Entanglement and anomalies

Amos Yarom

(Work in progress with T. Nishioka (Tokyo))

Entanglement

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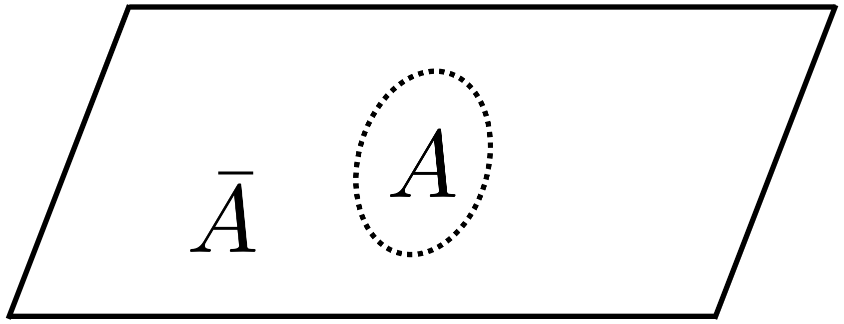

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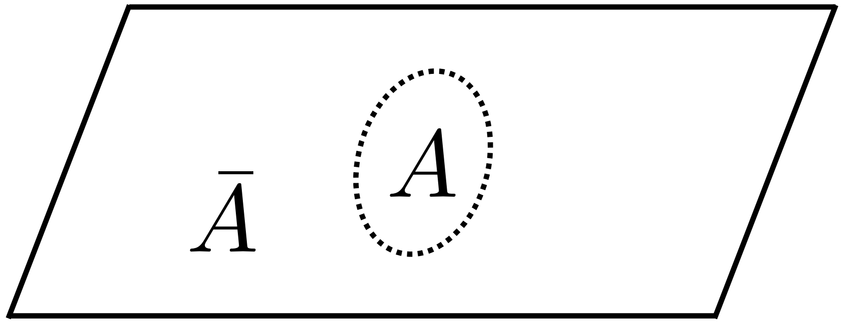

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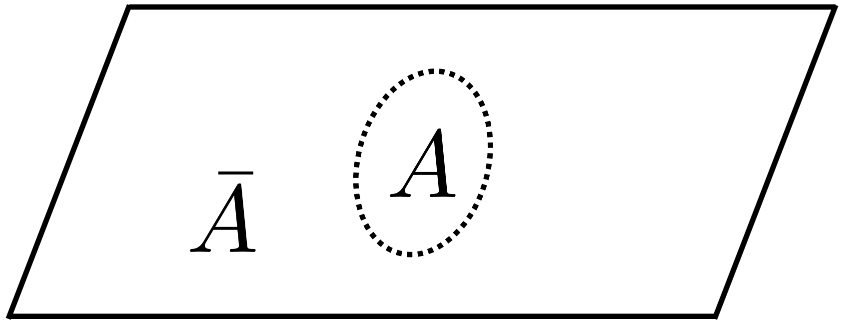

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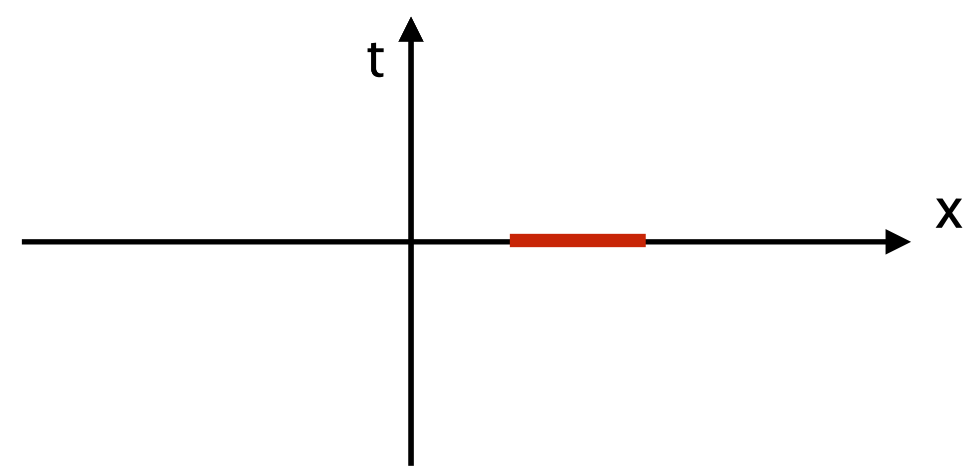

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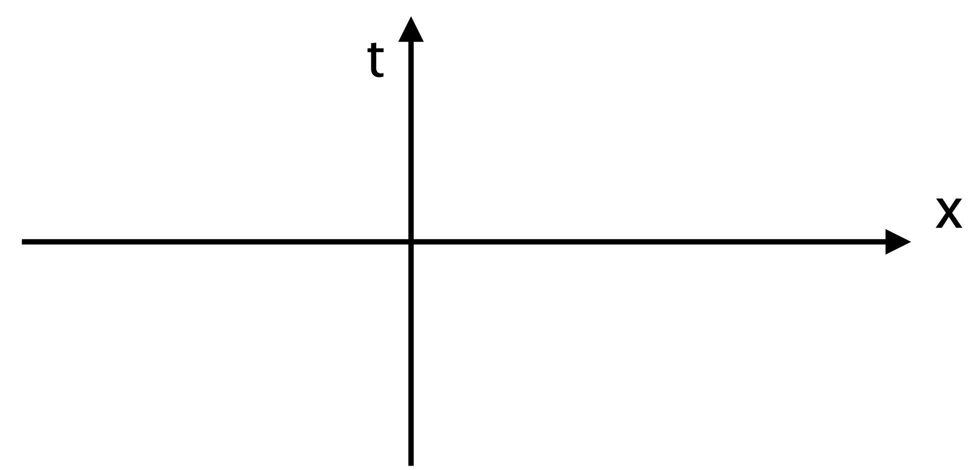

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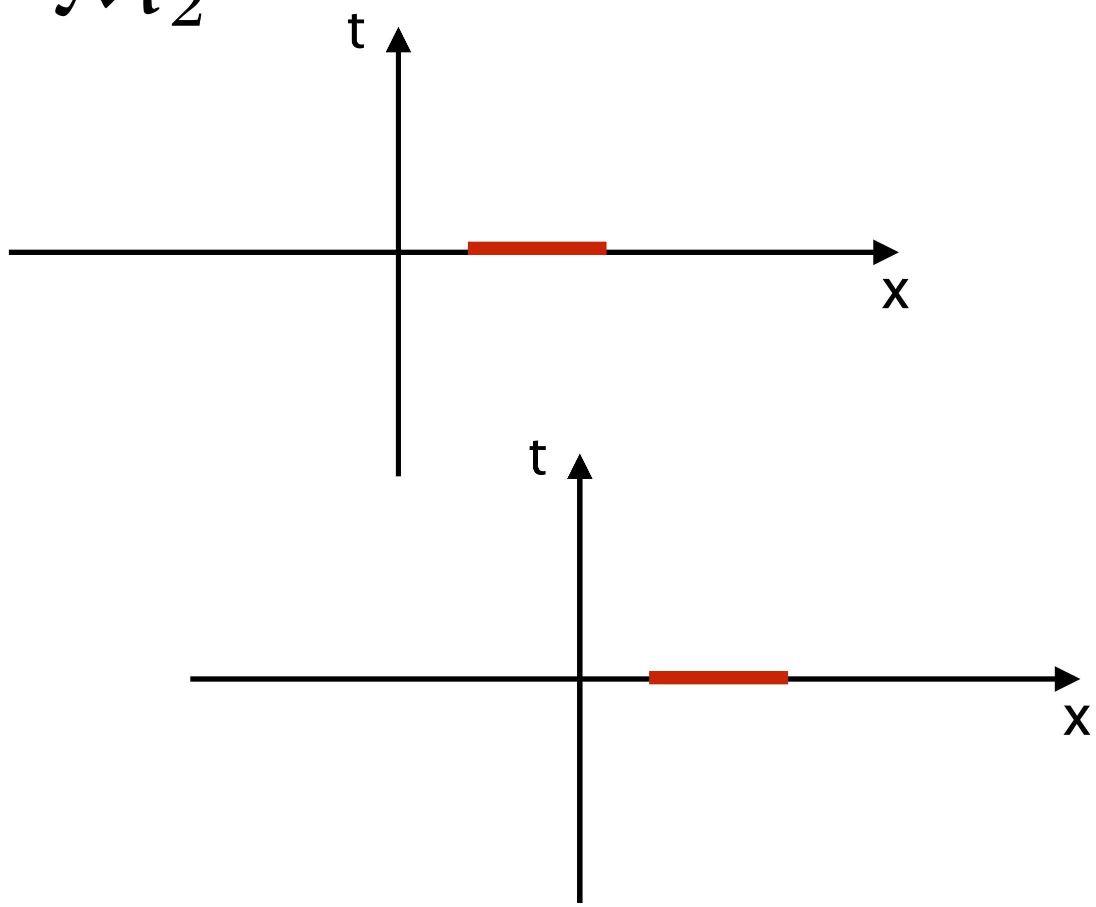

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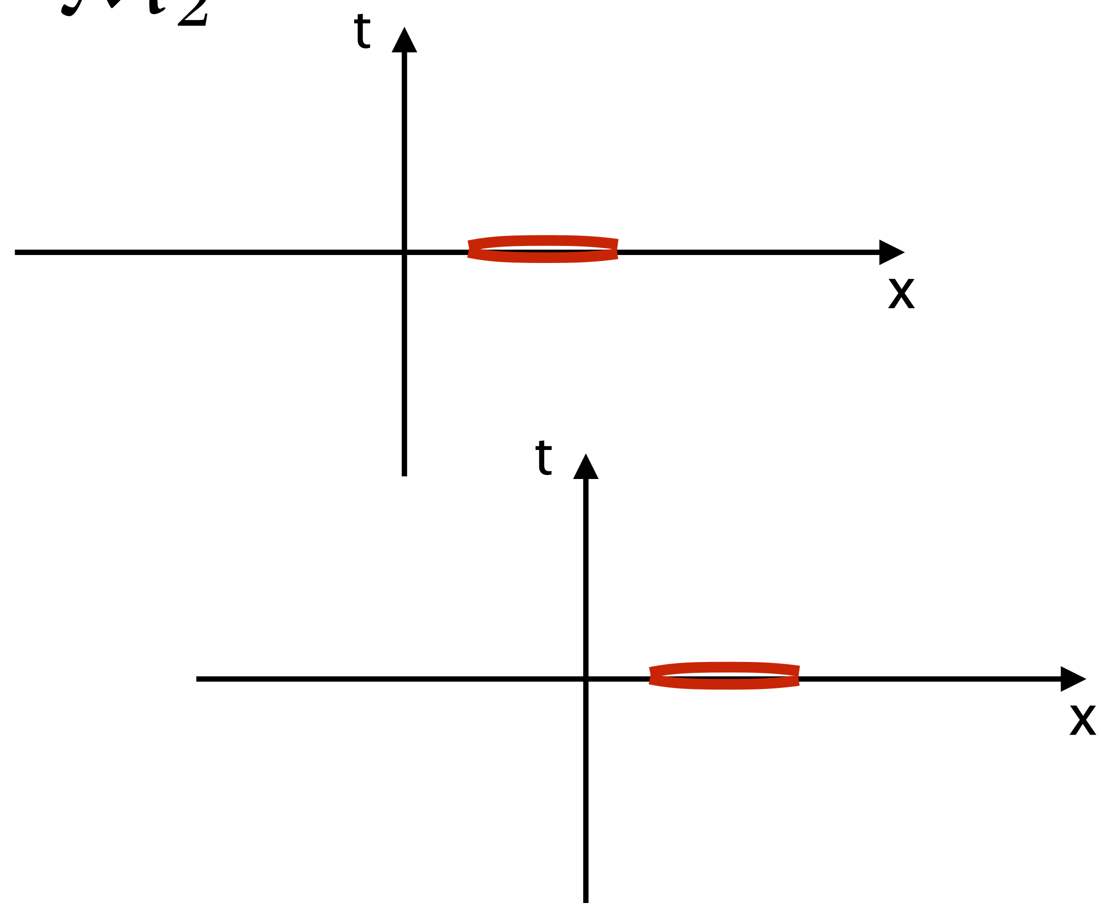

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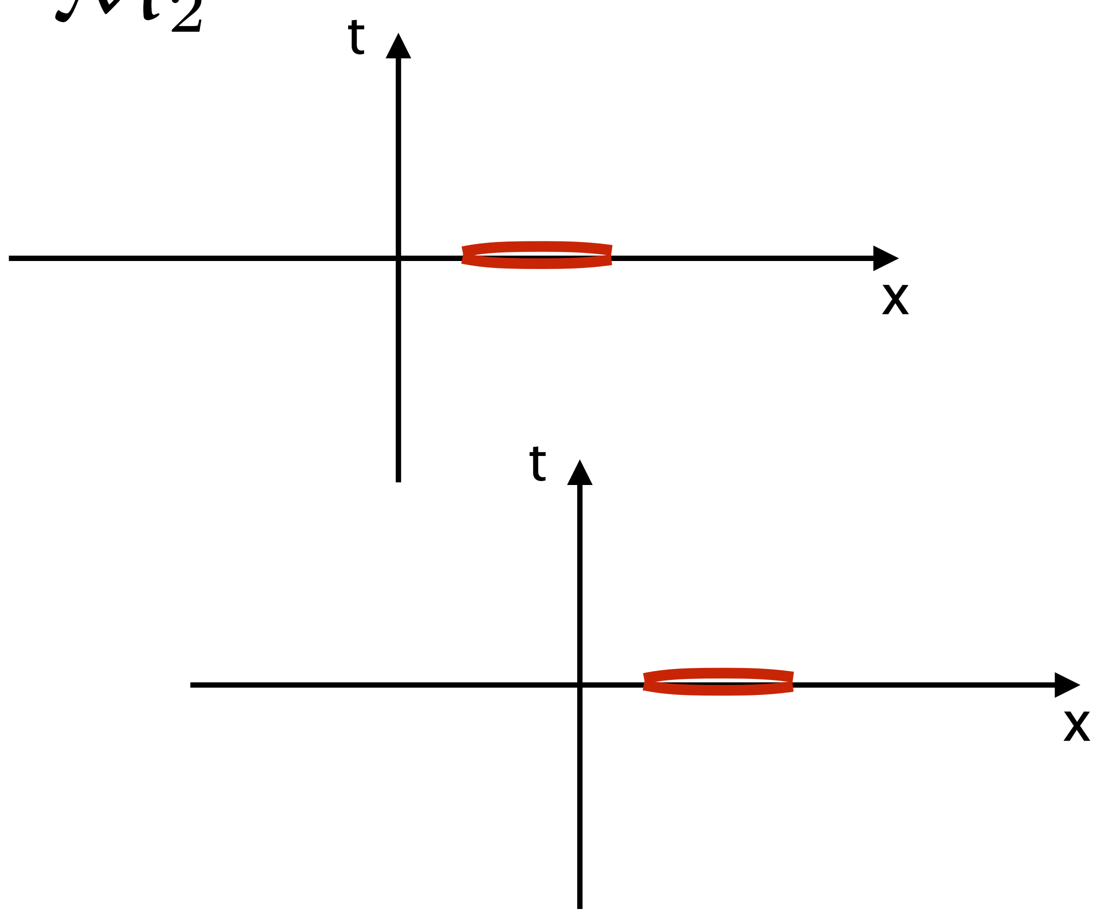

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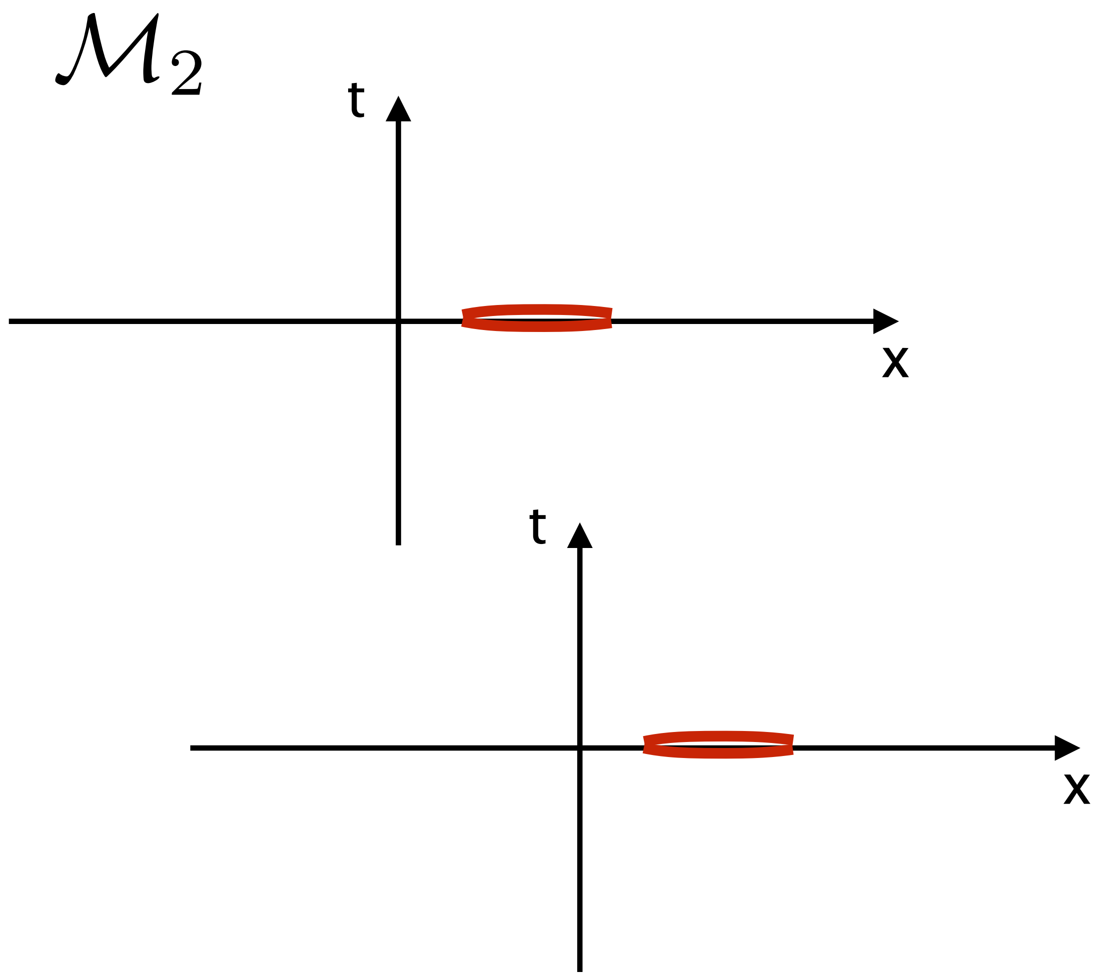

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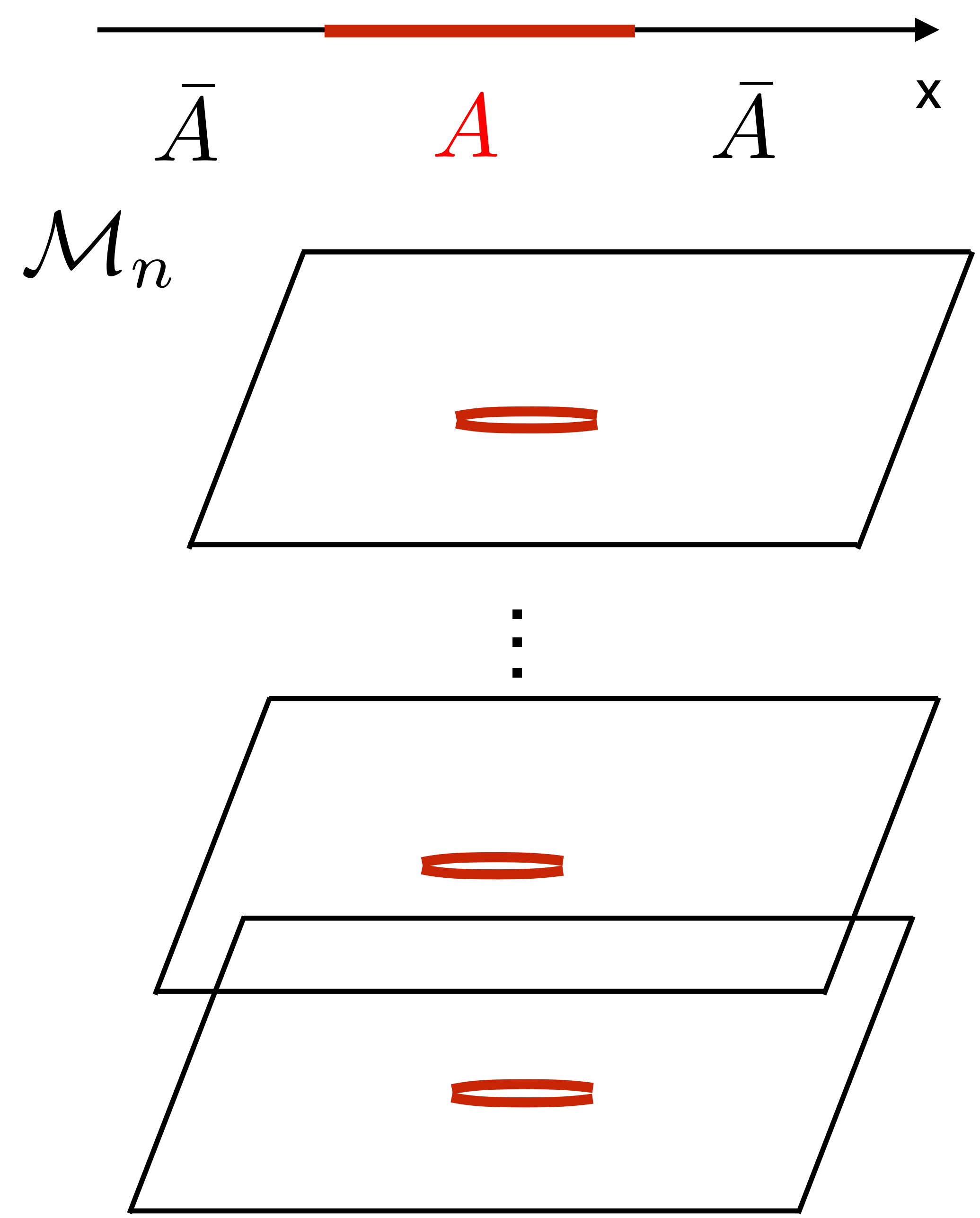

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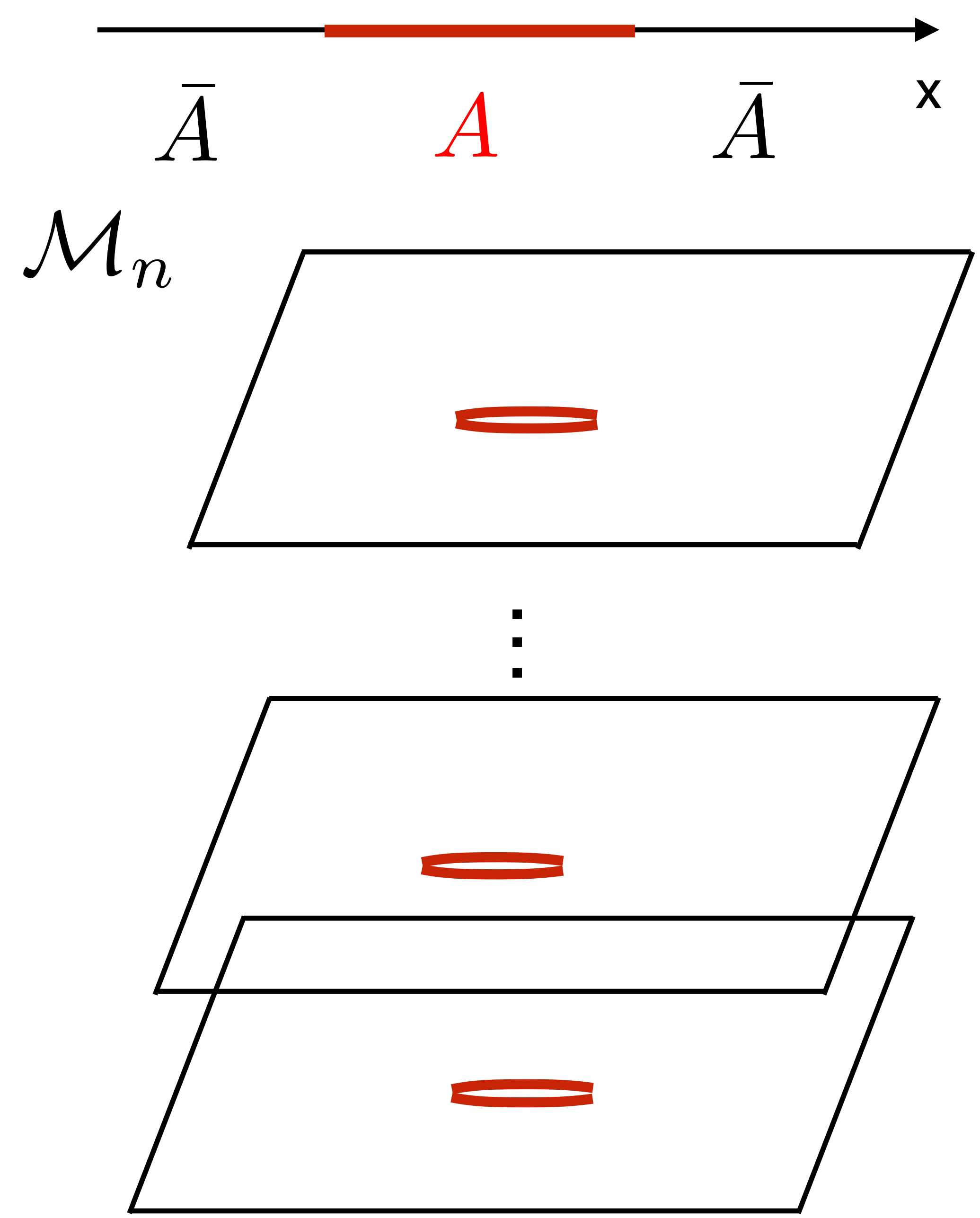

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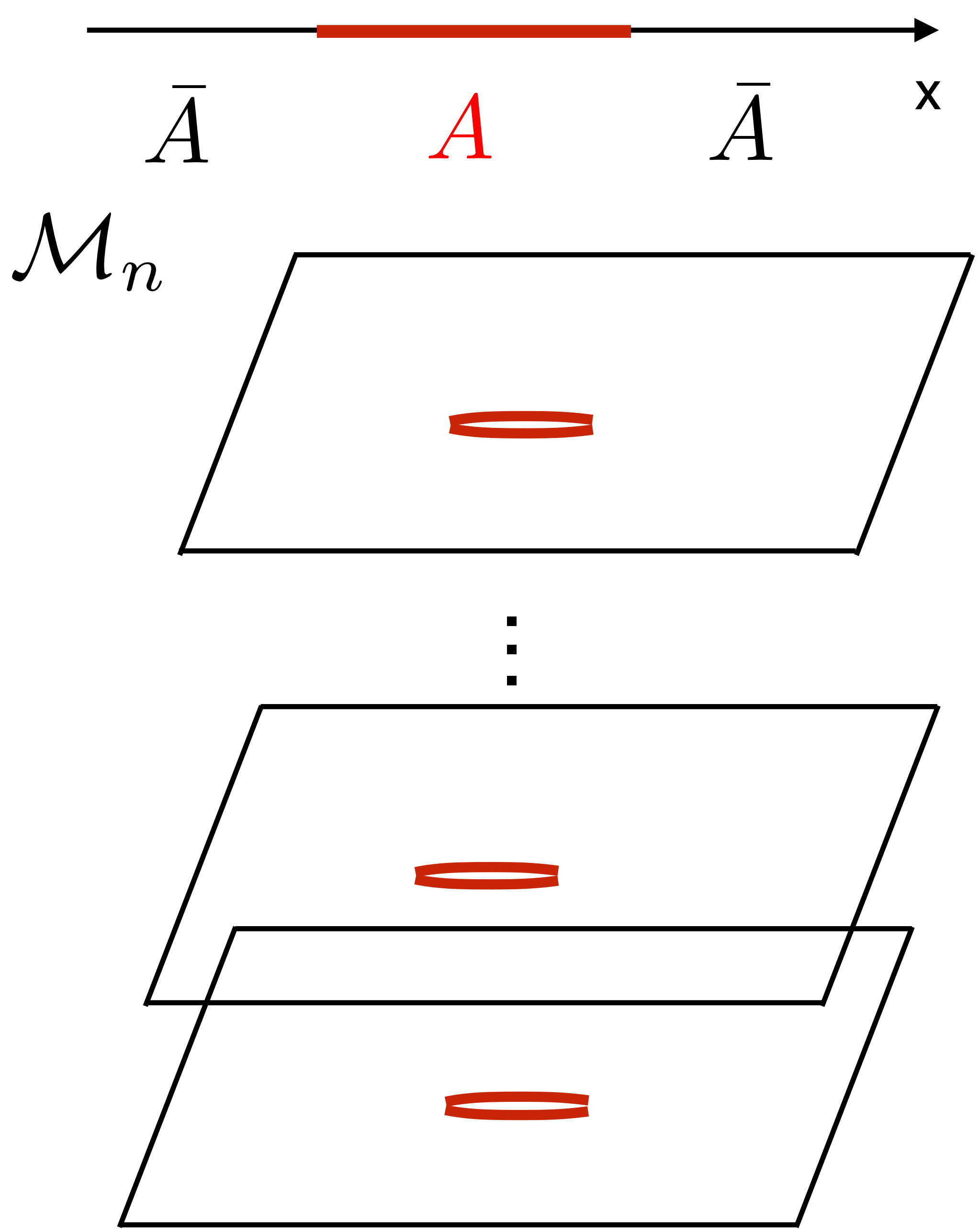

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Which can be shown to lead to

$$S_A = \frac{c}{3}\ln(L/\epsilon) + \ldots$$

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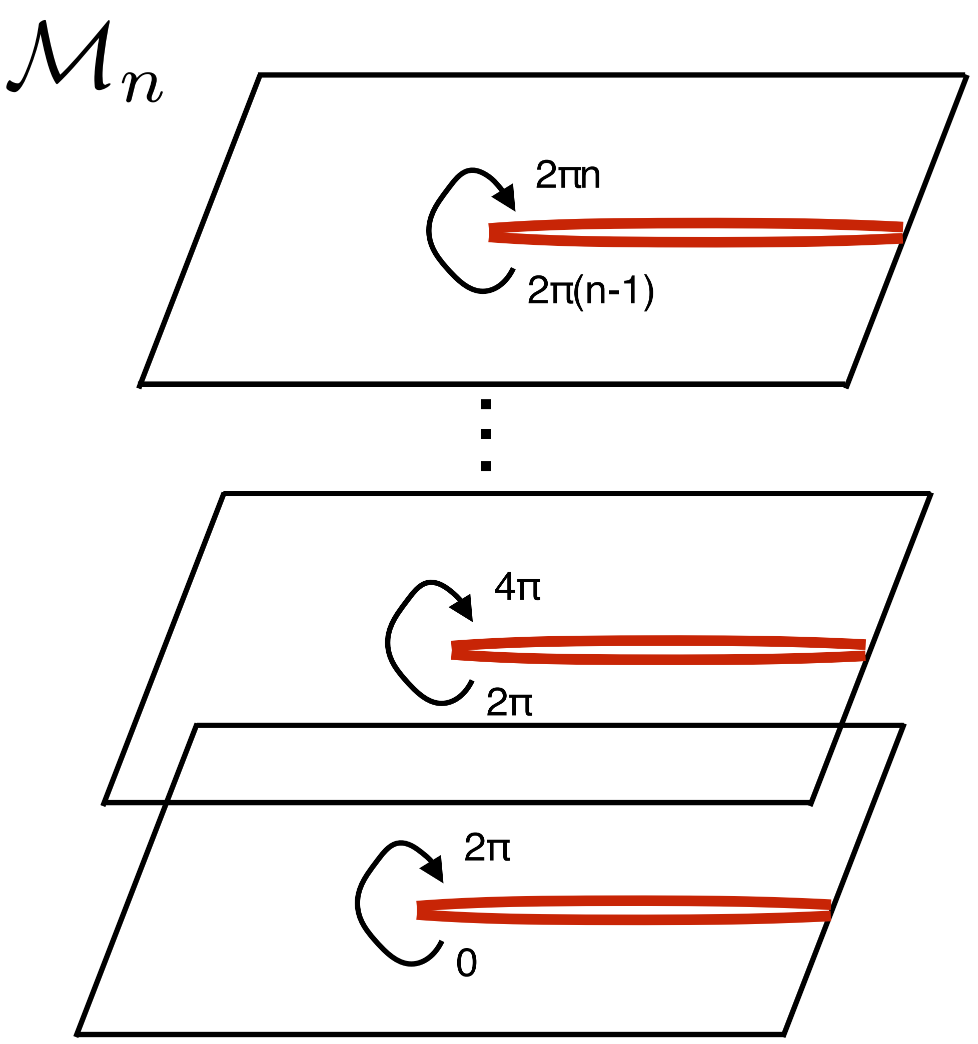

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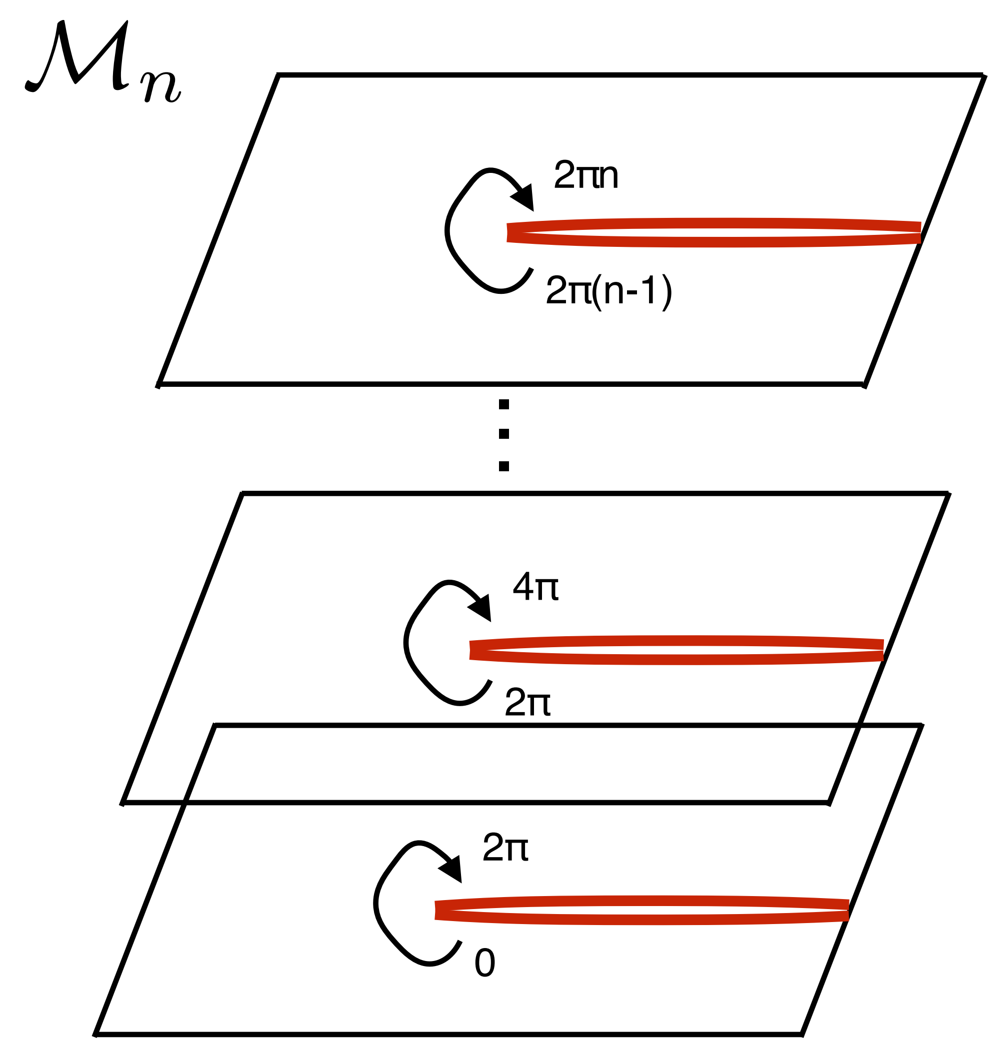

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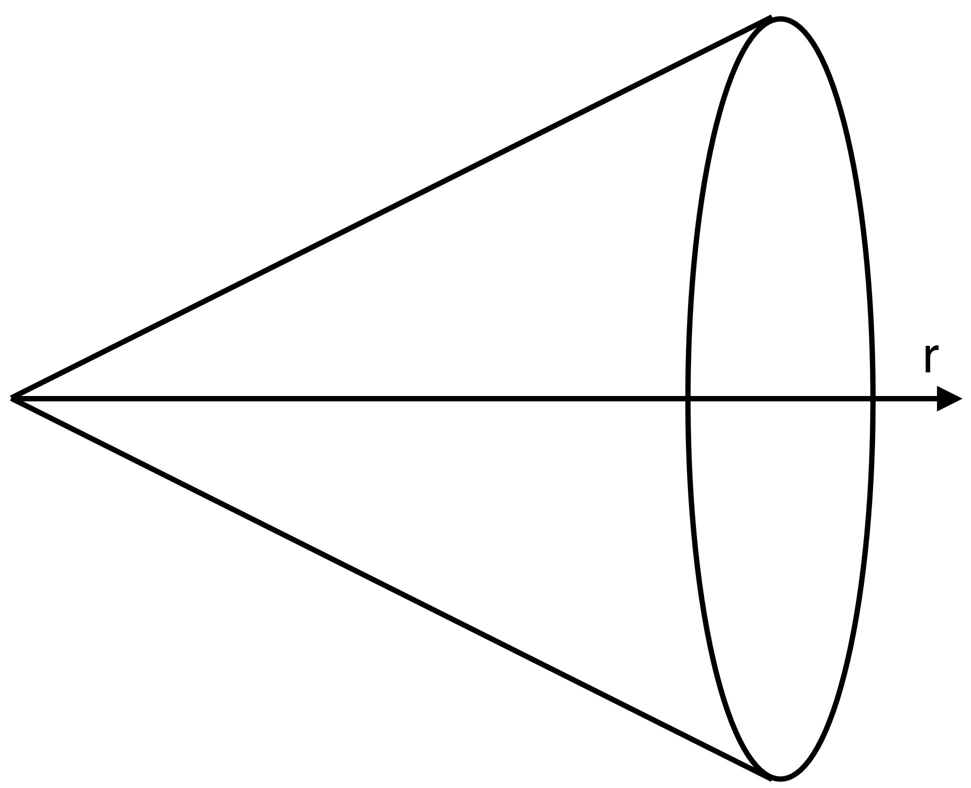

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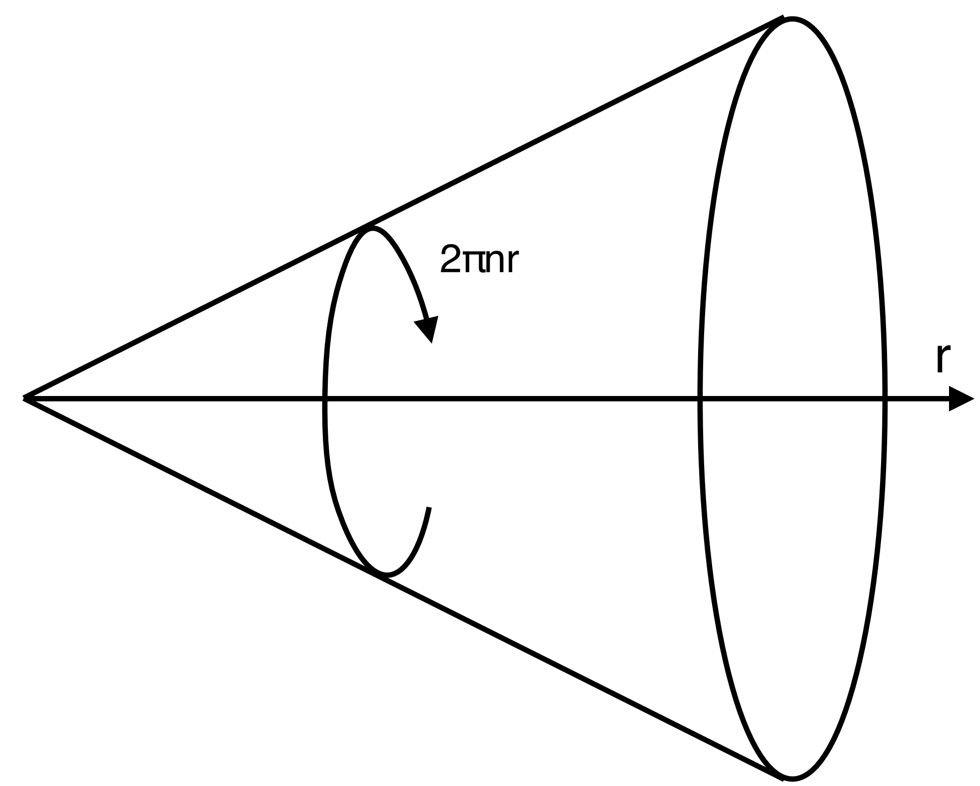

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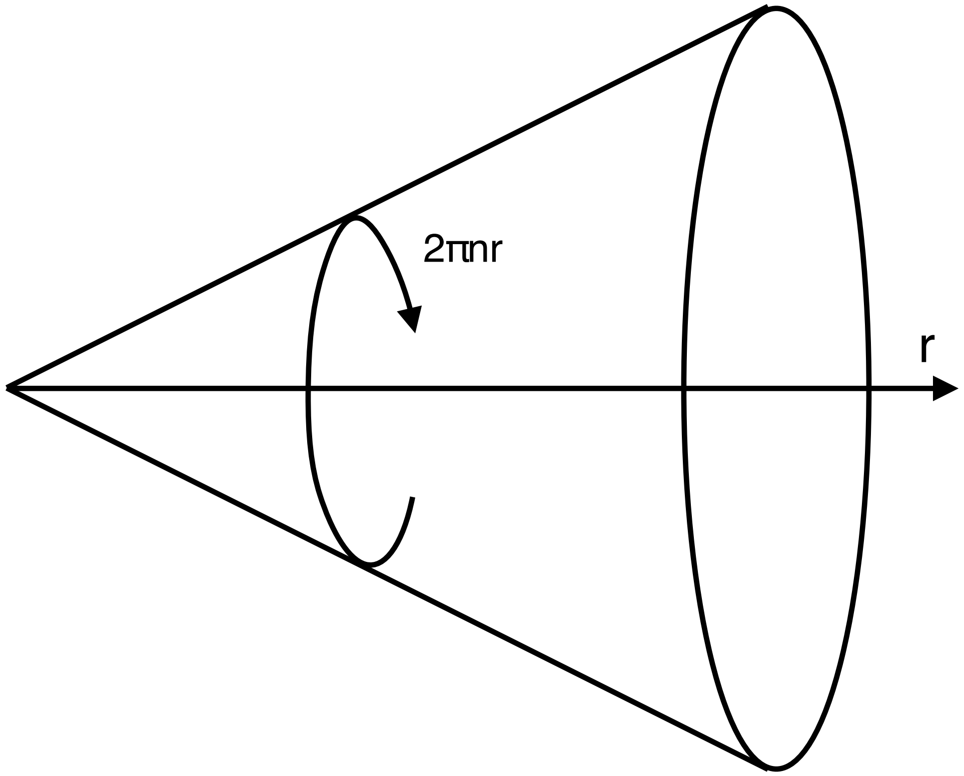

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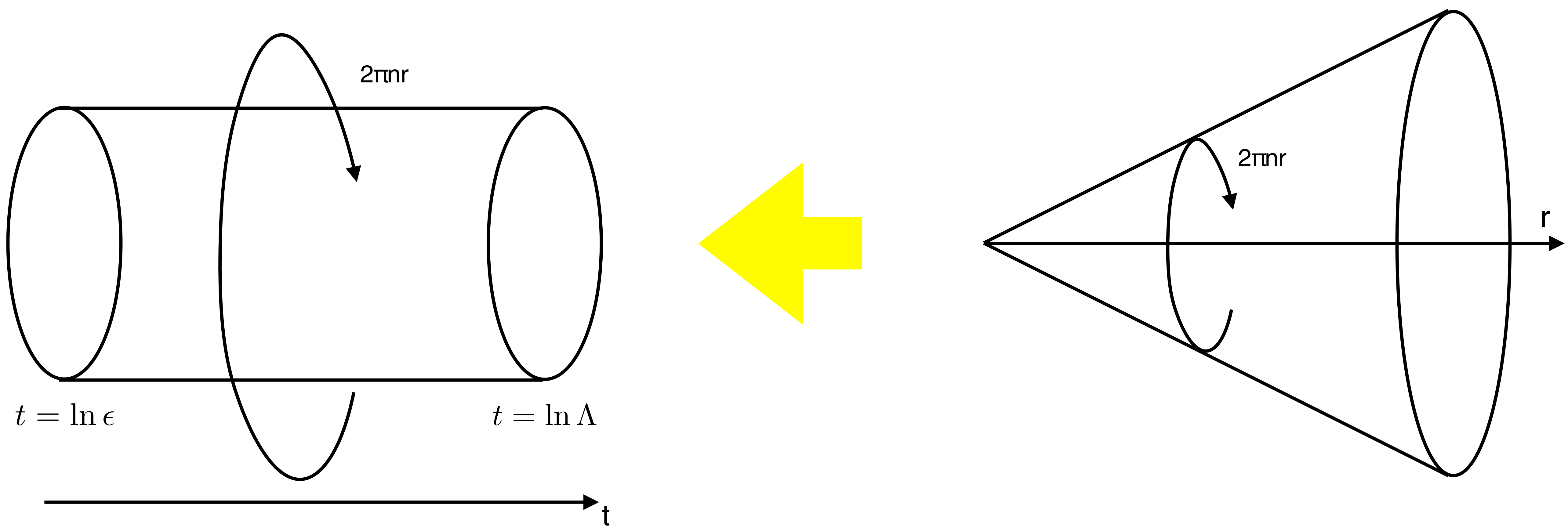

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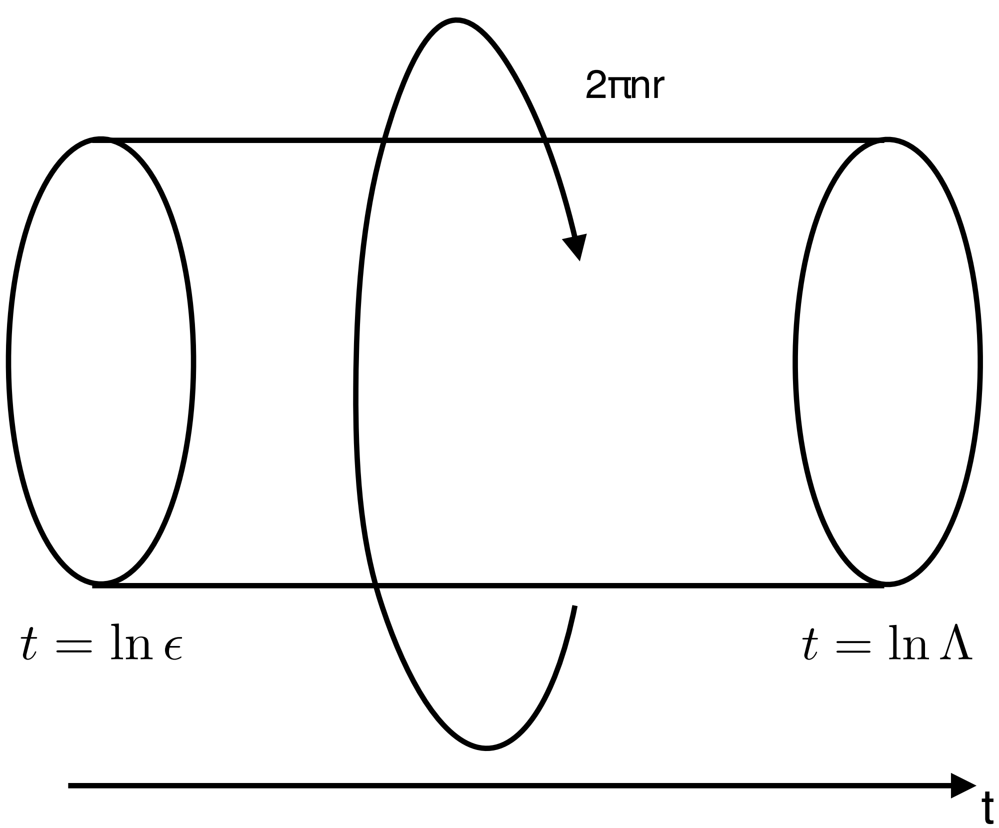

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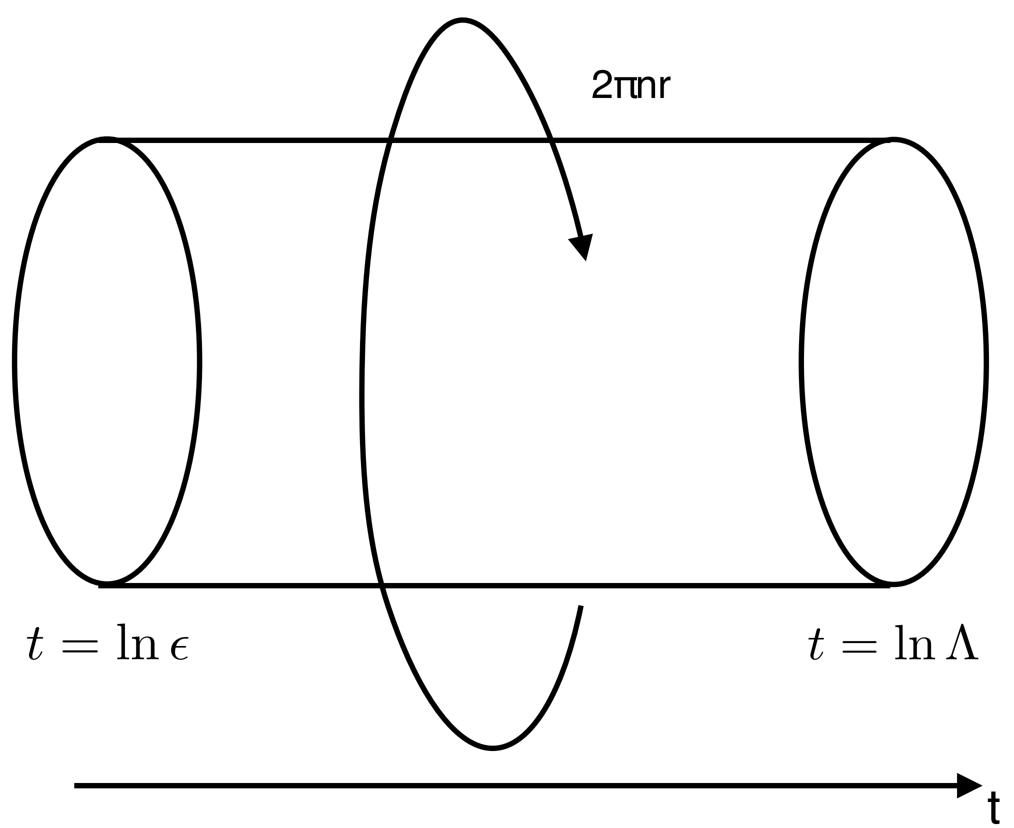

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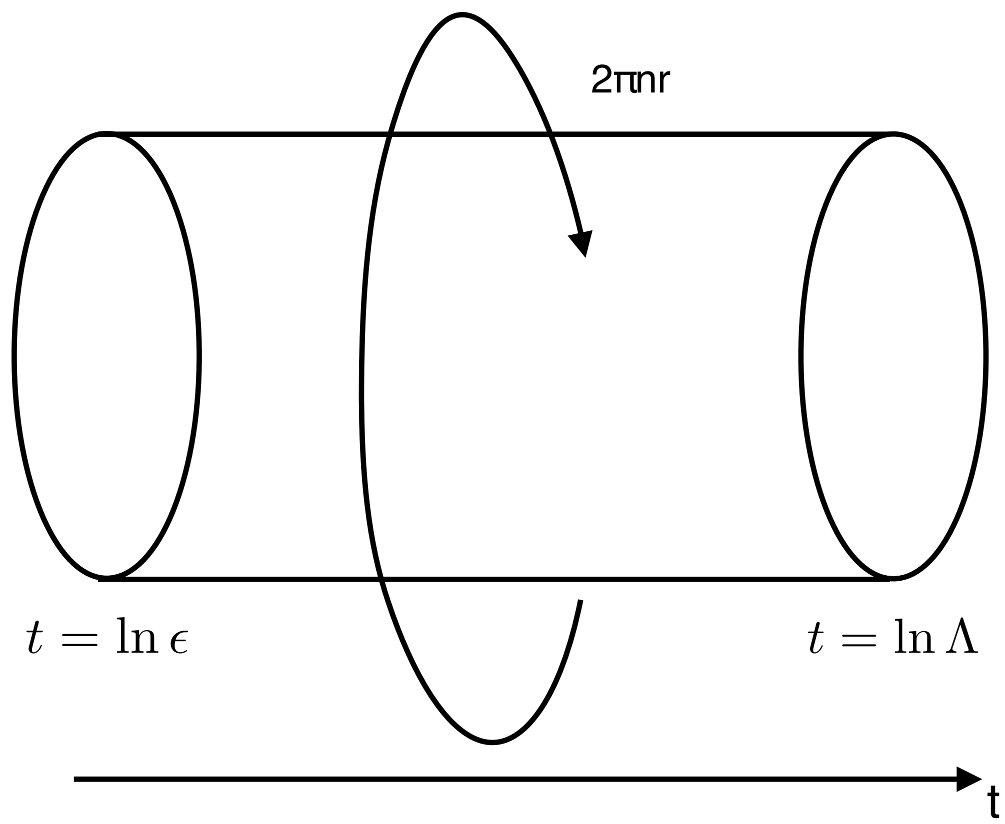

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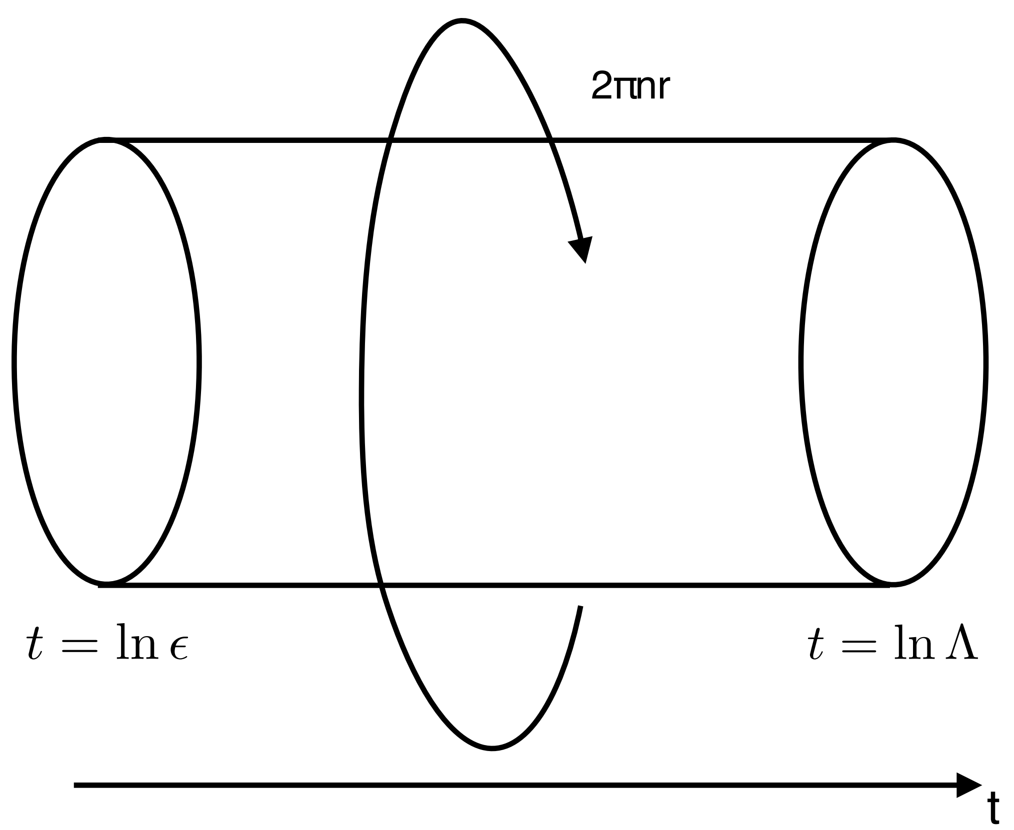

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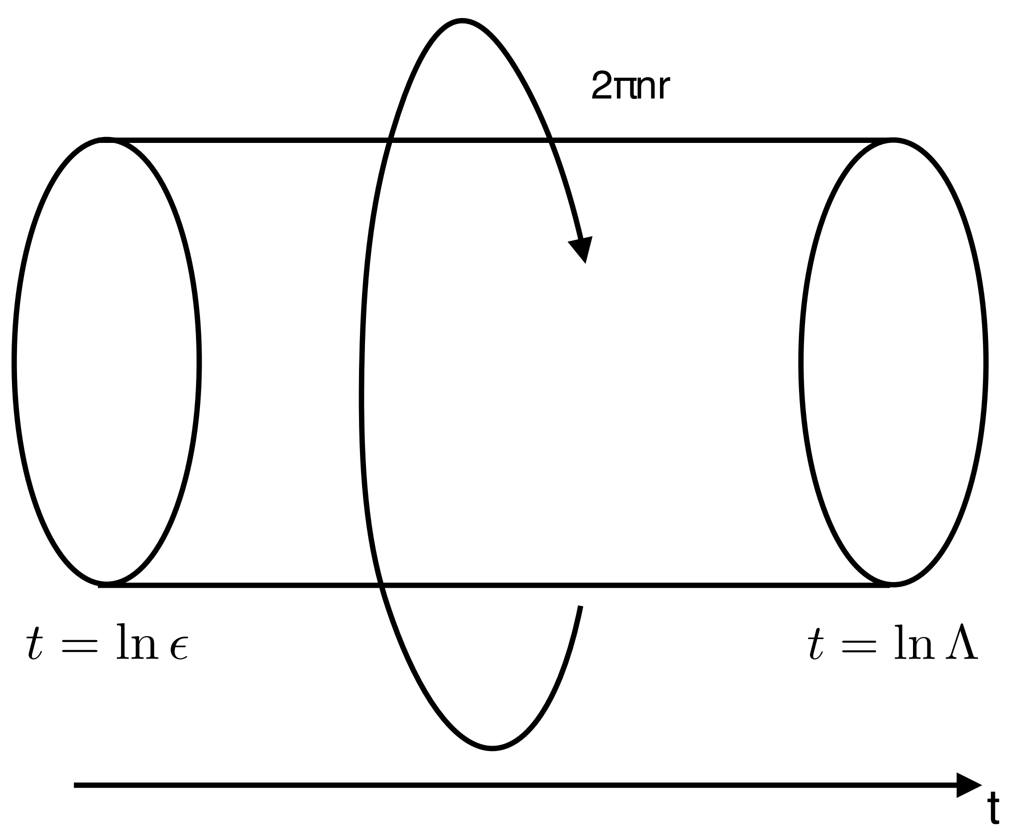

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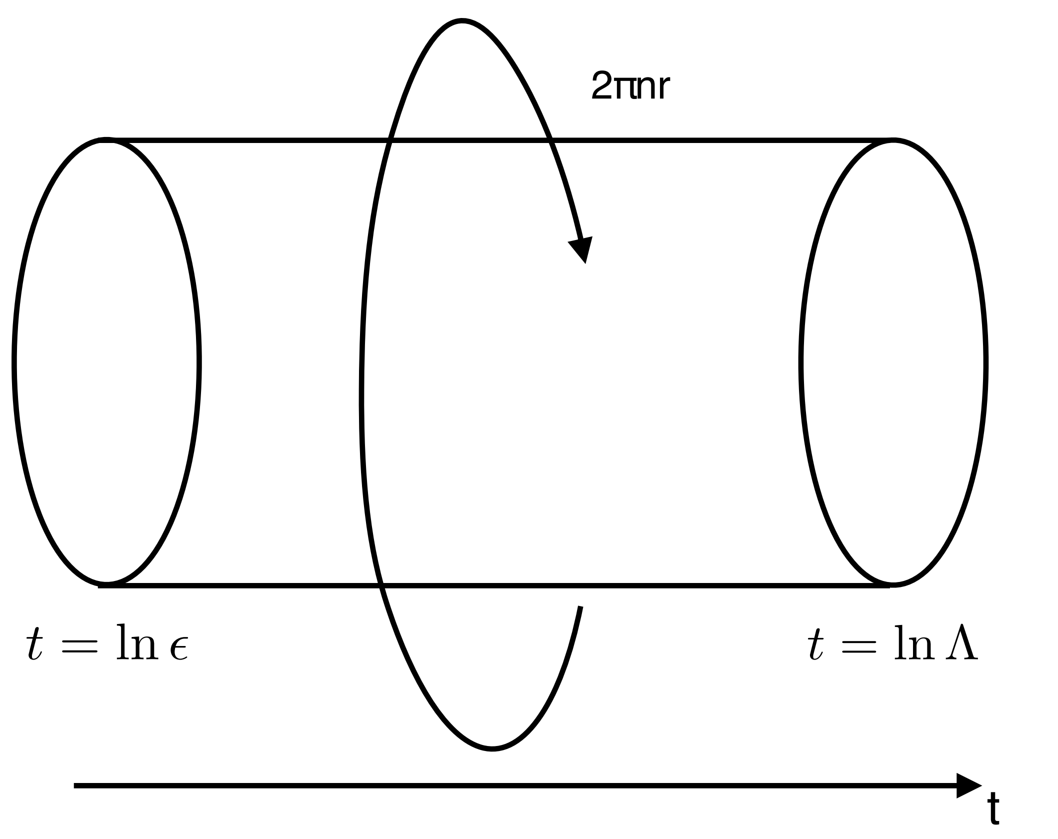

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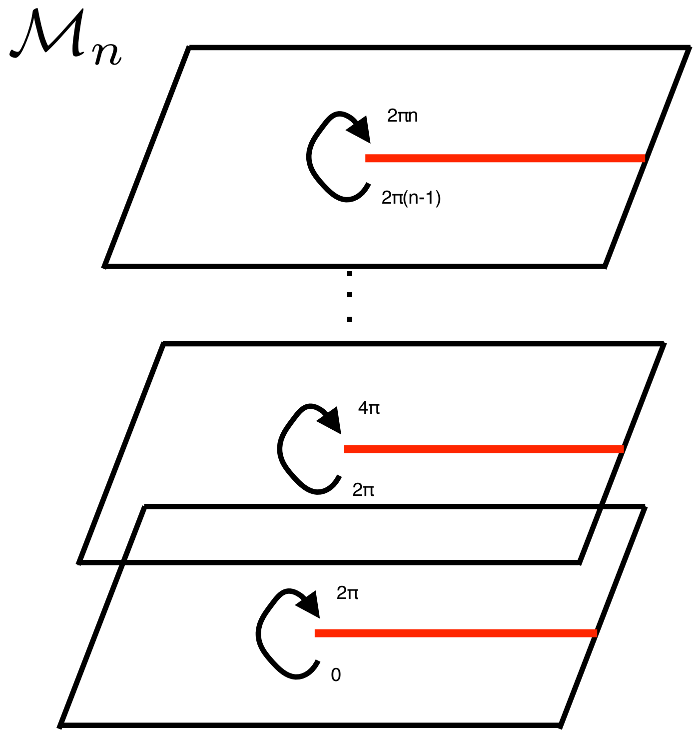
$$S_A = -\lim_{n\to 1}\left(\partial_n - 1\right) W_n$$

I would like to focus on entangling regions in which space is split in two.

Method 1: We can map the cone into a cylinder of radius n via $t + i\sigma = \ln(re^{i\phi})$

We find:

$$S_A = \frac{c_L + c_R}{12}\left(\ln \Lambda/\epsilon\right)$$



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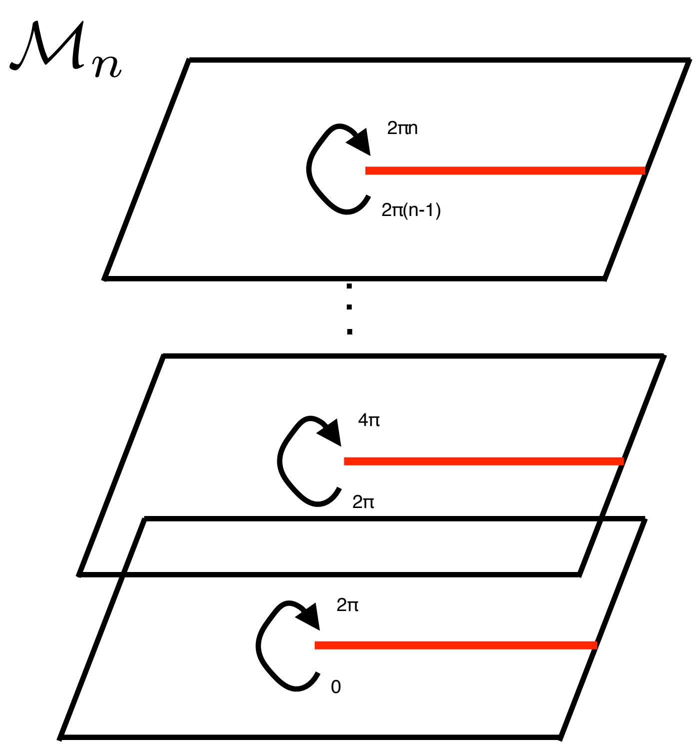

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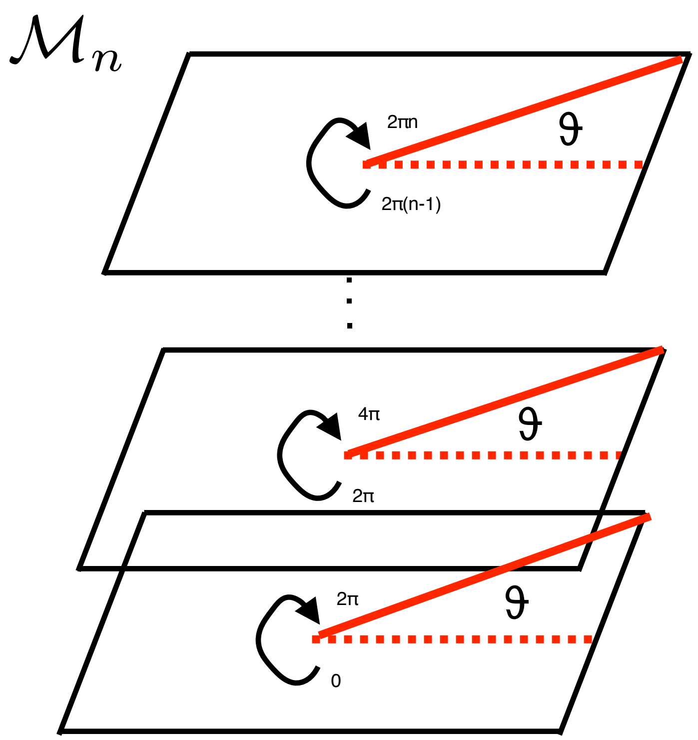

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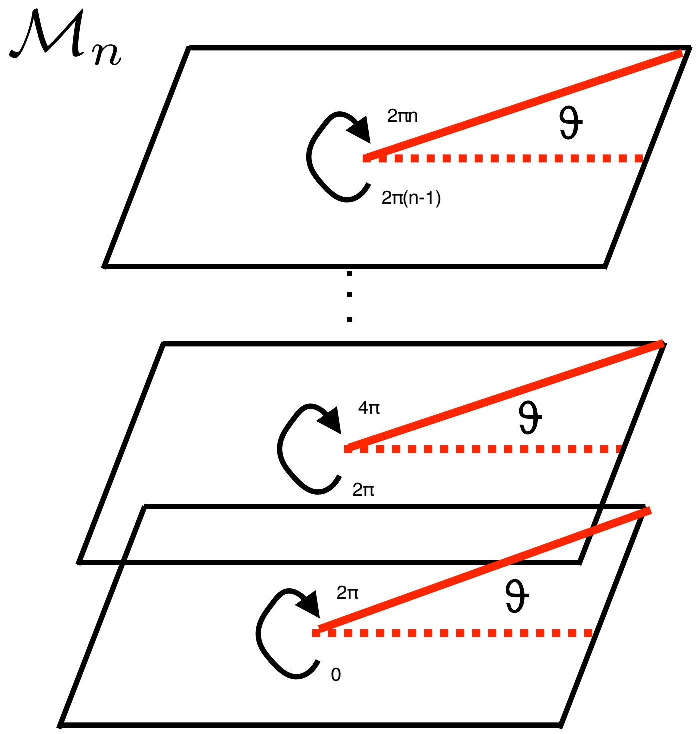

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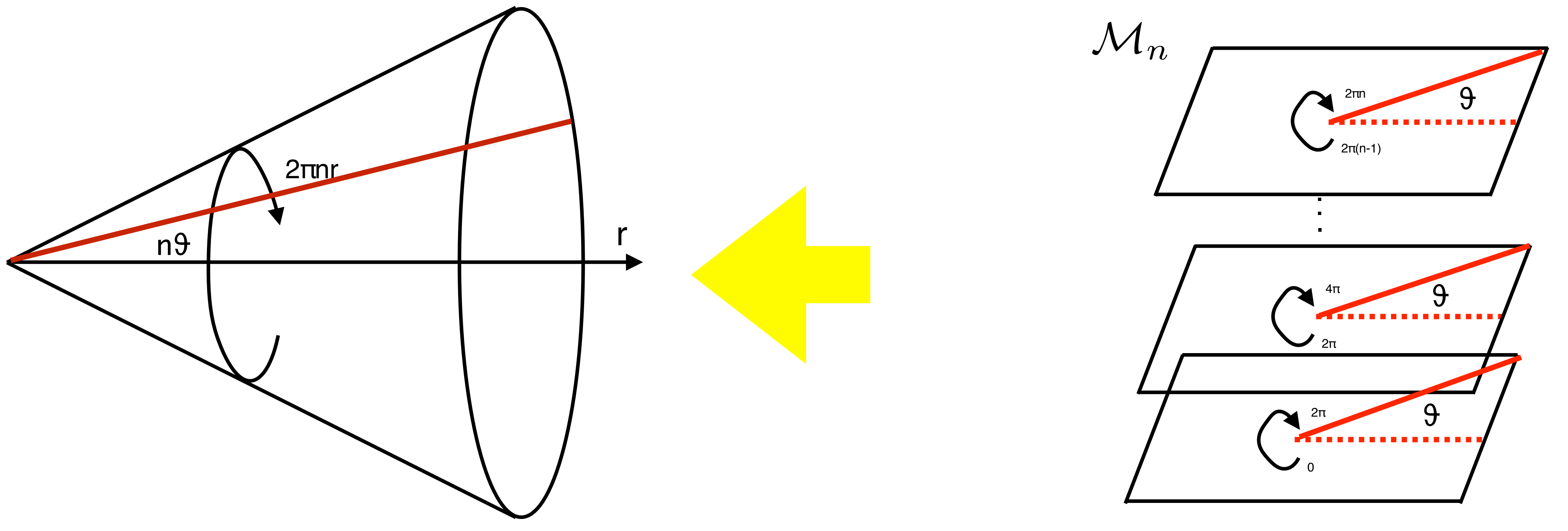

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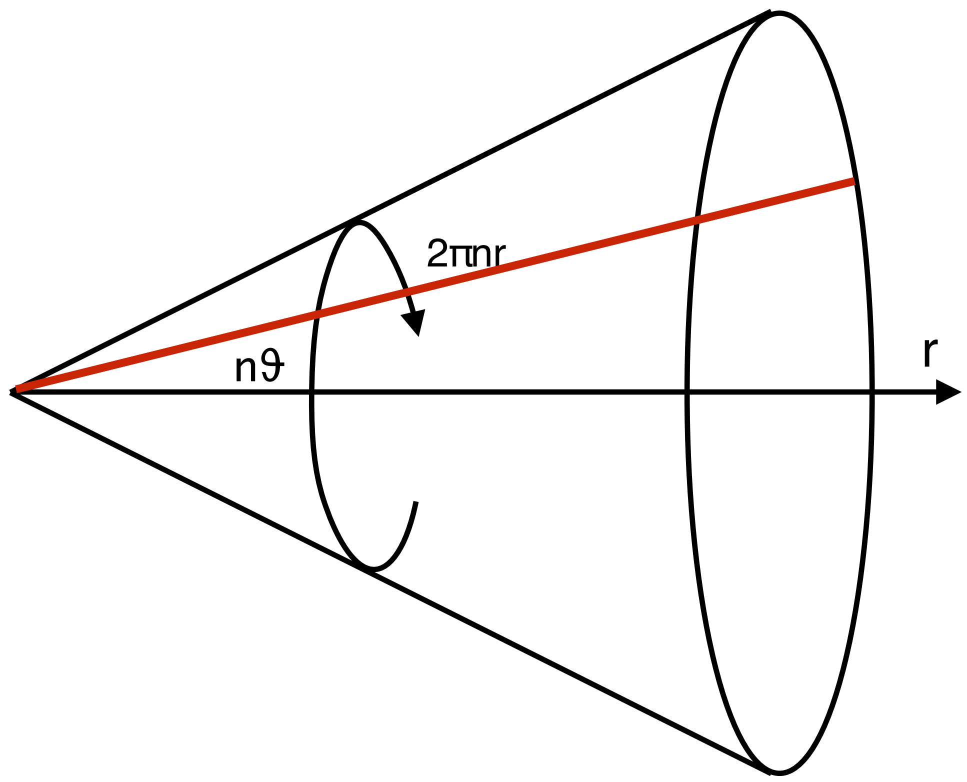

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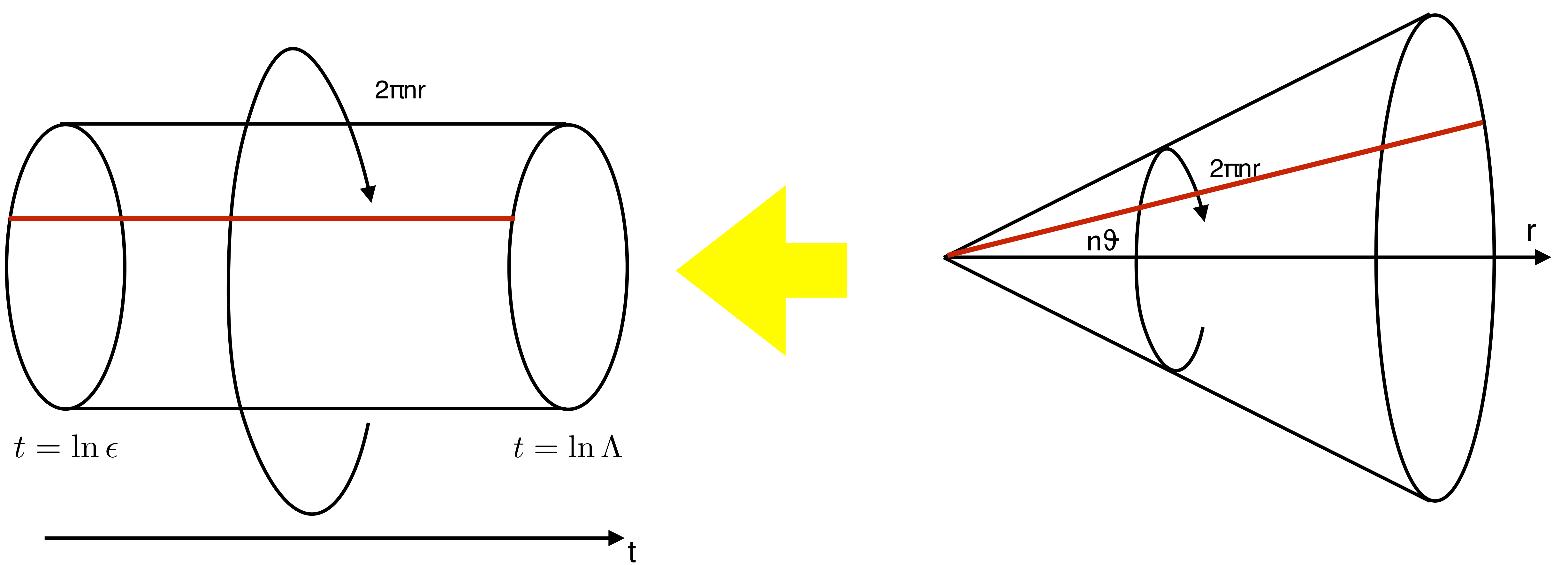

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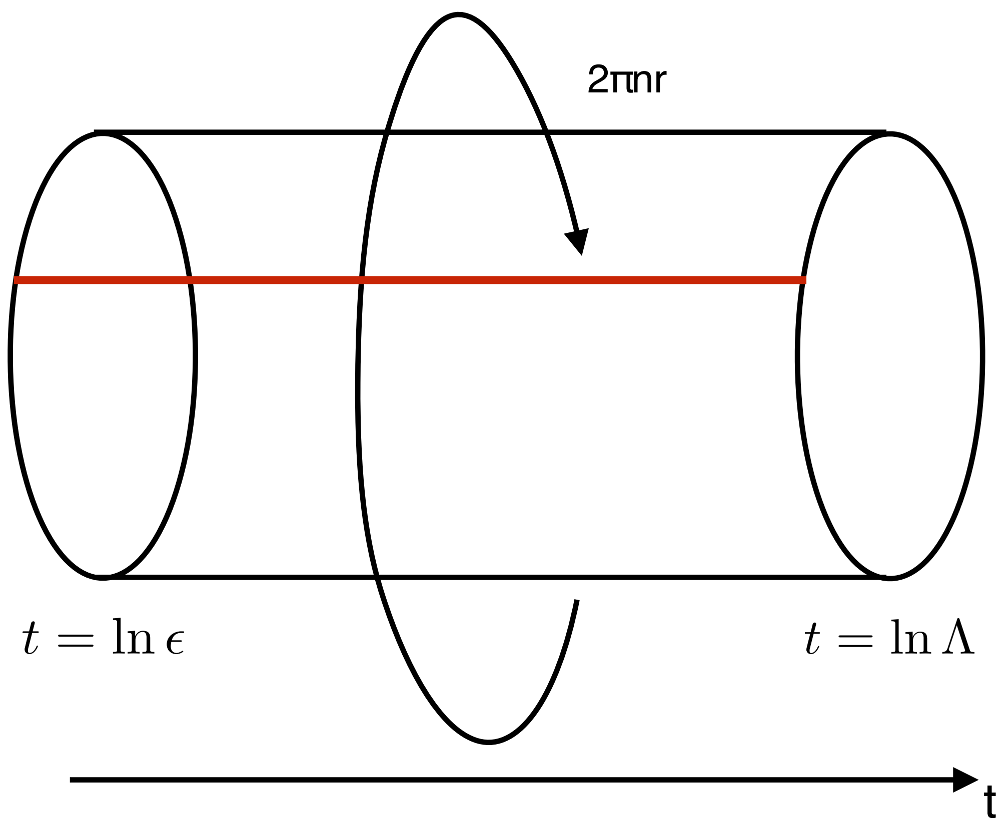

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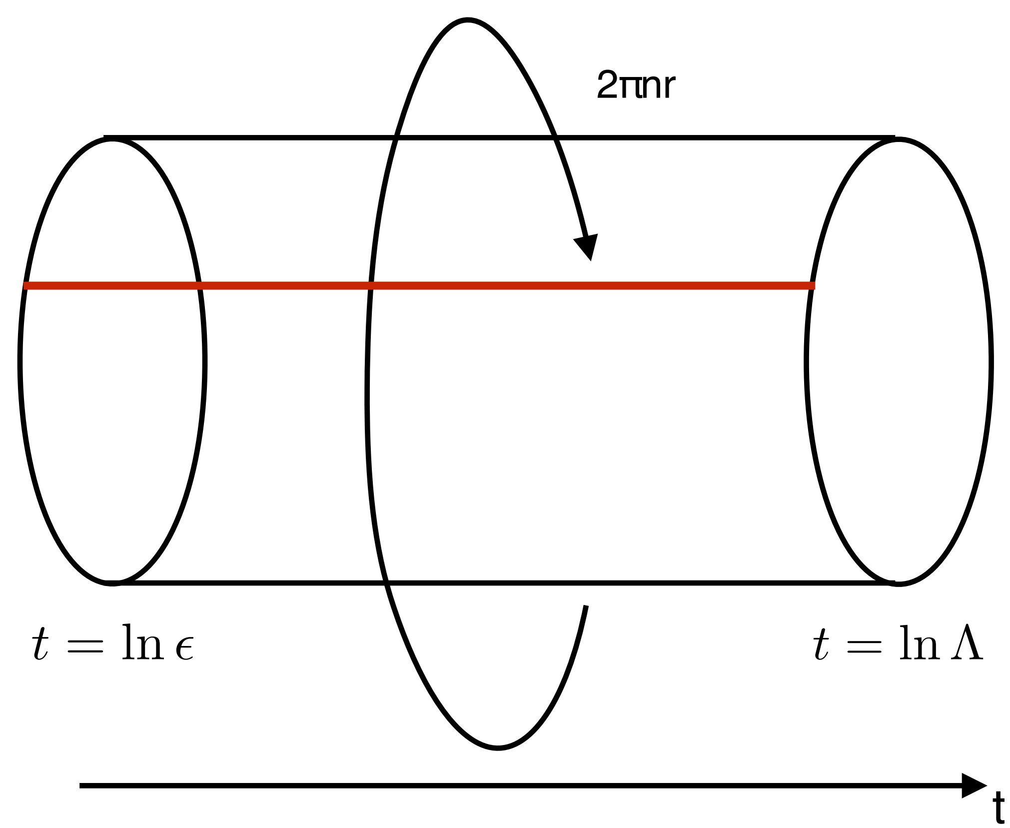
$$Z[\mathcal{M}_n] = \langle B_\epsilon | e^{-\ell H + in\theta P} | B_\Lambda \rangle$$



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(A related result has been obtained previously by Castro, Detournay, Iqbal, Perlmutter, 2014)

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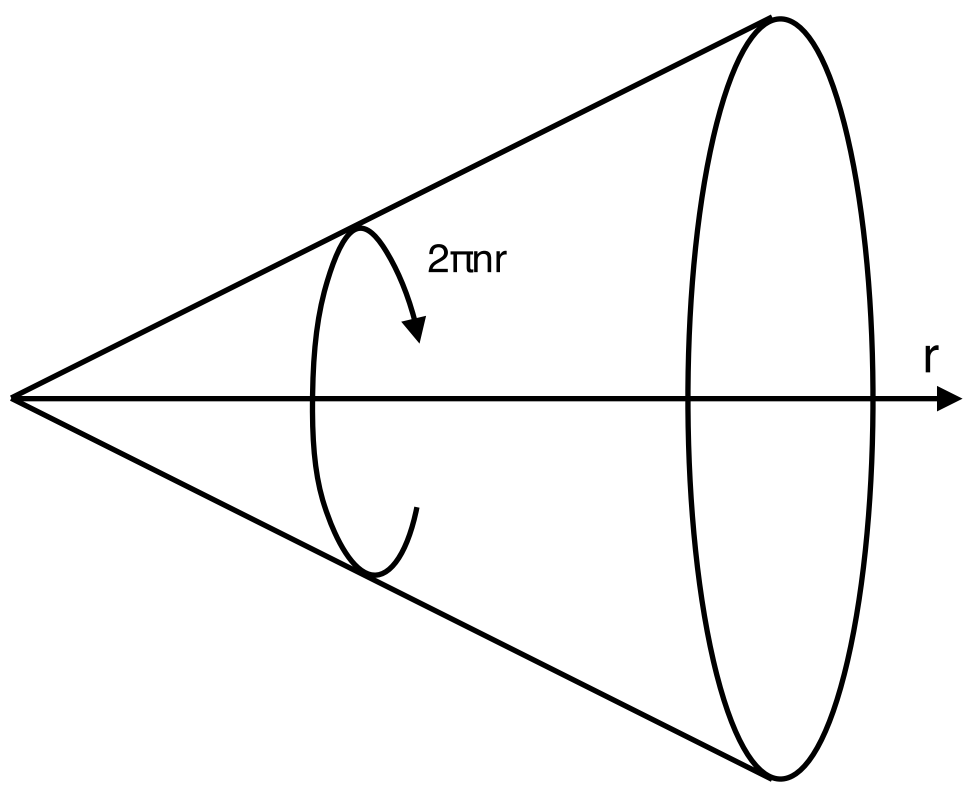
Method 2: One can think of Z_n as the thermodynamic partition function on a semi-infinite line with non uniform temperature.

$$T^{-1} = 2\pi n r$$



W_n is the generating function for connected correlators in such a state.

Constructing W_n

In a two dimensional conformal field theory and in the absence of global charges we define

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Then: (Jensen, Loganayagam, AY, 2012)

$$W = \int d^2x \sqrt{-g}\Bigg(\frac{\pi}{12}(c_R + c_L)T^2 - \frac{\pi}{12}(c_R - c_L)\beta^{-1} T \epsilon^{0\nu} u_\nu$$

$$+ \frac{c_L + c_R}{48\pi} u^\beta \partial_\beta u_\gamma u^\alpha \partial_\alpha u^\gamma + \frac{c_R - c_L}{96\pi} u_\alpha u^\beta \epsilon^{\mu\nu} \partial_\mu \Gamma^\alpha{}_{\beta\nu} \Bigg)$$

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$$S_A = - \lim_{n \to 1} \left(\partial_n - 1 \right) W_n$$

Unsurprisingly, we find:

$$S_A = \frac{c_L + c_R}{12} \ln(\Lambda / \epsilon)$$

Adding anomalies

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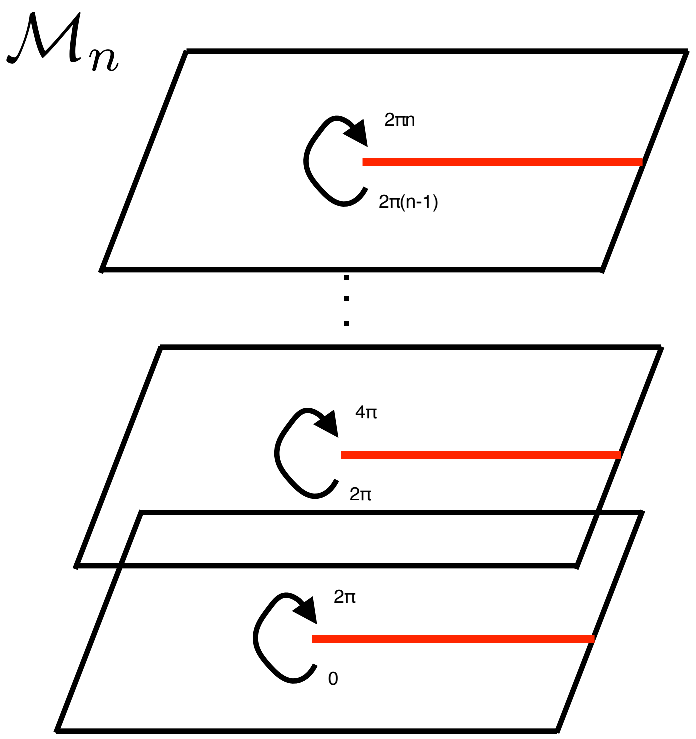

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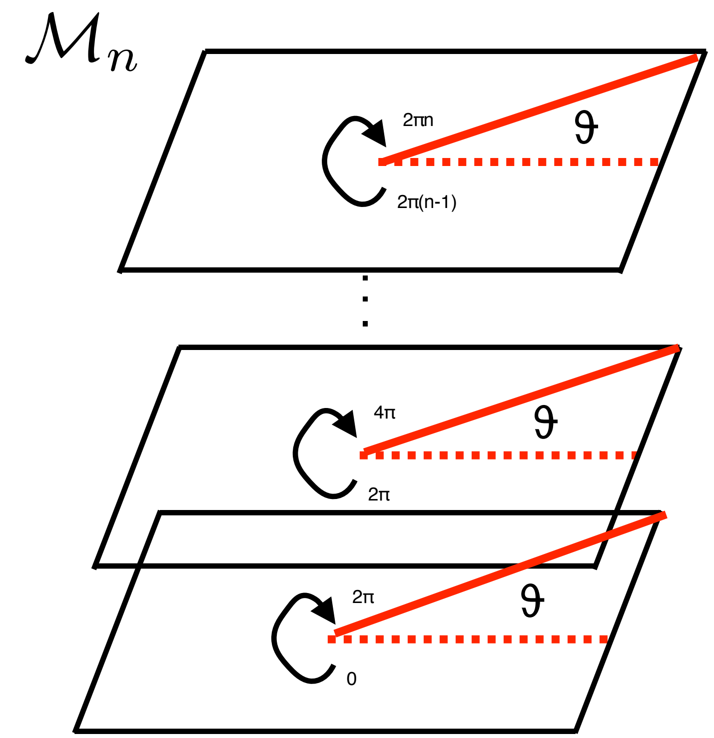

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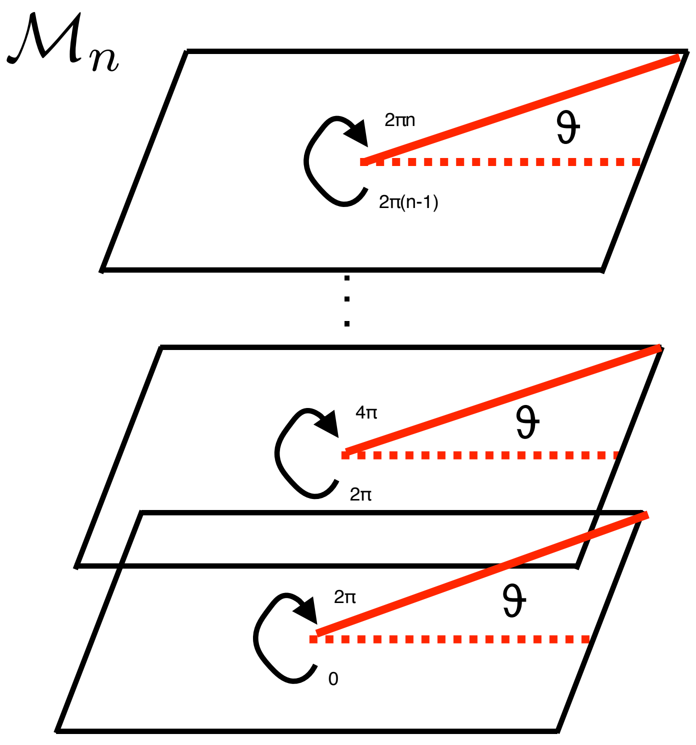

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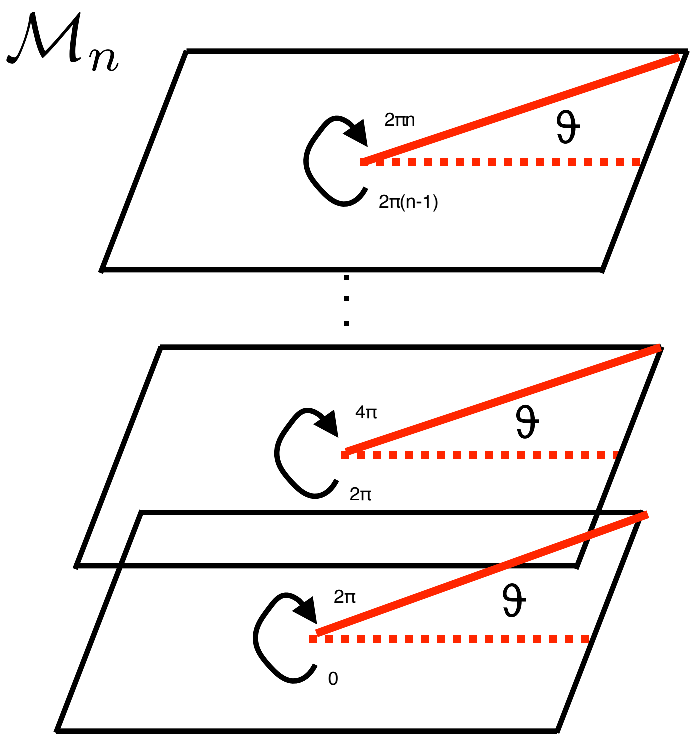

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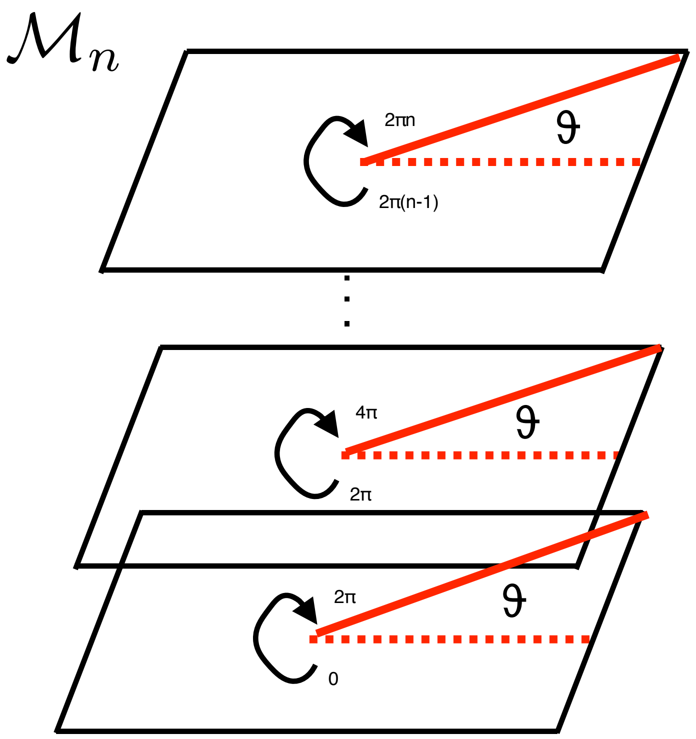

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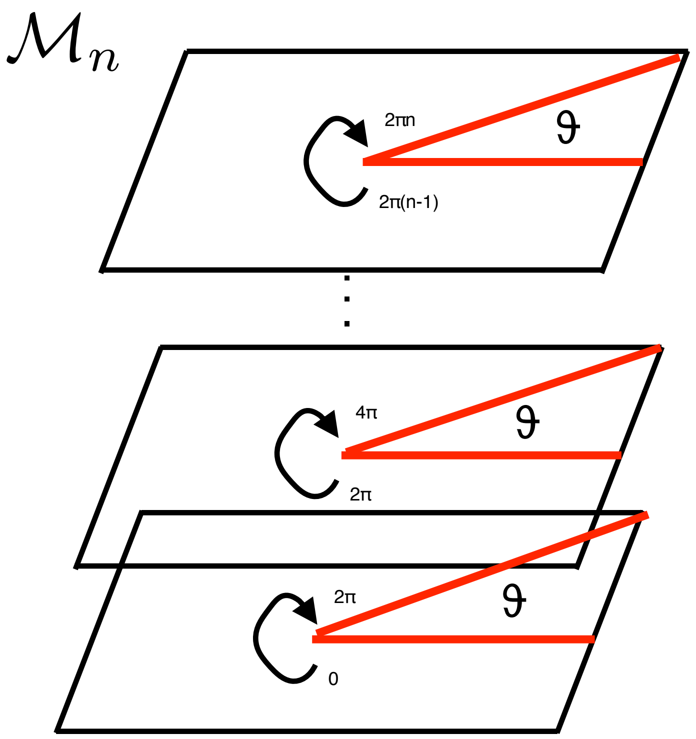

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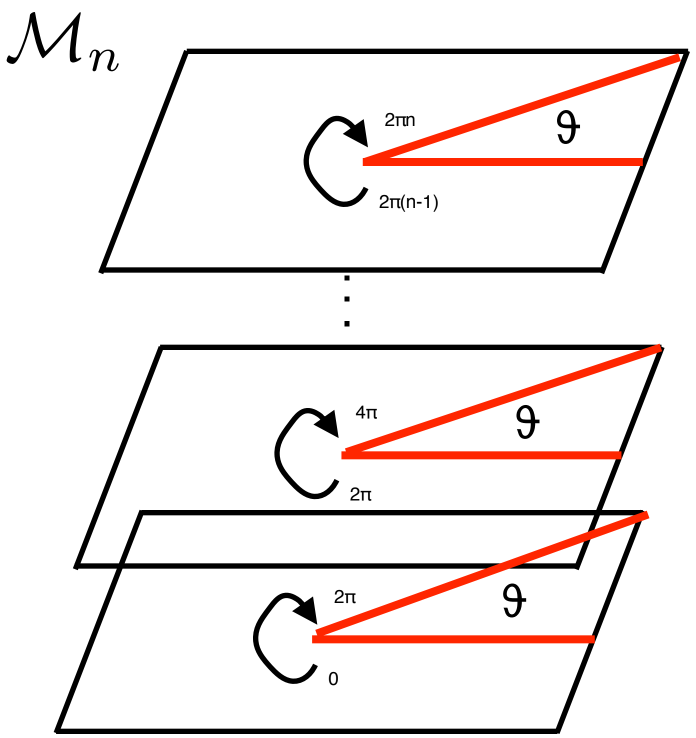

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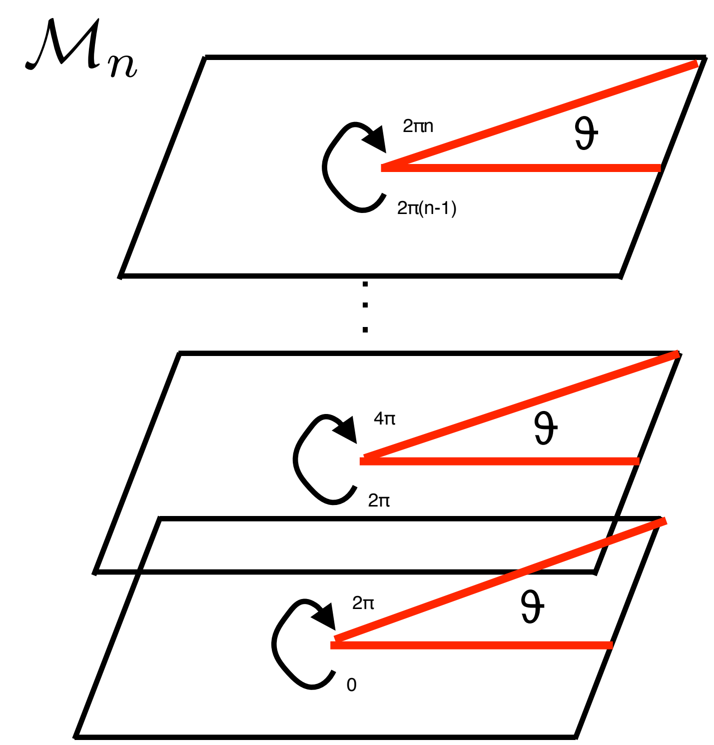

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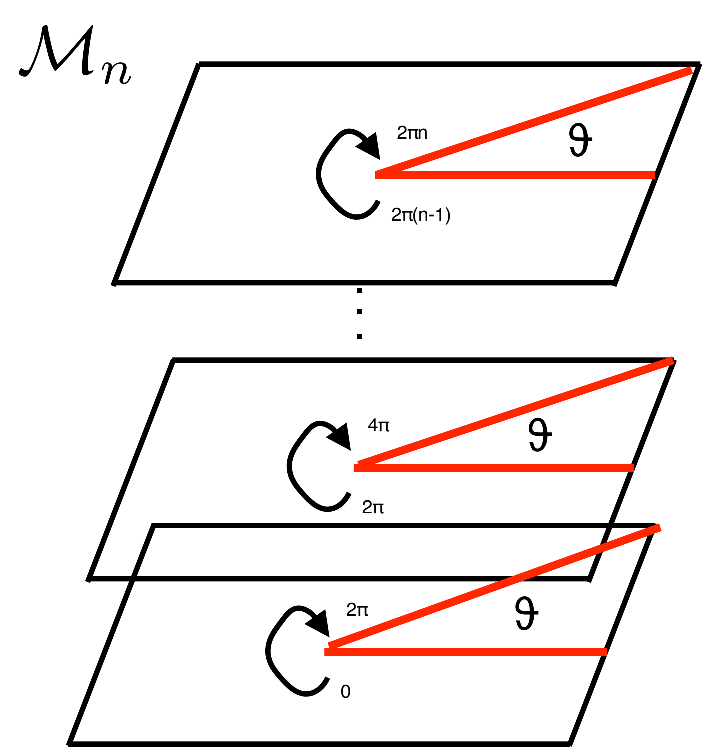

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Acting with a coordinate transformation on W gives the derivative of the stress tensor

$$\delta_\xi W_n = - \int d^d x \sqrt{g}\, \xi_\mu \partial_\nu T^{\mu\nu}$$

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In the presence of anomalies, the stress tensor is not conserved,

$$\partial_\nu T^{\mu\nu} = \tau^\mu$$

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The form of τ is completely fixed by the Wess-Zumino consistency conditions

e.g., in 2d:

$$\tau^\nu = -c_g g^{\mu\nu} \frac{1}{\sqrt{g}} \partial_\lambda \left(\sqrt{g}\epsilon^{\alpha\beta} \partial_\alpha \Gamma^\lambda{}_{\nu\beta} \right)$$

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Thus,

$$\partial_\theta S_A \big|_{\theta=0} = -\int d^d x \sqrt{g} \tau^\theta$$

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In 4 dimensions there isn't a gravitational anomaly, but there is a mixed gauge-gravitational anomaly.

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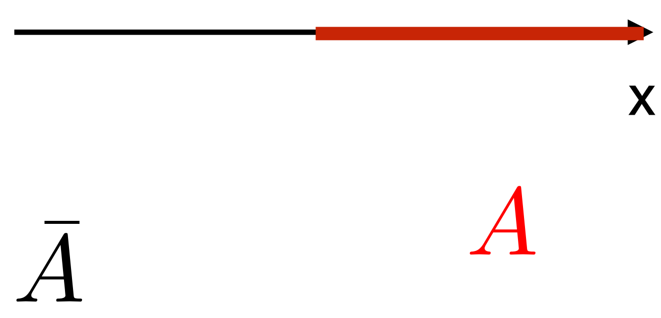

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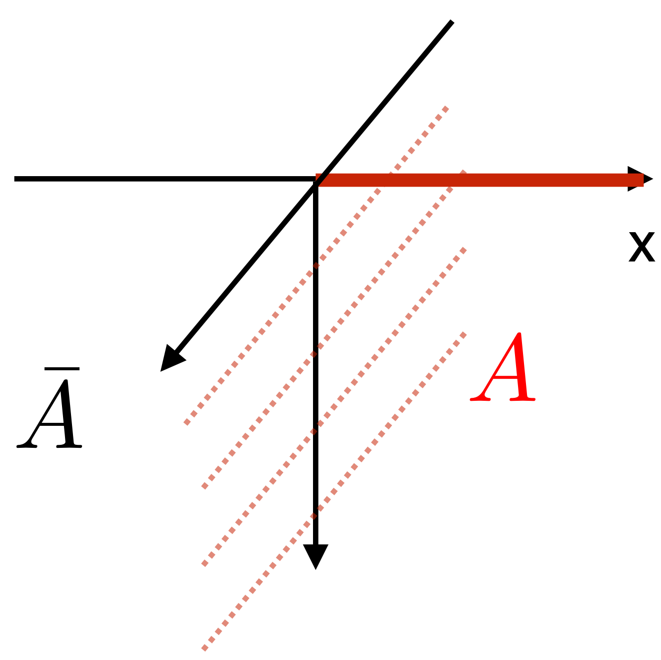

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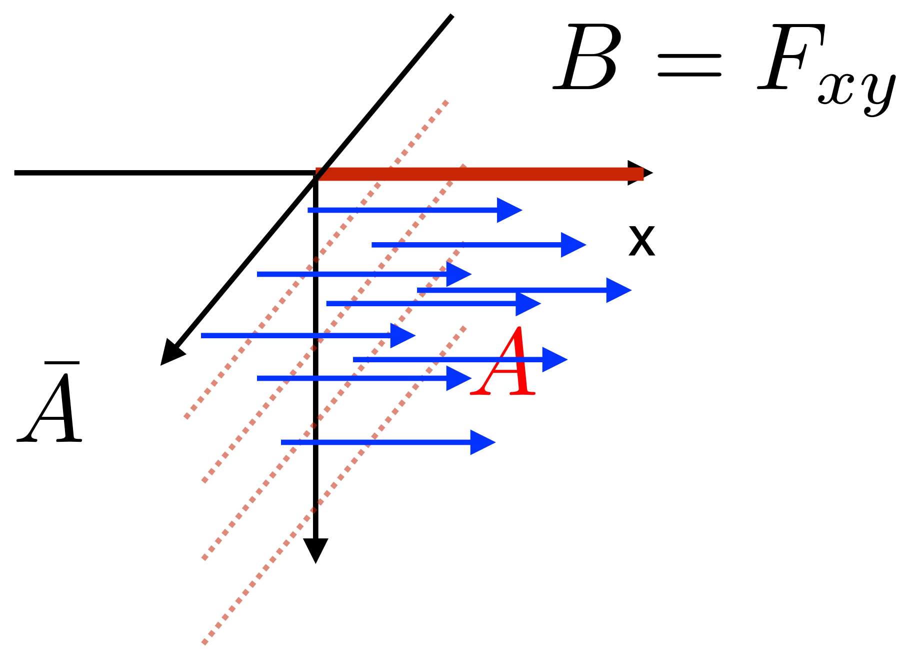

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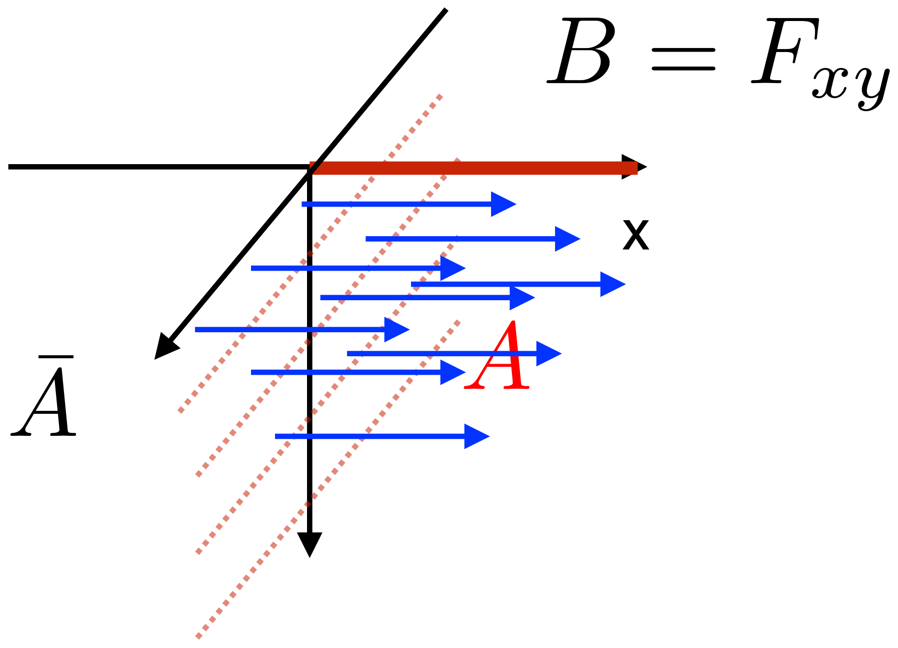

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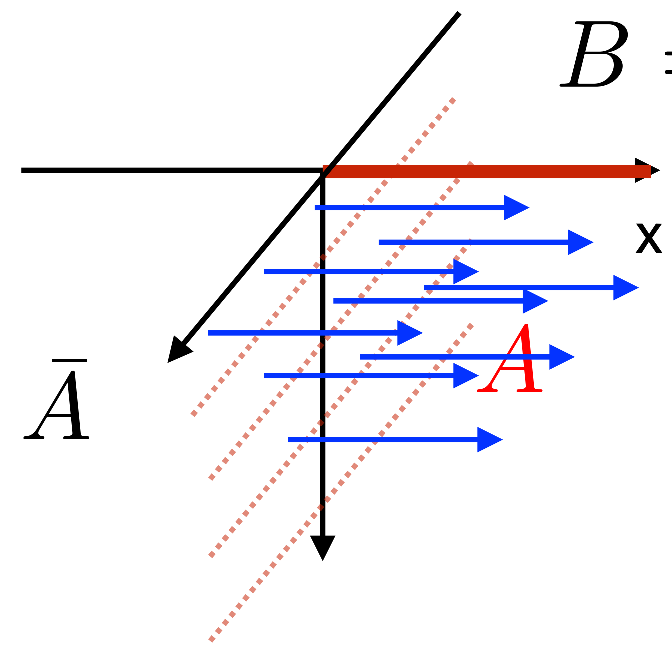

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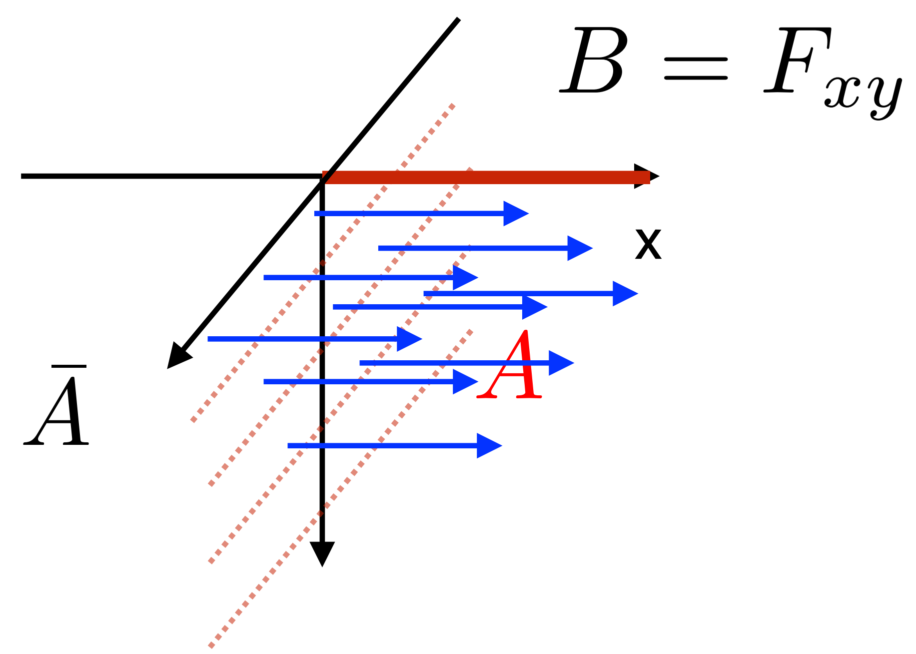

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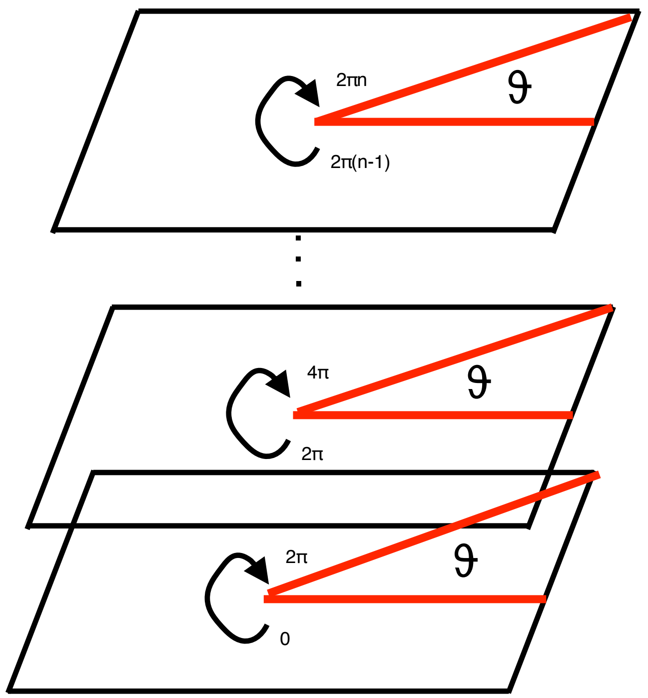

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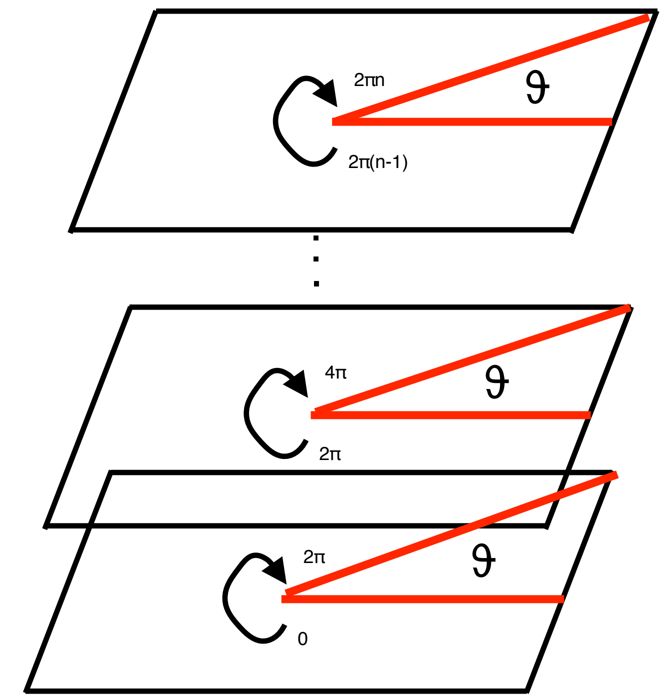

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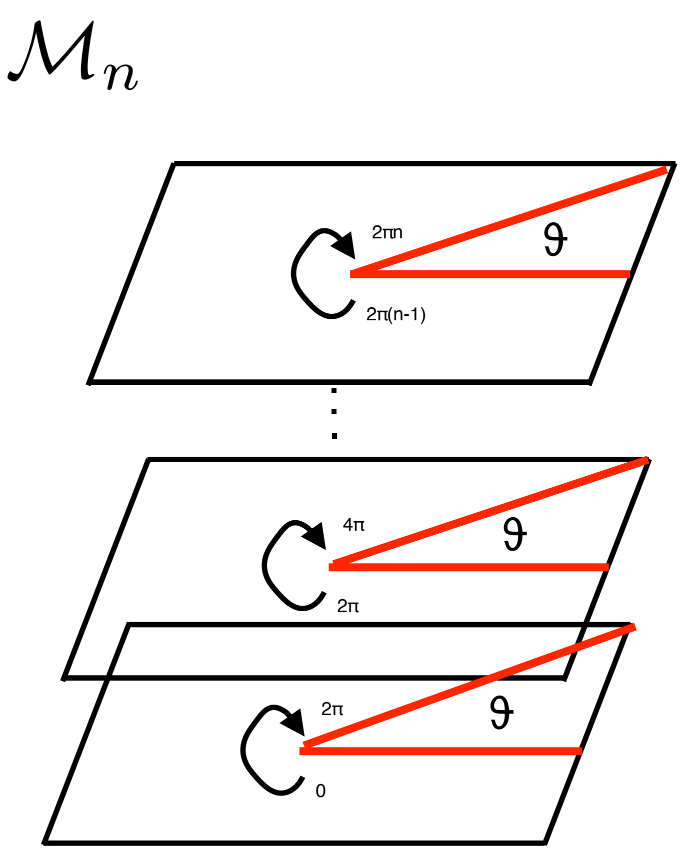

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Thank you

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