

“Conductivity at strong coupling”

Talk at “Eighth Crete Regional Meeting In String Theory”

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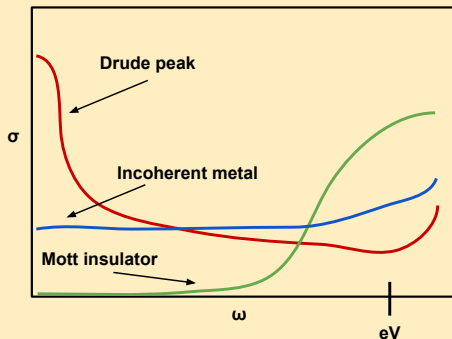
July 2015

Based on work with J. P. Gauntlett:

arXiv: 1311.3292, 1401.5077, 1506.01360, 1507.00234

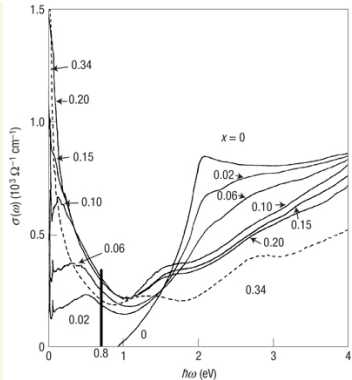
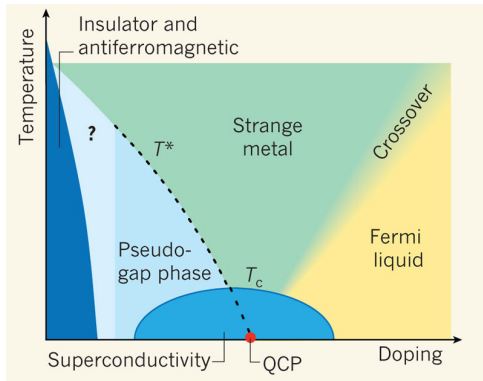
- 1 Introduction/Motivation
- 2 The Holographic Lab
 - Incoherent transport
 - Anomalous scaling of Hall angle
 - DC conductivities from BH horizons
- 3 Summary / Outlook

Charge transport in real materials



- Materials with charged d.o.f. can be
 - Coherent metals with a well defined Drude peak
 - Insulators
 - Incoherent conductors of electricity
- Interactions expected to become important in the incoherent phase → Possible description in AdS/CFT?

The Cuprates



The Cuprates are real life example of :

- Incoherent transport
- Anomalous scaling of conductivity and Hall angle with T

$$\sigma_{DC}^{B=0} \propto T, \quad \theta_H \propto T^{-2}$$

Perfect Holographic Conductor

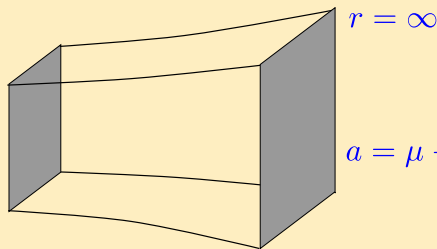
Do it in $D = 4$ Einstein-Maxwell with AdS asymptotics:

$$\mathcal{L}_{EM} = R - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} + 12$$

$$ds_4^2 = -U(r) dt^2 + U(r)^{-1} dr^2 + r^2 (dx_1^2 + dx_2^2)$$

$$A = a(r) dt$$

$r = r_+$

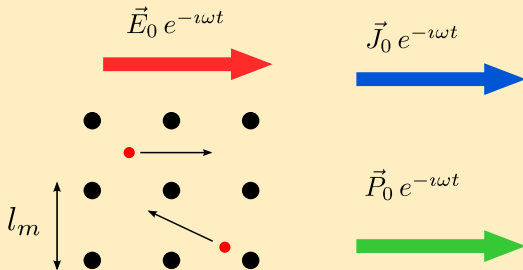


$r = \infty$

$$a = \mu - q r^{-1} + \dots$$

Background black hole has temperature T , energy E , pressure P , entropy s and charge q .

Classical Drude model



- Average momentum obeys

$$\langle \dot{p} \rangle = qE - \frac{1}{\tau} \langle p \rangle \Rightarrow$$
$$\sigma = \frac{nq^2}{m} \frac{\tau}{1 - i\omega\tau}$$

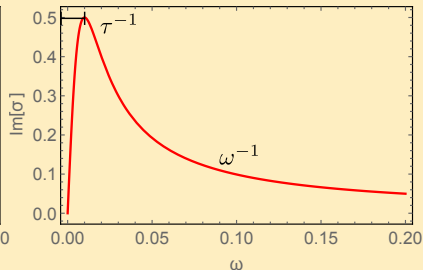
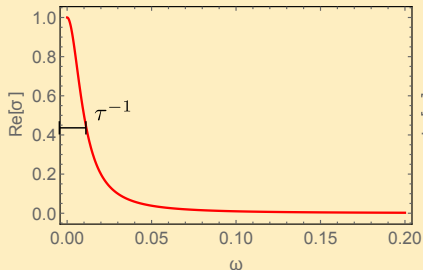
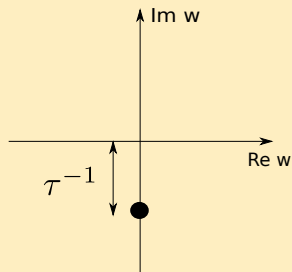
- Without collisions $\tau \rightarrow \infty \Rightarrow \sigma = \frac{nq^2}{m} \left(\delta(\omega) + \frac{i}{\omega} \right)$

Classical Drude model

Don't need quasi-particles to have Drude physics.

Coherent metals arise when momentum relaxation is slow with dominant pole on real axis.

[Hartnoll, Hofman]

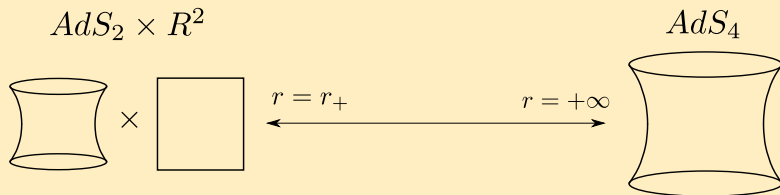


- Apart from electric currents we have a thermal current
 $Q^i = T^{ti} - \mu J^i$
- Transport coefficients are packaged in Ohm/Fourier law

$$\begin{pmatrix} J \\ Q \end{pmatrix} = \begin{pmatrix} \sigma & \alpha T \\ \bar{\alpha} T & \bar{\kappa} T \end{pmatrix} \begin{pmatrix} E \\ -(\nabla T)/T \end{pmatrix}$$

- With ∇T a temperature gradient

Holographic Lattice



To add momentum dissipation introduce a UV - IR benign lattice:

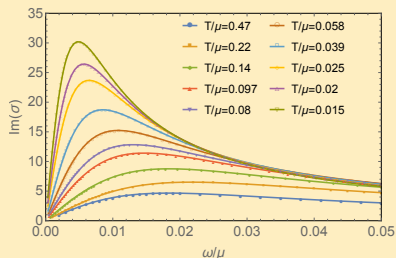
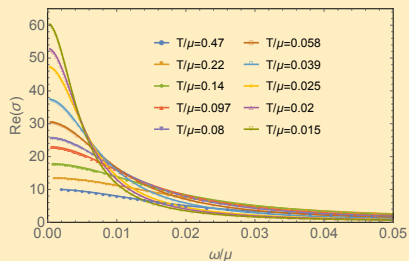
- Keep UV fixed point \Rightarrow relevant deformation $\mathcal{O}(x)$
- Drude physics $\Rightarrow T = 0$ horizon restores translations
- Charge density is a universal relevant operator \Rightarrow Impose
 $A_t = \mu(x) - J^t(x) r^{-1} + \dots$

[Hartnoll, Hofman][Horowitz, Santos, Tong][AD, Gauntlett]

$$\mu(x) = \mu_0 + A(x), \quad \langle A \rangle_L = 0$$

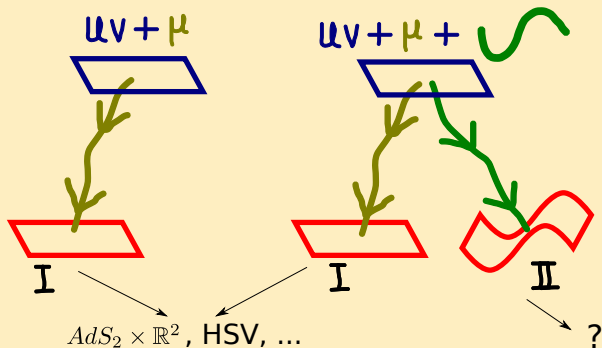
- $\mu_0 \Rightarrow$ chemical potential, $A'(x) \Rightarrow$ periodic electric field

Inhomogeneous Lattices



- Drude peaks are there
- Get rid of them!

RG/Holographic picture



- I Charge dominated RG flows, translations restored in IR \rightarrow Coherent transport
- II Lattice dominated RG flows, translations broken in IR \rightarrow Incoherent transport

[AD, Hartnoll] [AD, Gauntlett]

Consider a simple model with a global $U(1)$ in addition to the gauged one [AD, Gauntlett]

$$S = \int d^4x \sqrt{-g} \left[R + 6 - \frac{1}{4} F^2 - |\partial\phi|^2 - m^2 |\phi|^2 \right]$$

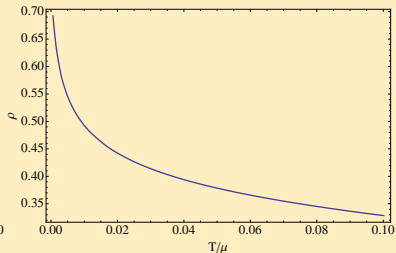
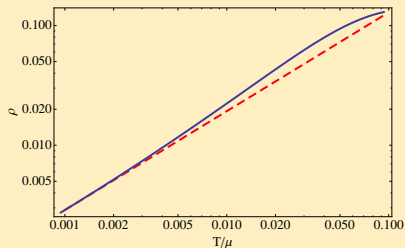
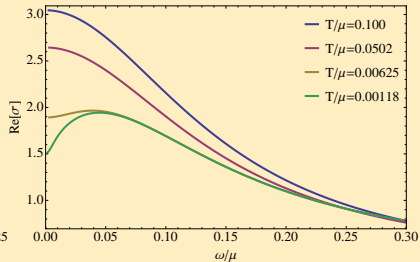
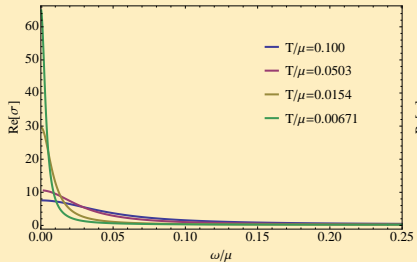
along with the ansatz

$$ds^2 = -U(r) dt^2 + U(r)^{-1} dr^2 + e^{2V_1(r)} dx_1^2 + e^{2V_2(r)} dx_2^2$$

$$A = a(r) dt, \quad \phi = e^{ikx_1} \varphi(r)$$

- Leads to ODEs both for background and perturbation
- Two real scalars with $\mathcal{O}_1 \sim \cos(kx)$, $\mathcal{O}_2 \sim \sin(kx)$
- RN still a solution
- Lattice operator relevant for $k < k_c$

Conductivity from Q-lattices



- Can model Metal - Insulator transitions
- Similar story for inhomogeneous lattices

[Rangamani, Rozali, Smyth]

More general Q-lattices

Consider more general situation where

$$\mathcal{L} = R - \frac{1}{2} \left[(\partial\varphi)^2 + \Phi_1(\varphi) (\partial\chi_1)^2 + \Phi_2(\varphi) (\partial\chi_2)^2 \right] \\ + V(\varphi) - \frac{Z(\varphi)}{4} F^2$$

The background is then

$$ds_4^2 = -U(r) dt^2 + U(r)^{-1} dr^2 + e^{2V_1(r)} dx_1^2 + e^{2V_2(r)} dx_2^2$$

$$A = a(r) dt, \quad \phi = \phi(r)$$

$$\chi_1 = k_1 x_1, \quad \chi_2 = k_2 x_2$$

Rich class of new ground states that break translations

- Insulators
- Novel metals

- Perturbative argument to determine horizon at low T
- Analytic argument to express DC transport coefficients in terms of bh horizon data

$$\sigma_{DC} = \left[\frac{Z(\phi)s}{4\pi e^{2V_1}} + \frac{4\pi q^2}{k_1^2 \Phi_1(\phi)s} \right]_{r=r_+} = \sigma_{ccs} + \sigma_{dis}$$

$$\bar{\kappa}_{DC} = \left[\frac{4\pi sT}{k_1^2 \Phi_1(\phi)} \right]_{r=r_+}, \quad \alpha_{DC} = \bar{\alpha}_{DC} = \left[\frac{4\pi q}{k_1^2 \Phi_1(\phi)} \right]_{r=r_+}$$

- Conductivity can be dominated by either σ_{ccs} or σ_{dis}
- Notice that

$$\sigma_{ccs} = \sigma_{Q=0} = \sigma - T \alpha \bar{\kappa}^{-1} \alpha$$

- Can include background magnetic field B
- For $B^{1/2} \ll T \ll \mu$

$$\theta_H \propto \frac{B}{q} \sigma_{dis}^{B=0}$$
$$\sigma_{DC}^{B=0} = \sigma_{ccs}^{B=0} + \sigma_{dis}^{B=0}$$

- It is possible to have

$$\theta_H \propto \frac{B}{q} \sigma_{dis}^{B=0}$$
$$\sigma_{DC}^{B=0} \propto \sigma_{ccs}^{B=0}$$

- Not possible to get with weakly coupled fermions.

- Powerful
- Reveal interesting physics
- Have been found for more examples of homogeneous lattices
e.g. Bianchi VII_0
[Iqbal, Liu] [Blake, Tong] [Andrade, Withers] [Gouteraux] [AD, Gauntlett]
[AD, Gouteraux, Kiritsis] [AD, Gauntlett, Pantelidou]
- Where do they come from?

DC conductivities from BH horizons

- Back to Einstein-Maxwell
- Consider electrically charged, static black branes

$$ds^2 = -UG dt^2 + \frac{F}{U} dr^2 + ds^2(\Sigma_d), \quad A = a_t dt$$
$$ds^2(\Sigma_d) = g_{ij}(r, x) dx^i dx^j$$

- Asymptotically, $r \rightarrow \infty$

$$U \rightarrow r^2, \quad F \rightarrow 1, \quad G \rightarrow \bar{G}(x),$$
$$g_{ij}(r, x) \rightarrow r^2 \bar{g}_{ij}(x), \quad a_t(r, x) \rightarrow \mu(x).$$

For the perturbation write

$$\begin{aligned}\delta(ds^2) &= \delta g_{\mu\nu}(r, x) dx^\mu dx^\nu - 2tGU\zeta_i dt dx^i, \\ \delta A &= \delta a_\mu(r, x) dx^\mu - tE_i dx^i + ta_t \zeta_i dx^i\end{aligned}$$

- $E(x^i)$ and $\zeta(x^i)$ are closed forms
- ζ is boundary temperature gradient
- E is boundary electric field

DC conductivities from BH horizons

Consider radial Hamiltonian and examine constraints close to the horizon.

- Define

$$v_i \equiv \delta g_{it}^{(0)}, \quad w \equiv \delta a_t^{(0)},$$
$$p \equiv 4\pi T \frac{\delta g_{rt}^{(0)}}{G^{(0)}} + \delta g_{it}^{(0)} g_{(0)}^{ij} \nabla_j \ln G^{(0)}$$

- To find

$$\nabla_i v^i = 0,$$

$$\nabla^2 w - v^i \nabla_i (a_t^{(0)}) = -\nabla_i E^i$$

$$2 \nabla^i \nabla_{(i} v_{j)} - a_t^{(0)} \nabla_j w - \nabla_j p = 4\pi T \zeta_j + a_t^{(0)} E_j$$

- Solve for a Stokes flow on the curved black hole horizon!
- Notice no hydro limit

Electric Current

Define

$$J^i = \sqrt{-g} F^{ir}$$

- At $r \rightarrow \infty$ gives field theory current densities
- Anywhere in the bulk

$$\partial_i J^i = 0, \quad \partial_r J^i = \partial_j (\sqrt{-g} F^{ji})$$

Heat Current

Let $k = \partial_t$ and define

$$G^{\mu\nu} = \nabla^{[\mu} k^{\nu]} + \frac{1}{2} k^{[\mu} F^{\nu]\sigma} A_\sigma + \frac{1}{4} (\phi - \theta) F^{\mu\nu}$$

Let

$$Q^i = -2\sqrt{-g}G^{ir}$$

- At $r \rightarrow \infty$ gives field theory heat current densities
- Anywhere in the bulk

$$\partial_i Q^i = 0, \quad \partial_r Q^i = -\partial_j (2\sqrt{-g}G^{ji})$$

DC conductivities from BH horizons

- Solutions for v^i , w and p are uniquely fixed by sources E and ζ
- Then

$$J^i|_{r=r_h} = \frac{s}{4\pi} (\partial^i w + E^i) - \rho v^i$$

$$Q^i|_{r=r_h} = -T s v^i, \quad s = 4\pi \sqrt{g_{(0)}}, \quad \rho = \sqrt{g_{(0)}} a_t^{(0)}$$

- Exploit

$$\partial_r J^i = \partial_j (\sqrt{-g} F^{ji}), \quad \partial_r Q^i = -\partial_j (2\sqrt{-g} G^{ji})$$

- To find field theory currents \bar{J}^i and \bar{Q}^i in e.g. $d = 2$

$$\bar{J}^1 = \int dx^2 J^1, \quad \bar{J}^2 = \int dx^1 J^2$$

- Conductivities determined solely by BH horizon data!

Examples

- Can recover earlier results for e.g. Q-lattices and 1-dim lattices
- Perturbative, periodic lattices about AdS-RN black brane

Let λ be the expansion parameter

The black hole horizon is a small expansion about flat space

$$g_{(0)ij} = g \delta_{ij} + \lambda h_{ij}^{(1)} + \lambda^2 h_{ij}^{(2)} + \dots$$

$$a_t^{(0)} = a + \lambda a_{(1)} + \lambda^2 a_{(2)} + \dots$$

$$G^{(0)} = f_{(0)} + \lambda f_{(1)} + \dots$$

Solve Navier-Stokes perturbatively in λ

DC conductivities from BH horizons

At leading order in λ we find

$$\alpha_{ij} = \bar{\alpha}_{ij} = \frac{L_{ij}^{-1}}{\lambda^2} 4\pi\rho + \dots, \quad \bar{\kappa}_{ij} = \frac{L_{ij}^{-1}}{\lambda^2} 4\pi sT + \dots$$

$$\sigma_{ij} = \frac{L_{ij}^{-1}}{\lambda^2} \frac{4\pi\rho^2}{s} + \dots$$

Where $L_{ij} = \int_H l_{ij} \left(h_{kl}^{(1)}, a^{(1)} \right)$

Consistent with memory matrix formalism

[Barkeshli, Hartnol, Mahajan]

- Holography is a tool to study transport in strongly coupled systems
- No assumption of quasiparticles
- Understand better the physics of the new ground states
- Fluid/gravity can be used to obtain DC thermoelectric conductivities
- Connection with fluid/gravity beyond DC?