Quantum Local Quench, AdS/BCFT

and

Yo-Yo String

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Motivations and Setup

Time dependent processes are of special interest (and usually difficult to study).

An example of such a process is $Quench$.

This is when a system with the Hamiltonian H_0 is prepared in a pure state, say the ground state $|\psi_0\rangle$, at time $t = 0$ and evolves unitarily by a $different$ Hamiltonian H for times $t > 0$.

H_0 and H might be related by changing parameters of the system, external sources, boundary conditions...

Quench can be global (homogeneous, inhomogeneous), or local depending on how H_0 and H are related.

The question of interest is how various quantities; correlation functions, expectation values ..... change in time.

• In this work we study an example of a local quench in a $1+1$ dimensional system;

Two identical one dimensional systems, each living on a half line, and prepared in their ground states, are attached to one another at their boundaries and make up a system on the infinite line

• We further require the two systems to be *Conformally Invariant* with a *Free Boundary Condition* at their boundaries.

• We express each of the two Boundary Conformal Field Theories (BCFT's) in terms of their gravitational duals via the AdS/BCFT proposal.

• We suggest that the process of quench is holographically described by the formation of a closed string in the gravitational dual. Subsequent motion of the string governs the time dependence.

• We provide some tests for the model.

- Quench by Field Theory
- AdS/BCFT
- Quench by Holography
- Entanglement Entropy (EE) after Quench

Some References

• Calabrese, Cardy • Eisler, Peschel • Asplund, Bernamonti •.....

• Ryu, Takayanagi • Casini, Huerta, Myers • Hartman, Maldacena • Aparicio, Lopez • Albash, Johnson • Nozaki, Numasawa, Takayanagi • Ugajin • ....

• Takayanagi

• Affleck, Laflorencie, Sorensen • Jensen, O'Bannon •...

• Ficnar, Gubser

• ....

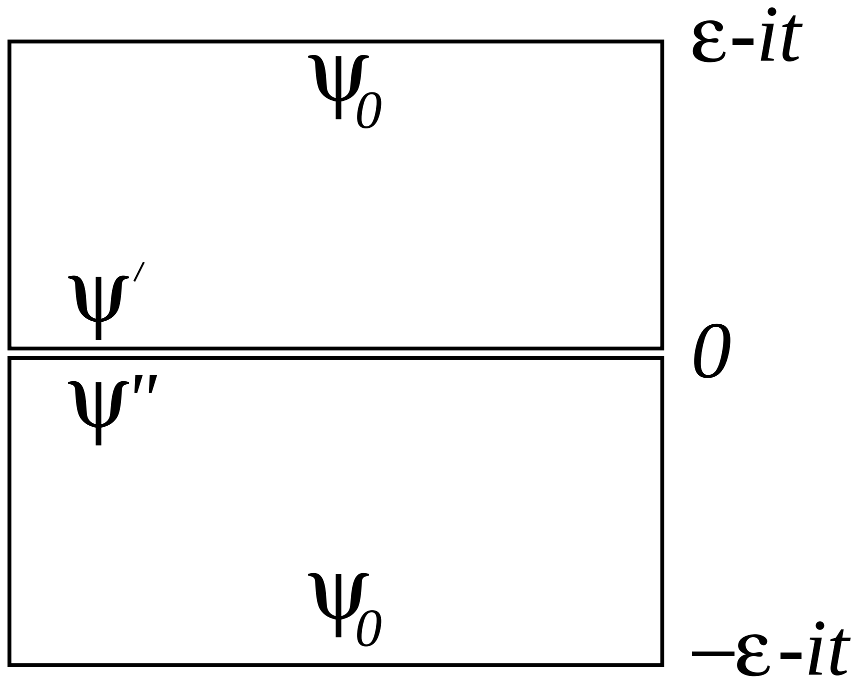
Quench by Field Theory

Consider the state $|\psi_0\rangle$ that evolves from $t = 0$ by the Hamiltonian H. The density matrix ρ at time t will be

$$\rho(t) \sim e^{-itH-\epsilon H}|\psi_0\rangle\langle\psi_0|e^{itH-\epsilon H}$$

$e^{-\epsilon H}$ is inserted for convergence. Upon analytical continuation, $\tau = it$, the matrix element can be shown by a path integral on a strip with the edges being in the state ψ_0 at the Euclidean times of $\tau_1 = -\epsilon - it$ and $\tau_2 = \epsilon - it$

$$\langle \psi'' | \rho(\tau) | \psi' \rangle \sim$$



(from Calabrese and Cardy, '09)

Any quantity can hence be found by inserting proper operators $\mathcal{O}$ on this manifold, sewing the two edges at $\tau = 0$ and calculating $tr(\rho\mathcal{O})$. That is

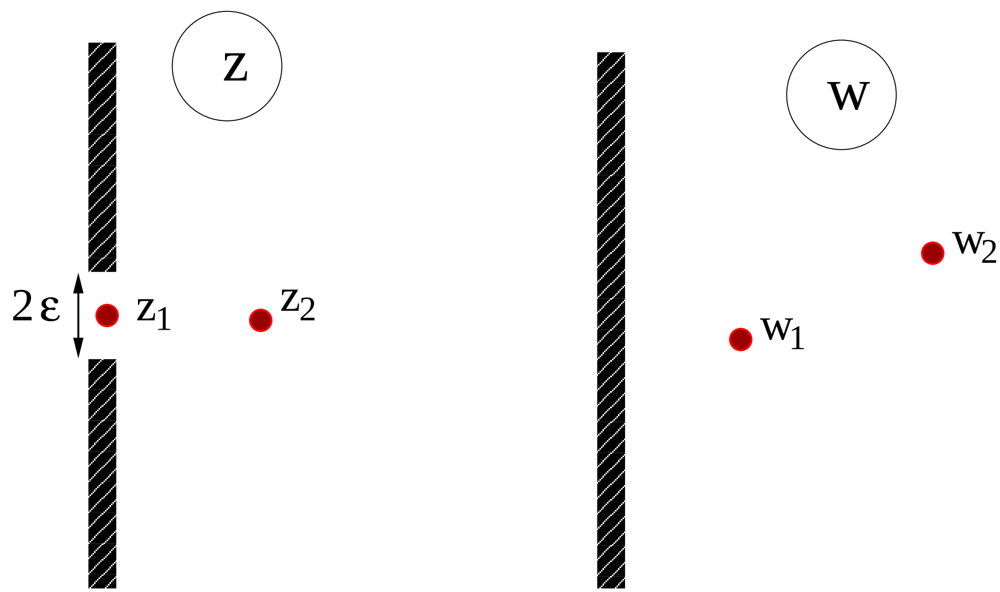
$$tr(\rho\mathcal{O}) = \langle\mathcal{O}\rangle_{strip}$$

The case which is of interest in this work is when theories on half lines join together at their boundaries at time $t = 0$. The system then evolves for a time period T at which we would like to calculate various quantities.

One can again make an analytic continuation, $\tau = it$, and construct the density matrix at time T. This amounts to having a manifold with two slits parallel to the imaginary time axis one from $\tau = -\infty$ to $\tau = -\epsilon$ and one from $\tau = +\epsilon$ to $\tau = +\infty$.

There are two edges at $\tau = iT$ where one puts the states ψ' and ψ'' to get the matrix element of the density matrix $\langle\psi''|\rho(T)|\psi'\rangle$.

One can again insert proper operators, sew the edges together and calculate the desired quantities.



(from Calabrese and Cardy, '09)

The conformal transformation

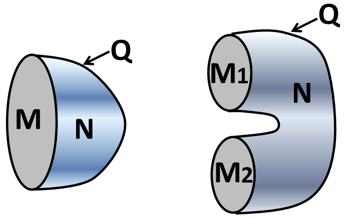
$$w = \frac{z}{\epsilon} + \sqrt{(\frac{z}{\epsilon})^2 + 1}$$

takes one from the worldsheet $(z, \bar{z})$, $z = \sigma + i\tau$, to the upper half plane parametrized by $(w, \bar{w})$. Again, expectation values after quench, $tr(\rho\mathcal{O})$, are mapped to correlation functions in a $BCFT$.

AdS/BCFT

This is a conjectured duality between CFT's with a boundary (BCFT's) and gravity. We focus on $AdS_3/BCFT_2$.

The BCFT is defined on a manifold $\mathcal{M}$ with a boundary $\partial\mathcal{M}$. The holographic dual is constructed by extending $\mathcal{M}$ to a 3-dimensional assymptotically AdS manifold $\mathcal{N}$. The boundary $\partial\mathcal{M}$ is extended to a 2-dimensional manifold Q such that $\partial\mathcal{M} = \partial Q$ and $\partial\mathcal{N} = Q \cup \mathcal{M}$.



(a) (b) (from Takayanagi, '11)

The holographic description consists of $\mathcal{N} \cup Q$. The action that describes this system is

$$S = S_{\mathcal{N}} + S_Q$$

where

$$S_{\mathcal{N}} = \frac{1}{16\pi G_N} \int_{\mathcal{N}} d^3x \sqrt{-G}(R - 2\Lambda)$$

$$S_Q = \frac{1}{8\pi G_N} \int_Q d^2x \sqrt{-h}(\mathcal{K} - T)$$

In these expressions h_{ij} and $\mathcal{K}_{ij}$ are the induced metric and extrinsic curvature of Q respectively and $\mathcal{K} = h^{ij}\mathcal{K}_{ij}$. The constant T is the tension of Q.

One should also include the boundary action associated with $\mathcal{M}$

$$S_{\mathcal{M}} = \frac{1}{8\pi G_N} \int_{\mathcal{M}} d^2x \sqrt{-g} K \ ,$$

where again g_{ij} and K_{ij} are the induced metric and extrinsic curvature of $\mathcal{M}$, respectively and $K = g^{ij} K_{ij}$.

We choose the usual *Dirichlet boundary condition* on $\mathcal{M}$ but *Neumann boundary condition* on Q. Then variational principle gives us Einstein equation in the bulk region, $\mathcal{N}$, as well as the following constraint on the boundary Q

$$\mathcal{K}_{ij} = h_{ij}(\mathcal{K} - T)$$

Using the Poincare coordinate for AdS

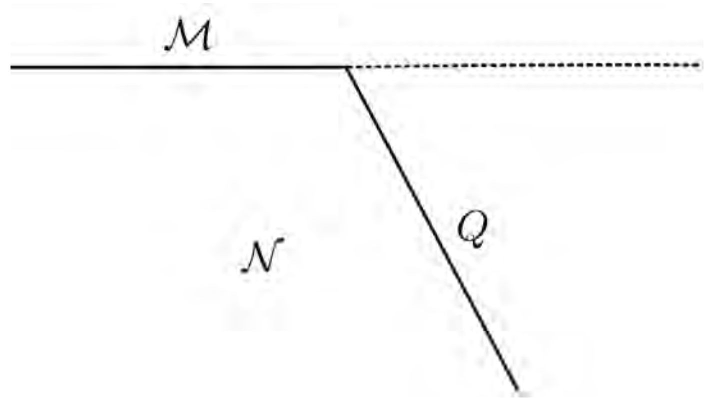
$$ds^2 = \frac{L^2}{z^2}\left(-dt^2 + dz^2 + dx^2\right)$$

we can parameterize The hypersurface Q as $x_Q = x_Q(z)$. The unit normal on this surface reads

$$n^\mu = \left(n^t, n^z, n^x\right) = \frac{z}{L\sqrt{1+x'^2(z)}}\left(0, -x'(z), 1\right)$$

and the profile of Q is obtained as

$$x_Q(z) = \frac{TL}{\sqrt{1-T^2L^2}}\, z$$



• The boundary state of a BCFT, $|B\rangle$, defines a *Boundary Entropy*, S_B, through

$$S_B = \log g_B \ , \qquad g_B = \langle 0|B\rangle$$

It can be shown that in the holographic setup

$$S_B \sim \tan^{-1}(LT)$$

Quench by Holography

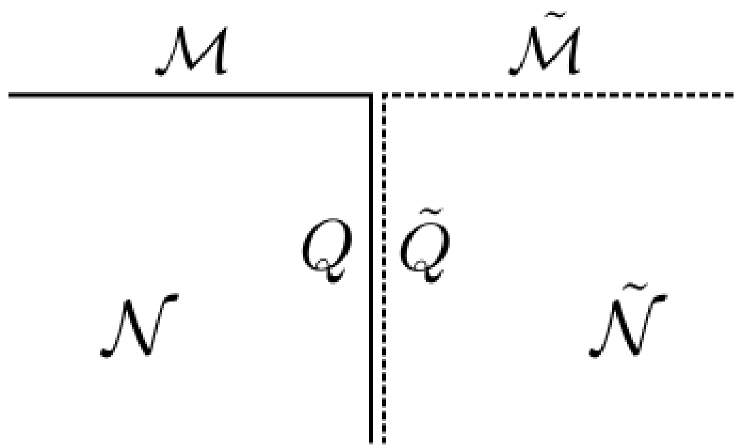
Consider a CFT living on a semi-infinite line $\mathcal{M}$ with $x < 0$. Specify to the situation when the tension on Q is zero. In the field theory language this means that the boundary entropy for this system is zero. this is what we mean by a *free boundary condition*.

A key observation is that for $T = 0$, the surface Q is simply the world sheet of an open string which satisfies the classical equations of motion for a bosonic Polyakov string. The string attaches the boundary at $z = 0$ with the usual Neumann boundary condition

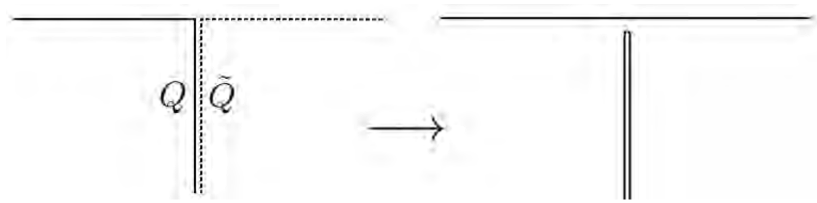
$$\partial_z x_Q|_{z=0} = 0 \quad \text{when} \quad T = 0$$

The string, however, is tensionless; $\alpha' \to \infty$.

Now consider an identical field theory which lives on the semi-infinite line $\tilde{\mathbb{M}}$ with $x > 0$. Again the same situation holds as before



Local quench occurs as a result of manipulations at and around the point $x = 0$ which joins the two disconnected field theories into a single one. Our proposal is that the bulk version corresponds to detaching each of the open strings from the boundary and attaching the two ends together



This will produce a folded closed string that can now propagate in the bulk

The Yo-Yo String

This is a classical configuration of the Polyakov string. It describes a closed folded string where the folding points move on light-like geodesics.

The folding point changes direction at

$$z_* = \frac{2LT}{\pi E}$$

where E is the total energy of the string.

This is called the snapback point.

The folded string that is formed after quench is a Yo-Yo string at the snapback point.

Being tensionless, one can consider the following limit of this solution

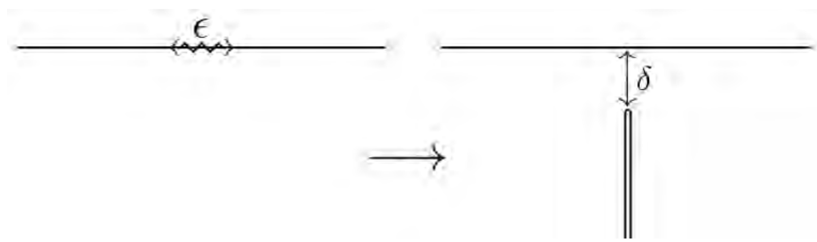
$$E \to 0\ , \quad T \to 0$$

which defines the snapback point as

$$\lim_{T\to 0, E\to 0} z_* = \lim_{T\to 0, E\to 0} \frac{2LT}{\pi E} \equiv \delta$$

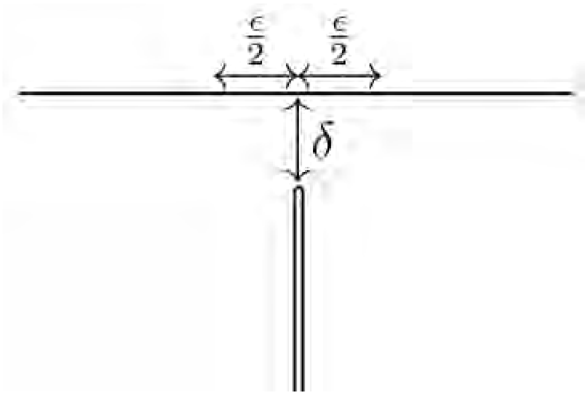
To interpret the length δ in the field theory setup one must address the question of how local the quench has taken place. In other words to what extent have the points around $x = 0$ have been disturbed or excited during quench.

Assume that the process of attaching the two field theories has affected a region of size ϵ. We wish to relate ϵ and δ



A natural identification for our holographic picture will be

$$\delta = \frac{1}{2}\,\epsilon$$

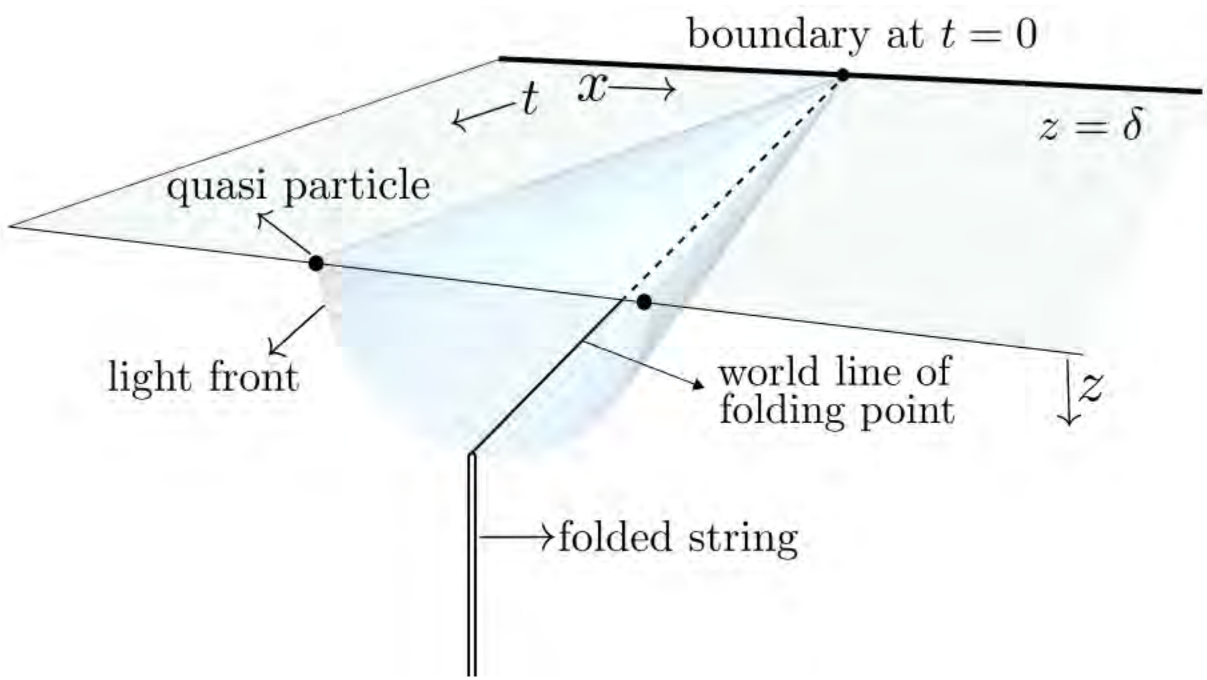


Some General Features

Dealing with zero boundary entropy for the initial BCFT's, the whole process is governed by causal effects rather than energy transfer. In the field theory language this is a result of propagation of *quasi-particles* in both direction on the line which deliver the message of manipulations at $x = 0$. These quasi-particles have an arbitrarily small energy.

In the bulk, the folded string falls freely away from the boundary into the bulk. The folding point falls on a light-like geodesic. The string has an arbitrarily small energy and does not back react on the geometry, again all that comes into play is the causal effects.

The message of formation of the folded string propagates into the bulk along the light-cone which divides bulk points into those who *know* of the manipulations and those who do not. The light front is identified as the bulk version of quasi-particles



Entanglement Entropy After Quench

We are using EE as a probe to see how the state of the system changes in time. Denote the entangling region by A and its EE by S_A.

The example we pick up for illustration is when A is entirely inside one of the original BCFT's. The endpoints of A are located at ℓ_2 and ℓ_1 with $\ell_1 > \ell_2 > 0$. The two BCFT's are joined at $x = t = 0$.

In field theory, $S_A(t)$ is calculated by the BCFT techniques

$$S_A(t < \ell_2) = \frac{c}{6} \log \frac{(\ell_1 - \ell_2)^2}{a^2} + \frac{c}{6} \log \frac{4\ell_1 \ell_2}{a^2} - \frac{c}{6} \log \frac{(\ell_1 + \ell_2)^2}{a^2}$$

$$S_A(\ell_2 < t < \ell_1) = \frac{c}{6} \; \log \frac{\ell_1 - \ell_2}{\ell_1 + \ell_2} \frac{\ell_1 - t}{\ell_1 + t} \; \frac{t^2 - \ell_2^2}{\epsilon a^2} \; 4\ell_1$$

$$S_A(t > \ell_1) = \frac{c}{3} \log \frac{\ell_1 - \ell_2}{a}$$

a is a short distance regulator.

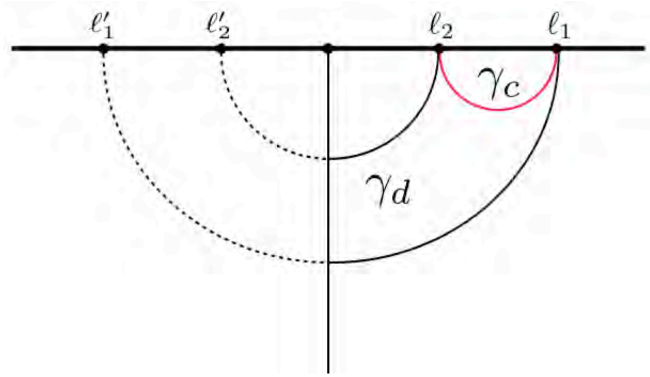
In the gravity side and before quench we have the Ryu-Takatyanagi proposal to calculate S_A. Denoting the endpoints of A by (x_i, x_j), the holographic value for S_A is given by the RT curve through

$$S_A = \frac{\mathcal{A}(\gamma_{x_i x_j})}{4G_N}$$

where G_N is the newton constant, $\gamma_{x_i x_j}$ is the geodesic curve in the bulk that is homologous to A and $\mathcal{A}(\gamma_{x_i x_j})$ is the length of $\gamma_{x_i x_j}$.

In presence of a boundary Q, curves can end on the boundary as well. This can be taken care of by introducing *image points* on $\mathcal{M}$.

For the case at hand, this prescription is summarised as follows



with

$$S_A = Min\{\mathcal{A}(\gamma_c), \mathcal{A}(\gamma_d)\}$$

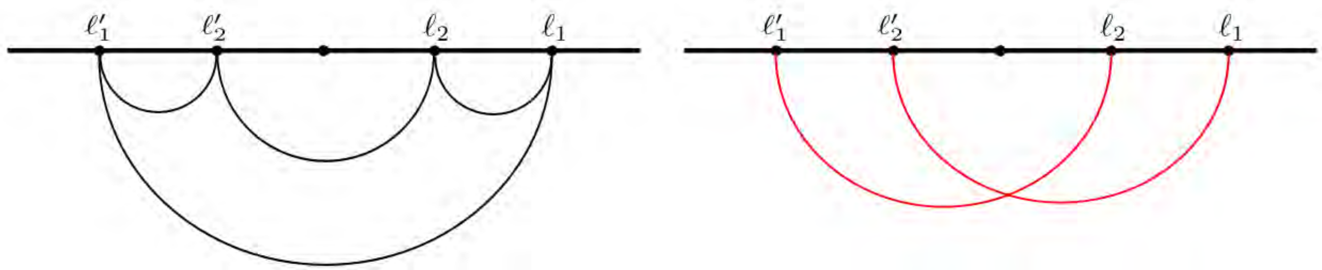
$$\mathcal{A}(\gamma_c) = \frac{c}{6} \log \frac{(\ell_1 - \ell_2)^2}{a^2} \ , \qquad \mathcal{A}(\gamma_d) = \frac{c}{6} \log \frac{4\ell_1 \ell_2}{a^2}$$

The negative contribution is missing in the standard RT prescription. In a time dependent situation, for example when the entangling region moves with respect to the boundary, this part plays an important role; it smoothly interpolates between disconnected and connected pieces.

In the limit of $\ell_2 << \ell_1 - \ell_2$, the negative part candles the connected piece and the disconnected part dominates.

In the limit of $\ell_2 >> \ell_1 - \ell_2$, the negative part cancels the disconnected piece and the connected part dominates.

We thus propose the following modification of the RT prescription



(Hubeny and Rangamani; 2007)

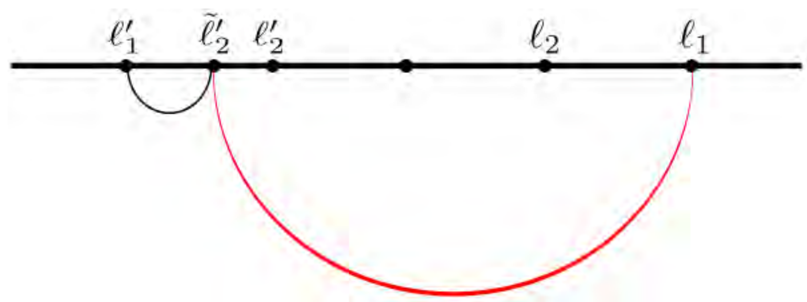
In absence of any back reactions, variations of EE in time can only come through the *deformations* and *modifications* of the RT curves.

To motivate the modification we note that the complementary region to A, denoted by $\bar{A}$, changes in time. At early times when A has not yet received the message of quench, its reduced density matrix ρ_A is unaffected and hence S_A remains unchanged. After the quasi-particle has penetrated A (say at time $t = t_*$), part of the region, denoted by $\alpha \subset A$, receives the message and the entanglement pattern begins to change. This continues until the quasi-particle exits the interval when S_A assumes its steady state value. We are only interested in the intermediate times $\ell_2 < t < \ell_1$ in the following.

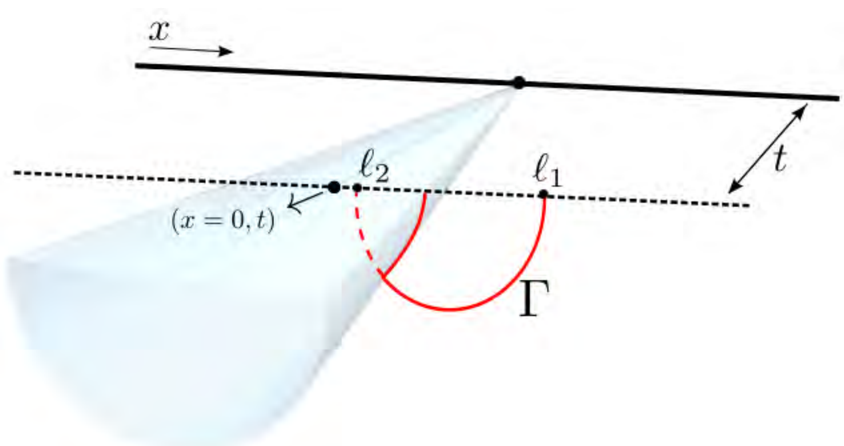
Entanglement entropy changes by two competing contributions; one that decreases S_A and one that increases it. Some of the existing entanglements disappear and some new ones form. In other words, the entanglement between A and $\bar{A}(t < t_*)$ transfers to that between A and $\bar{A}(t > t_*)$.

As the quasi-particle pair travel in both directions, the degrees of freedom in α find new counterparts to entangle with. These are those degrees of freedom which have been swept by the pair. This in turn transfers part of entanglement between A and $\bar{A}(t < t_*)$ to α and $\bar{A}(t > t_*)$.

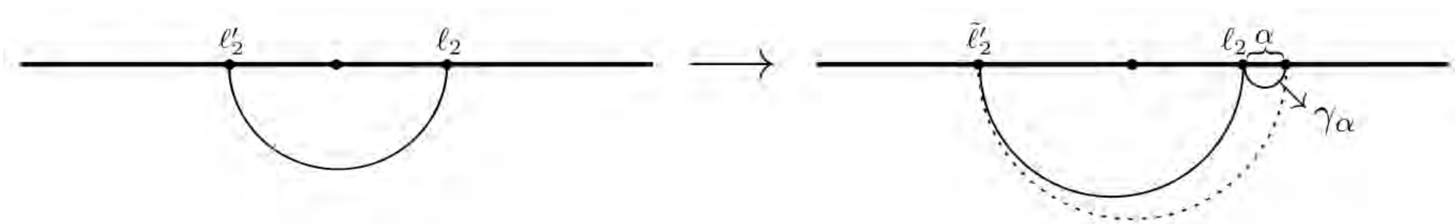
The decreasing contribution can be best understood by focusing on the image point ℓ_2'. This point is swept off to the left to $\tilde{\ell}_2'$ as the quasi-particle passes through it. In terms of the RT curves, this decreases the positive contribution of $\mathcal{A}(\gamma_{\ell_1'\ell_2'}) \to \mathcal{A}(\gamma_{\ell_1'\tilde{\ell}_2'})$ and increases the negative contribution of $\mathcal{A}(\gamma_{\ell_1\ell_2'}) \to \mathcal{A}(\gamma_{\ell_1\tilde{\ell}_2'})$. The rest of the curves are unaffected

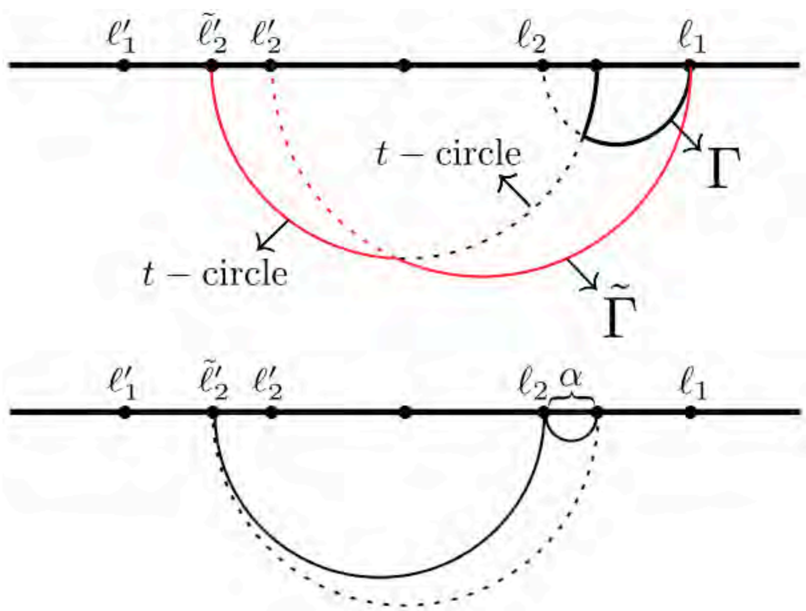


We suggest the following modification



The increasing contribution is a result of the new entanglements that the subset α finds with those degrees of freedom that have already received the quench message. We suggest the following modification





$$S_D = \mathcal{A}(\Gamma) - \mathcal{A}(\tilde{\Gamma}) = \frac{c}{6} \ \log \frac{\ell_1 - \ell_2}{\ell_1 + \ell_2} \ \frac{\ell_1 - t}{\ell_1 + t}$$

$$S_I = \frac{1}{2}\mathcal{A}(\gamma_{\ell_2 \tilde{\ell}_2'}) + \frac{1}{2}\mathcal{A}(\gamma_\alpha) = \frac{c}{6} \log \frac{t^2 - \ell_2^2}{a\delta}$$ (δ light-cone regulator)

$$S_A = S_D + S_I + \frac{1}{2}\mathcal{A}(\gamma_{\ell_1 \ell_1'}) = \frac{c}{6} \ \log \frac{\ell_1 - \ell_2}{\ell_1 + \ell_2} \ \frac{\ell_1 - t}{\ell_1 + t} \ \frac{t^2 - \ell_2^2}{a\delta} \ \frac{2\ell_1}{a}$$

Summary

- We used AdS/BCFT to describe Local Quench by Holography

- We learned that Yo-Yo string forms

- For free boundary condition all that comes into play is causal effects

- We proposed how the light-cone of the string deforms RT curves and reproduced field theory results for EE as a function of time

- The case of a non-zero boundary entropy, backreaction,....

- A transformation in gravity side that parallels conformal transformation of CC

- Using the CHM map to find a time dependent thermal entropy

- ....