Electric Field Quench, Equilibration and Universal Behavior

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"Electric Field Quench, Equilibration and Universal Behavior,"
Phys. Rev. D **91**, no. 12, 126007 (2015)[arXiv:1504.03559 [hep-th]].

QUENCH

Quench $\mathcal{L}_0 \to \mathcal{L}_0 + \lambda(t)\,\mathcal{O}_\Delta$

Our goal $E(t)$

A. Buchel, L. Lehner, R. C. Myers and A. van Niekerk,
"Quantum quenches of holographic plasmas,"
JHEP **1305**, 067 (2013) [arXiv:1302.2924 [hep-th]].

A. Buchel, R. C. Myers and A. van Niekerk,
"Nonlocal probes of thermalization in holographic quenches with spectral methods,"
JHEP **1502**, 017 (2015) [arXiv:1410.6201 [hep-th]].

Background Metric

Dp-brane background

$$ds^2 = H^{-1/2}(-dt^2 + dx_p{}^2) + H^{1/2}(du^2 + u^2 d\Omega_{8-p}^2),$$
$$d\Omega_{8-p}^2 = d\theta^2 + \sin^2\theta d\Omega_k^2 + \cos^2\theta d\Omega_{7-p-k}^2,$$

$$H(u) = (\frac{R}{u})^{7-p}, \quad e^{\phi} = H^{\frac{3-p}{4}}, \quad C_{01..p} = H^{-1}$$

Change of coordinate

$$\rho = u\sin\theta,$$
$$\sigma = u\cos\theta,$$

New background

$$ds^2 = H^{-\frac{1}{2}}(-dt^2 + dx_p^2) + H^{\frac{1}{2}}\left(d\rho^2 + \rho^2 d\Omega_k^2 + d\sigma^2 + \sigma^2 d\Omega_{7-p-k}^2\right)$$

N. Itzhaki, J. M. Maldacena, J. Sonnenschein and S. Yankielowicz,
Supergravity and the large N limit of theories with sixteen supercharges,
"Phys. Rev. D 58, 046004 (1998) [hep-th/9802042].

D-BRANE ACTION

Dp–Dq system
Static gauge

	t	x_1	..	x_d	x_{d+1}	..	x_p	ρ	Ω_k	σ	Ω_{7-p-k}
Dp	$\times$	$\times$	$\times$	$\times$	$\times$	$\times$	$\times$				
Dq	$\times$	$\times$	$\times$	$\times$				$\times$	$\times$		,

↓

Dp–background + Dq–brane

Dq–brane action

$$S = S_{DBI} + S_{CS},$$

$$S_{DBI} = -\tau_q \int d^8\xi e^{-\phi} \sqrt{-\det(g_{ab} + B_{ab} + (2\pi\alpha')F_{ab})},$$

$$S_{CS} = \tau_q \int \mathrm{P}[C^{(n)} e^B] e^{(2\pi\alpha')F}.$$

Induced metric

$$g_{ab} = G_{MN} \partial_a X^M \partial_b X^N,$$

$$B_{ab} = B_{MN} \partial_a X^M \partial_b X^N.$$

A. Karch and E. Katz,
Adding flavor to AdS / CFT,
" JHEP 0206,043 (2002) [hep-th/0205236].

Time Dependent Gauge Field

Gauge field

$$A_x(t,z) = -\int^t E(t')dt' + a(t,z)$$

Asymptotic expansion

$$a(t,z) = a_0(t) + \frac{\langle J^x(t)\rangle}{2\mathcal{N}(2\pi\alpha')^2} z^2 + O(z^4)$$

$$z = \tfrac{2}{5-p} R^{-\frac{p-7}{2}} u^{\frac{p-5}{2}}$$

Time dependent current

$$\langle J^x(t)\rangle \propto \lim_{z\to 0} \partial_z^2 a(t,z)$$

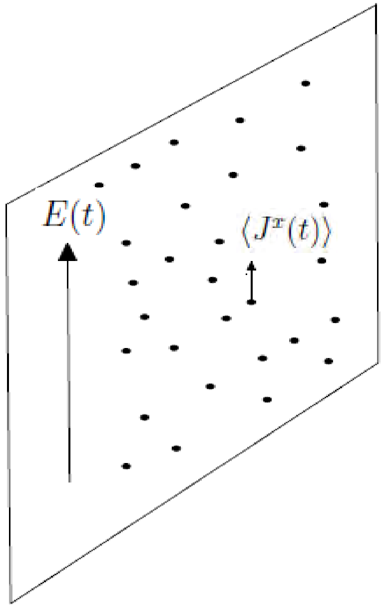
A. Karch and A. O'Bannon,
Metallic AdS/CFT,
" JHEP 0709, 024(2007) [arXiv:0705.3870 [hep-th]].

Gauge Theory Side

1. Strongly coupled field theory + massless fundamental matters
2. Electric filed
3. Current

Gravity Side

1. Dp-backgrounds + Dq-brane
2. Spatial component of gauge field.
3. $\langle J^x(t) \rangle$



Karch and A.O'Bannon,
"Metallic AdS/CFT,''
[arXiv:0705.3870 [hep-th]].

EQUATION OF MOTION

Lagrangian

$$\mathcal{L} \propto z^{\beta}\sqrt{1-\gamma(2\pi\alpha')^2(\frac{z}{R})^{\alpha}[(\partial_t A_x)^2-(\partial_z A_x)^2]}$$
$$= z^{\beta}\sqrt{w},$$

$$\alpha = \frac{2(p-7)}{p-5}, \quad \beta = \frac{q-2p+9}{p-5}, \quad \gamma = (\frac{p-5}{2})^{\alpha}$$

Equation of motion for the gauge field

$$\partial_z\left(\frac{z^{\alpha+\beta}\partial_z A_x}{\sqrt{w}}\right) - \partial_t\left(\frac{z^{\alpha+\beta}\partial_t A_x}{\sqrt{w}}\right) = 0$$

Gauge field

$$A_x(t,z) = -\int^t E(t')dt' + a(t,z)$$

Time dependent electric field

$$E(t) = \frac{E_0}{2}\left(1+\tanh(\frac{t}{k})\right),$$

$$E(t) = E_0\begin{cases} 0 & \text{if } t \le 0, \\ \frac{1}{2}\left(1-\cos(\frac{\pi t}{k})\right) & \text{if } 0 \le t \le k, \\ 1 & \text{if } t \ge k, \end{cases}$$

CONSTANT ELECTRIC FIELD

Constant electric field $F_{01} = \partial_t A_x(t,z) = E_0$

Equation of motion for the gauge field $\partial_z \left(\frac{z^{\alpha+\beta} \partial_z A_x}{\sqrt{w}} \right) = 0$

Current $\langle J^x_{eq} \rangle = C \frac{z^{\alpha+\beta} \partial_z A_x}{\sqrt{w}}$

On-shell action $w = \frac{1 - \frac{\gamma(2\pi\alpha')^2}{R^\alpha} z^\alpha E_0^2}{1 - \frac{\gamma(2\pi\alpha')^2}{C^2 R^\alpha} z^{-\alpha-2\beta} \langle J^x_{eq} \rangle^2}$

Current $\langle J^x_{eq} \rangle = C \gamma^{-\frac{\beta}{\alpha}} E_0^{-1-2\frac{\beta}{\alpha}} \longrightarrow$

$D3 - D7$	$\langle J^x_{eq} \rangle \propto E_0^{3/2}$
$D3 - D5$	$\langle J^x_{eq} \rangle \propto E_0$
$D4 - D6$	$\langle J^x_{eq} \rangle \propto E_0^{4/3}$
$D2 - D4$	$\langle J^x_{eq} \rangle \propto E_0^{11/9}$

A. Karch and A. O'Bannon,
Metallic AdS/CFT,
" JHEP 0709, 024(2007) [arXiv:0705.3870 [hep-th]].

Thermalization on the brane(D3-D7 system)

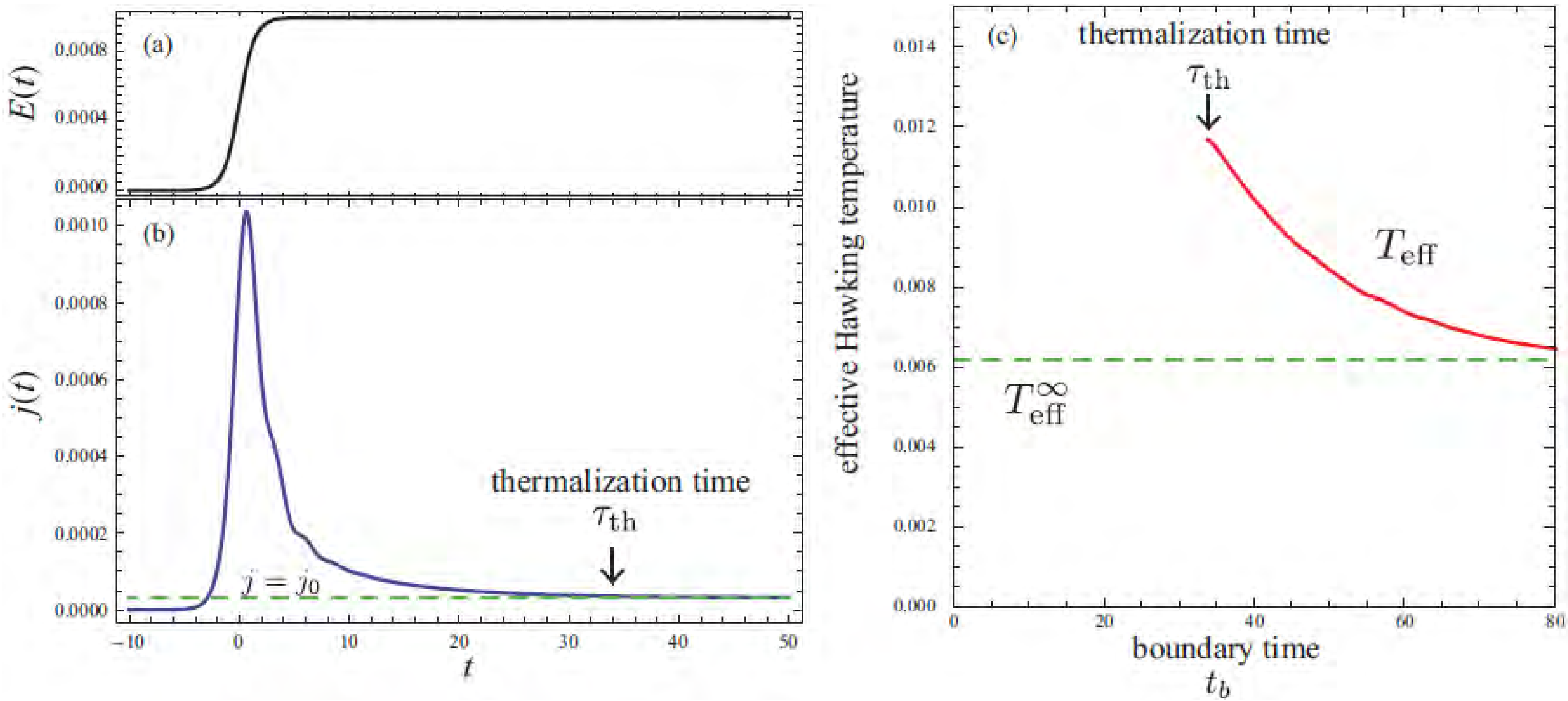
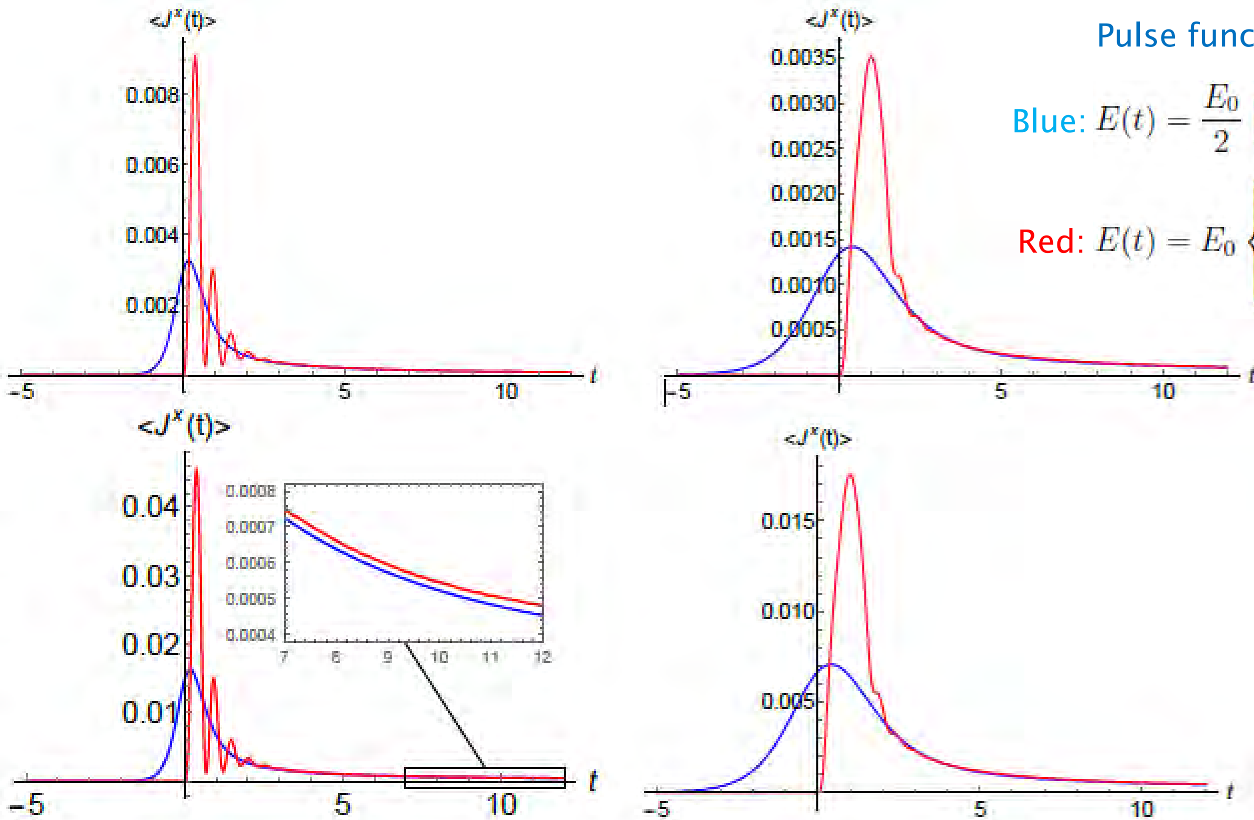


Figure 10: (a) The electric field $E(t)$. (b) Induced current $j(t)$. It converges to the steady state value $j = j_0$ (2.15). (c) The effective Hawking temperature (5.7) plotted against the boundary time. The "thermalization time" is defined as the boundary time of the horizon formation. In the long time limit, $T_{\rm eff}$ converges to the steady state value $T_{\rm eff}^{\infty}$. Same parameters as Fig. 9 is used.

K. Hashimoto and T. Oka,
Vacuum Instability in Electric Fields via AdS/CFT:
Euler-Heisenberg Lagrangian and Planckian Thermalization,
" JHEP 1310, 116 (2013) [arXiv:1307.7423 [hep-th]].

NUMERICAL RESULTS



Pulse functions

Blue: $E(t) = \frac{E_0}{2}\left(1 + \tanh(\frac{t}{k})\right),$

Red: $E(t) = E_0 \begin{cases} 0 & \text{if } t \leq 0, \\ \frac{1}{2}\left(1 - \cos(\frac{\pi t}{k})\right) & \text{if } 0 \leq t \leq k, \\ 1 & \text{if } t \geq k, \end{cases}$

Figure 1: Numerical results for current as a function of time for different values of electric fields, transition times and pulse functions.
top: $E_0 = 0.001$ with $k = 0.5$ (left) or $k = 1.5$ (right),
bottom: $E_0 = 0.005$ with $k = 0.5$ (left) or $k = 1.5$ (right).

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Equilibration time

Equilibrium value

$$R\langle J^x_{eq}\rangle = (2\pi\alpha' E)^{\frac{3}{2}}$$

Relative error for the current

$$\epsilon(t) \equiv \left|\frac{\langle J^x_{eq}\rangle - \langle J^x(t)\rangle}{\langle J^x_{eq}\rangle}\right|$$

Equilibration time

$$\epsilon(t_{eq}) < 0.05$$

Fast quench

$$k < 1$$

Slow quench

$$k > 1$$

Pulse functions

$$E(t) = E_0 \begin{cases} 0 & \text{if } t \leq 0, \\ \frac{1}{2}\left(1 - \cos(\frac{\pi t}{k})\right) & \text{if } 0 \leq t \leq k, \\ 1 & \text{if } t \geq k, \end{cases}$$

SLOW QUENCH

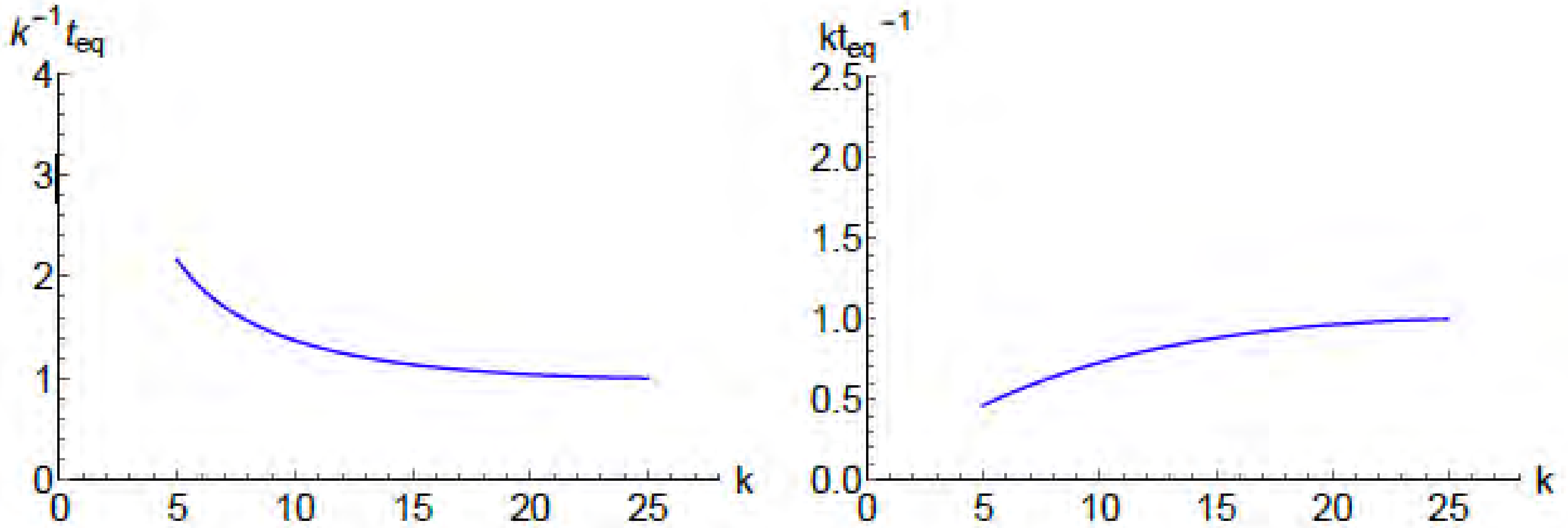

Figure 2: The left panel shows the resacled equilibration time versus the transition time. The right panel indicates that $(k^{-1}t_{eq})^{-1}$ goes to zero for smaller values of transition times, as we shall see for fast quenches.

$$\delta J = |\langle J^x(t)\rangle - \langle J^x_{eq}\rangle|$$

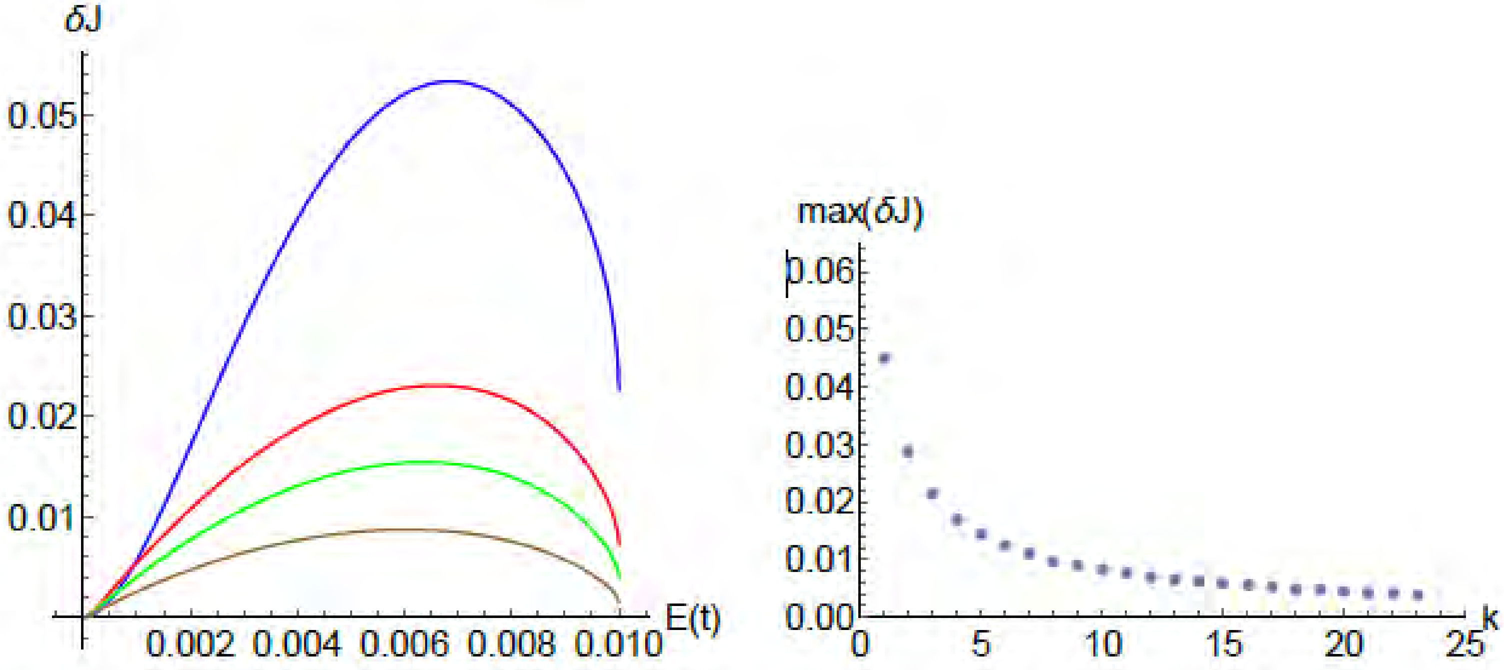

Figure 3: Left: $\delta J = |\langle J^x(t)\rangle - \langle J^x_{eq}\rangle|$ in terms of $E(t)$ has been plotted for $k = 1$ (blue), $k = 3$ (red), $k = 5$ (green) and $k = 10$ (brown) (top to bottom). Right: the maximum of δJ versus k.

An adiabatic process

FAST QUENCH

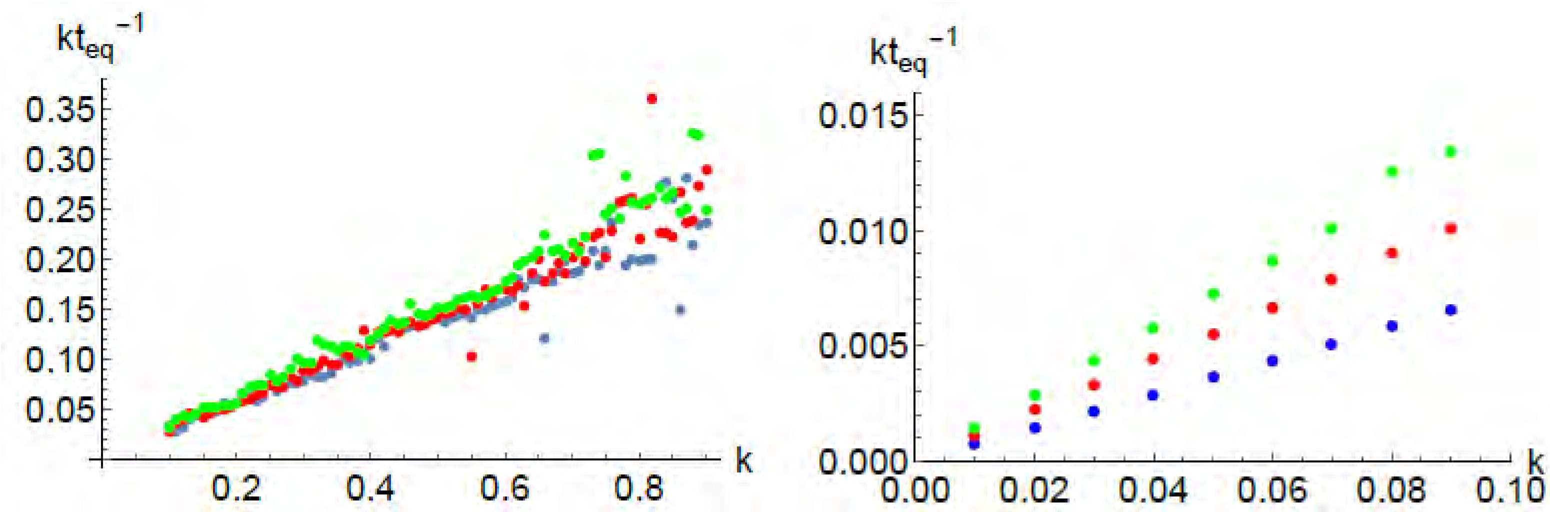

Figure 4: The inverse of the resacled equilibration time versus the transition time for $E_0 = 0.24$ (blue), $E_0 = 0.27$ (red) and $E_0 = 0.31$ (green).

GENERALIZATION

1. Dp-Dq system — D2-D4, D3-D5, D4-D6

2. New pulse function

$$E(t) = E_0 \begin{cases} 0 & \text{if } t \leq 0, \\ 6(\frac{t}{k})^5 - 15(\frac{t}{k})^4 + 10(\frac{t}{k})^3 & \text{if } 0 \leq t \leq k, \\ 1 & \text{if } t \geq k. \end{cases}$$

FINAL RESULTS

- Our calculations indicate that a universal behavior emerges when $k < 1$, i.e. *fast electric field quenches.* More precisely, by universality we mean the rescaled equilibration time $k^{-1}t_{eq}$ becomes independent of the final value of the electric field for small enough k.
- For $k > 1$, i.e. *slow electric field quench*, the rescaled equilibration time decreases as one raises k at fixed final electric field value. In fact, it seems that for large enough values of k, the relaxation of the system to its final equilibrium is an adiabatic process.

Thank you very much for your attention