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Stringy $\mathbf{N} = 1$ Super no Scale Models

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Based on:

Costas Kounnas and Hervé Partouche, in print and
Alon Faraggi, Costas Kounnas and Hervé Partouche arXiv:1409.7076 [hep-th]

1. Introduction

Superstring theory is the only known theory where all known interactions, including quantum gravity, are consistently unified.

Thirty years of effort led to several $\mathbf{N} = 1$ string constructions having:
i) quasi-realistic spectrum with a net number of three chiral families,
ii) containing the standard model interactions at low energies.

Among them, are those obtained in 2d-fermionic construction with an initial $SO(10)$ gauge symmetry which is broken at the string level by discrete Wilson lines to the Pati-Salam gauge group, $G_{\rm PS} = SU(4) \times SU(2)_L \times SU(2)_R$, and which is further broken to the standard model gauge group, $G_{\rm SM} = SU(3) \times SU(2)_L \times U(1)_Y$, by a conventional Higgs mechanism.
Antoniadis Bachas Kounnas, Kawai, Lewellen Tye '86, Antoniadis, Leontaris Rizos '10, Faraggi Kounnas Rizos '07 +collaboratos '10-'11, Ibanez Kim Nilles Quevedo '87, Lebedev Nilles Raby Ramos-Sanchez Ratz Vaudrevange Wingerter '05

All of the quasi-realistic string models that have been constructed to date possess an $\mathbf{N} = 1$ space time supersymmetry, and the question is how this symmetry is broken and what are the implication to the cosmological term.

The proposed SUSY breaking mechanisms are either perturbative or non-perturbative. All proposed susy breakings give rise to an effective no-scale $\mathbf{N} = 1$ supergravity with zero cosmological term at the classical level. Ferrara Kounnas Zwirner '94

$$V_{eff} = 0, \quad m^2_{3/2} = \frac{w_0^2}{z_1 z_2 z_3}, \qquad \{z_A, A = 1,2,3\} \subset \{S, T_I, U_I, \; I = 1,2,3\} \; :$$

• A non-perturbative breaking via gaugino condensation is qualitatively described by an effective no-scale supergravity. The string predictability is partially lost having only an effective parametrization of the susy breaking parameters proportional to the gaugino condensation scale. $\{z_A, A = 1,2,3\} = \{T_I, \; I = 1,2,3\}, \quad w_0^2 = \frac{\Lambda^3}{M_P}$

Nilles, Ferrara Girardello Nilles '83, Derendinger, Ibáñez Nilles '85, Dine Rohm Seiberg Witten '85, Kounnas Porrati '87, Derendinger Kounnas Petropoulos '06

• Perturbative and/or non-perturbative flux compactifications, where internal fluxes are introduced and break susy suitably and using the non-perturbative S, T, U-dualities between the heterotic, Type IIA, Type IIB and orientifold superstring vacua.
Here also the string predictability of the effective no-scale supergravity is partially lost in most of the cases. $\{z_A\} = \{T_I\}$ either $\{z_A\} = \{U_I\}$ or $\{z_A\} = \{S, T_1, U_1\}$ or ...

• An interesting example of geometrical fluxes is the one associated to a Stringy Scherk-Schwarz susy breaking, having the advantage to be implemented at the perturbative string level. The SUSY breaking parameters are obtained directly from the perturbative string theory with: Kounnas Porrati '87, Ferrara Kounnas Porrati '88 + Zwirner '89, Kounnas Kiritsis '95

$$m^2_{\frac{3}{2}} \sim \left(\frac{M_P}{ST_1U_1}\right)^2 = \left(\frac{M_{Str}}{R_1}\right)^2 = \mathcal{O}\left(1 - 10\ TeV\right)^2 \quad \longrightarrow \quad R_1 \sim \mathcal{O}(10^{13})$$

R_1 is a typical radius of the internal compact coordinates. $\longrightarrow$ Low energy susy breaking implies that some of the internal dimensions are extremely large.

This problem is known as the *decompactification problem.*
Kounnas '90, Antoniadis '91, Kounnas Kiritsis '96 +Petropoulos Rizos Gregory '96-
The large internal dimensions give rise to tower of states, charged under low-energy gauge groups, that populate the energy range between the susy breaking scale and the Plank scale. They induce large threshold corrections, that contribute to the running of the gauge couplings, Yukawa couplings and soft susy breaking parameters.

In field theory the *decompactification problem* is avoided in non-chiral models with spontaneously broken $\mathbf{N} = 4 \to \mathbf{N} = 0$. The thresholds of different spin states cancel among each other at large energies due to the restoration of the $\mathbf{N} = 4$ supersymmetry at large energy scales $Q \gg m_{3/2}$:

$$\frac{16\,\pi^2}{g_i^2(Q)} = k^i \frac{16\,\pi^2}{g_{\rm s}^2} - \sum b^i_{Bos} \log\left(\frac{Q^2}{Q^2 + M^2_{Bos}}\right) - \sum b^i_{Fer} \log\left(\frac{Q^2}{Q^2 + M^2_{Fer}}\right)$$

$$\sum b^i_{Bos} = -\sum b^i_{Fer} = -\frac{8}{3} C(\mathcal{A}^i)$$

There is no way to obtain in field theory framework chiral models based on $\mathbf{N} = 4$ theories with spontaneously broken susy.

In string theories however this may be possible thanks to "miraculous" organization of the string spectrum in sectors with different amount of supersymmeties and net number of chiral states implying fermion-boson mass degeneracy even in susy broken phases.

This reorganization of the mass spectrum is related to the properties of stringy sub-sectors with spontaneously broken non-allgned $\mathbf{N} = 2$ and $\mathbf{N} = 4$ supersymmetries.

In the $N = 1$ orbifolds models, with broken susy à la Stringy Scherk-Schwarz, the partition function contains extra $\mathbf{N} = 0$ sector. However, thanks to the organization of stringy states only few relevant sectors survive in the large radius limit, $R \gg 1$:

(i) one $\mathbf{N} = 4$ susy sector with $V_{eff} = 0$

(ii) one $\mathbf{N} = 4 \to \mathbf{N} = 0$ sector with $V_{eff} = -\xi(2 + d_G - n_F) m_{3/2}^4$

(iii) one, two, three or even four non aligned $\mathbf{N} = 2$ susy sectors with $V_{eff} = 0$

(iv) several sectors $\mathbf{N} = 2, 1 \to \mathbf{N} = 0$ with $V_{eff} = m_{3/2}^2 \, M_{str}^2 \; \mathcal{O}\left(e^{-cR}\right)$

Thanks to the stringy reorganization of the spectrum the expected field theory terms $m_{3/2}^2 \, M_{str}^2$ turns out to be exponentially suppressed!!

The leading terms scale like $m_{3/2}^4$!!

2. $\mathbb{Z}_2\times\mathbb{Z}_2$ fermionic models with perturbative broken susy

The above extraordinary properties concerning V_{eff} can be derived from the explicit expressions of the generic form of the partition function of $Z_2 \times Z_2$ moduli deformed fermionic models where $\mathbf{N} = 1 \to \mathbf{N} = 0$ spontaneously broken susy "a la Stringy Scherk-Schwarz".

$$Z(v,\bar{v}^i) = \frac{1}{\tau_2|\eta|^4}\,\frac{1}{2}\sum_{a,b}\,\frac{1}{4}\sum_{H_I,G_I}(-)^{a+b+ab}\,\frac{\theta[^a_b](2v)}{\eta}\,\frac{\theta[^{a+H_2}_{b+G_2}]}{\eta}\,\frac{\theta[^{a+H_1}_{b+G_1}]}{\eta}\,\frac{\theta[^{a+H_3}_{b+G_3}]}{\eta}\;\times$$

$$\frac{1}{2^N}\sum_{h^i_I,g^i_I} S\begin{bmatrix} a, h^i_I, H_I \\ b, g^i_I, G_I\end{bmatrix} Z_{2,2}\begin{bmatrix} h^i_1 \\ g^i_1\end{bmatrix}\!\!\begin{vmatrix} H_2 \\ G_2\end{vmatrix}\; Z_{2,2}\begin{bmatrix} h^i_2 \\ g^i_2\end{bmatrix}\!\!\begin{vmatrix} H_1 \\ G_1\end{vmatrix}\; Z_{2,2}\begin{bmatrix} h^i_3 \\ g^i_3\end{bmatrix}\!\!\begin{vmatrix} H_3 \\ G_3\end{vmatrix}\; Z_{0,16}(\bar{v}^i)\begin{bmatrix} h^i_I, H_I \\ g^i_I, G_I\end{bmatrix}$$

- The contribution of the six internal coordinates, shifted by (h^i_I, g^i_I) and twisted by (H_I, G_I) is given in terms of three $Z_{2,2}\begin{bmatrix} h^i_I \\ g^i_I\end{bmatrix}\!\!\begin{vmatrix} H_I \\ G_I\end{vmatrix}$ conformal blocks.

$$Z_{2,2}\left[{}^{h^1,\,h^2}_{g^1,\,g^2}\Big|{}^{H}_{G}\right] = \frac{\Gamma_{2,2}\left[{}^{h^1,\,h^2}_{g^1,\,g^2}\right](T,U)}{(\eta\bar\eta)^2}\,, \qquad \text{when} \quad (H,G)=(0,0)\,,$$

$$Z_{2,2}\left[{}^{h^1,\,h^2}_{g^1,\,g^2}\Big|{}^{H}_{G}\right] = \frac{4\,\eta\bar\eta}{\theta\left[{}^{1-H}_{1-G}\right]\bar\theta\left[{}^{1-H}_{1-G}\right]}\ \delta_{0,(Gh^{1,2}-Hg^{1,2})}\,, \qquad \text{when} \quad (H,G)\neq(0,0)\,,$$

- $Z_{0,16}\left[{}^{h^i_I,\ H_I}_{g^i_I,\ G_i}\right]$ denotes the contribution of the 32 extra right-moving worldsheet fermions.

- The contribution of the left-moving 2d-fermionic superpartners twisted by (H_I, G_I)

$$\frac{1}{2}\sum_{a,b}\ (-)^{a+b+ab}\,\frac{\theta\left[{}^a_b\right](2v)}{\eta}\,\frac{\theta\left[{}^{a+H_2}_{b+G_2}\right]}{\eta}\,\frac{\theta\left[{}^{a+H_1}_{b+G_1}\right]}{\eta}\,\frac{\theta\left[{}^{a+H_3}_{b+G_3}\right]}{\eta}$$

will play the most important role in the sector by sector decomposition of the theory.

• $S\begin{bmatrix} a, h_I^i, H_I \\ b, g_I^i, G_I \end{bmatrix}$ is a phase that implement the breaking of the $\mathbf{N}=1 \to \mathbf{N}=0$.

When $S\left[{}^{a,\, h_I^i,\, H_I}_{b,\, g_I^i,\, G_I}\right] \equiv 1$, the theory is $\mathbf{N}=1$ supersymmetric.
The supersymmetry can be broken spontaneously "à la Stringy Scherk-Schwarz" once the 10-dimensional helicity characters (R-parity charges)

$$\binom{a}{b}\,, \quad \binom{a+H_1}{b+G_1}\,, \quad \binom{a+H_2}{b+G_2}\,, \quad \binom{a+H_3}{b+G_3}$$

are correlated with the lattice charges, $i.e.$with some shift pairs (h_I^i, g_I^i).
Kounnas Porrati '87, Ferrara Kounnas Porrati '88 + Zwirner '89, Kounnas Kiritsis '95
• In the absence of twists and shifts $Z_{0,16}$ is either the partition function associated to the $E_8 \times E_8$ or $SO(32)$. The non-trivial shifts and twists generate non-zero discrete and continuous Wilson lines which break the initial gauge group to lower subgroups.

• The choice of shifts and twists must be such that the gauge group contains an $SO(10)$ chiral factor, which is further broken to a subgroup that contains the desired standard model gauge group coupled to acceptable particle content in three generations.

The genus-1 effective potential is nothing but the integrated partition function:

$$V_{\text{eff}} = -\frac{1}{(2\pi)^4}\int_{\mathcal{F}}\frac{d^2\tau}{2\tau_2^2}\, Z(v,\bar{v}^i)|_{v=0,\bar{v}^i=0}$$

The gauge coupling corrections Δ^i:

$$\frac{16\,\pi^2}{g_i^2(Q)} = k^i\frac{16\,\pi^2}{g_s^2} + \Delta^i$$

$$\Delta^i = \int_{\mathcal{F}_{\text{reg}}}\frac{d^2\tau}{\tau_2}\left(\frac{1}{2}\sum_{a,b}\mathcal{Q}_v\left(\mathcal{P}_{\bar{v}^i}^2 - \frac{k^i}{4\pi\tau_2}\right)\tau_2 Z(v,\bar{v}^i)\right)_{v=0,\bar{v}^i=0}$$

By insertion of : The helicity operator $\mathcal{Q}_v$, acting as derivative operator $\mathcal{Q}_v \sim \frac{\partial^2}{\partial v^2} \sim \frac{\partial}{\partial \tau}$ on the fermionic left-moving sector.

The modular covariant $(\text{charge})^2$ gauge operator, $\mathcal{P}_{\bar{v}^i}^2 \sim \frac{\partial^2}{\partial \bar{v}^{i\,2}} \sim \frac{\partial}{\partial \bar{\tau}}$ acting as derivative on the right-moving sector.

In a recent work by Faraggi, Kounnas and Partouche, arXiv:1409.7076 [hep-th] a solution to the decompactification problem was provided in $\mathbf{N} = 0$ theories where the three planes are treated asymmetrically: (0,1,2)-chirality per plane.

- In 1^{st} non-chiral plane, the $\mathbb{Z}_2$ twist is freely acting and is correlated with a shift to the 1^{st} $\Gamma_{2,2}(T_1, U_1)$ lattice even in the absence of the SSS-breaking phase S.
This ensures that the breaking of $\mathbf{N} = 4 \rightarrow \mathbf{N}_C = 2$ "C-sector" is spontaneous.

- A second shift of the lattice in the 1^{st} plane is associated to the SSS spontaneous breaking of supersymmetry which is also freely acting.
$\mathbf{N} = 4 \ \rightarrow \ \mathbf{N} = 0$ "B-sector",
AND
$\mathbf{N} = 4 \rightarrow \mathbf{N}_D = 2$ "D-sector" (when the twist is equal to the lattice shift).

There no-fixed points and neither chiral representations coming from the 1^{st} plane in both $\mathbf{N} = 1$ and $\mathbf{N} = 0$ realizations of the theory.

• In the 2^{nd} and 3^{rd} plane the coordinates of the 1^{st} plane, associated to $\Gamma_{2,2}(T_1, U_1)$ lattice, are twisted and therefore the correlation of the SSS phase is inactive. This implies that the 2^{nd} and 3^{rd} planes are effectively similar to that of $\mathbf{N} = 1$ susy theories with two non-aligned $\mathbf{N}_{2,3} = 2$ supersymmeries giving rise in certain cases to $\mathbf{N} = 0$ susy with chiral spectrum.

• It is also of main importance that the moduli $T_{2,3}, U_{2,3}$ has to be closed to their self-dual points because of the requirement of the decompactification problem.

• In a very recent work (in print) by Kounnas and Partouche, we claim that the most natural and *dynamically stable* values of the moduli $T_{2,3}, U_{2,3}$ are those described by the fermionic points. More precisely the point where the initial $\mathbf{N} = 4$ right-moving gauge symmetry is extended to $SO(4) \times SO(4)$. At this point big surprise accrue. The one loop cosmological term is exponentially suppressed in all sectors and looks like a miracle! Is that an accident or there is a more profound reason?

3. The stringy STU-Super-no-Scale model

The only substantial contribution to V_{eff} comes from the "B-sector" where the "mother" susy theory $\mathbf{N}=4 \to \mathbf{N}=0$ is spontaneously broken.

All other sectors are either $N=2$ supersymmetric:
$N_{C,D}=2$, $N_{2,3}=2$ giving vanishing contribution to V_{eff}
or
from the $\mathbf{N}=1,0$ sectors ("E, F-sectors") with exponentially suppressed contribution in the large radius limit, ${V_{eff}}_{E,F}=\mathcal{O}(e^{-cR})$.

It is therefore sufficient to examine the contribution of the "mother" $\mathbf{N}=4$

$$(\mathbf{N}=4, G_{\mathrm{Right}}) \qquad \longrightarrow \qquad (\mathbf{N}=0, \hat{G}_{\mathrm{Right}})$$

$$G_{\mathrm{Right}}=U(1)^2\times SO(4)^2\times E_8^2 \quad \longrightarrow \quad \hat{G}_{\mathrm{Right}}=U(1)^2\times SO(4)^2\times SO(16)^2$$

In the absence of the SSS-breaking phase the "mother" $N = 4$ partition function takes a factorized form written in terms of left- and right-moving characters:

$$Z_{\mathbf{N}=4} = O^0_{x^\mu}\, O^1_{22}\, O^2_{22}\, O^3_{22} \{V_8 - S_8\}\, \{O_{16} + C_{16}\}\, \{O'_{16} + C'_{16}\},$$

• $O^0_{x^\mu}$ denotes the contribution of the $(2_L, 2_R)$ light-cone space-time coordinates.

• O^I_{22}, $I = 1, 2, 3$ stands for the contribution of the six internal coordinates

$$O^0_{x^\mu} = \frac{\mathrm{Im}\tau^{-1}}{\eta^2 \bar{\eta}^2}, \qquad O^I_{22} = \frac{\Gamma^I_{22}(T_I, U_I)}{\eta^2 \bar{\eta}^2}\,.$$

• V_8 and S_8 are characters of $SO(8)$ constructed with the two (light-cone) Ψ^μ left-moving superpartners of x^μ and the six internal left-moving $\chi^i = 1, \ldots, 6$ left-moving super-partners of the six internal compact coordinates:

$$V_8 = \frac{\theta_3^4 - \theta_4^4}{2\eta^4}, \quad S_8 = \frac{\theta_2^4 - \theta_1^4}{2\eta^4}, \quad O_8 = \frac{\theta_3^4 + \theta_4^4}{2\eta^4}, \quad C_8 = \frac{\theta_2^4 + \theta_1^4}{2\eta^4}\,.$$

• O_{16}, C_{16} and O'_{16}, C'_{16} are the characters of $SO(16)$ and $SO'(16)$ constructed with the $16+16'$ extra right moving fermions

$$V_{16} = \frac{\bar\theta_3^8 - \bar\theta_4^8}{2\bar\eta^4}, \qquad S_{16} = \frac{\bar\theta_2^8 - \bar\theta_1^8}{2\bar\eta^4}, \qquad O_{16} = \frac{\bar\theta_3^8 + \bar\theta_4^8}{2\bar\eta^4}, \qquad C_{16} = \frac{\bar\theta_2^8 + \bar\theta_1^8}{2\bar\eta^4}\,.$$

Similarly for the V'_{16}, S'_{16}, O'_{16}, C'_{16} of the $SO'(16)$.

The combinations,

$$O(E_8) \equiv \{\, O_{16} + C_{16} \,\} = \frac{1}{2}\sum_{\gamma,\delta} \frac{\theta[^\gamma_\delta]^8}{\eta^8}, \quad O(E'_8) \equiv \{\, O'_{16} + C'_{16} \,\} = \frac{1}{2}\sum_{\gamma',\delta'} \frac{\theta[^{\gamma'}_{\delta'}]^8}{\eta^8}$$

are the E_8 (E'_8) characters written in terms of the $SO(16)$ ($SO'(16)$) ones.

There are several ways to realize the breaking of $\mathbf{N}=4 \rightarrow \mathbf{N}=0$
depending of the choice os the SSS-breaking phase.

The miracles appear when the SSS-breaking susy phase is chosen to be:

$$S = e^{i\pi\left[\, g(a+\gamma+\gamma')+h(b+\delta+\delta')+gh \,\right]}$$

correlating the left-moving helicity characters (a, b), the spinor characters (γ, δ), (γ', δ') of the right-moving gauge groups E_8 and E_8' with the lattice shifts (h, g) of the 1^{st} complex plane $\Gamma \left[{h \atop g}\right] (T_1, U_1)$.

The $\{a + \gamma + \gamma'\}$-breaking generalizes the so called "temperature-like" breaking where

$$S = e^{i\pi(ga+hb+hg)}$$

acts only on the left-moving helicity charges giving susy breaking masses to all space-time fermions while keeping massless the space-time bosons.

Contrary, the $\{a + \gamma + \gamma'\}$-breaking give non-zero masses to both bosons and fermions as soon as $\{a + \gamma + \gamma'\}$ =1 mod 2.

The susy is spontaneously broken in both $\{a+\gamma+\gamma'\}$-breaking and $\{a\}$-breaking. The genus-1 effective potential is given in terms of the volume, T_1 and complex structure U_1 moduli of the 1^{st} plane in terms of the "shifted real analytic Eisenstein series" .

$$V_{\text{eff}} = -\frac{8(2+d_G-n_F)}{16\pi^7}\frac{1}{(\text{Im}T)^2}\,E_{(g_1,g_2)}(U_1|\,3) + \mathcal{O}\left(e^{-c\sqrt{\text{Im}T_1}}\right)$$

$$E_{(g_1,g_2)}(U|\,s) = \sum_{\tilde{m}_1,\tilde{m}_2}{}' \frac{(\text{Im}U)^s}{|\tilde{m}_1+\frac{g_1}{2}+(\tilde{m}_2+\frac{g_2}{2})U|^{2s}}\,.$$

or

$$V_{\text{eff}} = -\xi(U_1)\;8(2+d_G-n_F)\,m_{3/2}^4 + \mathcal{O}\left(e^{-c\sqrt{\text{Im}T_1}}\right)$$

In the "temperature like breaking" $n_F=0$ since all ferrmionic modes become massive.

In the $\{a+\gamma+\gamma'\}$-breaking however n_F is not trivial which would compensate the bosonic contribution in the effective potential once $\underline{2+d_G-n_F=0}$.

The vanishing of V_{eff} is happening when the four internal coordinates of the 2^{nd} and 3^{rd} plane are at the fermionic points with an enhanced gauge group $SO(4) \times SO(4)$.

Our interest is for large T_1, (small $m_{3/2}$). However it is equally important to examine the behavior of the model for any value of T_1. This would be relevant for a dynamical determination of the susy breaking scale towards to hierarchically small values compare to the string scale.

For generic values of the $(T_2, U_2; T_3, U_3)$-moduli of the 2^{nd} and 3^{rd} complex plane, the unbroken gauge group is $\hat{G}_{\rm Right} = SO(16) \times SO(16)' \times U(1)^2 \times H_R$ with $H_R = U(1)^4$.

For special values of the moduli H_R can be extended to a non-abelian gauge group of rank-4 like for instance $H_R = SO(4)^2$.
(This is the most natural extension of the $\mathbb{Z}_2 \times \mathbb{Z}_2$ fermionic models.)

The partition function is tachyon free for any value of the radius R and for any H_R !

$$Z_{\mathbf{N}=0}^{(a+\gamma+\gamma')} = O^0_{x^\mu}\, O^2_{22}\;\, O^3_{22}\times$$

$$\{\; O^1_{22}\left[{0 \atop 0}\right]\{V_8\,(O_{16}\,O'_{16} + C_{16}\,C'_{16}) - S_8(O_{16}\,C'_{16}\; + C_{16}O'_{16})\} +$$

$$O^1_{22}\left[{0 \atop 1}\right]\{V_8\,(O_{16}\,C'_{16}\; + C_{16}\,O'_{16}) - S_8(O_{16}\,O'_{16} + C_{16}\,C'_{16})\}+$$

$$O^1_{22}\left[{1 \atop 0}\right]\{O_8\,(\,V_{16}\,S'_{16} + S_{16}\,V'_{16}\,) - C_8(\,V_{16}\,V'_{16}\; + S_{16}\,S'_{16}\,)\}+$$

$$O^1_{22}\left[{1 \atop 1}\right]\{O_8\,(\,V_{16}\,V'_{16}\; + S_{16}\,S'_{16}\,) - C_8\,(\,V_{16}\,S'_{16} + S_{16}\,V'_{16}\,)\}\;\;\}.$$

$$O_{22}\left[{0 \atop 0}\right] = \frac{\Gamma_{22}\left[{0 \atop 0}\right] + \Gamma_{22}\left[{0 \atop 1}\right]}{2\eta^2\bar{\eta}^2} \;:\quad \left[{n \atop m}\right] = \left[{\text{even} \atop \text{even}}\right]\,, \qquad O_{22}\left[{0 \atop 1}\right] = \frac{\Gamma_{22}\left[{0 \atop 0}\right] - \Gamma_{22}\left[{0 \atop 1}\right]}{2\eta^2\bar{\eta}^2} \;:\quad \left[{n \atop m}\right] = \left[{\text{even} \atop \text{odd}}\right]\,,$$

$$O_{22}\left[{1 \atop 0}\right] = \frac{\Gamma_{22}\left[{1 \atop 0}\right] + \Gamma_{22}\left[{1 \atop 1}\right]}{2\eta^2\bar{\eta}^2} \;:\quad \left[{n \atop m}\right] = \left[{\text{odd} \atop \text{even}}\right]\,, \qquad O_{22}\left[{1 \atop 1}\right] = \frac{\Gamma_{22}\left[{1 \atop 0}\right] - \Gamma_{22}\left[{1 \atop 1}\right]}{2\eta^2\bar{\eta}^2} \;:\quad \left[{n \atop m}\right] = \left[{\text{odd} \atop \text{odd}}\right]\,.$$

$O^1_{22}[{}^{\alpha}_{\beta}]$ are the characters of the shifted (2,2)-lattice associated to the 1^{st} complex plane:

For comparison reasons the partition function of the Temperature-like breaking is:

$$Z^{(\mathrm{a})}_{\mathbf{N}=0} = O^0_{x^\mu}\, O^2_{22}\; O^3_{22}\; (O_{16}+C_{16})\,(O'_{16}+C'_{16}) \times$$

$$\{\, O^1_{22}[{}^0_0]\, V_8 - O^1_{22}\,[{}^0_1]\, S_8 - O^1_{22}[{}^1_0]\, C_8 + O^1_{22}[{}^1_1]\, O_8 \,\}\,.$$

which become tachyonic in the range $\frac{\sqrt{3}-1}{2} < R_1 < \frac{\sqrt{3}+1}{2}$

The non-supersymmetric theory based on $Z^{(a+\gamma+\gamma')}_{\mathbf{N}=0}$ has several *remarkable properties*:

• T**-duality**, $\frac{R}{R_0} \leftrightarrow \frac{R_0}{R}$ which is translated to a **dual susy breaking :**

$$m^2_{\frac{3}{2}} = \frac{M_s^2}{2}\,\frac{R_0^2}{R^2} \qquad \longleftrightarrow \qquad m^2_{\frac{3}{2}} = \frac{M_s^2}{2}\,\frac{R^2}{R_0^2}\,, \qquad \text{with} \qquad R_0 = \frac{1}{\sqrt{2}}\,.$$

- $Z^{(a+\gamma+\gamma')}_{\mathbf{N}=0}$ **is tachyon free** in the whole range of $0 < R < \infty$.

* **Massless fermions**. They appear in $\{S_8\,(O_{16}\,C'_{16} + C_{16}\,O'_{16})\}$:

$$d(\text{Fermions}) = d(V_8)\ (d(C_{16}) + d(C'_{16})) = 8 \times (2 \times 2^7) = \underline{8 \times 16 \times 16}\,.$$

Their multiplicity is independent of H_R and is unchanged for any R.

* **Massive bosons with light mass** $\sim m_{\frac{3}{2}}$, $(R >> R_0)$.

They appear in $\{O^0_{x^\mu}\,O^2_{22}\ O^3_{22}\,O^1_{22}[^0_1]V_8\,(O_{16}\,C'_{16}\ + C_{16}\,O'_{16})\}$

Their number is independent of H_R like the massless fermions.

There is a tower of states given by $O^1_{22}[^0_1]$ with a multiplicity

$$d(\text{Bosons}; m_{\frac{3}{2}}) = d(V_8)\ (d(S_{16}) + d(S'_{16})) = 8 \times (2 \times 2^7) = \underline{8 \times 16 \times 16}\,.$$

* **Massless bosons**. They appear in $\{O^0_{x^\mu}\,O^2_{22}\ O^3_{22}\,O^1_{22}[^0_0]\,V_8\,O_{16}\,O'_{16}\}$:

$$d(\text{Bosons}) = d(V_8)\ \left(d(O^0_{x^\mu}) + d(O^1_{22}[^0_0]) + d(O^2_{22}) + d(O^3_{22}) + d(O_{16}) + d(O'_{16})\right)$$
$$= 8 \times (2 + 2 + d(H_R) + 8 \times 15 + 8 \times 15) = \underline{8 \times 16 \times 16} + \underline{8(d(H_R) - 12)}\,.$$

Their multiplicity is equal to the massless fermions when $H_R = SO(4)^2$!

*** Massive fermions with light mass** $\sim m_{\frac{3}{2}}$ **$(R >> R_0)$** .

They appear in $\left\{O^0_{x^\mu}\, O^2_{22}\ O^3_{22}\, O^1_{22}\left[{0 \atop 0}\right] V_8\, O_{16}\, O'_{16}\right\}$, in the same representations as the massless bosons. Their multiplicity depends on H_R

$$d(\text{Fermions}; m_{\frac{3}{2}}) = \underline{8 \times 16 \times 16} + \underline{8(d(H_R) - 12)}\,.$$

Their multiplicity is also equal to the massless bosons when $H_R = SO(4)^2$!

**** Extra massless bosons at the self-dual radius** $R = R_0$.

They appear in $\left\{O^1_{22}\left[{1 \atop 1}\right] O_8\, V_{16}\, V'_{16}\ \right\}$. These extra states become massless only at the self-dual fermionic point with $P_L^2 = \frac{1}{2}$ and $P_R^2 = 0$. with multiplicity,

$$d(\text{Bosons};\ \text{massless at } R = R_0) = \underline{2 \times 16 \times 16}\,.$$

3. A. Magic degeneracies in weighted supertraces

The bosons-fermion degeneracies appearing at the massless and at the light-masse level are surprising. They imply the suppression of V_{eff} beyond the $\mathrm{O}(m_{\frac{3}{2}}^4)$ which is naturally expected in spontaneously broken $\mathbf{N} = 4$ supersymmetry, based on the helicity supertraces arguments :

$$\mathrm{Str}\ M^n = \mathrm{Tr}(-)^F M^n = 0 \text{ for } \ n < \mathbf{N}, \quad \mathrm{Tr}(-)^F M^n = c\, m_{\frac{3}{2}}^{\mathbf{N}} \ \text{ for } \ n = \mathbf{N} = 4\,.$$

In the "temperature like" breaking the effective potential is proportional to:

$$V_{\text{eff}} = -c\, \mathrm{Tr}(-)^F M^4 = -\xi(U)\, 8(2 + d_G)$$

as expected.

In more involved susy breaking the relevant quantity appearing in V_{eff} generalizes the naive Str formula. Actually, in the case of $\{a + \gamma + \gamma'\}$-breaking V_{eff} becomes proportional to a *weighted* Str :

$$V_{\rm eff} = -c\,{\rm Tr}\,(-)^{(F+Q+Q')}\ M^4\,,$$

where Q and Q' are odd for the $\frac{1}{2}$-integer representations, (spinor or anti-spinor) of $SO(16)$ and $SO(16)'$ respectively.

$$V_{\rm eff} = -c\,{\rm Tr}\,(-)^{(F+Q+Q')}\ M^4\ = -\xi(U)\,8\,(\,d(H_R) - 12\,)\,m^4_{\frac{3}{2}}\,.$$

$H_R = SO(4)^2$ is very special since it give rise to a zero weighted-trace $(d(H_R) = 12)$.

When H_R is broken by moving T_I, U_I away from their fermionic point, V_{eff} becomes positive which indicates that the fermionic point is an attractor at least in the large radius limit.

The $\mathbf{N} = 0$ model under investigation, must not be confused with the ten-dimensional tachyon-free $SO(16) \times SO(16)'$ proposed in the early literature.
Alvarez-Gaumé Ginsparg Moore Vafa '86, Dixon Harvey '86

The $SO(16) \times SO(16)'$ model considered here is defined only in space-time dimensions lower or equal to 5.

The reasons are simple but of main impotence:

(i) the necessity of non-trivial extended gauge group request T_2, U_2, T_3, U_3 to be closed to their self-dual fermionic points.

(ii) at least one extra compact dimension is necessary to correlate in a non-triviall way the R-charges to the lattice charges breaking spontaneously the supersymmetry.

(iii) In $\mathbb{Z}_2 \times \mathbb{Z}_2$ orbifolds with a solution to the decompactification problem the maximal space-time dimension is 4 while

$$V_{eff}\,|_{\mathbb{Z}_2\times\mathbb{Z}_2} = \frac{1}{4}\, V_{eff}^{(a+\gamma+\gamma')}$$

since all other sectors give vanishing or exponentially suppressed contribution !

3.B. Gauge coupling corrections

The gauge coupling corrections can be expressed in terms of the energy scale Q, and the string scale $M_{\rm s}$ and the characteristic scales appearing in the individual sectors:

$$\frac{1}{M_B^2} = \frac{1}{M_{\rm s}^2}\,|\theta_4(U)|^4 {\rm Im}T_1{\rm ImU}_1 \quad \text{“}N=0, \quad B-\text{sector”}$$

$$\frac{1}{M_C^2} = \frac{1}{M_{\rm s}^2}\,|\theta_2(U)|^4 {\rm Im}T_1{\rm ImU}_1 \quad \text{“}N_C=2, \quad C-\text{sector”},$$

$$\frac{1}{M_D^2} = \frac{1}{M_{\rm s}^2}\,|\theta_3(U)|^4 {\rm Im}T_1{\rm ImU}_1 \quad \text{“}N_D=2, \quad D-\text{sector”},$$

$$\frac{1}{M_2^2} = \frac{16}{M_s^2}\,|\eta(T_I)|^4\,|\eta(U_I)|^4 {\rm Im}T_2{\rm Im}U_2\,, \quad \text{“}N=2, \quad 2^{nd}\text{ complex plane”},$$

$$\frac{1}{M_2^2} = \frac{16}{M_s^2}\,|\eta(T_I)|^4\,|\eta(U_I)|^4 {\rm Im}T_3{\rm Im}U_2\,, \quad \text{“}N=2, \quad 3^{rd}\text{ complex plane”}.$$

Since T_2, U_2, T_3, U_3 are closed to their self dual fermionic point $M_{1,2} = \mathcal{O}(M_{str})$

In terms of Q, M_B, M_C, M_D and M_1, M_2, the gauge couplings at genus-1 are:

$$\frac{16\,\pi^2}{g_i^2(Q)} = k^i \frac{16\,\pi^2}{g_{\rm s}^2}$$

$$-\frac{1}{4} b_B^i \log\left(\frac{Q^2}{Q^2 + M_B^2}\right) - \frac{1}{4} b_C^i \log\left(\frac{Q^2}{Q^2 + M_C^2}\right) - \frac{1}{4} b_D^i \log\left(\frac{Q^2}{Q^2 + M_D^2}\right)$$

$$-\frac{1}{2} b_2^i \log\left(\frac{Q^2}{M_2^2}\right) - \frac{1}{2} b_3^i \log\left(\frac{Q^2}{M_3^2}\right)$$

The sector by sector β-coefficients are :

$$b^i_B = -\frac{8}{3}\left\{C((\mathcal{A}_{\mathcal{B}}) - \mathcal{C}(\mathcal{R}_{\mathcal{B}})\right\},$$

$$b^i_C = -2\left\{C((\mathcal{A}_{\mathcal{C}}) - \mathcal{C}(\mathcal{R}_{\mathcal{C}})\right\},$$

$$b^i_D = -2\left\{C((\mathcal{A}_{\mathcal{D}}) - \mathcal{C}(\mathcal{R}_{\mathcal{D}})\right\},$$

$$b^i_2 = -2\left\{C(\mathcal{A}_2) - C(\mathcal{R}_2)\right\},$$

$$b^i_3 = -2\left\{C(\mathcal{A}_3) - C(\mathcal{R}_3)\right\}.$$

The β-function coefficient of the $\mathbf{N} = 1 \rightarrow \mathbf{N} = 0$ model, for Q smaller than all threshold scales is:

$$b^i = \frac{1}{4}(b^i_B + b^i_C + b^i_D) + \frac{1}{2}(b^i_2 + b^i_3)$$

4. Conclusions

• In the context of $\mathbb{Z}_2 \times \mathbb{Z}_2$ moduli deformed $\mathbf{N} = 1$ fermionic models we implemented a low scale supersymmetry breaking while maintaining the validity of gauge coupling perturbation theory.

• The moduli of the 2nd and 3rd plane do not participate in the susy breaking.

• From the effective field theory view point, the SSS-breaking gives rise to a specific $\mathbf{N} = 1$ supergravity no-scale model the so called "ST_1U_1"- no-scale model. FKZ '94

• We develop a sector by sector analysis of the $\mathbf{N} = 0$ models and analyze systematically the associated induced threshold corrections. There are five relevant sectors:

- The sector B, describes the $\mathbf{N} = 0$ broken phase of the $\mathbf{N} = 4$ initial spectrum. It is the only non-supersymmetric sector that is relevant for the gauge couplings and effective potential corrections. It is solely responsible for the cosmological term $\mathcal{O}(m_{\frac{3}{2}})^4$.

- The sectors C and D are both sub-sectors of the non-chiral $1^{\rm st}$ complex plane, preserving $\mathbf{N}_C = 2$ and $\mathbf{N}_D = 2$ susy, respectively.

- The $2^{\rm nd}$ and $3^{\rm nd}$ chiral complex planes preserve $\mathbf{N}_2 = 2$ and $\mathbf{N}_3 = 2$ non-aligned supersymmetries.

We derived the complete dependance of the effective gauge couplings in terms of the energy scale Q, up to $M_{\rm s}$, including when Q crosses the thresholds scales $M_{B,C,D}$.

- Our results, take a universal form depending only on the β-function coefficients of the five relevant sectors.

- It is remarkable that in the breaking we considered here the $m_{\frac{3}{2}}^2\, M_s^2$ term are absent thanks to the underlying $\mathbf{N} = 4 \to \mathbf{N} = 0$ supersymmetry imposing that the genus-1 effective potential scales like $m_{\frac{3}{2}}^4$.

• There are even ST_1U_1 -SUPER NO SCALE models where both $m_{\frac{3}{2}}^2 M_{str}^2$ and $m_{\frac{3}{2}}^4$-terms are absent at genus-1.

$$V_{eff} = -c\mathrm{Tr}(-)^{(F+Q+Q')}M^4 = -\xi(U_1)\,2\,(d(H_{R,B}) - 12)\,m_{3/2,B}^4$$

$V_{eff} = 0$ if $d(H_{R,B}) = 12$ indicating the dynamical determination of $H_{R,B} \to SO(4)^2$.

• For other type of chiral $\mathbf{N} = 0$ String constructions can be found in a recent paper by: S. Abel, K. Dienes, E. Mavroudi '15 arXiv:1502.03087 [hep-th]
"Towards a Non-Supersymmetric String Phenomenology".
Those construction however suffer generically from the decompactification problem which still remains an open problem. !

Thank you for your attention

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