Double Field Theory and Stringy Geometry

EIGHTH CRETE REGIONAL MEETING IN STRING THEORY

Double Field Theory

Hull & Zwiebach

- From sector of String Field Theory. Features some stringy physics, including T-duality, in simpler setting
- Strings see a doubled space-time
- Necessary consequence of string theory
- Needed for non-geometric backgrounds
- What is geometry and physics of doubled space?

Strings on a Torus

- States: momentum p, winding w
- String: Infinite set of fields $\psi(p, w)$
- Fourier transform to doubled space: $\psi(x, \tilde{x})$
- "Double Field Theory" from closed string field theory. Some non-locality in doubled space
- Subsector? e.g. $g_{ij}(x, \tilde{x}),\ b_{ij}(x, \tilde{x}),\ \phi(x, \tilde{x})$

Double Field Theory

- Double field theory on doubled torus
- General solution of string theory: involves doubled fields $\psi(x, \tilde{x})$
- *Real* dependence on *full* doubled geometry, dual dimensions not auxiliary or gauge artifact. Double geom. *physical* and *dynamical*
- *Strong constraint* restricts to subsector in which extra coordinates auxiliary: get conventional field theory locally. Recover Siegel's duality covariant formulation of (super)gravity

Strings on T^d

$$X = X_L(\sigma + \tau) + X_R(\sigma - \tau), \qquad \tilde{X} = X_L - X_R$$

X conjugate to momentum, $\tilde{X}$ to winding no.

$$dX = *d\tilde{X} \qquad \partial_a X = \epsilon_{ab}\partial^b \tilde{X}$$

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Need "auxiliary" $\tilde{X}$ for interacting theory

i) Vertex operators $e^{ik_L \cdot X_L}, \quad e^{ik_R \cdot X_R}$

ii) String field Kugo & Zwiebach $\Phi[x, \tilde{x}, a, \tilde{a}]$

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X conjugate to momentum, $\tilde{X}$ to winding no.

$$dX = *d\tilde{X} \qquad \partial_a X = \epsilon_{ab}\partial^b \tilde{X}$$

Strings on torus see **DOUBLED GEOMETRY**!

T-duality group $O(d,d;\mathbb{Z})$

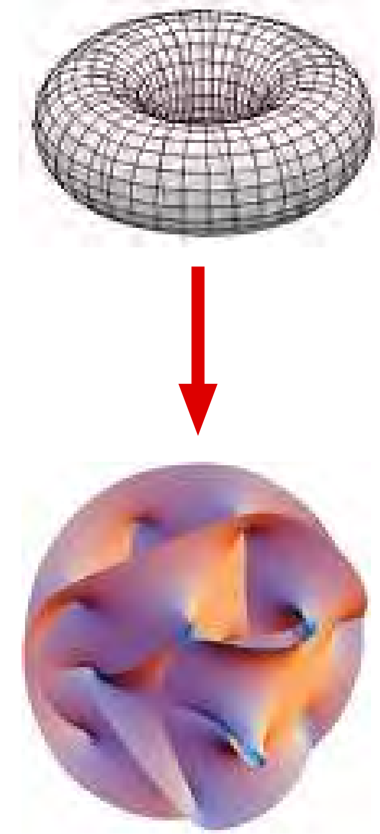
Doubled Torus 2d coordinates

Transform linearly under $O(d,d;\mathbb{Z})$

$$X \equiv \begin{pmatrix} \tilde{x}_i \\ x^i \end{pmatrix}$$

Sigma model on doubled torus Tseytlin; Hull

T-Duality



- Space has d-torus fibration
- G,B on fibres
- T-Duality O(d,d;Z), mixes G,B
- Mixes Momentum and Winding
- Changes geometry and topology

$$E \to (aE+b)(cE+d)^{-1}$$

$$h = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \in O(d,d;Z) \qquad E_{ij} = G_{ij} + B_{ij}$$

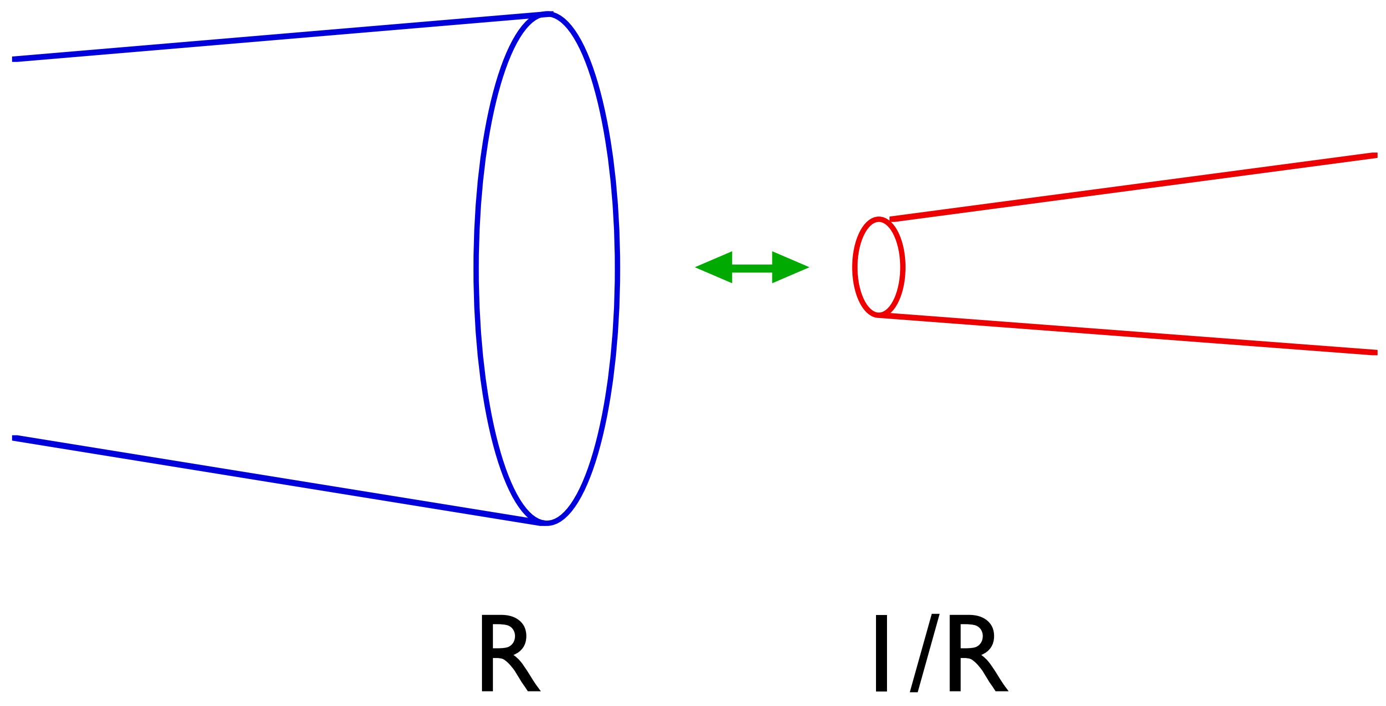
On circle, radius R: $\quad O(1,1;\mathbb{Z}) = \mathbb{Z}_2 : R \mapsto \dfrac{1}{R}$

T-Folds

Hull 2004

- Spacetime constructed from local patches
- All symmetries of physics used in patching
- Patching with diffeomorphisms, gives manifold
- Patching with gauge symmetries: bundles
- String theory has new symmetries, not present in field theory. New non-geometric string backgrounds e.g. for torus fibrations
- Patching with T-duality: **T-FOLDS**
- Patching with U-duality: **U-FOLDS**

T-fold patching

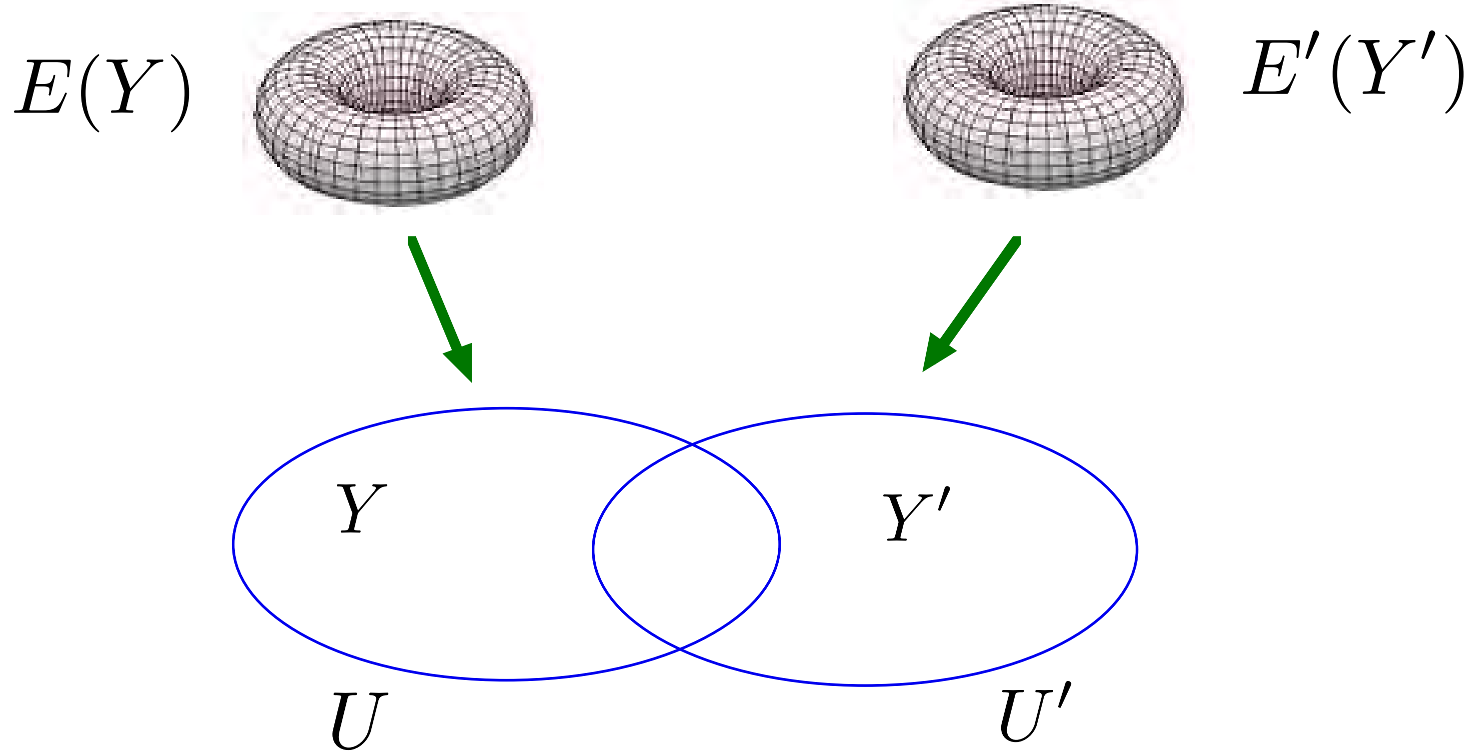


Glue big circle (R) to small (1/R)

Glue momentum modes to winding modes

(or linear combination of momentum and winding)

Not conventional smooth geometry



Torus fibration

Geometric background: G, H=dB tensorial

T-fold: Transition functions involve T-dualities (as well as diffeomorphisms and 2-form gauge transformations)

E=G+B Non-tensorial

$O(d,d;\mathbb{Z})$ $\qquad E' = (aE+b)(cE+d)^{-1} \;\; \text{in} \;\; U \cap U'$

Glue using T-dualities also → **T-fold**

Physics smooth, as T-duality a symmetry

Not conventional smooth geometry

Doubled Geometry for T-fold

Hull

T^d torus fibres have doubled coords $\mathbb{X}^I = \begin{pmatrix} X^i \\ \widetilde{X}_i \end{pmatrix}$ $I = 1, ..., 2d$

Transforms linearly under $O(d,d;\mathbb{Z})$
T-fold transition: mixes $X, \tilde{X}$
No global way of separating "real" space coordinate X from "auxiliary" $\widetilde{X}$

Duality covariant formulation in terms of $\mathbb{X}$
Transition functions $O(d,d;\mathbb{Z}) \subset GL(2d;\mathbb{Z})$
can be used to construct bundle with fibres T^{2d}

Doubled space is smooth manifold!

Sigma Model on doubled space. T-duality manifest.

- More general non-geometric backgrounds. Gives uplift of GENERIC gauged Sugras

Dabholkar & Hull 2005 Shelton, Taylor & Wecht 2005

- Explicit doubled geometries constructed for T-folds and "spaces with R-flux"

Hull & Reid-Edwards 2008-9

- Sigma models on doubled spaces; constraints from gauging. Quantisation.

Hull 2004-6

- Other approaches to quantisation

Tseytlin; Berman, Thompson, Copland; Hackett-Jones & Motsopoulos
Lust et al; Bakas & Lust,....

String Field Theory on Minkowski Space

Closed SFT: Zwiebach

String field $\qquad \Phi[X(\sigma), c(\sigma)]$

$$X^i(\sigma) \to x^i, \text{ oscillators}$$

Expand to get infinite set of fields

$$g_{ij}(x), b_{ij}(x), \phi(x), \ldots, C_{ijk\ldots l}(x), \ldots$$

Integrating out massive fields gives field theory for

$$g_{ij}(x), b_{ij}(x), \phi(x)$$

String Field Theory on a torus

String field $$\Phi[X(\sigma), c(\sigma)]$$

$$X^i(\sigma) \to x^i, \tilde{x}_i, \text{ oscillators}$$

Expand to get infinite set of double fields

$$g_{ij}(x,\tilde{x}), b_{ij}(x,\tilde{x}), \phi(x,\tilde{x}), \ldots, C_{ijk\ldots l}(x,\tilde{x}), \ldots$$

Seek **double field theory** for

$$g_{ij}(x,\tilde{x}), b_{ij}(x,\tilde{x}), \phi(x,\tilde{x})$$

Free Field Equations (B=0)

$$L_0 + \bar{L}_0 = 2$$

$$p^2 + w^2 = N + \bar{N} - 2$$

$$L_0 - \bar{L}_0 = 0$$

$$p_i w^i = N - \bar{N}$$

Free Field Equations (B=0)

$$L_0 + \bar{L}_0 = 2$$

$$p^2 + w^2 = N + \bar{N} - 2$$

Treat as field equation, kinetic operator in doubled space

$$G^{ij} \frac{\partial^2}{\partial x^i \partial x^j} + G_{ij} \frac{\partial^2}{\partial \tilde{x}_i \partial \tilde{x}_j}$$

$$L_0 - \bar{L}_0 = 0$$

$$p_i w^i = N - \bar{N}$$

Treat as constraint on double fields

$$\Delta \equiv \frac{\partial^2}{\partial x^i \partial \tilde{x}_i} \qquad (\Delta - \mu)\psi = 0$$

Free Field Equations (B=0)

$$L_0 + \bar{L}_0 = 2$$

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Treat as field equation, kinetic operator in doubled space

$$G^{ij} \frac{\partial^2}{\partial x^i \partial x^j} + G_{ij} \frac{\partial^2}{\partial \tilde{x}_i \partial \tilde{x}_j}$$

Laplacian for metric

$$ds^2 = G_{ij} dx^i dx^j + G^{ij} d\tilde{x}_i d\tilde{x}_j$$

$$L_0 - \bar{L}_0 = 0$$

$$p_i w^i = N - \bar{N}$$

Treat as constraint on double fields

$$\Delta \equiv \frac{\partial^2}{\partial x^i \partial \tilde{x}_i} \qquad (\Delta - \mu)\psi = 0$$

Laplacian for metric

$$ds^2 = dx^i d\tilde{x}_i$$

$$g_{ij}(x,\tilde{x}), b_{ij}(x,\tilde{x}), \phi(x,\tilde{x})$$

$$N = \bar{N} = 1$$

$$p^2 + w^2 = 0$$

$$p \cdot w = 0$$

"Double Massless"

DFT gives O(D,D) covariant formulation

O(D,D) Covariant Notation

$$X^M \equiv \begin{pmatrix} \tilde{x}_i \\ x^i \end{pmatrix} \qquad \partial_M \equiv \begin{pmatrix} \tilde{\partial}^i \\ \partial_i \end{pmatrix}$$

$$\eta_{MN} = \begin{pmatrix} 0 & I \\ I & 0 \end{pmatrix} \qquad M = 1, ..., 2D$$

$$\Delta \equiv \frac{\partial^2}{\partial x^i \partial \tilde{x}_i} = \frac{1}{2}\partial^M \partial_M$$

Constraint $\partial^M \partial_M A = 0$

on all fields and parameters

Weak Constraint or weak section condition

Arises from string theory constraint

$$(L_0 - \bar{L}_0)\Psi = 0$$

- Weakly constrained DFT non-local. Constructed to cubic order Hull & Zwiebach
- ALL doubled geometry dynamical, evolution in all doubled dimensions
- Restrict to simpler theory: STRONG CONSTRAINT
- Fields then depend on only half the doubled coordinates
- Locally, just conventional SUGRA written in duality symmetric form

Strong Constraint for DFT   Hohm, H &Z

$$\partial^M \partial_M (AB) = 0 \qquad\qquad (\partial^M A)\,(\partial_M B) = 0$$

on all fields and parameters

If impose this, then it implies weak form, but product of constrained fields satisfies constraint.

This gives **Restricted DFT,** a subtheory of DFT

Locally, it implies fields only depend on at most half of the coordinates, fields are restricted to null subspace N.
Looks like conventional field theory on subspace N

- If fields supported only on submanifold N of doubled space M, recover Siegel's duality covariant form of (super)gravity on N
- In general get this only locally. In each 2D-dim patch of doubled space, fields supported on D-dim sub-patch, but sub-patches don't fit together to form a manifold with smooth fields.
- DFT 'background independent' HHZ. Can write on doubling of any space. What is double if not derived from string theory?
- Extension to WZW models   Blumenhagen, Hassler & Lust

Generalised T-duality transformations: HHZ

$$X'^M \equiv \begin{pmatrix} \tilde{x}'_i \\ x'^i \end{pmatrix} = hX^M = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} \tilde{x}_i \\ x^i \end{pmatrix}$$

h in O(d,d;Z) acts on toroidal coordinates only

$$\mathcal{E}_{ij} = g_{ij} + b_{ij}$$

$$\mathcal{E}'(X') = (a\mathcal{E}(X) + b)(c\mathcal{E}(X) + d)^{-1}$$

$$d'(X') = d(X)$$

Buscher if fields independent of toroidal coordinates
Generalisation to case without isometries

$$X^M = \begin{pmatrix} \tilde{x}_m \\ x^m \end{pmatrix} \qquad \xi^M = \begin{pmatrix} \tilde{\epsilon}_m \\ \epsilon^m \end{pmatrix}$$

Linearised Gauge Transformations

$$\delta h_{ij} = \quad \partial_i \epsilon_j + \partial_j \epsilon_i \; + \; \tilde{\partial}_i \tilde{\epsilon}_j + \tilde{\partial}_j \tilde{\epsilon}_i \, ,$$

$$\delta b_{ij} = -(\tilde{\partial}_i \epsilon_j - \tilde{\partial}_j \epsilon_i) - (\partial_i \tilde{\epsilon}_j - \partial_j \tilde{\epsilon}_i) \, ,$$

$$\delta d \; = -\, \partial \cdot \epsilon + \tilde{\partial} \cdot \tilde{\epsilon} \, .$$

Invariance needs constraint

Diffeos and B-field transformations mixed.

If fields indep of $\tilde{x}_m$, conventional theory $g_{ij}(x), b_{ij}(x), d(x)$

ϵ^m parameter for diffeomorphisms

$\tilde{\epsilon}_m$ parameter for B-field gauge transformations

Generalised Metric Formulation

Hohm, H &Z

$$\mathcal{H}_{MN} = \begin{pmatrix} g^{ij} & -g^{ik}b_{kj} \\ b_{ik}g^{kj} & g_{ij} - b_{ik}g^{kl}b_{lj} \end{pmatrix} .$$

2 Metrics on double space $\quad \mathcal{H}_{MN}, \ \eta_{MN}$

$$\mathcal{H}^{MN} \equiv \eta^{MP}\mathcal{H}_{PQ}\eta^{QN}$$

Constrained metric $\quad \mathcal{H}^{MP}\mathcal{H}_{PN} = \delta^M{}_N$

Generalised Metric Formulation

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$$\mathcal{H}_{MN} = \begin{pmatrix} g^{ij} & -g^{ik}b_{kj} \\ b_{ik}g^{kj} & g_{ij} - b_{ik}g^{kl}b_{lj} \end{pmatrix} .$$

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Covariant O(D,D) Transformation

$$h^{P}{}_{M}h^{Q}{}_{N}\mathcal{H}'_{PQ}(X') = \mathcal{H}_{MN}(X)$$

$$X' = hX \qquad\qquad h \in O(D,D)$$

O(D,D) covariant action

$$S = \int dx d\tilde{x}\, e^{-2d}\ L$$

$$L = \frac{1}{8}\mathcal{H}^{MN}\partial_M\mathcal{H}^{KL}\,\partial_N\mathcal{H}_{KL} - \frac{1}{2}\mathcal{H}^{MN}\partial_N\mathcal{H}^{KL}\,\partial_L\mathcal{H}_{MK}$$
$$- 2\,\partial_M d\,\partial_N\mathcal{H}^{MN} + 4\mathcal{H}^{MN}\,\partial_M d\,\partial_N d$$

Gauge Transformation

$$\delta_\xi\mathcal{H}^{MN} = \xi^P\partial_P\mathcal{H}^{MN}$$
$$+ (\partial^M\xi_P - \partial_P\xi^M)\,\mathcal{H}^{PN} + (\partial^N\xi_P - \partial_P\xi^N)\,\mathcal{H}^{MP}$$

Write as "Generalised Lie Derivative"

$$\delta_\xi\mathcal{H}^{MN} = \widehat{\mathcal{L}}_\xi\mathcal{H}^{MN}$$

Generalised Lie Derivative

$$A_{N_1\ldots}^{M_1\ldots}$$

$$\widehat{\mathcal{L}}_\xi A_M{}^N \equiv \xi^P \partial_P A_M{}^N$$
$$+(\partial_M \xi^P - \partial^P \xi_M) A_P{}^N + (\partial^N \xi_P - \partial_P \xi^N) A_M{}^P$$

Usual Lie derivative, plus terms involving η_{MN}

$$\widehat{\mathcal{L}}_\xi A_M{}^N = \mathcal{L}_\xi A_M{}^N$$
$$- \eta^{PQ} \eta_{MR}\, \partial_Q \xi^R\, A_P{}^N$$
$$+ \eta_{PQ} \eta^{NR}\, \partial_R \xi^Q\, A_M{}^P$$

Strong Constraint: Gauge symm ~ diffeos and b-field trans

O(D,D) $X' = hX$

Symmetry for flat doubled space $M = \mathbb{R}^{2D}$

B-shifts and $GL(D, \mathbb{R})$ arise from local symmetries.

Isometries: if fields indep of some coords, more of O(D,D) can arise from local symmetries HHZ

Torus spacetime $N = \mathbb{R}^{n-1,1} \times T^d$ $M = \mathbb{R}^{2n-2,2} \times T^{2d}$

O(D,D) broken to subgroup containing B-shifts and

$$O(n,n) \times O(d,d;\mathbb{Z})$$

General Spacetime: No natural action of O(D,D)

DFT geometry

arXiv:1406.7794

- Simple explicit form of finite gauge transformations. Associative and commuative.
- Doubled space is a manifold, not flat, despite constant ‘metric’ η in DFT.
- Gives geometric understanding of ‘generalised tensors’ & relation to generalised geometry
- Transition functions give global picture
- T-folds: non-geometric backgrounds included

What is the Geometry of Generalised Tensors?

Doubled space coordinates $$X^M = \begin{pmatrix} x^m \\ \tilde{x}_m \end{pmatrix}$$

O(D,D) covariant vectors and tensors

$$V^M = \begin{pmatrix} v^m \\ \tilde{v}_m \end{pmatrix} \qquad \mathcal{H}_{MN}$$

Suggestive of tensors on doubled space, but transformations not those of diffeomorphisms on doubled space, as generated by generalised Lie derivative, not usual Lie derivative.

If not tensors on doubled space, what are they?

Constraint $$\partial^M \partial_M A = 0$$

Strong Constraint for restricted DFT

$$\partial^M \partial_M (AB) = 0 \qquad\qquad (\partial^M A)\,(\partial_M B) = 0$$

Generic solution in patch $\hat{U}$: fields and parameters independent of half the coordinates:

$$\tilde{\partial}^i = 0$$

$$X^M = \begin{pmatrix} x^m \\ \tilde{x}_m \end{pmatrix} \qquad \partial_M = \begin{pmatrix} \partial_m \\ \tilde{\partial}^m \end{pmatrix} \qquad \eta_{MN} = \begin{pmatrix} 0 & I \\ I & 0 \end{pmatrix}$$

Fields live on null patch U, coordinates x: $\phi(x^m)$

U ‘physical’ spacetime

Vectors

$$V^M = \begin{pmatrix} v^m \\ \tilde{v}_m \end{pmatrix}$$

Generalised Lie derivative

$$\widehat{\mathcal{L}}_V W^M \ = \ V^P \partial_P W^M + W^P \left(\partial^M V_P - \partial_P V^M \right)$$

Vectors

$$V^M = \begin{pmatrix} v^m \\ \tilde{v}_m \end{pmatrix}$$

Generalised Lie derivative

$$\widehat{\mathcal{L}}_V W^M \;=\; V^P \partial_P W^M + W^P \left(\partial^M V_P - \partial_P V^M\right)$$

has the components

$$(\widehat{\mathcal{L}}_V W)^m = \mathcal{L}_v w^m$$

$$(\widehat{\mathcal{L}}_V W)_m = \mathcal{L}_v \tilde{w}_m + w^p(\partial_m \tilde{v}_p - \partial_p \tilde{v}_m)$$

$\mathcal{L}_v$ is usual Lie derivative

$$\mathcal{L}_v w^m = v^p \partial_p w^m - w^p \partial_p v^m$$

$$\mathcal{L}_v \tilde{w}_m = v^p \partial_p \tilde{w}_m + \tilde{w}_p \partial_m v^p$$

Under infinitesimal transformation $\delta W^M = \widehat{\mathcal{L}}_V W^M$

$$\delta w^m = \mathcal{L}_v w^m$$
$$\delta \tilde{w}_m = \mathcal{L}_v \tilde{w}_m + w^p(\partial_m \tilde{v}_p - \partial_p \tilde{v}_m)$$

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Introduce a gerbe connection b with transformations

$$\delta_v b_{mn} = \mathcal{L}_v b_{mn} + \partial_m \tilde{v}_n - \partial_n \tilde{v}_m$$

Define $\hat{w}_m = \tilde{w}_m - b_{mn} w^n$

Under infinitesimal transformation $\delta W^M = \widehat{\mathcal{L}}_V W^M$

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Define $\hat{w}_m = \tilde{w}_m - b_{mn} w^n$

Then $\delta \hat{w}_m = \mathcal{L}_v \hat{w}_m$

Under infinitesimal transformation $\delta W^M = \widehat{\mathcal{L}}_V W^M$

$$\delta w^m = \mathcal{L}_v w^m$$

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Introduce a gerbe connection b with transformations

$$\delta_v b_{mn} = \mathcal{L}_v b_{mn} + \partial_m \tilde{v}_n - \partial_n \tilde{v}_m$$

Define $\hat{w}_m = \tilde{w}_m - b_{mn} w^n$

Then $\delta \hat{w}_m = \mathcal{L}_v \hat{w}_m$

$\hat{w}$ transforms as 1-form under v-transformations and is invariant under $\tilde{v}$ transformations!

COVARIANT TRANSFORMATIONS

Then given $$W^M = \begin{pmatrix} w^m \\ \tilde{w}_m \end{pmatrix}$$

can define $$\hat{W}^M = \begin{pmatrix} w^m \\ \hat{w}_m \end{pmatrix} = \begin{pmatrix} w^m \\ \tilde{w}_m - b_{mn} w^n \end{pmatrix}$$

$$\delta \hat{W}^M = \mathcal{L}_v \hat{W}^M$$

It is invariant under $\tilde{v}$ transformations

COVARIANT TRANSFORMATIONS

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$$\delta \hat{W}^M = \mathcal{L}_v \hat{W}^M$$

It is invariant under $\tilde{v}$ transformations

Gives finite transformations!

$$x \to x'(x) = e^{-v^m \partial_m} x$$

$$w'^m(x') = w^n(x) \frac{\partial x'^m}{\partial x^n} \qquad \hat{w}'_m(x') = \hat{w}_n(x) \frac{\partial x^n}{\partial x'^m}$$

Can also find the transformation of $\tilde{w}$

Standard finite transformations of gerbe connection:

$$b'_{mn}(x') = [b_{pq}(x) + (\partial_p \tilde{v}_q - \partial_q \tilde{v}_p)(x)] \frac{\partial x^p}{\partial x'^m} \frac{\partial x^q}{\partial x'^n}$$

gives

$$\tilde{w}'_m(x') = \left[\tilde{w}_n(x) + (\partial_n \tilde{v}_q - \partial_q \tilde{v}_n) w^q(x)\right] \frac{\partial x^n}{\partial x'^m}$$

$$w'^m(x') = w^n(x) \frac{\partial x'^m}{\partial x^n}$$

DFT and GENERALISED GEOMETRY

Consider case fields restricted to submanifold N of M

w transforms as a tangent vector on N and $\hat{w}$ transforms as a cotangent vector under diff(N).
Both invariant under $\tilde{v}$ transformations.

$w \oplus \hat{w}$ is a section of $(T \oplus T^*)N$

This is Hitchin's generalised tangent bundle on N

$$w \oplus \tilde{w}$$

is section of E, which is $T \oplus T^*$ twisted by a gerbe

$$0 \to T^* \to E \to T \to 0$$

Then ‘generalized vectors’ $W^M = \begin{pmatrix} w^m \\ \tilde{w}_m \end{pmatrix}$

are not really vectors on doubled space, but are sections of generalised tangent bundle over ‘physical space’ N, twisted by a gerbe

$v^m(x)$ symmetries are diffeomorphisms of N
$\tilde{v}_m(x)$ symmetries are b-field gauge transformations on N

Gauge symmetry of DFT same as that of string/sugra

$$\mathrm{Diff}(N) \ltimes \Lambda^2_{closed}(N)$$

Global O(D,D)

2D dimensional doubled space M, D dim. subspace N

3 kinds of vectors $V^M(X)$

1)Vector fields on M:

Sections of TM,

transform under diff(M)

2)Hatted generalised vector fields $\hat{W}$ on M:

Sections of $(T \oplus T^*)N$

transform under diff(N)

3)Generalised vector fields W on M

Sections of E(N)

transform under $\mathrm{Diff}(N) \ltimes \Lambda^2_{closed}(N)$

Extends to tensors, generalised tensors and untwisted generalised tensors

Generalised Metric

$$\mathcal{H}_{MN} = \begin{pmatrix} g_{mn} - b_{mk}g^{kl}b_{ln} & b_{mk}g^{kn} \\ -g^{mk}b_{kn} & g^{mn} \end{pmatrix}$$

Finite transformations give usual ones for g,b

Untwisted form of generalised metric

$$\hat{\mathcal{H}}_{MN} = \begin{pmatrix} g_{mn} & 0 \\ 0 & g^{mn} \end{pmatrix}$$

Natural metric on $T \oplus T^*$

Constant O(D,D) Metric

Matrix with constant components:

$$\eta_{MN} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

If this is tensor on M, then it is flat metric and this would greatly restrict possible M. Not invariant under Diff(M)

Constant O(D,D) Metric

Matrix with constant components:

$$\eta_{MN} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

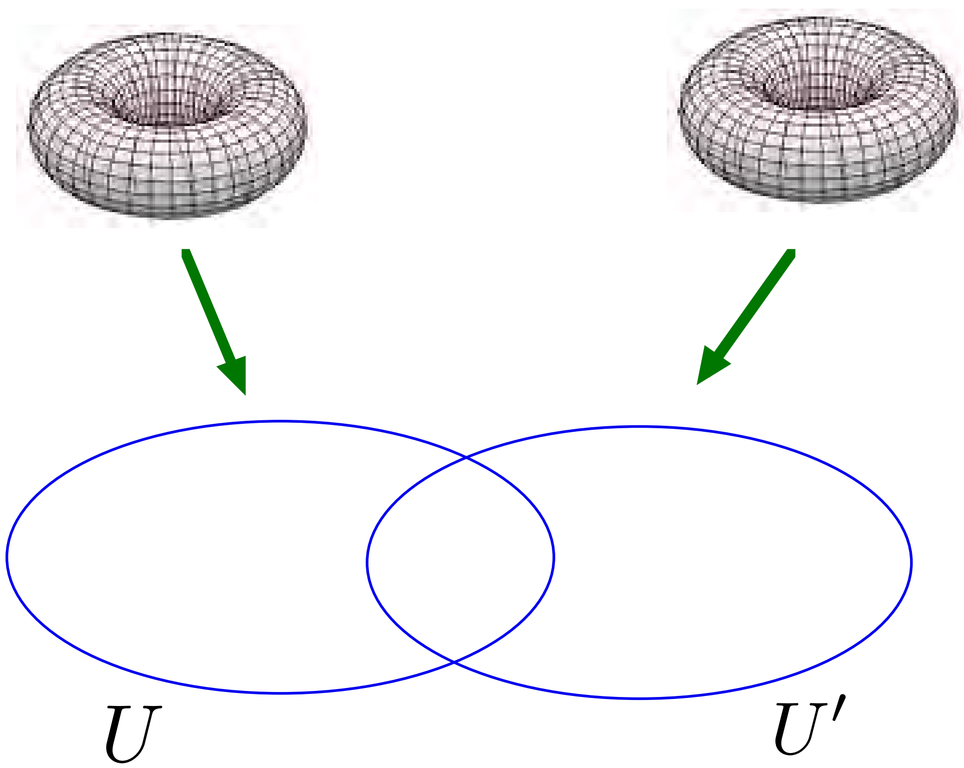
If this is tensor on M, then it is flat metric and this would greatly restrict possible M. Not invariant under Diff(M)

If it is generalised tensor, section of $E^* \otimes E^*(N)$

$$\hat{\eta}_{MN} = \eta_{MN}$$

Invariant under DFT gauge transformations, natural object in DFT. Metric for E(N), not T(M)
No restriction on geometry

Transition Functions and Non-Geometry



U, U' patches in $\mathbb{R}^{2n}$

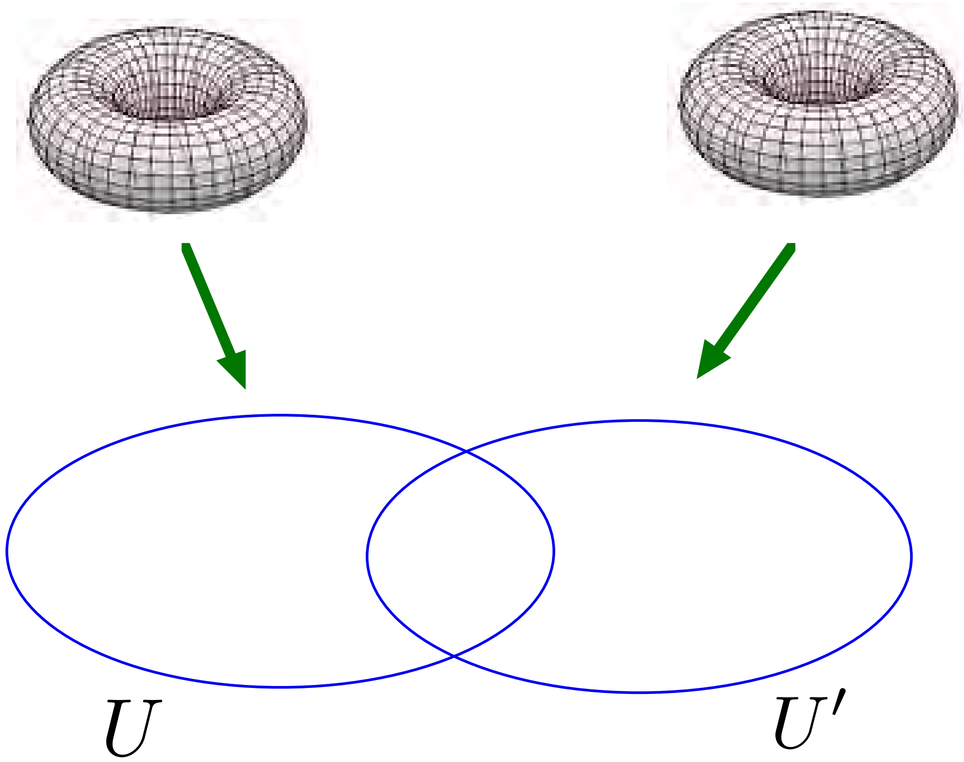
Fibres T^{2d}

Transition functions:

DFT gauge transformations and

$O(d, d; \mathbb{Z})$

Transition Functions and Non-Geometry

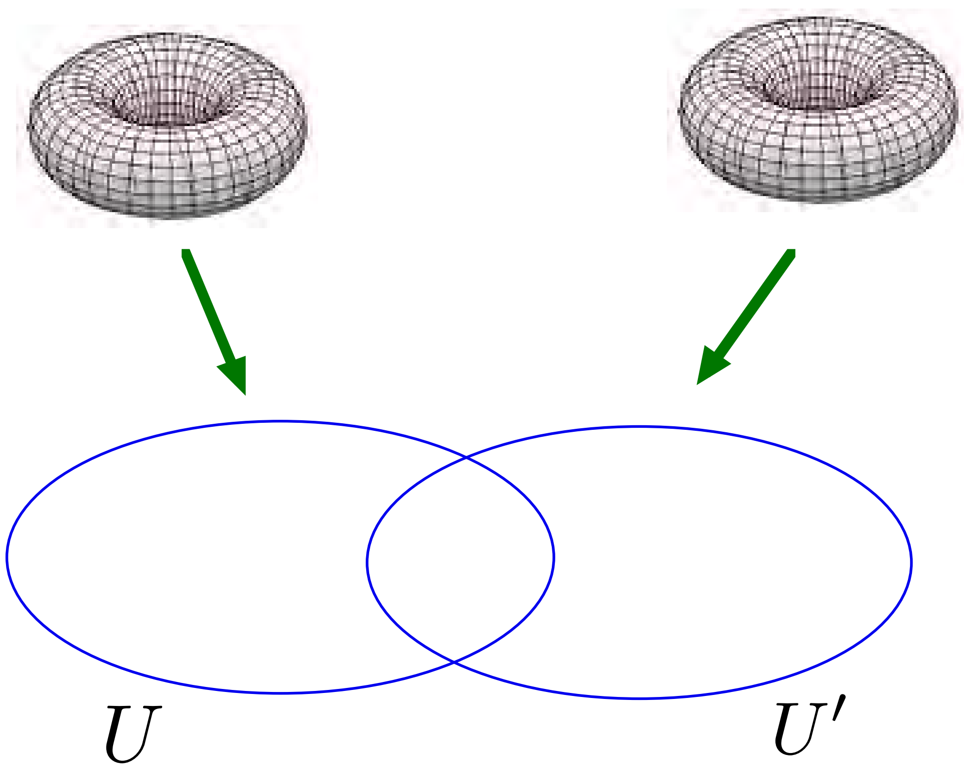


U, U' patches in $\mathbb{R}^{2n}$
Fibres T^{2d}

Transition functions:
DFT gauge transformations and $O(d,d;\mathbb{Z})$

If transition functions include T-duality, then can construct T-folds. As $O(d,d;\mathbb{Z}) \subset GL(2d;\mathbb{Z})$ coordinate transition functions are a diffeomorphism on doubled space, so doubled space is a manifold

Transition Functions and Non-Geometry



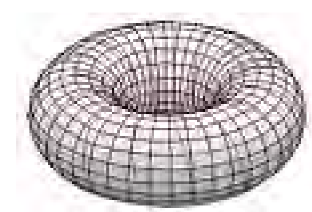
U, U' patches in $\mathbb{R}^{2n}$
Fibres T^{2d}

Transition functions:
DFT gauge transformations and
$O(d,d;\mathbb{Z})$

Can construct explicit doubled geometries of
Dabholkar & Hull; Hull & Reid-Edwards
in this way, including those with 'R-flux'

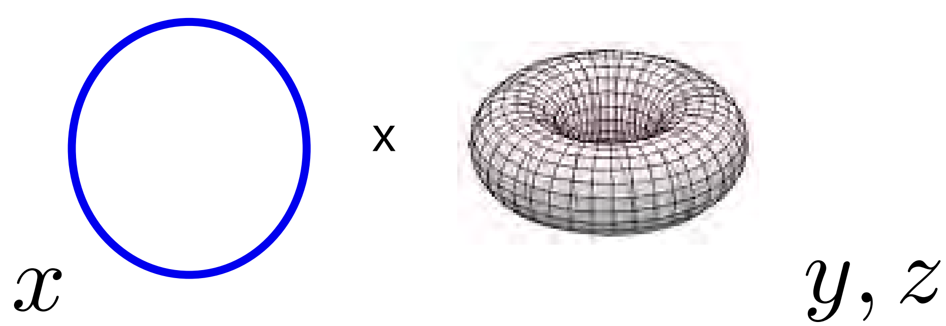
Example: T^3 with H-flux

$$H = N \times (Vol)$$



$$H_{xyz} = N$$

Regard as product $S^1 \times T^2$



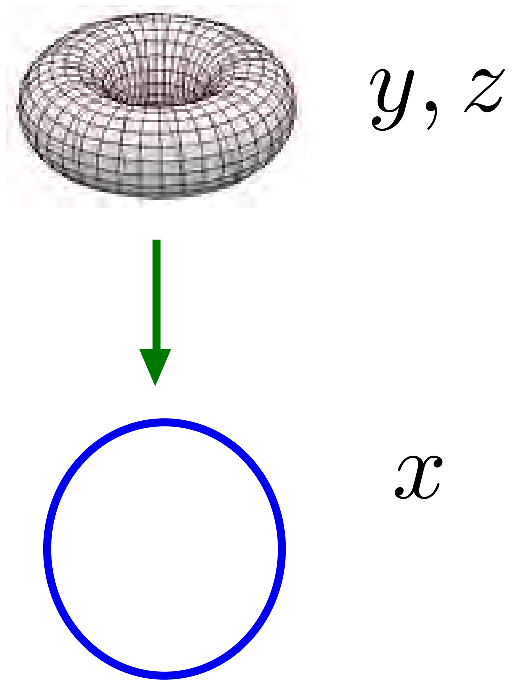
$$B_{yz} = B_0 + \frac{1}{2\pi} N x$$

T-dual on z-circle:

Torus bundle over circle, H=0

$$\tau(x) = \tau_0 + \frac{1}{2\pi} N x$$

Nilfold: Heisenberg group manifold identified under discrete subgroup



Next, T-dual on y-circle

No global Killing vector. Do fibrewise duality, use Buscher rules locally, using local gauging

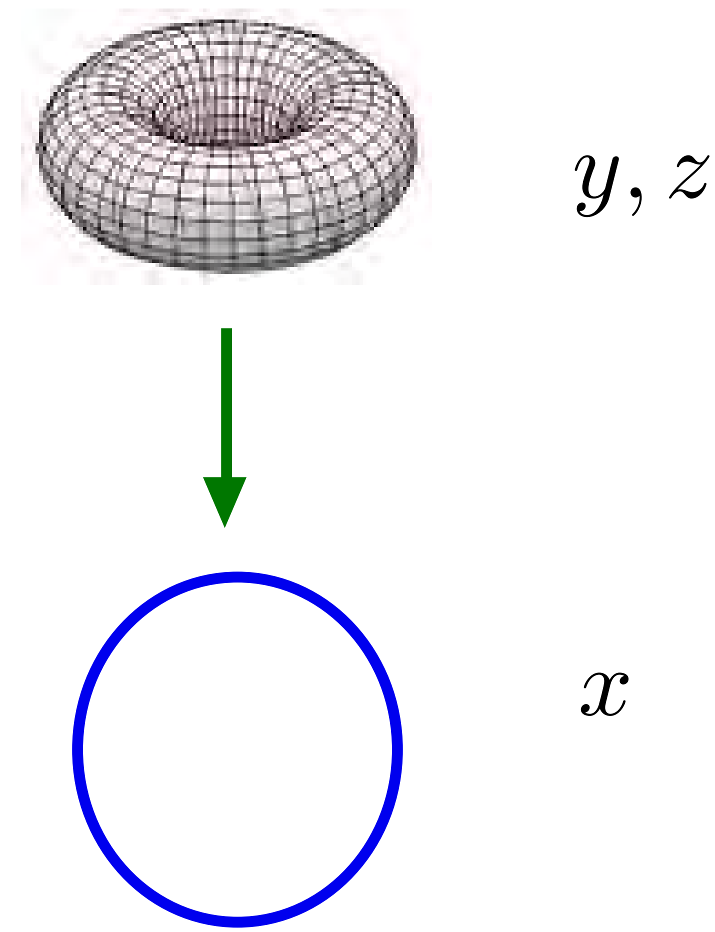
CMH

T-dual of T^3 with flux:

Torus bundle over circle?

$$ds^2 = dx^2 + \frac{1}{1+N^2x^2}\left(dy^2+dz^2\right)$$

$$B_{yz} = \frac{Nx}{1+N^2x^2}$$



But x periodic

$$E(x+2\pi) = (aE+b)(cE+d)^{-1}$$

Monodromy $\begin{pmatrix} a & b \\ c & d \end{pmatrix} \in O(2,2;Z)$ T-duality

T-fold. Realise as doubled torus T^4 bundle over S^1.

CMH

- Doubled geometry: non-compact group identified under discrete subgroup

CMH & Reid-Edwards

- Gives T-fold and its T-duals

- Transition functions are diffeomorphisms of base space + T-duality, so allowed in DFT

- Not solution. Solution obtained by adding dependence on a 4th dimension.

- Gives explicit DFT solution with patching by DFT symmetries.

Conclusions

- DFT: conventional sugra in duality symmetric formulation, using generalised geometry on N
- Covariant formulation of generalised geometry, indep. of choice of duality frame
- More generally, this applies locally in patches. Use DFT gauge and O(D,D) symmetries in transition functions. Get T-folds etc.

- DFT extends field theory to non-geometric spaces: T-folds, with T-duality transition functions.

- What is full theory with weak constraint?

- Winding modes: doubling of torus or torus fibres

- Other topologies may not have windings, or have different numbers of momenta and windings. No T-duality? No doubling?