Time-Dependent Meson Melting in External Magnetic Field

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Outline:

- Why Time-Dependence?
- Holographic Meson
- Time-Dependent Set-Up

Why Time-Dependence?

- system in equilibrium
  - non-dynamical, stationary
  - late-time state of a generic system
- time-dependent processes
  - perturbative methods, Linear Response Theory $H \to H + H_{pert}$
  - coarse-grained view of the system, low energy effective description, Hydrodynamics, Universal features
  - lack of adequate techniques for strongly correlated condensed matter theories, quark-gluon plasma
  - Focus on field theories with holographic dual. Two dual theories are just different descriptions of the same physical system.

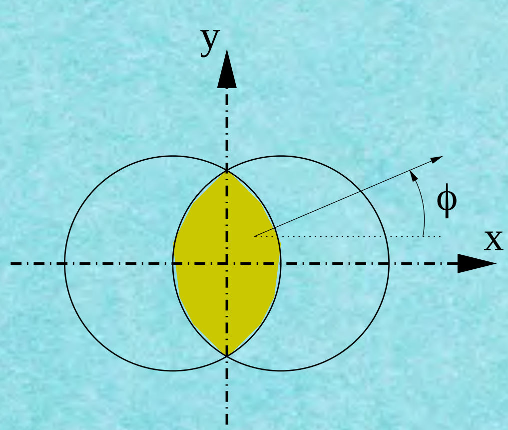
Thermalization in QGP

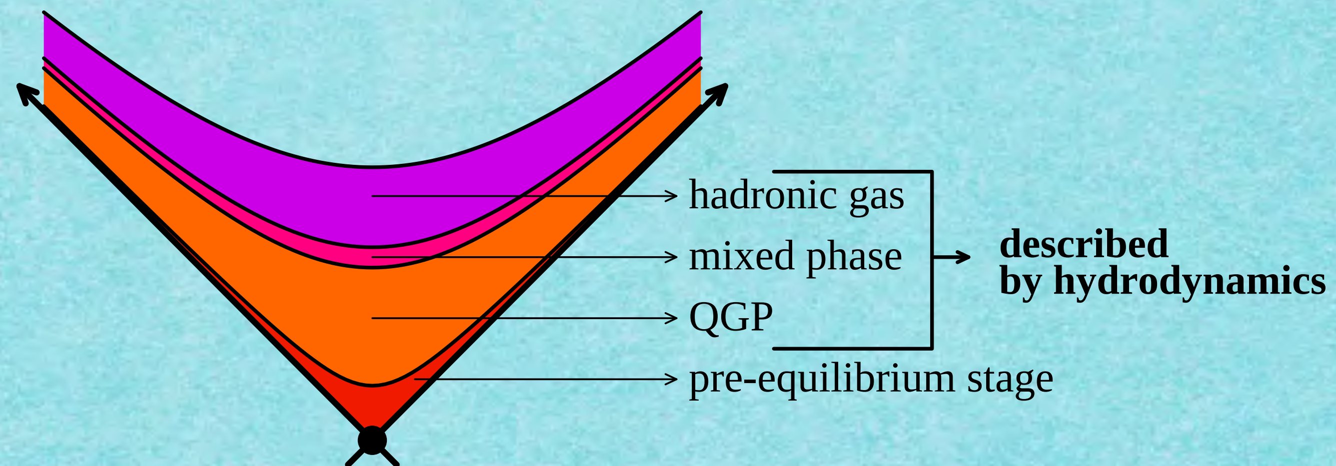
- Collision of two heavy nuclei (Gold or Lead) at the relativistic speed
- Production of an Anisotropic Plasma
- Hydrodynamics applies after a very short time-scale, 1 fm
  - Far-from-equilibrium effects at the early stages of QGP production
- Strongly Coupled Plasma

Shuryak, 2004; 2005

- Presence of a magnetic field at the early stages of QGP production

Kharzeev, McLerran, Warringa, 2008





Holography:

type IIB string theory on $AdS_5 \times S^5$ ⟺ N=4 su(N) Conformal SYM

$$\frac{R^4}{\alpha'^2} = 4\pi g_s N = g_{YM}^2 N = \lambda >> 1$$ ⟹ Strong/Weak Duality

Strongly Coupled CFT

⟶ Classical Gravity on Asymptotically AdS Space-time

- The dual gravitational description of a strongly coupled gauge theory provides an efficient way to study the thermodynamic properties of gauge theories.
- Correspondence both in static and dynamical situations: time-dependence on the boundary manifested by time-dependence in the bulk

Holographic Mesons

- Flavour added to the gauge theory by introducing probe D7-branes in the bulk   A. Karch and E. Katz, 2002

$N_f/N_c \to 0$

- Fluctuations of the fields on the brane produce the meson spectrum

$\phi = \Phi(z)$ $\psi = 0$

$$ds_{10}^2 = \frac{L^2}{z^2}\left[-f(z)dt^2 + \frac{dz^2}{f(z)} + d\vec{x}_3^2\right] + L^2(d\phi^2 + \cos^2\phi\, d\Omega_3^2 + \sin^2\phi\, d\psi^2)$$

$\Phi = \pi/2$
$\rho = 0$

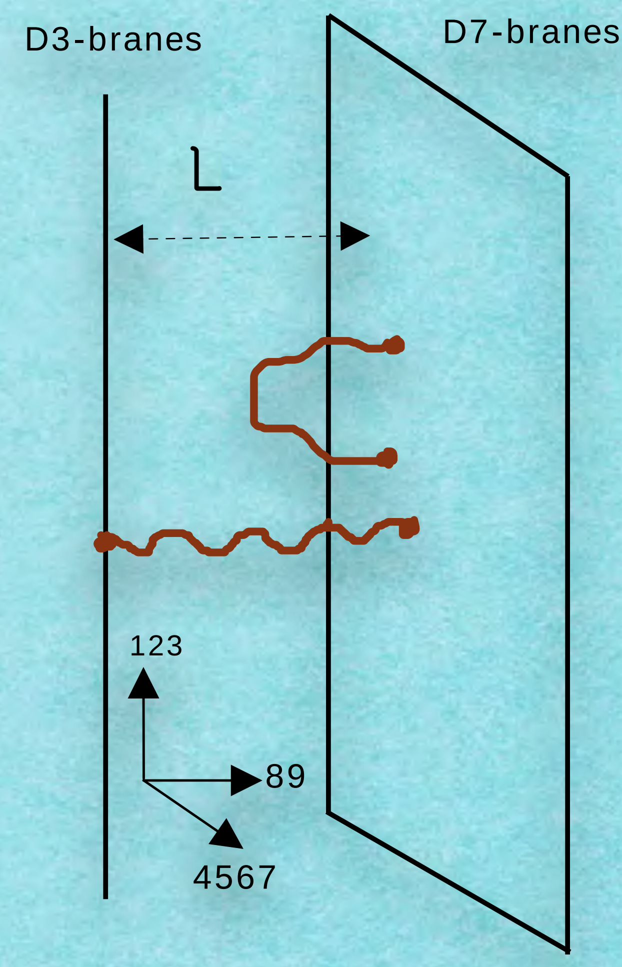
$$(w,\rho) = (L^2 z^{-1}\sin\phi, L^2 z^{-1}\cos\phi)$$

$$w = W(\rho)$$

$$S = -\tau_7 \int d^8\xi \sqrt{-g_{ab}} \qquad g_{ab} = \partial_a x^M \partial_b x^N G_{MN}$$

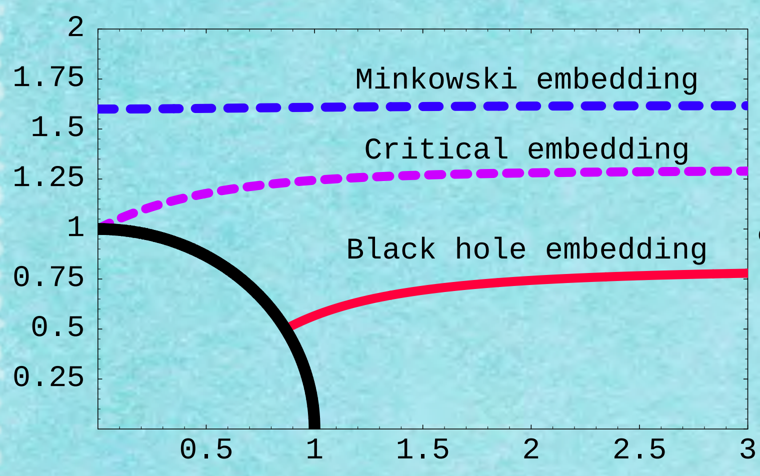
$$W(\rho) = m + \frac{c}{\rho^2} + \cdots$$

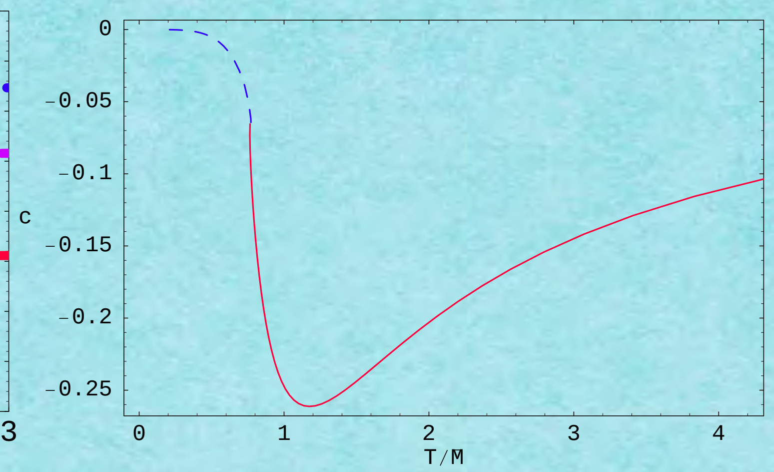
$$M_q = \frac{m}{2\pi\ell_s^2}\,, \qquad \langle \mathcal{O}_m \rangle = -\frac{N_f}{16\pi^4 g_s \ell_s^6} c$$

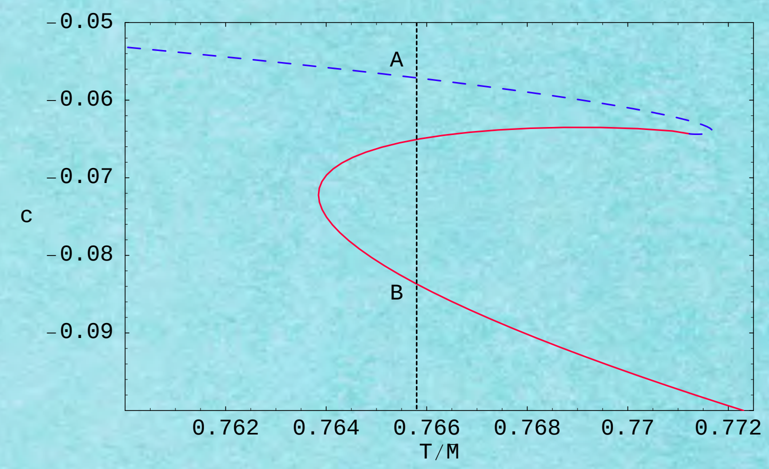


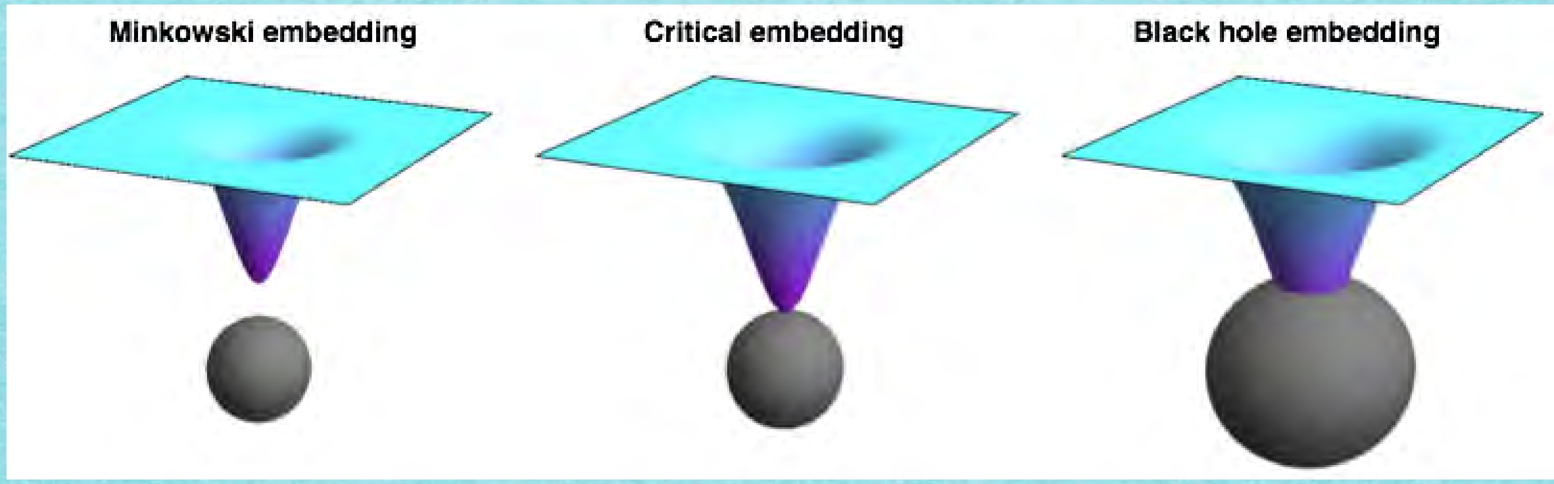
	0	1	2	3	4	5	6	7	8	9
$D3$	$\times$	$\times$	$\times$	$\times$						
$D7$	$\times$	$\times$	$\times$	$\times$	$\times$	$\times$	$\times$	$\times$		

$(t, z, \vec{x}_3, \Omega_3)$









mesonic
phase

melted
phase

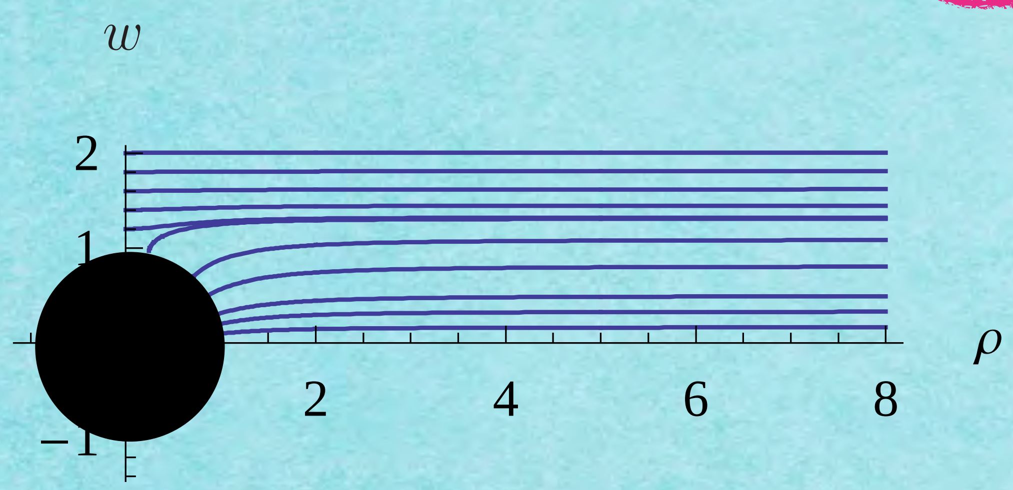
Mateos, Myers, Thomson, 2007

Non-Zero Magnetic Field:

Erdmenger, Meyers, Shock, 2007
Filev, Johnson, Rashkov, Viswanathan, 2007

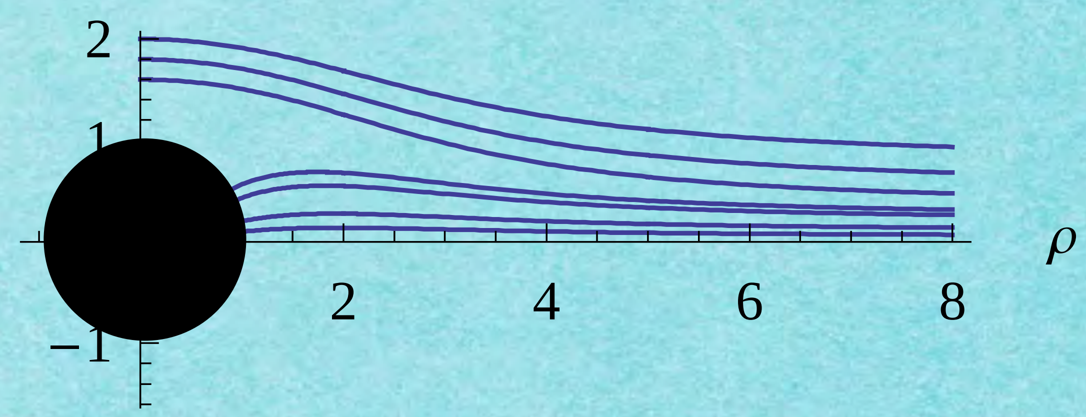
$$S = -\tau_7 \int d^8\xi \sqrt{-g_{ab} + (2\pi\alpha')F_{ab}}$$

$F_{x_1 x_2} = B$



B=0

B=10



Time-Dependent Set-Up

for zero magnetic field: Ishii, Kinoshita, Murata, Tanahashi; 2014

- AdS-Vaidya Background

Thermalization in the field theory is dual to black hole formation in gravity.

$$ds^2 = G_{MN}dx^M dx^N = \frac{1}{z^2}\left[-F(V,z)dV^2 - 2dVdz + d\vec{x}_3^2\right] + d\phi^2 + \cos^2\phi\, d\Omega_3^2 + \sin^2\phi\, d\psi^2$$

$$F(V,z) = 1 - M(V)z^4$$

Boundary Time

$$M(V) = M_f \begin{cases} 0 & V < 0, \\ \frac{1}{2}\left[1-\cos(\frac{\pi V}{\Delta V})\right] & 0 \leqslant V \leqslant \Delta V, \\ 1 & V > \Delta V, \end{cases}$$

Period of the Energy Injection

$$r_h = M_f^{\frac{1}{4}}$$

- Freedom to choose eight world-volume coordinates

$$(\xi^2, ..., \xi^7) = (\vec{x}_3, \Omega_3)$$

$$(\xi^0, \xi^1) = (u, v)$$

Null Coordinate on the Probe Brane

$$V = V(u,v), \quad z = Z(u,v), \quad \phi = \Phi(u,v), \quad \psi = 0$$

$$\Psi(u,v) \equiv \frac{\Phi(u,v)}{Z(u,v)}$$

gives the shape of the brane

Constraint equations to keep u,v null

$$FV_{,v}^2 + 2Z_{,v}V_{,v} - Z^2(Z\Psi)_{,v}^2 = 0$$
$$FV_{,u}^2 + 2Z_{,u}V_{,u} - Z^2(Z\Psi)_{,u}^2 = 0$$

$$S = -\tau_7 V_{\Omega_3} V_{\vec{x}} \int dudv \frac{\cos^3\Phi}{Z^3}\sqrt{\frac{1}{Z^4} + (2\pi\alpha')^2 B^2}\left(FV_{,u}V_{,v} + V_{,u}Z_{,v} + Z_{,u}V_{,v} - Z^2\Phi_{,u}\Phi_{,v}\right)$$

equations of motion:

$$V_{,uv} = \frac{Z\mathcal{B}_1^+}{2}(Z\Psi)_{,u}(Z\Psi)_{,v} + \frac{3}{2}\tan(Z\Psi)[(Z\Psi)_{,u}V_{,v} + (Z\Psi)_{,v}V_{,u}] + \frac{1}{2}\left(F_{,Z} - \frac{F\mathcal{B}_2}{Z}\right)V_{,u}V_{,v}\,,$$

$$Z_{,uv} = -\frac{FZ\mathcal{B}_1^+}{2}(Z\Psi)_{,u}(Z\Psi)_{,v} + \frac{3}{2}\tan(Z\Psi)[(Z\Psi)_{,u}Z_{,v} + (Z\Psi)_{,v}Z_{,u}] + \frac{\mathcal{B}_2}{Z}Z_{,u}Z_{,v} - \frac{F_{,V}}{2}V_{,u}V_{,v}$$
$$- \frac{1}{2}\left(F_{,Z} - \frac{F\mathcal{B}_2}{Z}\right)(FV_{,u}V_{,v} + V_{,u}Z_{,v} + V_{,v}Z_{,u})\,,$$

$$\Psi_{,uv} = \left(\frac{3\tan(Z\Psi)}{2Z} + \frac{\Psi F\mathcal{B}_1^+}{2}\right)(Z\Psi)_{,u}(Z\Psi)_{,v} - \frac{\mathcal{B}_1^- + 3Z\Psi\tan(Z\Psi)}{2Z^2}[(Z\Psi)_{,u}Z_{,v} + (Z\Psi)_{,v}Z_{,u}]$$
$$+ \frac{\Psi}{2Z}\left(F_{,Z} - \frac{F\mathcal{B}_2}{Z} + \frac{3\tan(Z\Psi)}{Z^2\Psi}\right)(FV_{,u}V_{,v} + V_{,u}Z_{,v} + V_{,v}Z_{,u}) - \frac{\Psi\mathcal{B}_1^+}{Z^2}Z_{,u}Z_{,v} + \frac{\Psi F_{,V}}{2Z}V_{,u}V_{,v}$$

$$\mathcal{B}_1^{\pm} = 1 \pm \frac{2}{1 + (2\pi\alpha')^2 Z^4 B^2}$$
$$\mathcal{B}_2 = 3 + \frac{2}{1 + (2\pi\alpha')^2 Z^4 B^2}$$

$$\Psi(V,Z)|_{Z\to 0} = m(V) + \left(c(V) + \frac{m(V)^3}{6}\right)Z^2 + \ldots.$$

- Second order, nonlinear partial differential equations: We use finite difference method to solve them.
- Rescaling the parameters to have m=1.
- Boundary and Initial Conditions:

Boundary Condition at the AdS Boundary, ME and BE

$$Z|_{u=v} = 0$$
$$\Psi|_{u=v} = m$$
$$V_0(v) = V|_{u=v}$$
$$\frac{d}{dv}V_0(v) = 2Z_{,u}|_{u=v}$$
$$Z_{,uv}|_{u=v} = 0$$

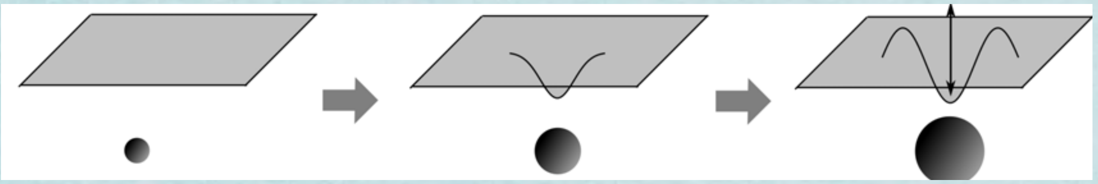
Boundary condition at the Pole, ME

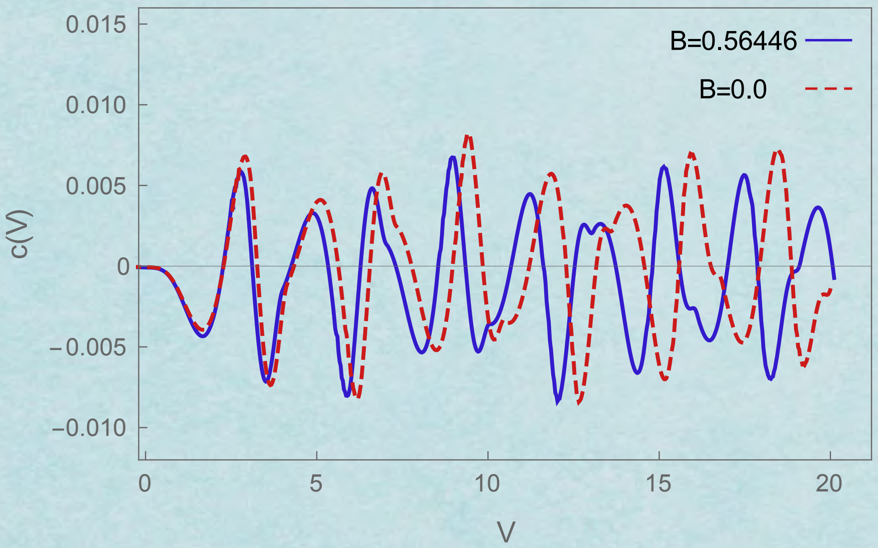
$$\Phi|_{u=v+\frac{\pi}{2}} = \frac{\pi}{2}$$
$$Z_{,u} = Z_{,v}$$
$$V_{,u} = V_{,v}$$

Initial data is the static solution to the e.o.ms for pure AdS with non-zero magnetic field.

Minkowski Embedding:

If the corresponding mass of the initial ME configuration is much larger than the final temperature of the system: $\frac{m}{T} \gg 1$

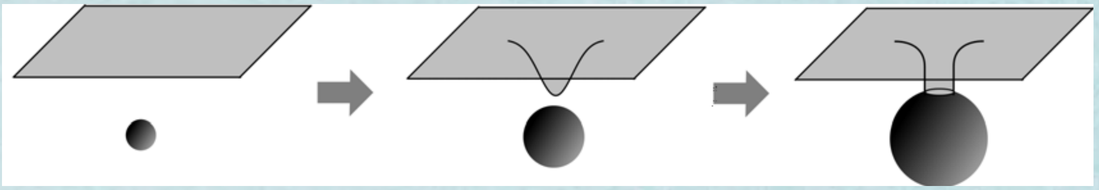


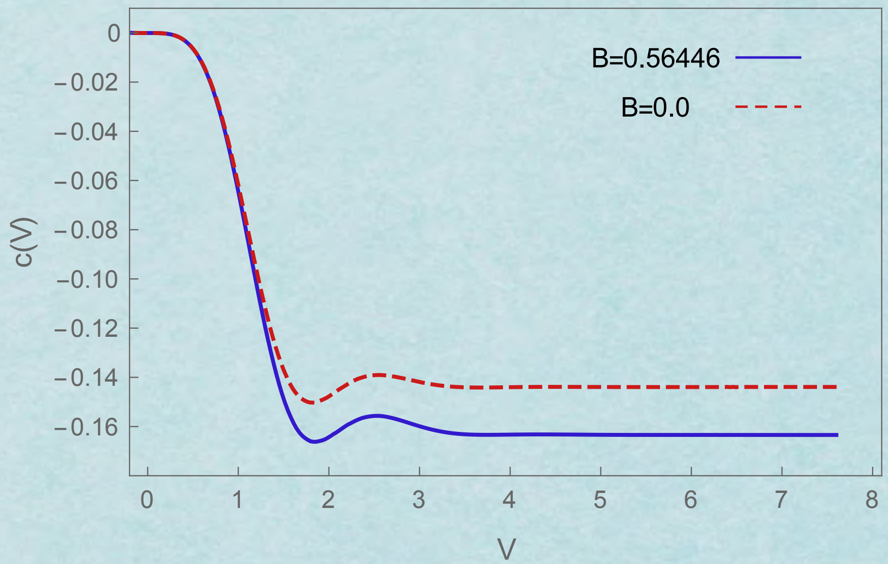


No Dissipation and thus Oscillatory Behavior

Black Hole Embedding:

If the corresponding mass of the initial ME configuration is less than the final temperature of the system: $\frac{m}{T} < 1$





$\Delta V = 1.0$

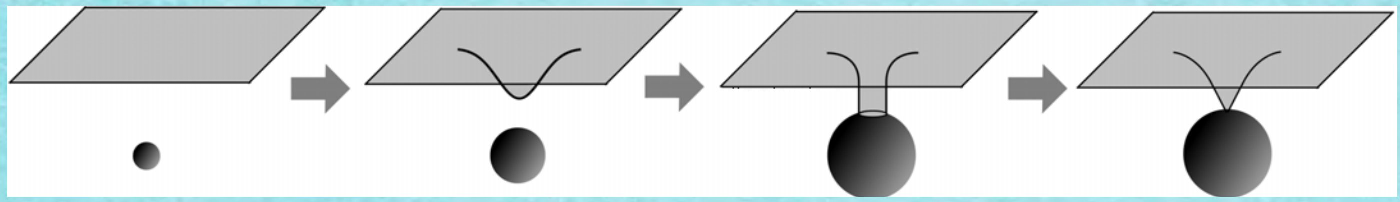
$r_h = 1.25$

Overeager:

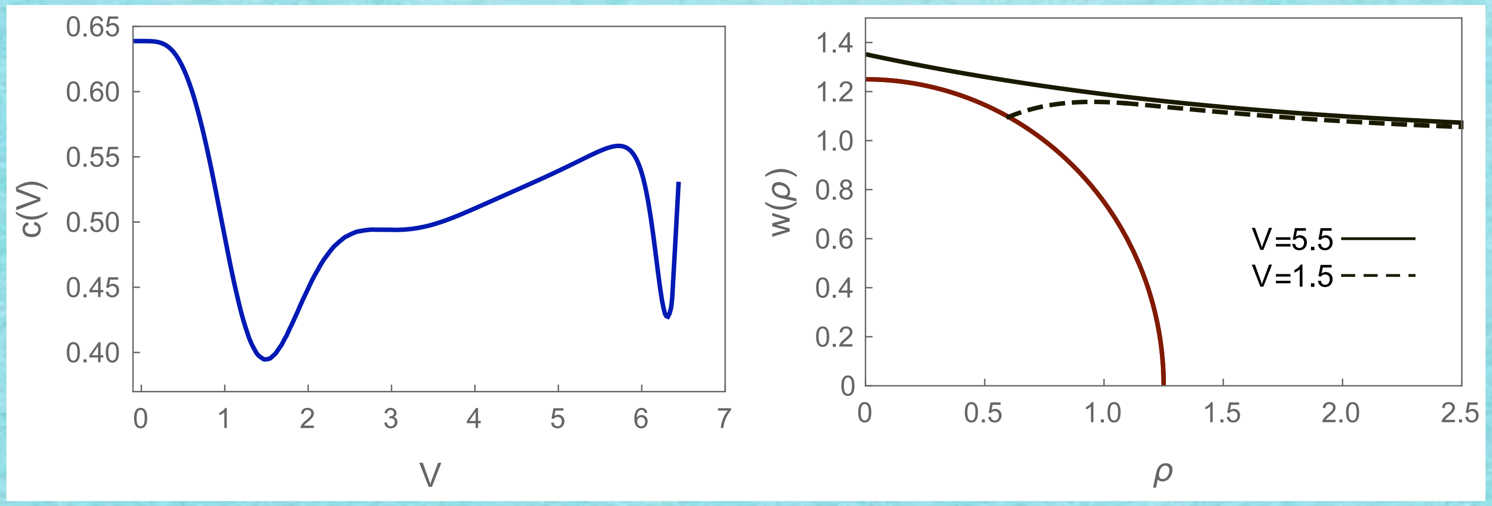
If the corresponding mass of the initial ME configuration is larger but close to the final temperature of the system, provided that the time-scale of the change in the temperature is small: Ishii, Kinoshita, Murata, Tanahashi; 2014

$$\frac{m}{T} \sim 1$$

$$\Delta V < 1$$



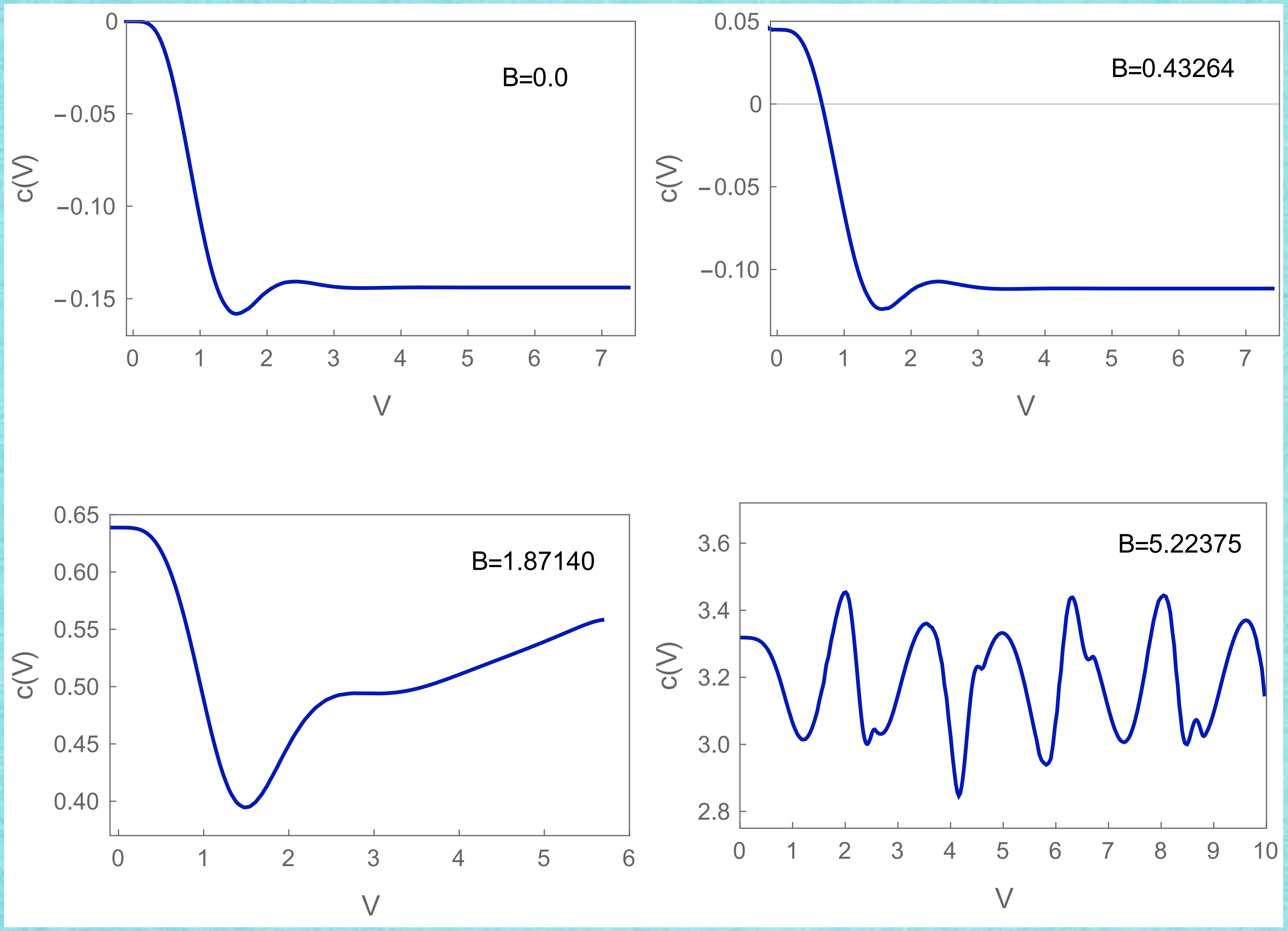
Brane Reconnection



$$\Delta V = 0.5$$

$$B = 1.87140$$

$$r_h = 1.25$$

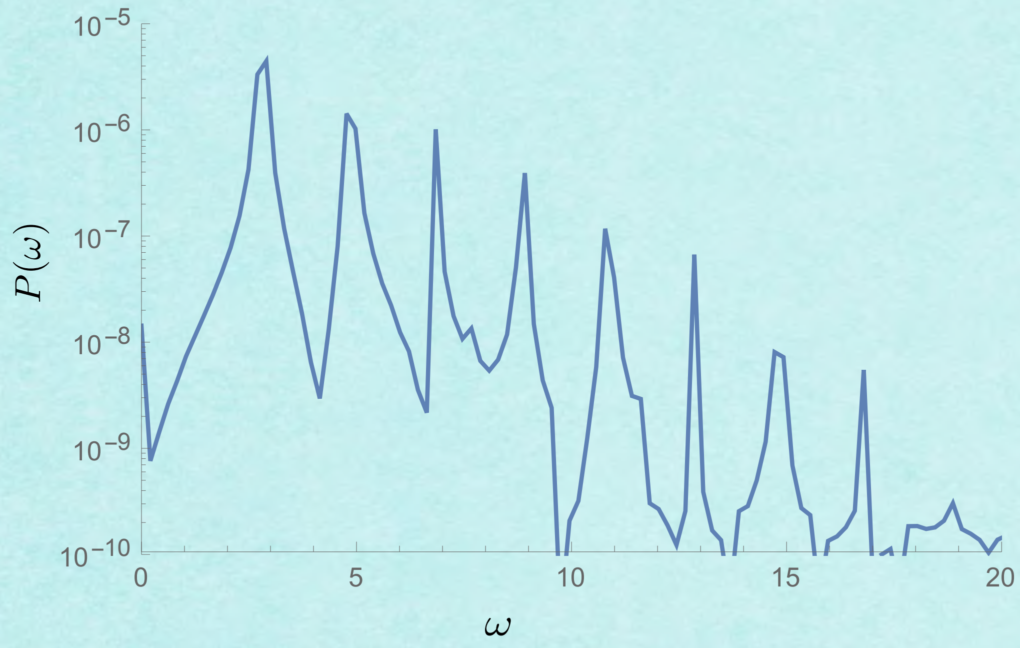


BE changes to ME by increasing the magnetic filed.

$$\Delta V = 0.5$$

$$r_h = 1.25$$

Power Spectrum of c(V)-c_{eq} for the ME:



The Fourier transform of the oscillations of c(V) has discrete spectrum.

Bound-States obtained from ME c(V) Power Spectrum

Stable Modes	$B = 0$	$B = 0.56446$
ω_1	2.82052	3.04731
ω_2	4.83518	5.18042
ω_3	6.84984	7.21553

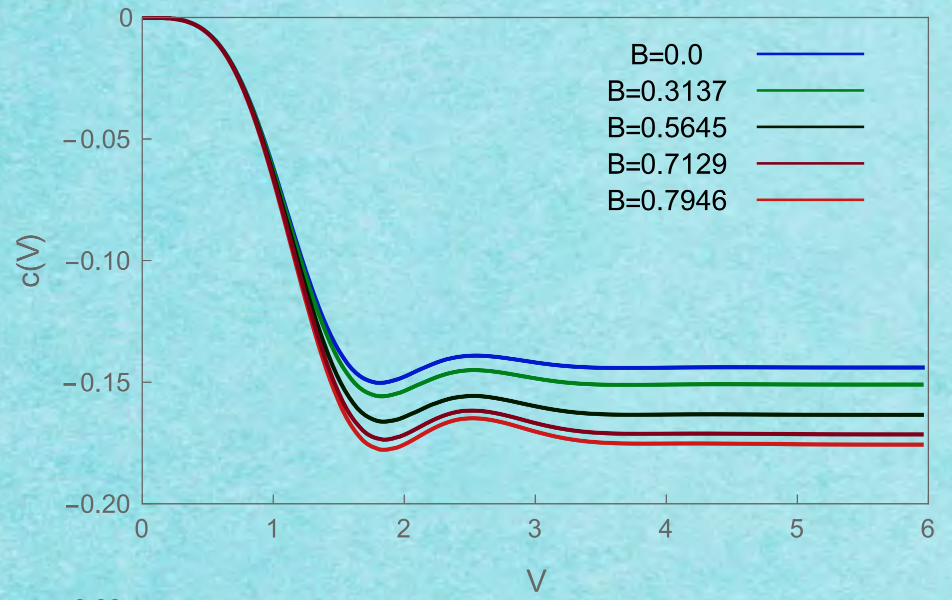
In the presence of magnetic field, mesons are more resistant to melting.

$$\Delta V = 1.0$$
$$r_h = 1.25$$

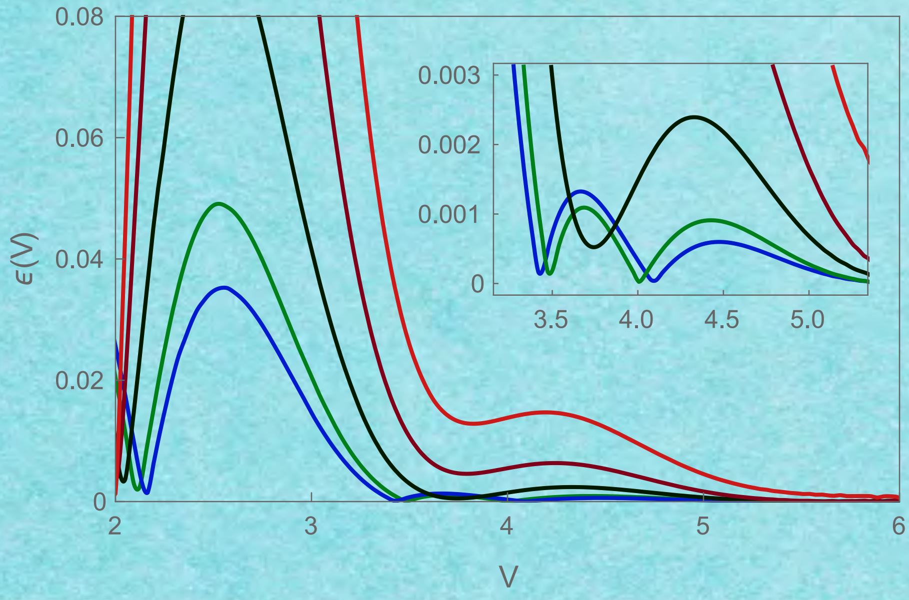
Equilibration Time:

$$\epsilon(V) = \left| \frac{c(V) - c_{eq}}{c(V)} \right|$$

is defined as the time which satisfies: $\epsilon(V_{eq}) < 0.003$ and for the time afterwards.



B	V_{eq}
0.0	3.27834
0.3137	3.33708
0.5645	3.49866
0.7129	4.8057
0.7946	5.15314



Equilibration happens later as the magnetic field is increased.

Ali-Akbari, Ebrahim, 2013

$\Delta V = 1.0$

$r_h = 1.25$

Summary and Future Direction:

- Mesons melt at higher temperatures due to the presence of magnetic field.
- The time-dependent system equilibrates later at non-zero magnetic field.
- studying the effect of anisotropy by having a time-dependent anisotropic background
- quark-antiquark bound-state (potential) on the time-dependent background
- looking for universal behaviour at very small energy injection time-scale

“Thank you”