4}\mathcal{H}(x)\nabla^\nu h(x)$$

However the above can be reinterpreted as an equation for deformed IR-CFT in Minkowski space

$$\Gamma^\mu_{\nu\rho}[t] = \frac{\gamma}{2Q_s^4}\left(\partial_\nu t^\mu{}_\rho + \partial_\rho t^\mu{}_\nu - \partial^\mu t_{\nu\rho}\right) + \mathcal{O}(t^2)$$

$$\partial_\mu \mathcal{T}^{\mu\nu} = \frac{\beta}{Q_s^4}\mathcal{H}\, g^{(\mathrm{b})\mu\nu}[t]\,\partial_\mu h - \Gamma^\gamma_{\alpha\gamma}[t]\mathcal{T}^{\alpha\nu} - \Gamma^\nu_{\alpha\beta}[t]\mathcal{T}^{\alpha\beta}$$

Forces generated by the hard sector

The full energy-momentum tensor is non-local:

$$
\begin{aligned}
t^{\mu\nu\,\mathrm{FULL}}(x) = \quad & \eta^{\mu\alpha}\; t^{\mathrm{YM}}_{\alpha\beta}(x)\; \eta^{\beta\nu} - \\
& - \frac{\beta}{Q_s^4} \int \mathrm{d}^4 y\; \mathcal{G}^{\mu\nu}(x,y)\; h(y) \;-\; \frac{\gamma}{Q_s^4} \int \mathrm{d}^4 y\; \mathcal{G}^{\mu\nu\rho\sigma}(x,y)\; t^{\mathrm{YM}}_{\mu\nu}(y) \;+ \\
& + \frac{\beta}{Q_s^4}\; \eta^{\mu\alpha}\; t^{\mathrm{YM}}_{\alpha\beta}(x)\; \eta^{\beta\nu}\; \mathcal{H}(x) \;+ \\
& + \frac{\gamma}{Q_s^4}\; \frac{2}{N_c} \cdot \mathcal{T}^{\alpha\beta}(x)\; \mathrm{Tr}\Big(F_\alpha{}^{\mu}(x) F_\beta{}^{\nu}(x) - \frac{1}{2}\, \eta^{\mu\nu}\; F_{\alpha\gamma}(x) F_\beta{}^{\gamma}(x) + \\
& + \frac{1}{4}\; \delta^\mu{}_\alpha \delta^\nu{}_\beta\; F_{\gamma\delta}(x) F^{\gamma\delta}(x) - \\
& - \frac{1}{2}\; \eta_{\alpha\beta}\; F^\mu{}_\gamma(x) F^{\nu\gamma}(x) + \frac{1}{8}\; \eta^{\mu\nu}\eta_{\alpha\beta}\; F_{\gamma\delta}(x) F^{\gamma\delta}(x) \Big)
\end{aligned}
$$

$$
\mathcal{G}^{\mu\nu}(x,y) = 2\; \frac{\delta \mathcal{H}(y)}{\delta g_{\mu\nu}(x)}\Big|_{g_{\mu\nu}=\eta_{\mu\nu}},
$$

$$
\mathcal{G}^{\mu\nu\rho\sigma}(x,y) = 2\; \frac{\delta \mathcal{T}^{\mu\nu}(y)}{\delta g_{\mu\nu}(x)}\Big|_{g_{\mu\nu}=\eta_{\mu\nu}}
$$

$\mathcal{G}^{\mu\nu}(x,y)$ and $\mathcal{G}^{\mu\nu\rho\sigma}(x,y)$ are the $\mathcal{T}^{\mu\nu}(x) - \mathcal{H}(y)$ and $\mathcal{T}^{\mu\nu}(x) - \mathcal{T}^{\rho\sigma}(y)$ non-eq. Feynman propagators in the self-consistent IR CFT state

Late-time simplification (Di-Lun Yang and AM)

The classical YM equations reduce to an attractor solution [*Berges, Venugopalan, et al 2013, Lappi 2006*]

The IR-CFT dynamics is a fluid living in a geometry determined by the Yang-Mills attractor — Bjorken flow goes like:

$$\boxed{\mathcal{E}(\tau) = \frac{\mathcal{E}_0 \tau_0^{4/3}}{\tau^{4/3}} \left(1 - 2\frac{\eta_0}{\mathcal{E}_f \tau_0^2}\left(\frac{\tau_0}{\tau}\right)^{2/3}\right) + \frac{\gamma \mathcal{E}_0 \epsilon_0 \tau_0^{4/3}}{3 Q_s^5 \tau^{7/3}} \times \frac{a + 4(Q_s \tau_0)^{4/3}}{a + 2(Q_s \tau_0)^{4/3}} + \cdots}$$

$$\text{Hydro} \sim \qquad \tau^{-\frac{4}{3} - \frac{2}{3}n}$$

$$\text{Forcing terms} \sim \qquad \tau^{-\frac{7}{3} - \frac{2}{3}n}$$

Is it an effective theory?

Non-minimalist deformations include adding multi-trace couplings (with right scalings in N_c), more bulk fields, a potential for dilation.

Do these have small effects when the QGP temperature is << 1 GeV and > 125 MeV suppressed by T/Q_s?

Partial successes achieved in collaboration with A. Rebhan, S. Stricker, F. Preis (IFT Vienna Institute of Technology) and E. Iancu.

Semi-holographic jet physics

We introduce a hybrid model for the heavy quark probe

$$S_{\text{string}} = \quad -T_0 \int \mathrm{d}^2\sigma \sqrt{-\det h_{\alpha\beta}} + g \int_{\sigma=0} \mathrm{d}\tau \, \mathcal{Q}_a(\tau) \, \frac{\mathrm{d}x^\mu(\tau)}{\mathrm{d}\tau} \, A^a_\mu(X(\tau))$$

$$h_{ab} = G^{(\mathrm{s})}_{MN} \partial_\alpha X^M \partial_\beta X^N, \quad G^{(\mathrm{s})}_{MN} = e^{\frac{\Phi}{2}} G_{MN}$$

Gauge invariance requires that Wong equation is satisfied:

$$\frac{\mathrm{d}\mathcal{Q}_a(\tau)}{\mathrm{d}\tau} + g f_{abc} \frac{\mathrm{d}x^\mu(\tau)}{\mathrm{d}\tau} A^b_\mu(x(\tau)) \mathcal{Q}^c(\tau) = 0$$

Dirichlet boundary conditions do not hold.

The trajectory of the end point of the string is determined from

$$\sqrt{-h_{\tau\tau}}\, n_\alpha P^\alpha_\mu + g Q_a \frac{\mathrm{d}x^\nu}{\mathrm{d}\tau} F^a_{\mu\nu} = 0$$

Initial condition: vertical string in pure AdS with all points moving with an uniform velocity.

Include “hybrid fragmentation” as suggested by Jorge Casalderrey Solana et. al. [arXiv:1405.3864]

Part 3: Hadrons and Nuclei

MIT bag model

Quarks are free inside a cavity — the size of the cavity is penalised by a bag constant (Chodos, Jaffe, Johnson, Thorn ’74)

$$T_{\mu\nu}^{\mathrm{MIT-Bag}} = T_{\mu\nu}^{\mathrm{free-quark}} - B\eta_{\mu\nu}$$

Boundary condition of the Dirac equation determined by energy-momentum conservation

Size of the bag determined by energy-minimisation

Gives a good estimate of proton charge radius, etc.

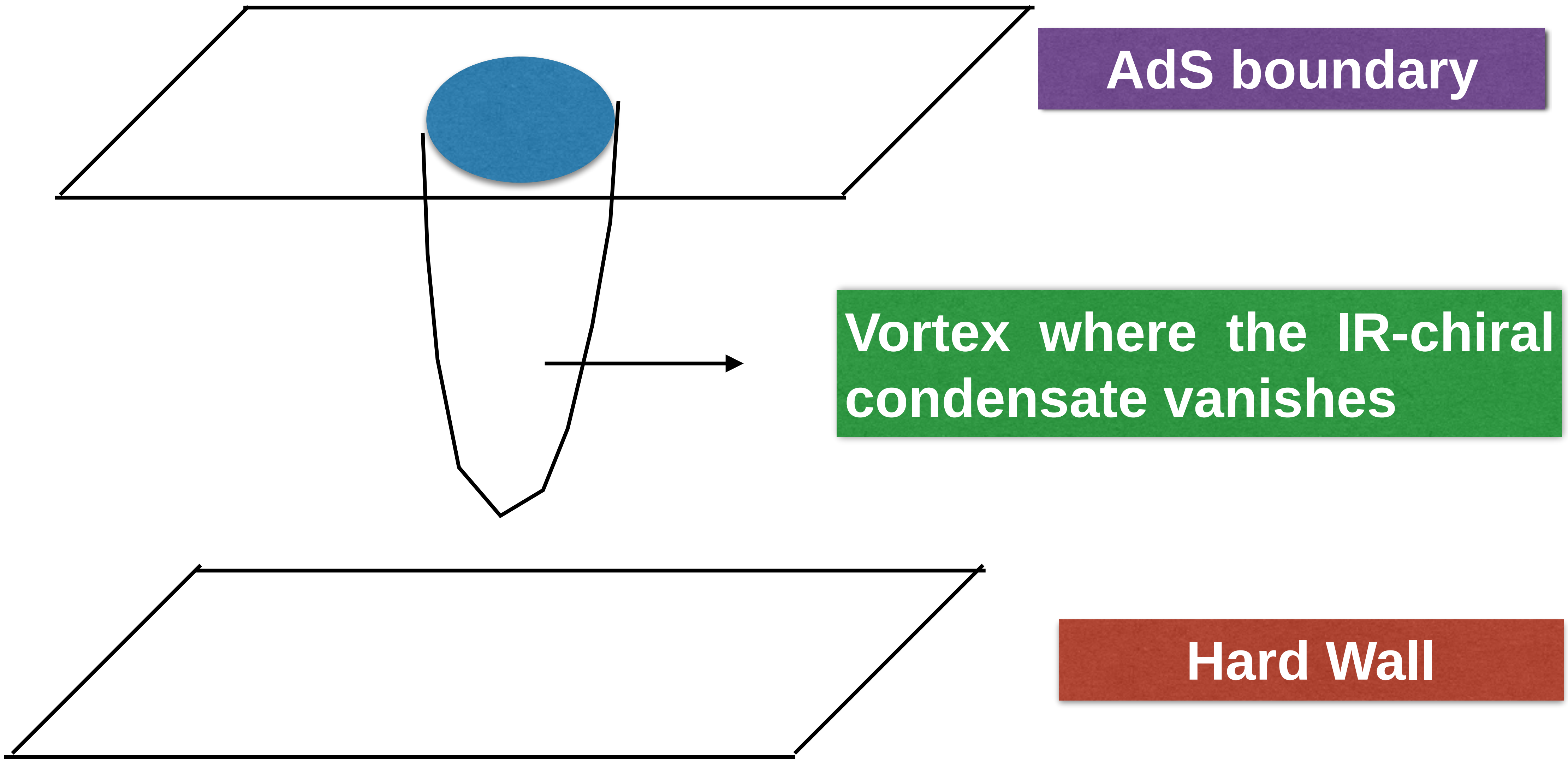
Semi-holographic model

Minimally coupled MIT bag model (with perturbative gluonic corrections) + Sakai-Sugimoto for the IR

Replace bag constant with hard-soft couplings

The bag is created dynamically via coupling to a vortex solution of 5D Yang-Mills equations

Bulk fields: Graviton (dual to em-tensor), SU(3)_L times SU(3)_R gauge fields (dual to flavour currents), bi-fundamental scalar (dual to chiral condensate)



Minimalist couplings:

$$j_\mu \mathcal{J}^\mu, \quad t_{\mu\nu} \mathcal{T}^{\mu\nu}, \quad \bar{\Psi}\Psi\Phi \quad \text{suppressing traces}$$

Capture aspects of parton distributions at Q^2 below saturation scale?

Pentaquark states: Bound states of semi-holographic bag mesons with holographic baryons ?

Note holographic baryons are solitons of 5D gauge field [Hata, Sakai, Sugimoto, Yamato 2007]

Work in progress with A. Rebhan and E. Iancu

Part 5:Outlook

The focus is first on proving the internal consistency of the semi-holographic framework for QCD as an effective theory

After this we can have robust predictions which can be matched with lattice data and experiments

Finally we can look for a derivation

Embed semi-holography in string theory?

“It's still magic even if you know how it's done”

–Terry Pratchett, *A Hat Full of Sky*

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