Turbulent strings in AdS/CFT

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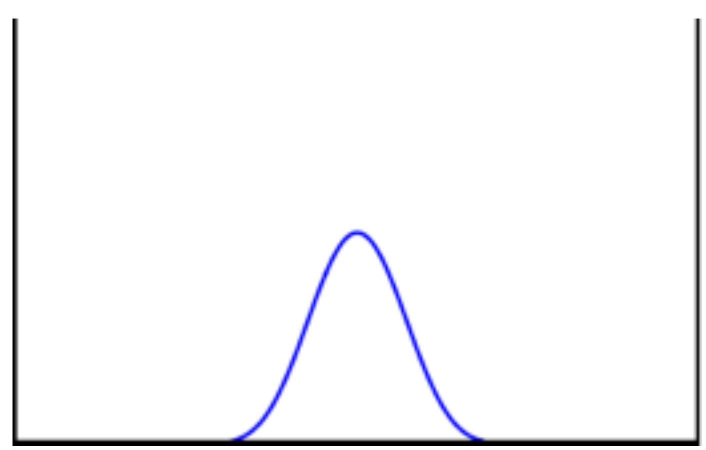
JHEP06(2015)086 [arXiv:1504.02190]
with Keiju Murata

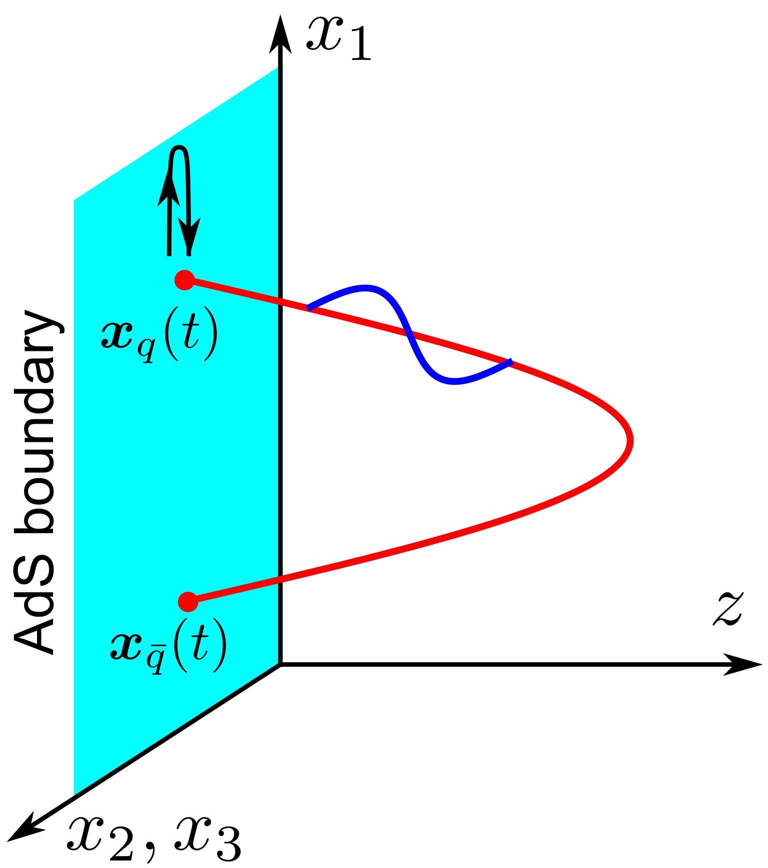


6 July 2015@Nafplion

What I will do

Perturb holographic quark-antiquark potential

Solve nonlinear time evolution



Motivations

- AdS turbulent instability
- Electric field quench on D7 [Hashimoto-Kinoshita-Oka-Murata]
- Dynamical meson melting [TI-Kinoshita-Murata-Tanahashi]
- Cosmic strings in flat space

Contents

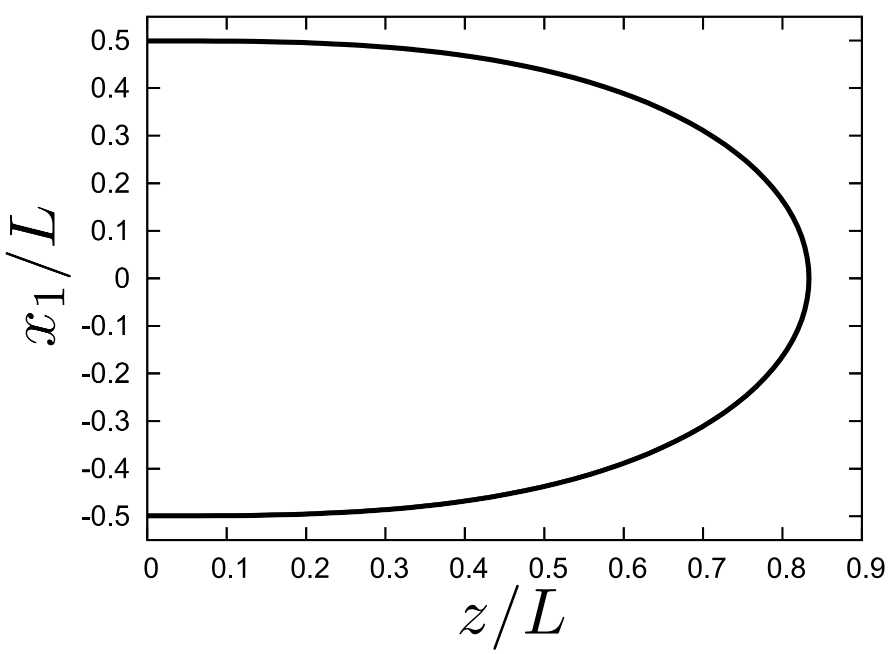
Holographic quark potential

[Maldacena, Rey-Yee]

AdS_5xS^5 $$ds^2 = \frac{\ell^2}{z^2}\left(-dt^2 + dz^2 + d\boldsymbol{x}^2\right) + \ell^2 d\Omega_5^2$$

Static gauge: $(\tau,\sigma)=(t,z)$
Target space embedding: $x_1=X_1(z)$

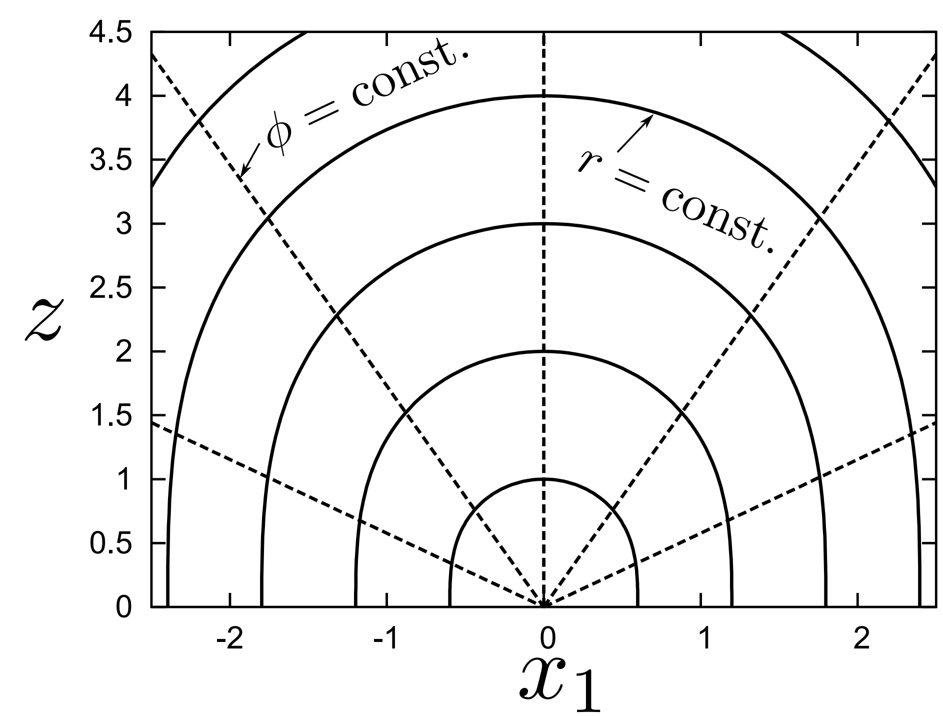
Solution with separation L



A polar parametrization

Use polar-like coordinates (r,φ) with $0 \le \phi \le \beta_0$

The static solution is at r=const.



$$z = rf(\phi) = r\,\mathrm{sn}(\phi; i)$$

$$x_1 = rg(\phi) = r \begin{cases} \phi - E(\mathrm{sn}(\phi; i); i) + \Gamma_0 & (\phi \le \beta_0/2) \\ \phi + E(\mathrm{sn}(\phi; i); i) - \Gamma_0 - \beta_0 & (\phi > \beta_0/2) \end{cases}$$

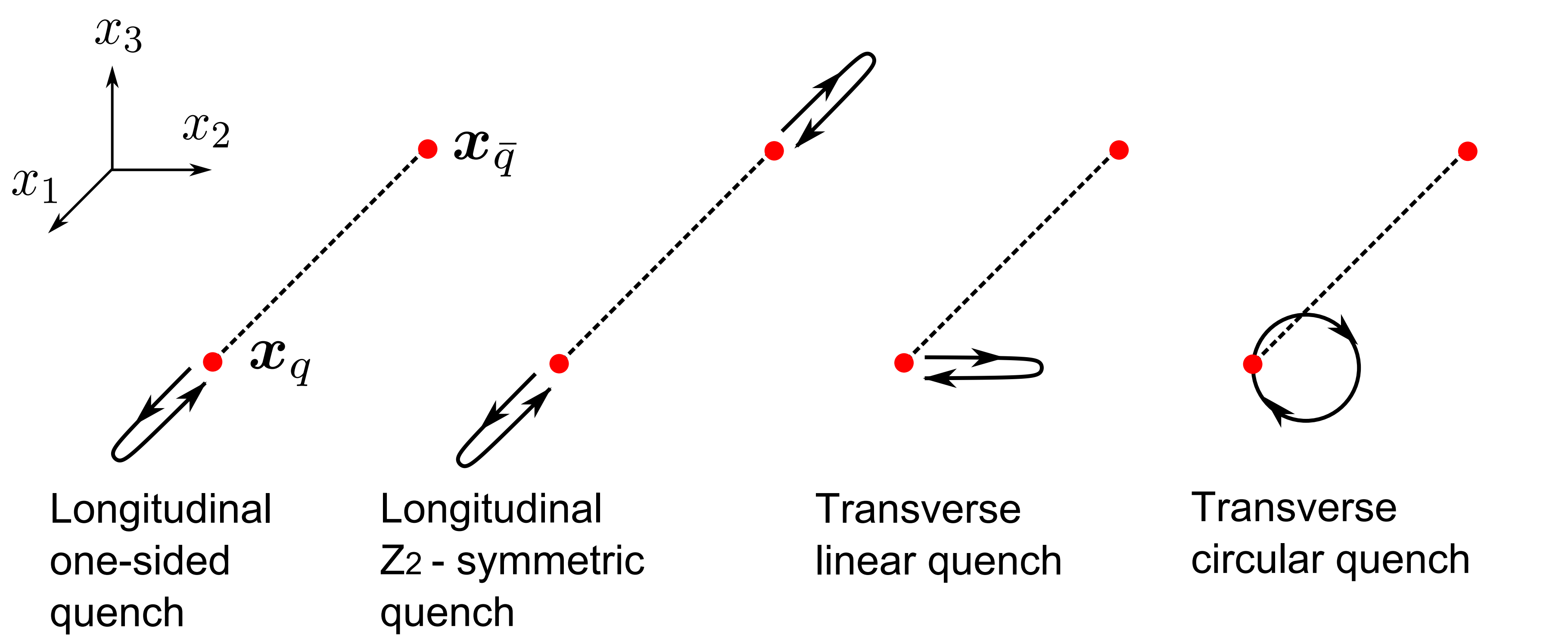
β_0~2.622

Γ_0~0.599

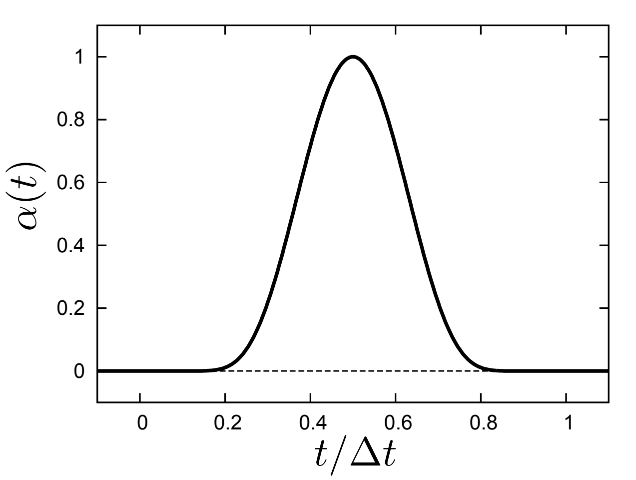
Contents

Perturb the string endpoints

We examine 4 representative patterns:



Quench profile: a Gaussian-ish compact C$^\infty$-function for $0 \le t \le \Delta t$



Worldsheet double null coordinates

Induced metric $ds^2_{F1} = -2\gamma_{uv}dudv$

Worldsheet: u,v

Target space: T(u,v), Z(u,v), $X_{1,2,3}$(u,v)

$$\gamma_{uv} = \frac{\ell^2}{Z^2}(-T_{,u}T_{,v} + Z_{,u}Z_{,v} + \boldsymbol{X}_{,u}\cdot\boldsymbol{X}_{,v})$$

Equations of motion

$$T_{,uv} = \frac{1}{Z}(T_{,u}Z_{,v} + Z_{,u}T_{,v})$$

$$Z_{,uv} = \frac{1}{Z}(T_{,u}T_{,v} + Z_{,u}Z_{,v} - \boldsymbol{X}_{,u}\cdot\boldsymbol{X}_{,v})$$

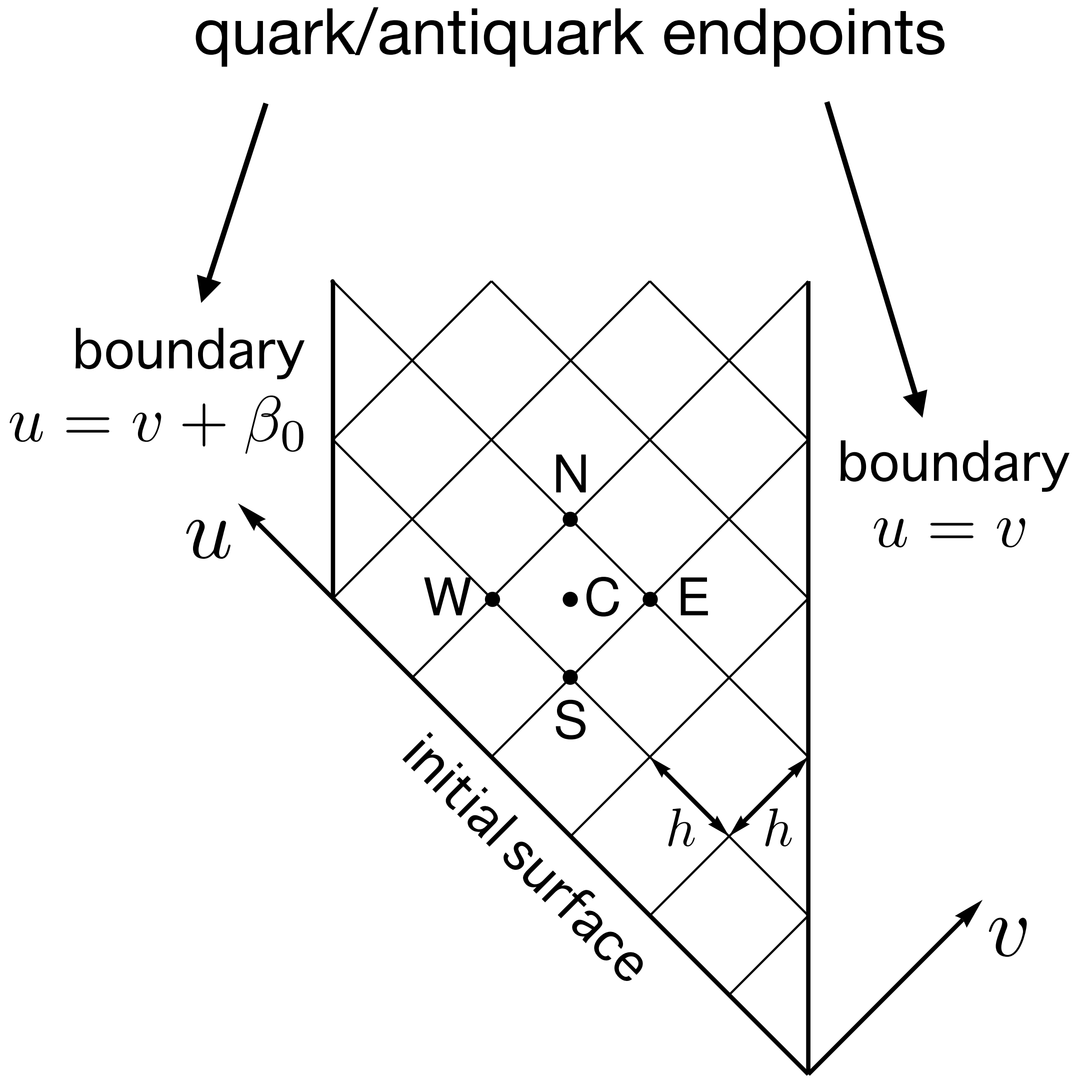
$$\boldsymbol{X}_{,uv} = \frac{1}{Z}(\boldsymbol{X}_{,u}Z_{,v} + Z_{,u}\boldsymbol{X}_{,v})$$

Constraints

$$\gamma_{uu} = \frac{\ell^2}{Z^2}(-T_{,u}^2 + Z_{,u}^2 + \boldsymbol{X}_{,u}^2) = 0$$

$$\gamma_{vv} = \frac{\ell^2}{Z^2}(-T_{,v}^2 + Z_{,v}^2 + \boldsymbol{X}_{,v}^2) = 0$$

Discretization



O(h^2) central finite differential

$$\Psi_{,uv}|_C = \frac{\Psi_N - \Psi_E - \Psi_W + \Psi_S}{h^2}$$

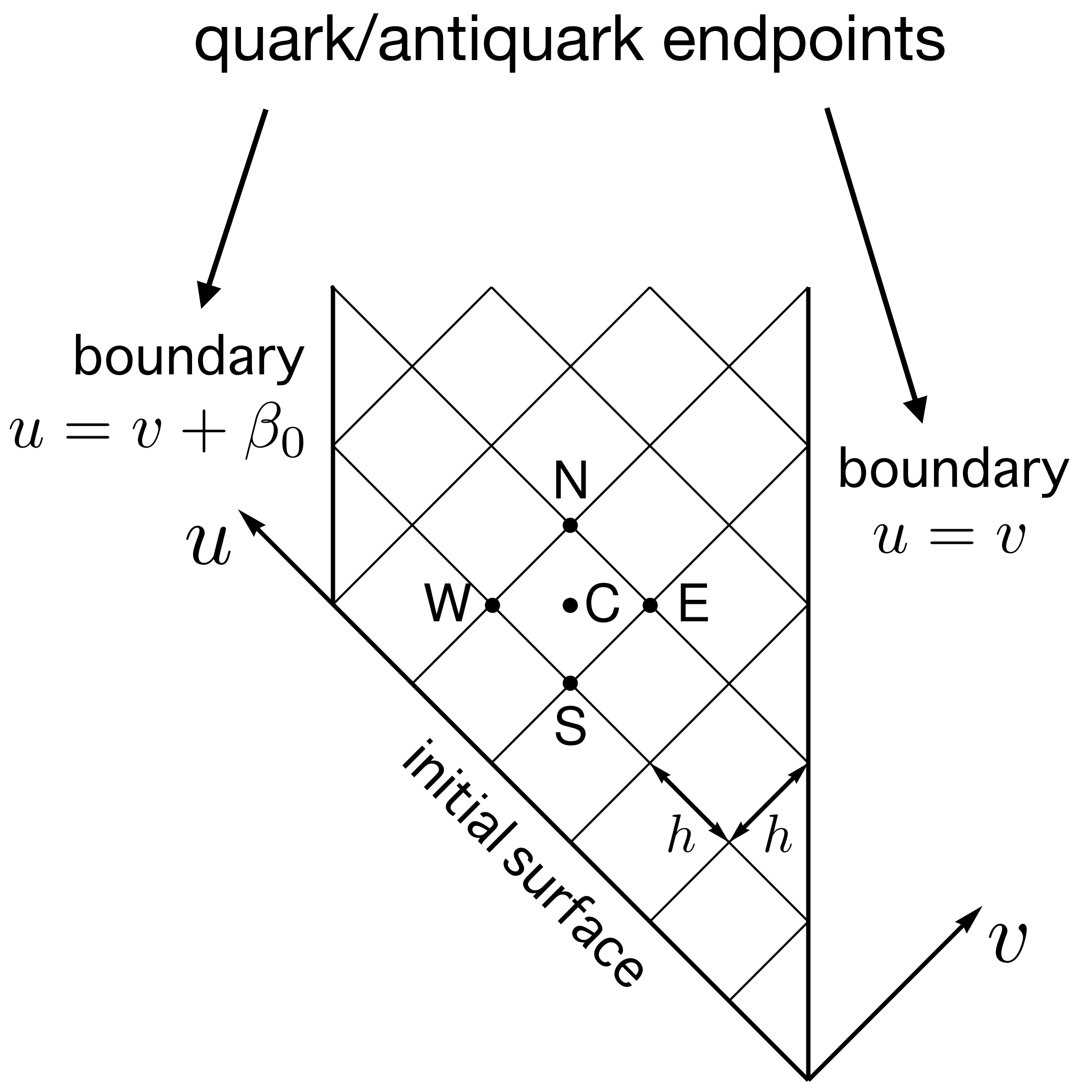
$$\Psi_{,u}|_C = \frac{\Psi_N - \Psi_E + \Psi_W - \Psi_S}{2h}$$

$$\Psi_{,v}|_C = \frac{\Psi_N + \Psi_E - \Psi_W - \Psi_S}{2h}$$

$$\Psi|_C = \frac{\Psi_E + \Psi_W}{2}$$

Compute N by using EWS data

Initial data (v=0)



- Gauge: ϕ=u at v=0
- Static solution: Z(u,0), **X**(u,0)
- Constraint then determines T(u,0)

$$\gamma_{uu} = \frac{\ell^2}{Z^2}(-T_{,u}^2 + Z_{,u}^2 + \boldsymbol{X}_{,u}^2) = 0$$

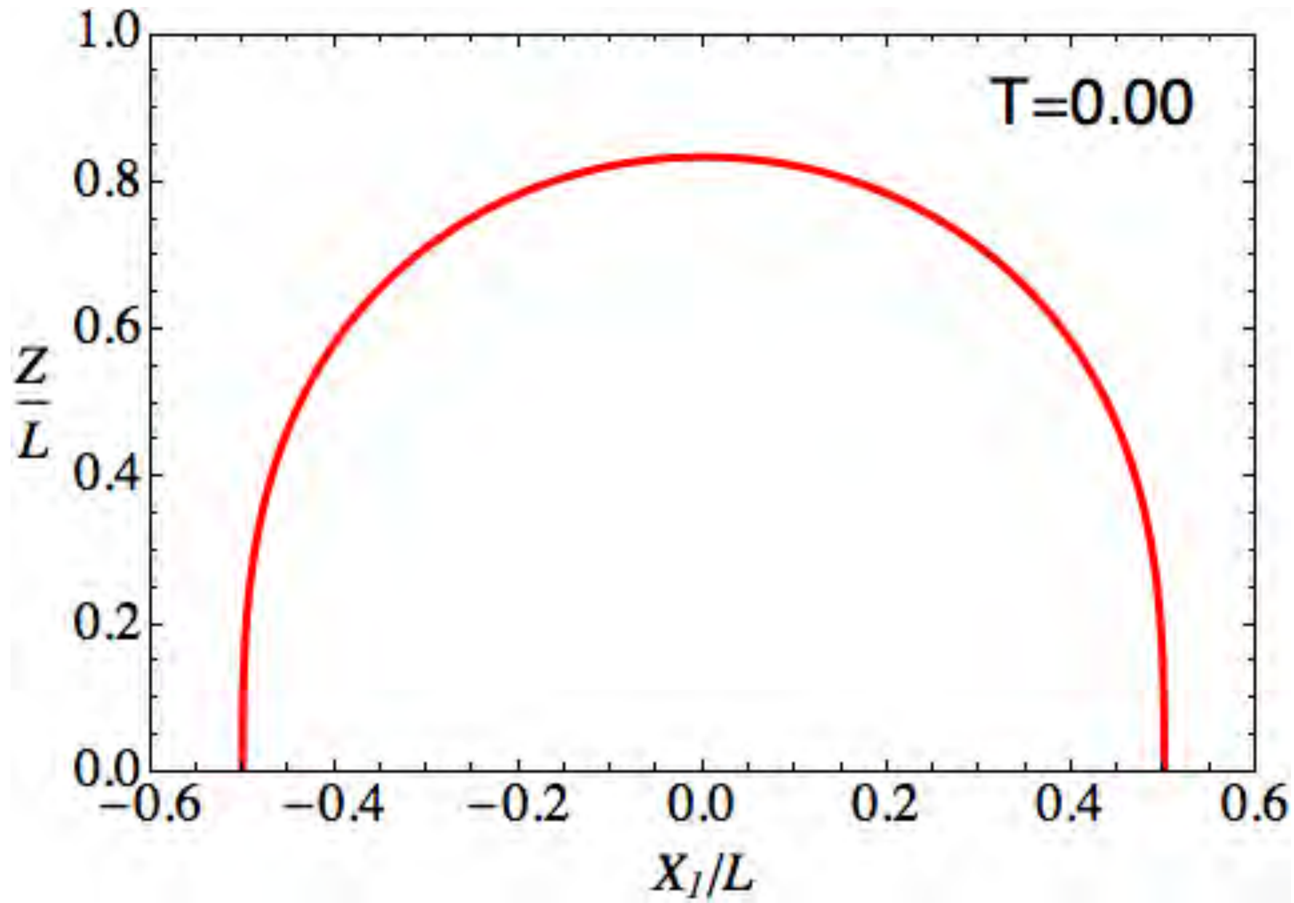
Result (analytic):

$$T(u,0) = z_0\, u$$
$$Z(u,0) = z_0\, f(u)$$
$$X_1(u,0) = z_0\, g(u)$$

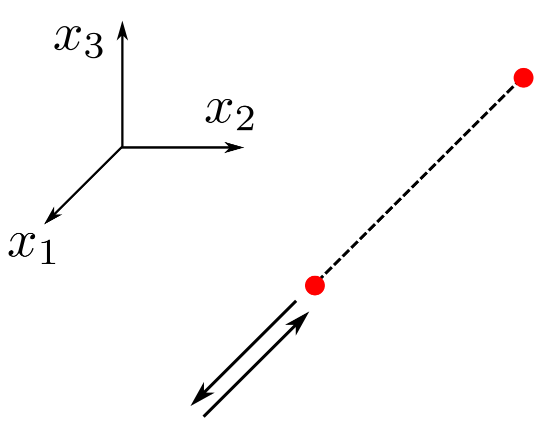
$$\frac{L}{2} = z_0 \Gamma_0$$

Contents

Longitudinal one-sided quench



ε=0.03, Δt/L=2

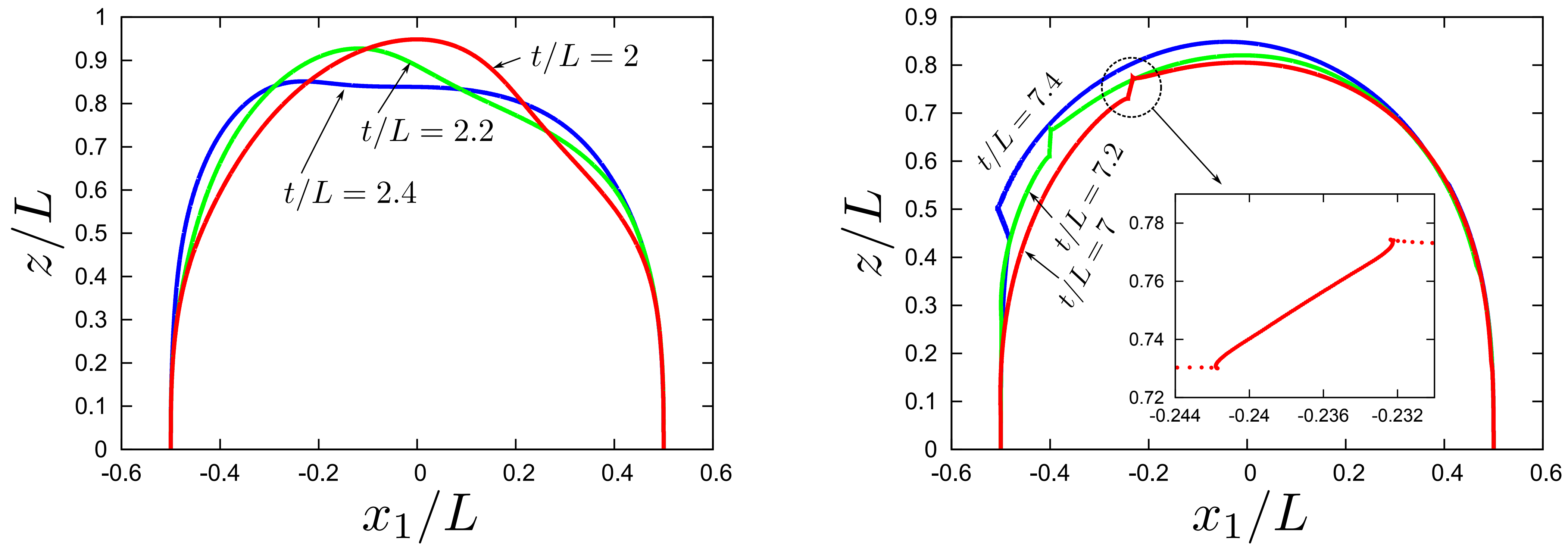


Two parameters:

Amplitude: ε=Δx/L

Duration: Δt/L

Cusp formation



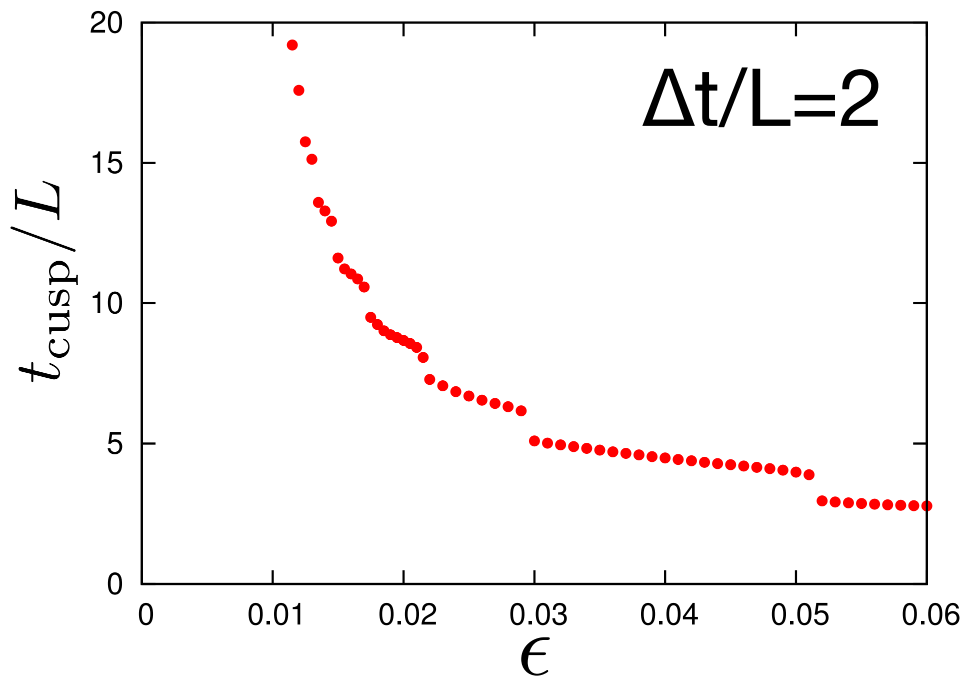
- Worldsheet T(u,v), Z(u,v), **X**(u,v) are all regular
- Target space plots show cusps
- Cusps are pair-created (here $t_{cusp}/L \sim 5$)

Analysis 1: Cusp detection

The conditions satisfied at the cusp:

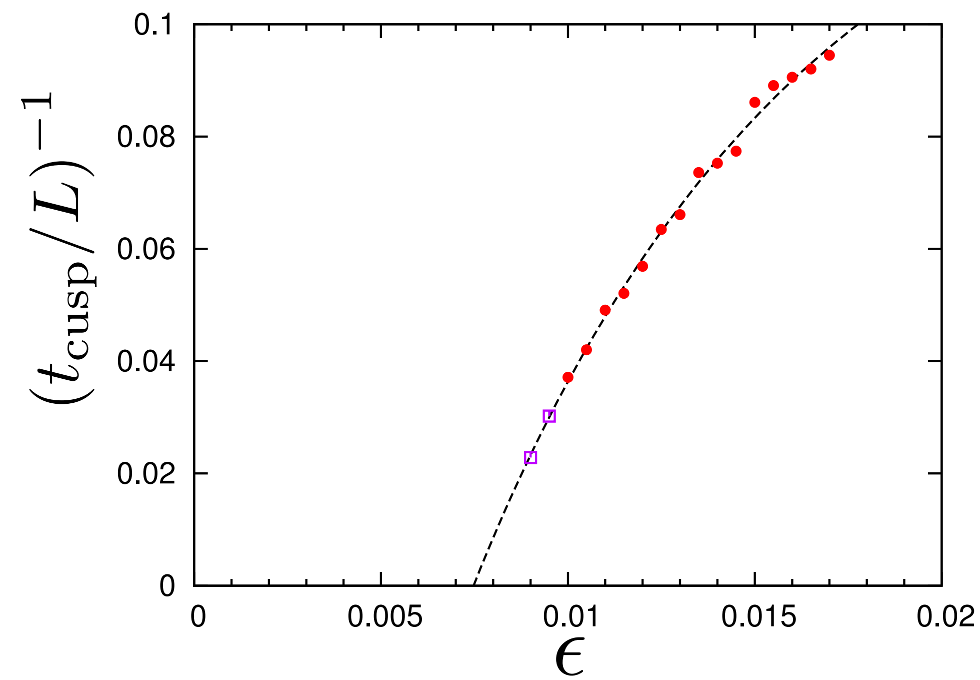
$$J_z \equiv T_{,u} Z_{,v} - T_{,v} Z_{,u} = 0$$

$$J_i \equiv T_{,u} X_{i,v} - T_{,v} X_{i,u} = 0$$



Cusp formation times
when amplitude ε is varied

flip



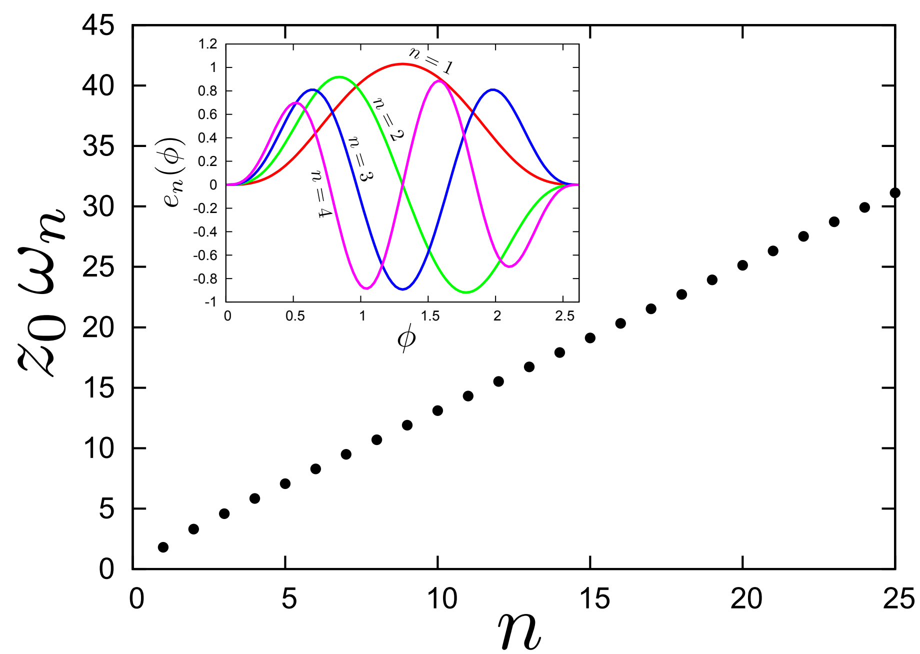
Minimal amplitude is nonzero
An extrapolation: ε_{crit}~0.075

Appendix: Linearized perturbations

[Callan-Guijosa, Klebanov-Maldacena-Thorn]

Linearized fluctuation $X_1 = X_{1(stat)} + \chi_1$

Eigenvalues/functions $(\partial_t^2 + \mathcal{H})\chi_1 = 0$



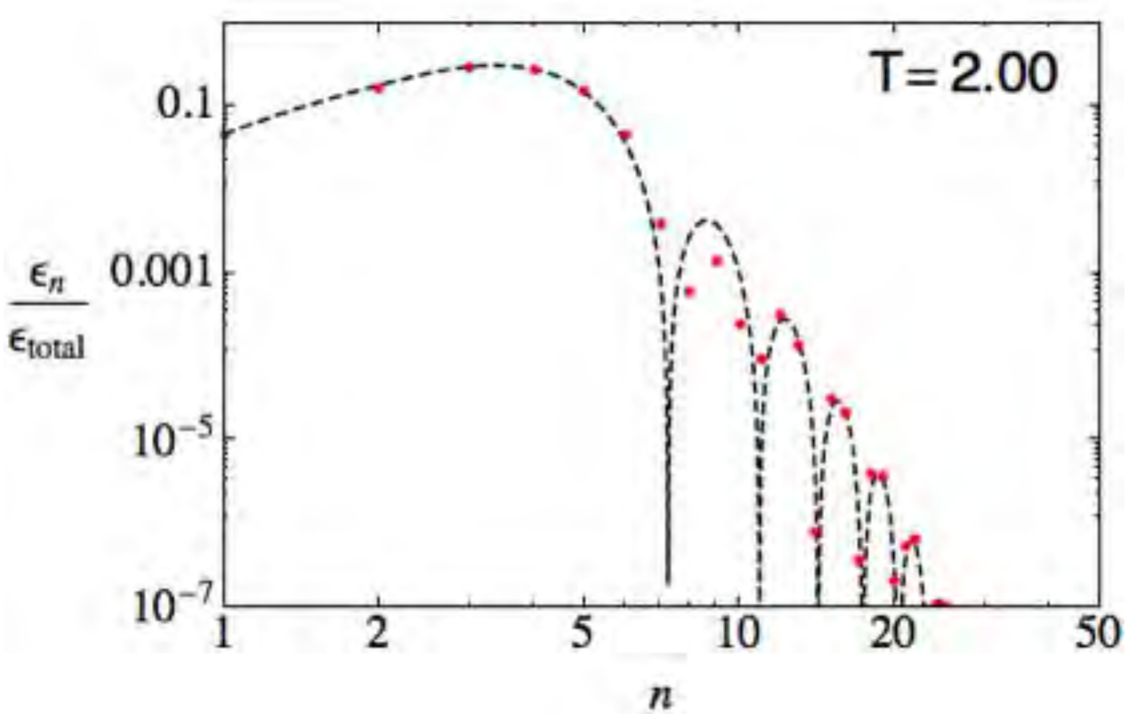
**Transverse modes (x_2,x_3) can be also computed

Analysis 2: Energy spectrum

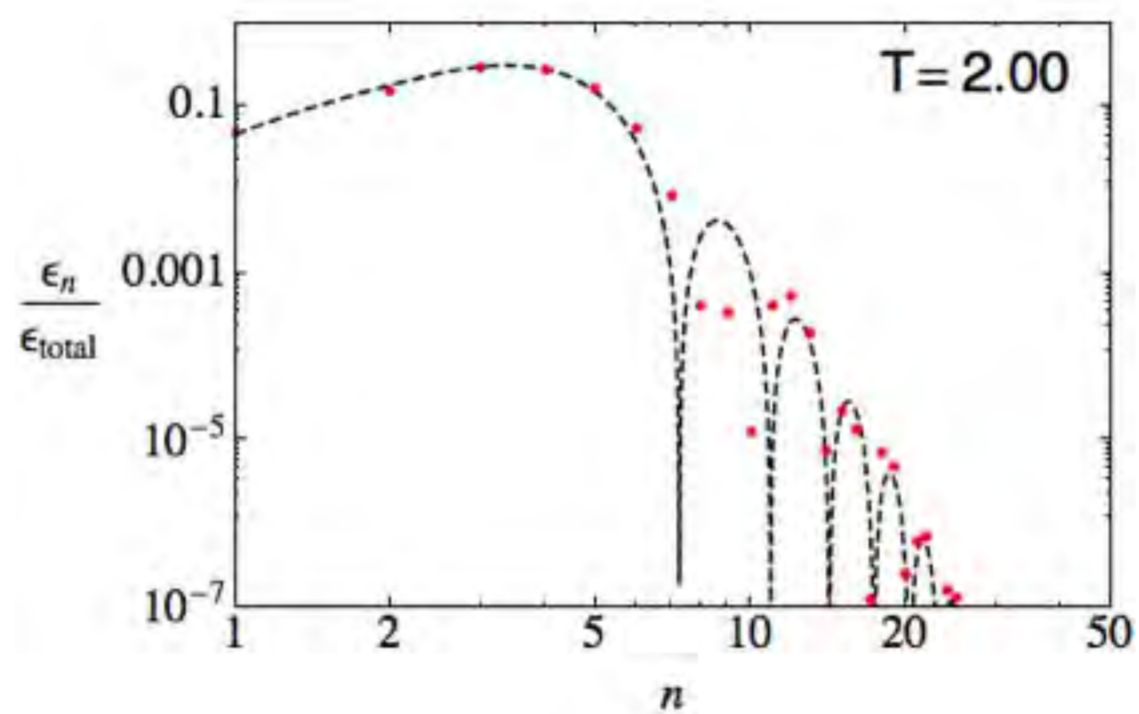
Decompose nonlinear solutions in linear eigenmodes e_n

$$\chi_1 = \sum_{n=1}^{\infty} c_n(t) e_n(\phi) \qquad \varepsilon_n(t) = \frac{\sqrt{\lambda} z_0}{4\pi} \left(\dot{c}_n{}^2 + \omega_n^2 c_n{}^2 \right)$$

Log-log plots



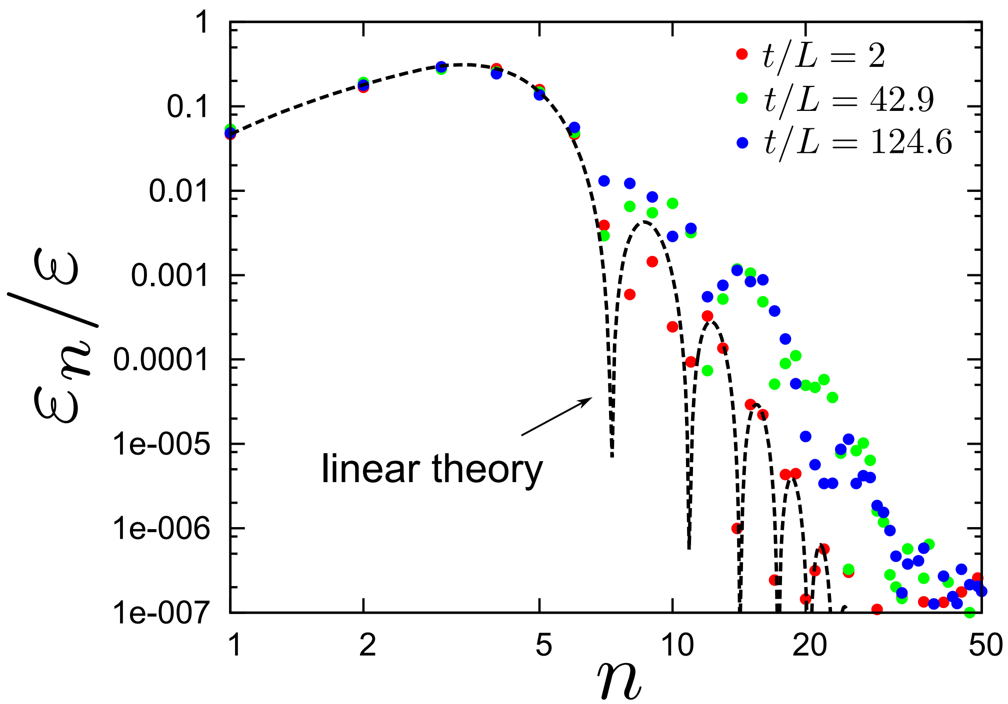
ε=0.005, Δt/L=2 (no cusp)



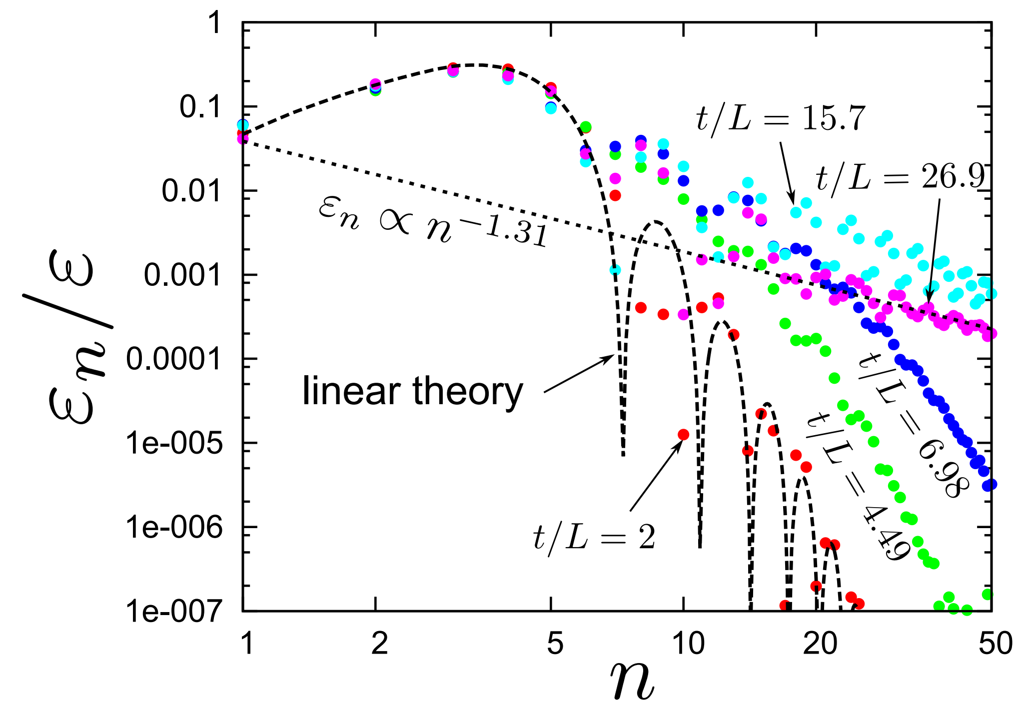
ε=0.01 (cusps T~27)

**Dashed lines: from linearized action

Energy cascade



ε=0.005, Δt/L=2 (no cusp)



ε=0.01 (cusps T~27)

Toward cusp formation: direct energy cascade

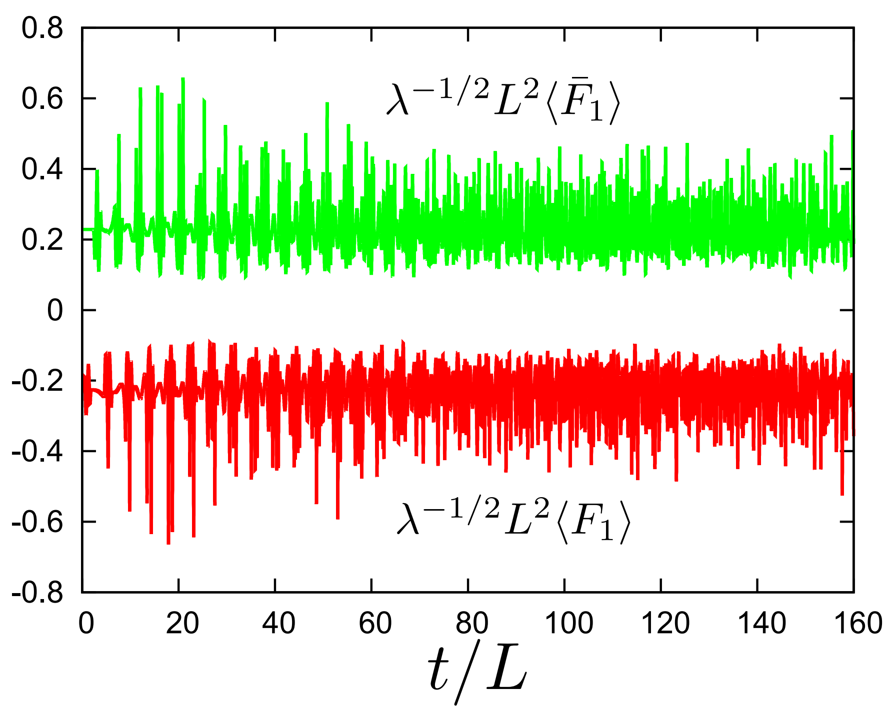
- A fit of right panel at T~27: $\varepsilon_n \sim n^{-1.3}$

No cusp (too small ε): no clear power law

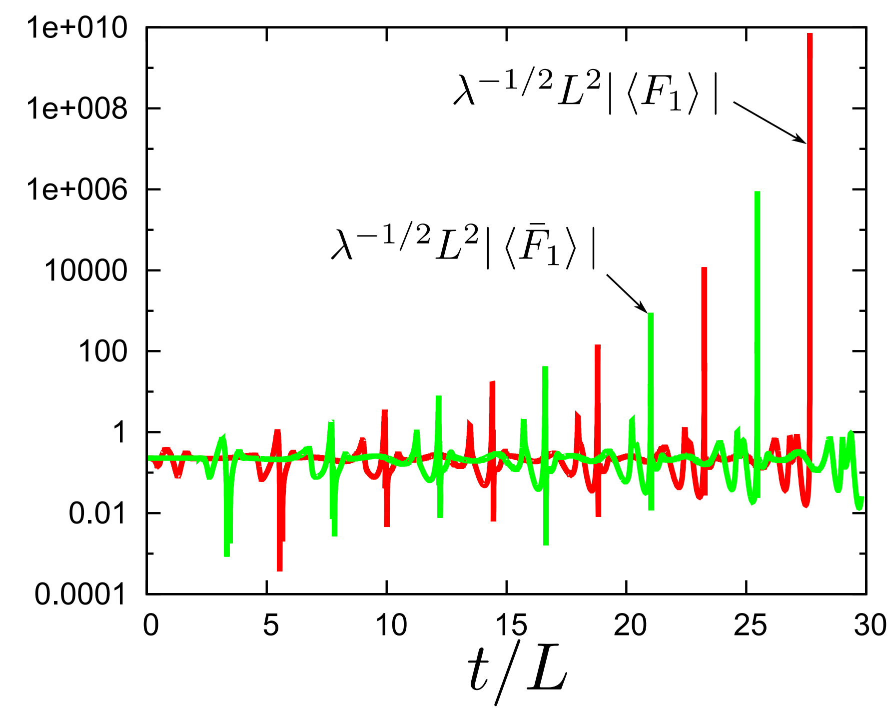
Analysis 3: Forces on the endpoints

$$\langle \boldsymbol{F}(t) \rangle = \frac{\delta S_{\text{on-shell}}}{\delta \boldsymbol{x}_q}$$

Force diverges when a cusp reaches the boundary



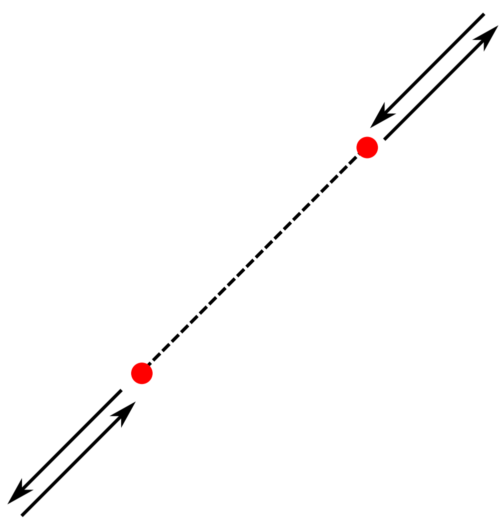
ε=0.005, Δt/L=2 (no cusp)

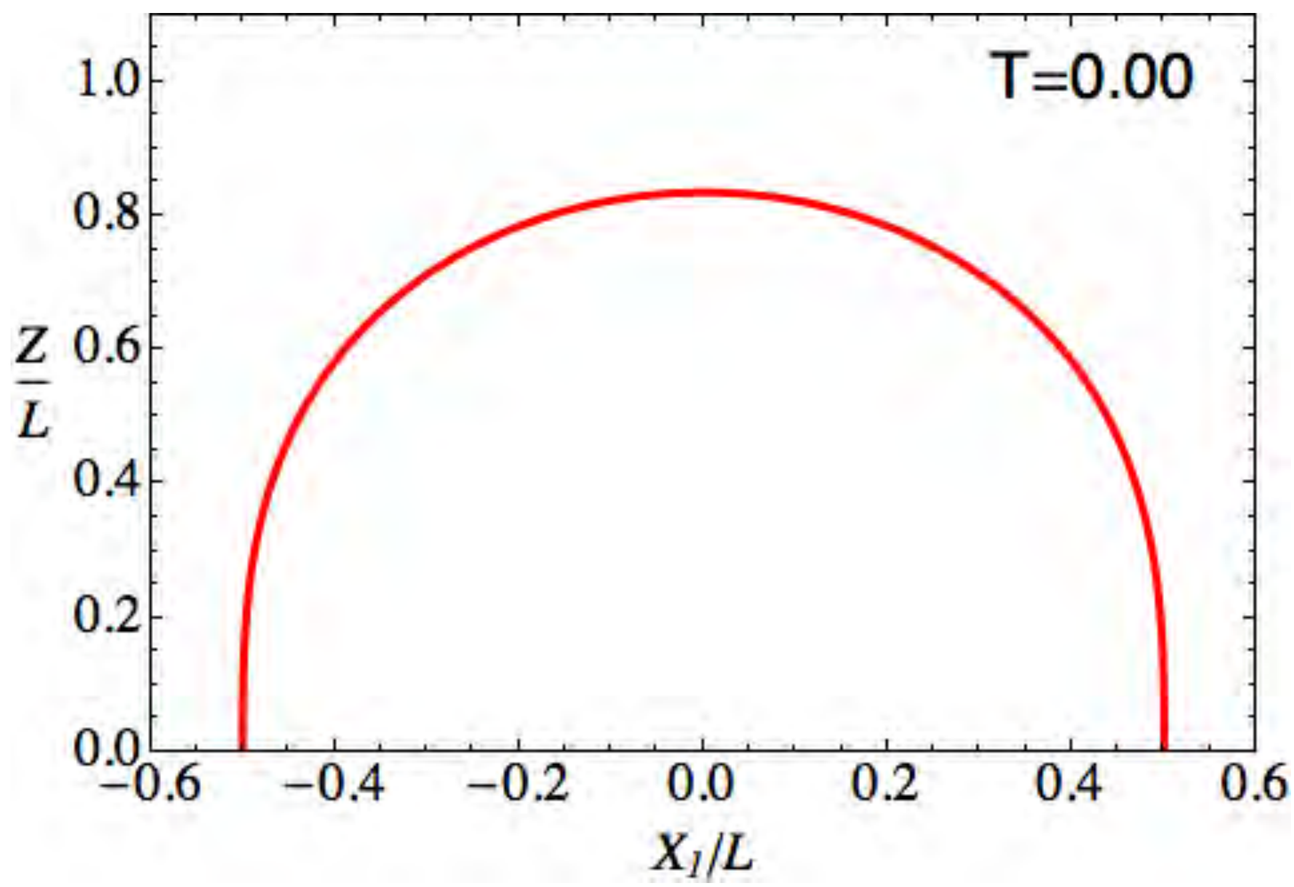


ε=0.01 (cusps T~27)

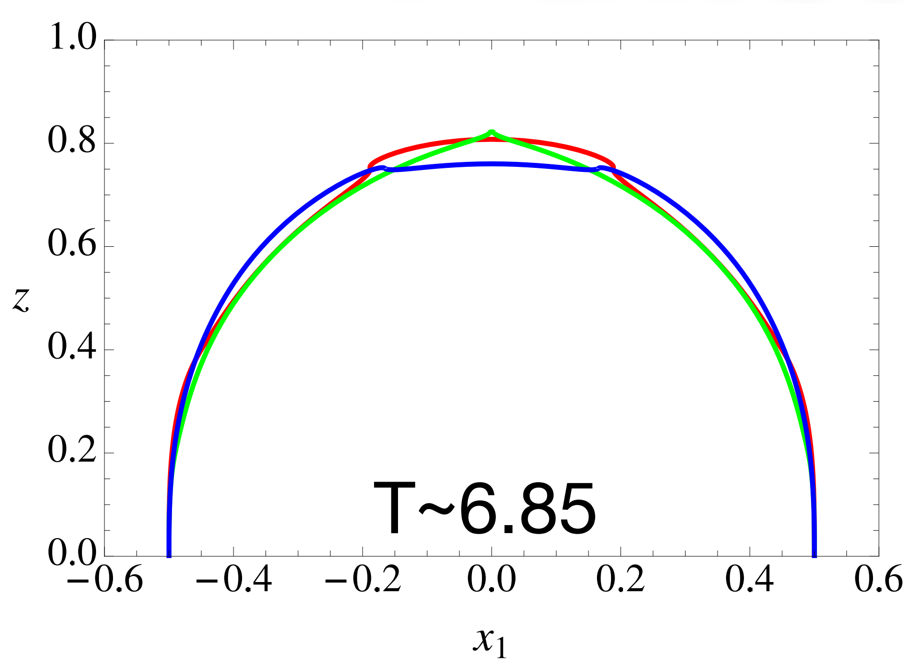
***Red: x=L/2, green: x=-L/2

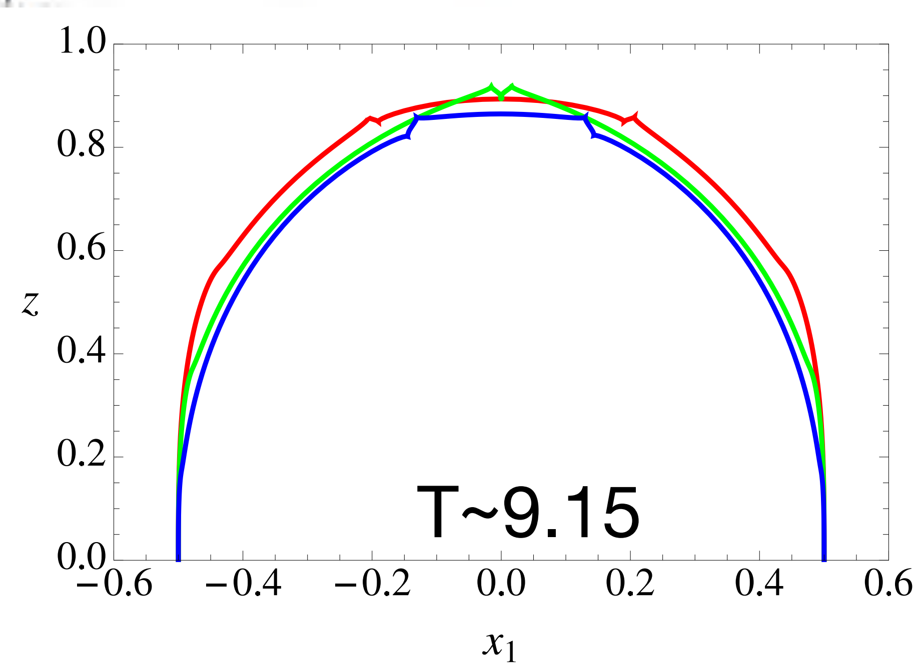
Z_2-symmetric quench



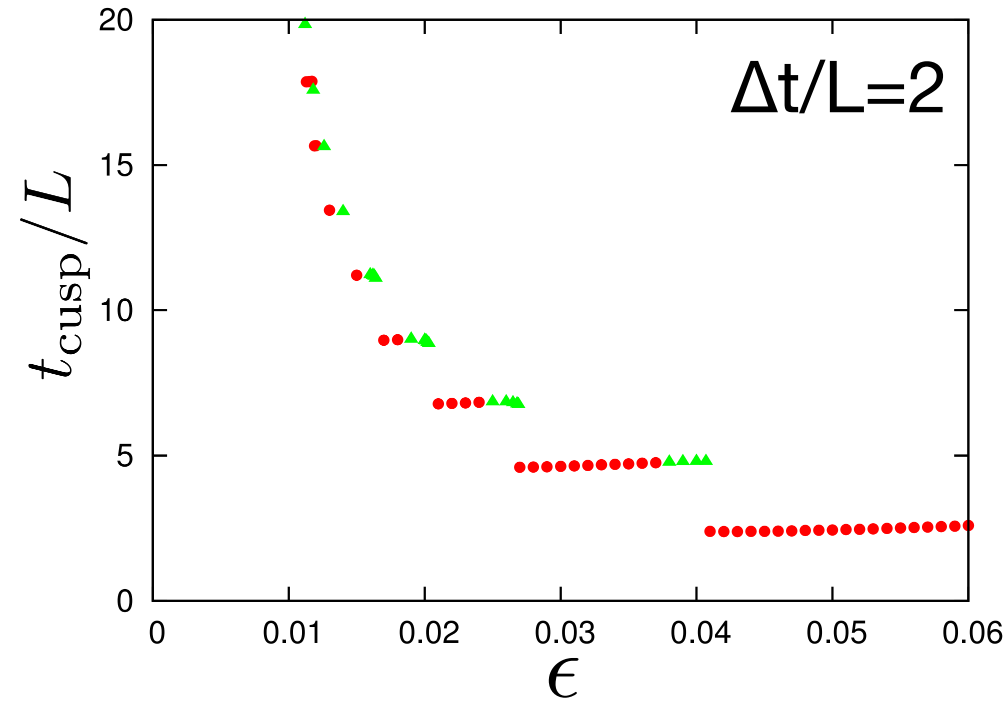


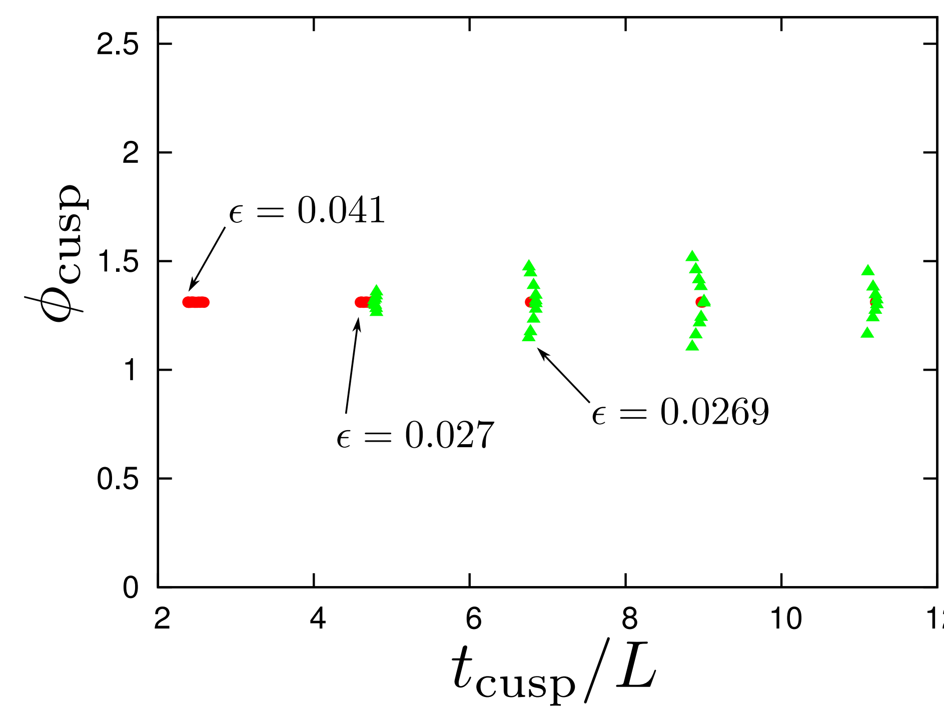
ε=0.025
Δt/L=2





Cusp formation in the Z_2-case



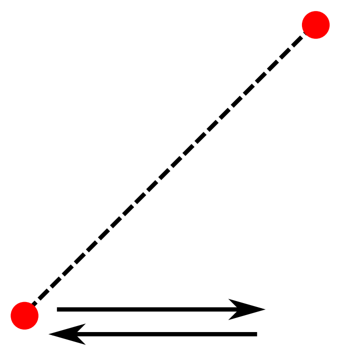


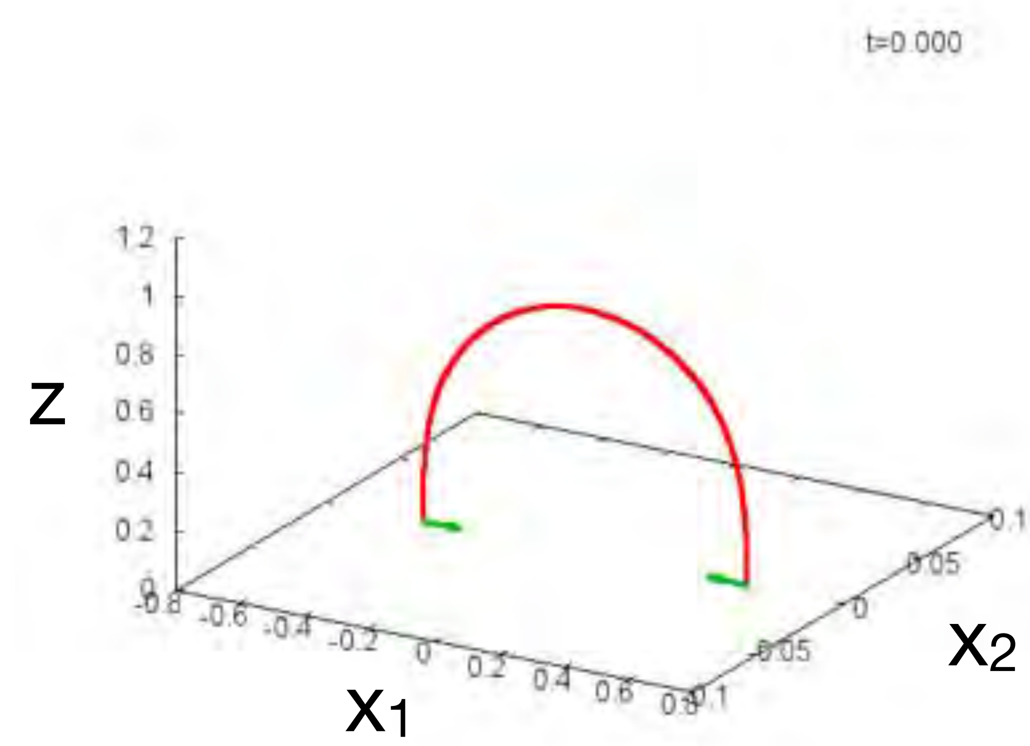
- Formation times are discretized by wave collisions

- First cusp formations on wave collisions (red ●). The cusps are pair-created and annihilated.

- Traveling cusps can be formed first (green ▲)

Transverse linear quench



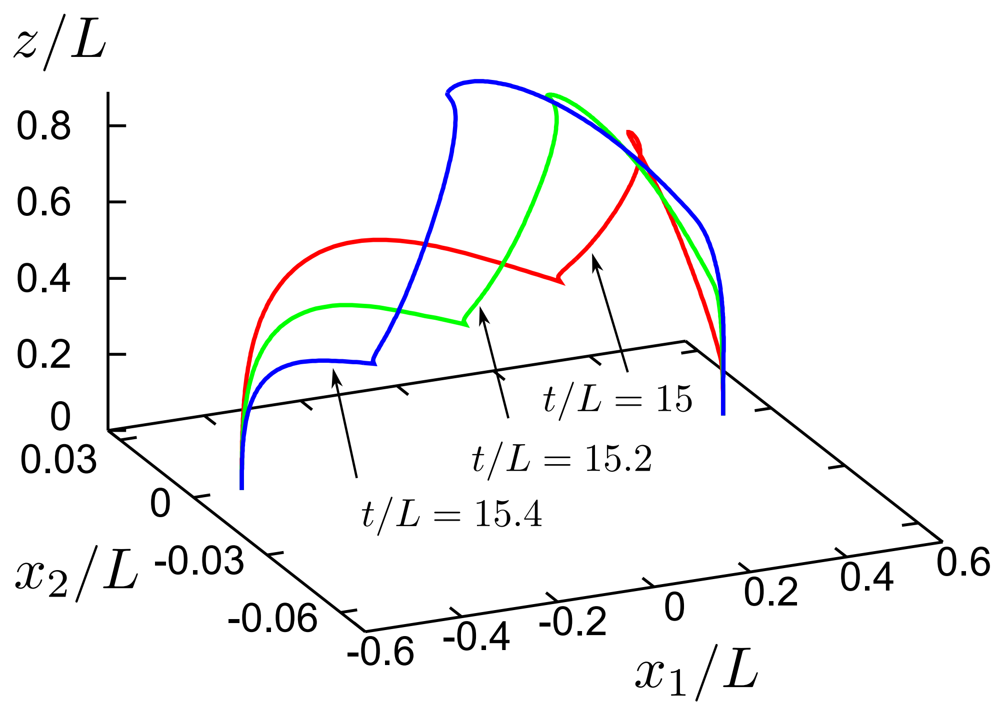


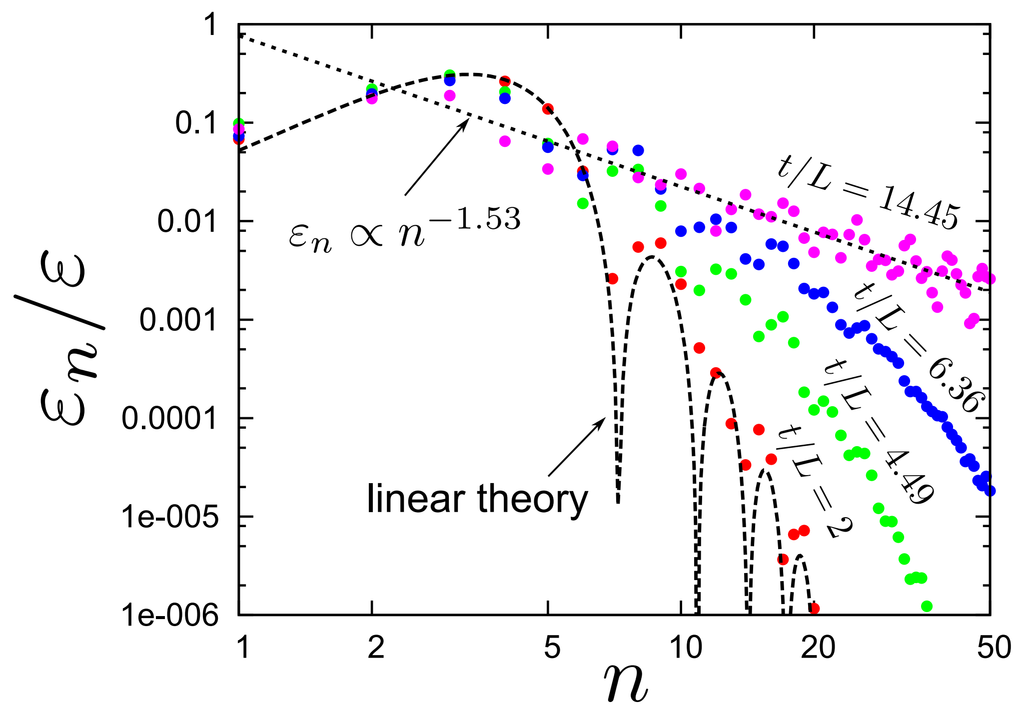
ε=0.03, Δt/L=2

***Green arrows: forces

String oscillates in 1+3 dim (t,z,x_1,x_2)

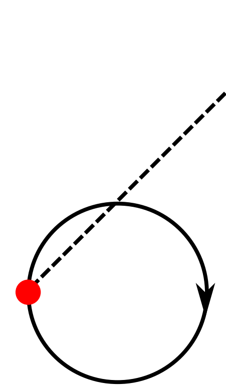
Transverse linear quench

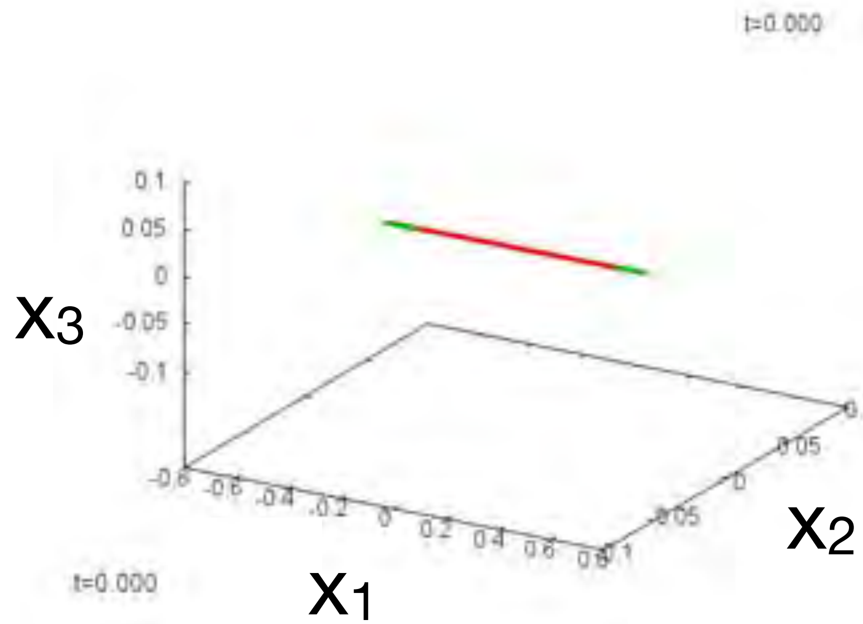




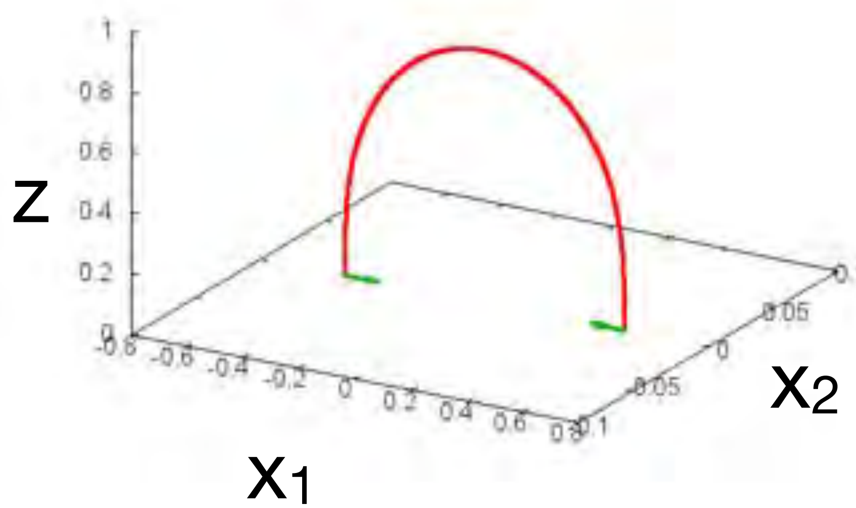
- Cusp formation at T~14.5 ($\varepsilon_n \sim n^{-1.5}$)
- Direct energy cascade

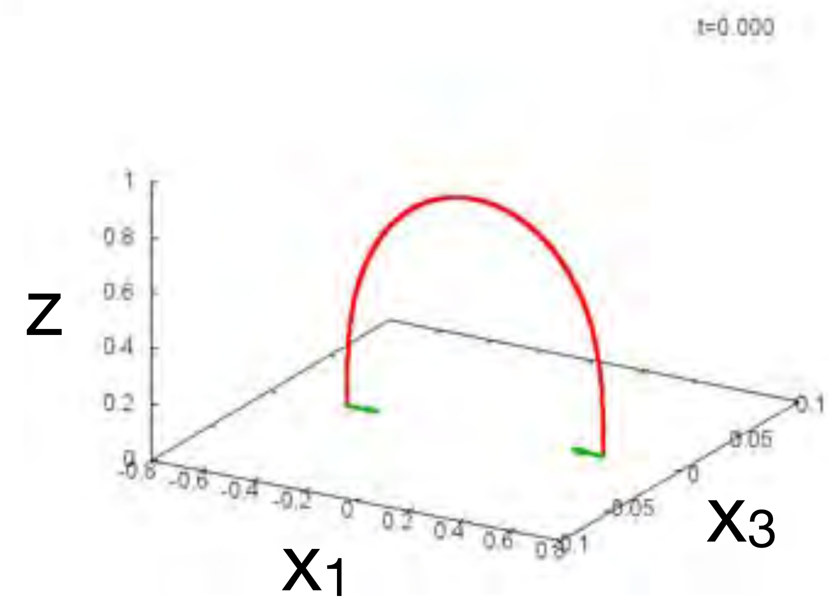
Transverse circular quench





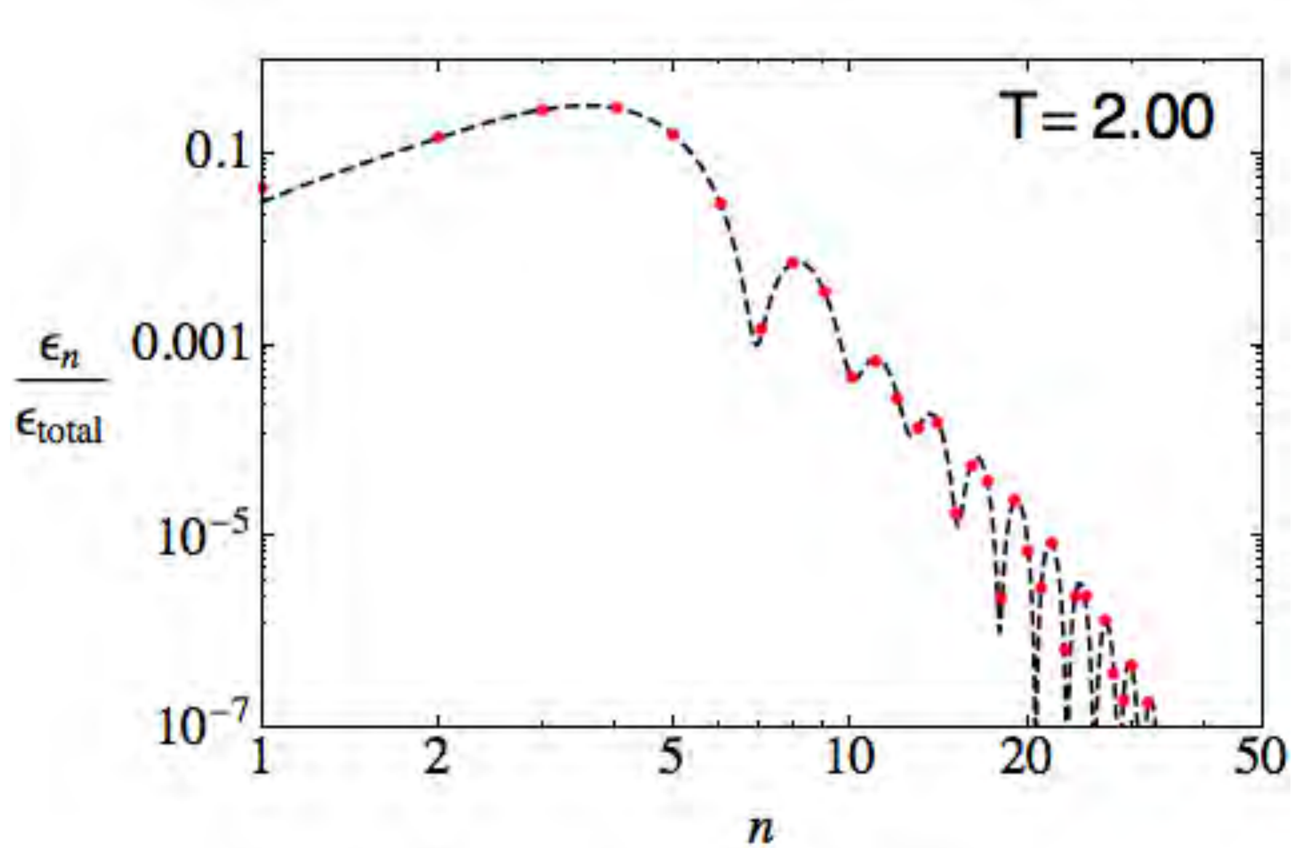
ε=0.02, Δt/L=2





String oscillates in all 1+4 dim (t,z,x_1,x_2,x_3)

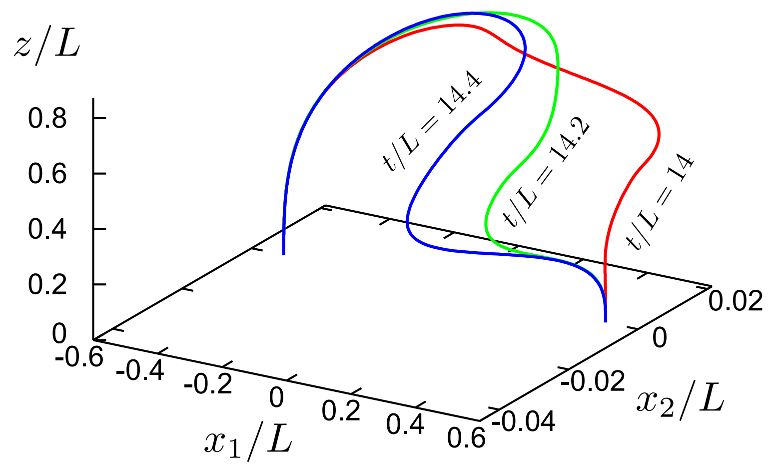
Transverse circular energy spectrum

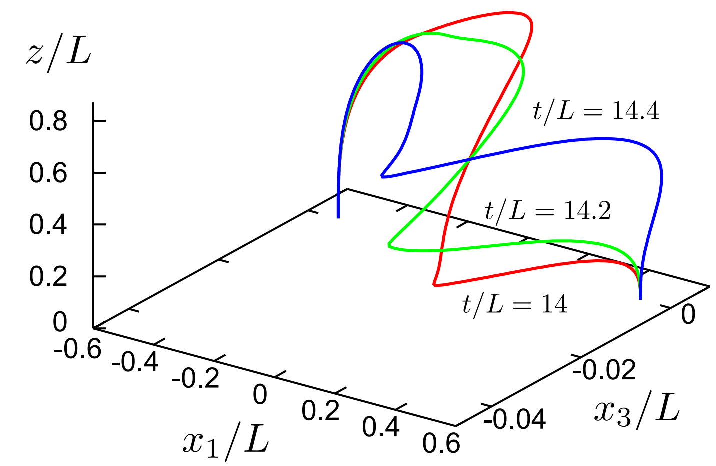


No cusp: no sustaining power law

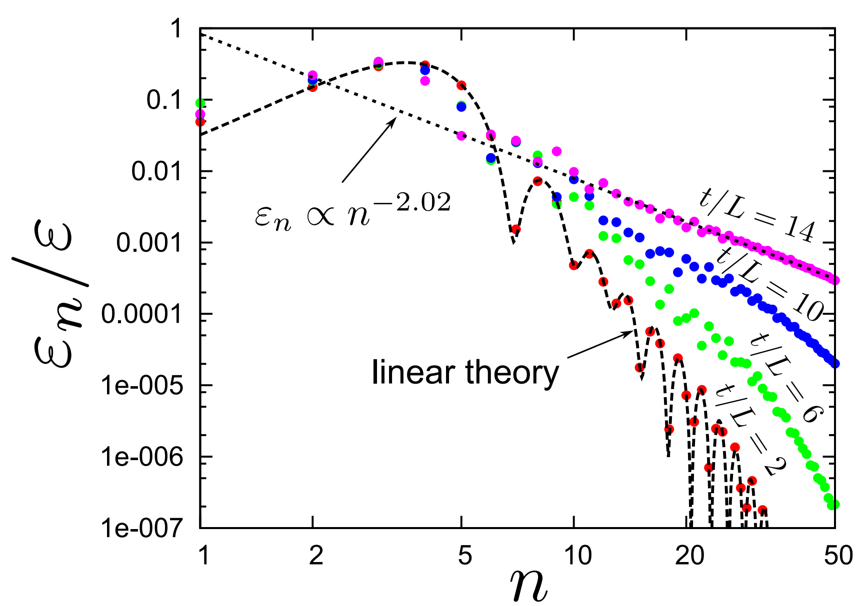
c.f.) Probability of cusp formation is zero if dim>4

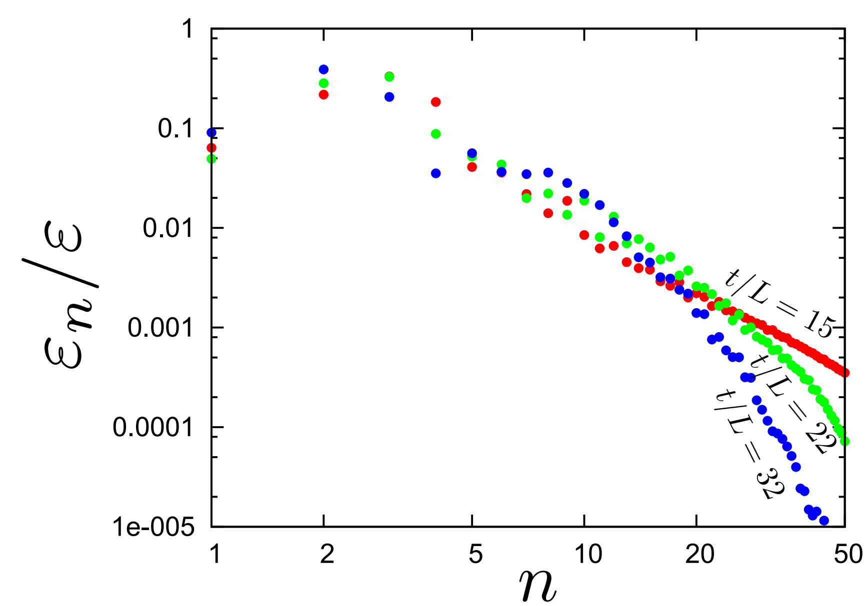
Transverse circular quench





Cuspy, but not real cusps





Direct cascade → ($\varepsilon_n \sim n^{-2}$) → inverse cascade

Summary

We computed nonlinear dynamics of the quark-antiquark fundamental string in AdS

- Cusps and turbulent behavior in $\leqq$ 1+3 dim
- No cusp and direct/inverse cascades in 1+4 dim

Boundary interpretation: Nonlinearity in YM flux tube might squeeze/de-squeeze the energy

Discussion

Backreaction may be necessary at (near) cusps

- Curvature diverges at the cusps
- AdS gravitational wave bursts?
- Boundary: gluon emission from the flux tube?

Future works

- Large amplitude/finite temperature
- Non-conformal backgrounds
- Application to drag force

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