Quantum state of the black hole

Ram Brustein

אוניברסיטת בן-גוריון

- Quantum state vs. geometry
- Quantum state: an outside view
  - Mean field approximation valid for BH
  - Radiation in a highly quantum state
- Quantum state: an inside view
  - Highly quantum bound state
  - Effective description $p = \rho$

RB, 1209.2686
+ Medved
1302.6086,1305.3139,…

Alberte, RB, Khmelnitsky, Medved 1502.02687

RB+ Medved
1505.07131+ to appear

Paradigm shift:
Unitary vs. non-unitary evolution ➔
What is the quantum state of the BH ?

$$\langle BH | e^{iHt} \hat{O} e^{-iHt} | BH \rangle$$

RB, 1209.2686

Description of BH needs to be modified

classical geometry → (Semiclassical) quantum state

Need to calculate:

$$\langle \mathcal{O}_M \rangle = \langle \Psi_{M,BH} | \mathcal{O}_M | \Psi_{M,BH} \rangle$$

Instead of:
QFT in curved space/
Effective field theory
on a fixed background

$$\langle \mathcal{O}_M \rangle_C = \langle \Psi_M(g_c) | \mathcal{O}_M | \Psi_M(g_c) \rangle$$

"Classicality parameter"

$$C_{BH} = \frac{1}{2\pi} \frac{\lambda_{BH}}{R_S} = 1/S_{BH}$$

Dvali & Gomez "1/N"
AdS/CFT $1/N^2$

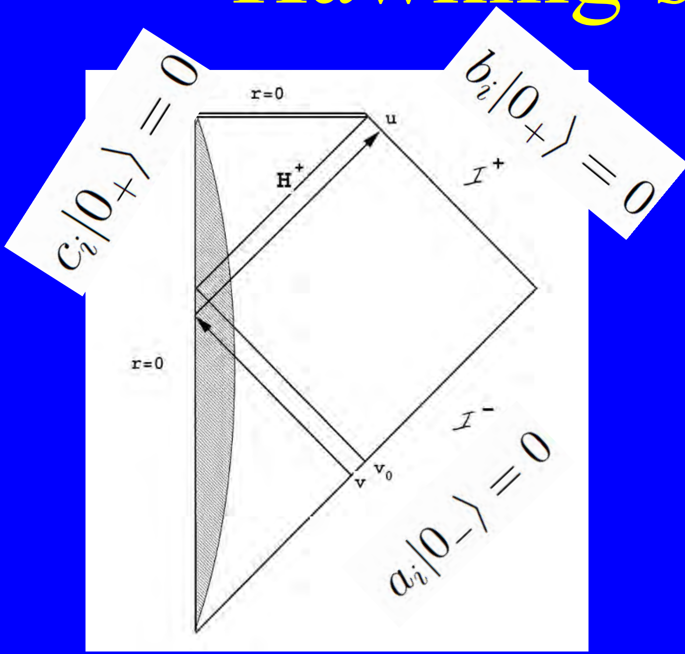
Paradigm shift:
Unitary vs. non-unitary evolution ➔
What is the quantum state of the BH ?

Q: How will we know the state of the BH ?
A: By knowing the state of the radiation it emits

Initial state (almost) pure *
➔ The radiation is the "purifier" of the BH
➔ Density matrix of the BH " = " Density matrix of the radiation

* For a solar mass BH
$S_{ini} \sim 10^{57}$
$S_{BH} \sim 10^{77}$

Hawking's model of BH radiance



Hawking PRD 1976

$$b^i = \sum_j \left(\bar{\alpha}^i{}_j a^j - \bar{\beta}^{ij} a^+_j\right)$$

$$b^+_i = \sum_j \left(\alpha_i{}^j a^+_j - \beta_{ij} a^j\right)$$

$$\widehat{\rho}_{\text{vac}} \equiv |0_-\rangle\langle 0_-|$$

$$\widehat{\rho}^{\ \text{out}} = \rho^{\text{out}}{}_{ac} |a_{\text{out}}\rangle\langle c_{\text{out}}|$$

$$\mathcal{O}_{\text{out}}{}^{ca} = \langle c_{\text{out}}|\widehat{\mathcal{O}}_{\text{out}}|a_{\text{out}}\rangle$$

$$\langle n_j\rangle \equiv \langle 0_-|b^+_j b_j|0_-\rangle = \sum_a n_{ja}\, \rho^{\text{out}}{}_{aa}$$

RB,Alberte,Khmelnitsky,Medved 1502.02687

Properties of $\hat{\rho}^{out} \equiv \hat{\rho}_{rad} \equiv \hat{\rho}$

Single-particle density matrix /two-point function

$$\rho^i{}_j = \langle 0_- | b_j^+ b^i | 0_- \rangle$$

$$\rho^i{}_j = \sum_k \bar{\beta}^{ik} \beta_{jk}$$

$$b^i = \sum_j \left(\bar{\alpha}^i{}_j a^j - \bar{\beta}^{ij} a_j^+ \right)$$

$$b_i^+ = \sum_j \left(\alpha_i{}^j a_j^+ - \beta_{ij} a^j \right)$$

Full density matrix

$$\widehat{\rho}^{\,\mathrm{out}} = \frac{1}{Z} e^{-b_i^+ \Omega^i{}_j b^j}$$

$$\rho = \frac{1}{e^{\Omega} - 1}$$

$$Z = \det \left[\frac{1}{1 - e^{-\Omega}} \right] = \det \left[1 + \rho \right]$$

$$\mathrm{tr} \left[\widehat{\rho}^{\,2} \right] = \det \left[\frac{1}{1 + 2\rho} \right]$$

Hawking: Two-point function diagonal in frequency

$$\rho_H(\omega, \widetilde{\omega}) \propto \frac{1}{e^{\frac{\hbar\omega}{T_H}} - 1}\delta(\omega - \widetilde{\omega})$$

$$\hat{\rho} = \frac{1}{Z} e^{-b_i^+ \Omega^i{}_j b^j}$$

$$Z = \det\left[\frac{1}{1 - e^{-\Omega}}\right]$$

$$\rho = \frac{1}{e^{\Omega} - 1}$$

$$\Omega^i{}_j = e^{-\frac{\hbar\omega_i}{T_H}} \delta^i{}_j$$

Density matrix diagonal in frequency and particle number → **mixed state**

Breakdown of predictability in gravitational collapse

Also, Bekenstein ’75, Wald ’75, Parker ‘75

State of the emitted radiation

The Hawking model

$$\hat{\rho} \sim \begin{array}{c} \mathcal{N}'=1 \rightarrow \\ \mathcal{N}'=2 \rightarrow \\ \mathcal{N}'=3 \rightarrow \\ \mathcal{N}'=4 \rightarrow \\ \vdots \\ \\ \end{array} \begin{array}{c} \begin{pmatrix} \beta\bar{\beta} & 0 & \bar{\alpha}\bar{\beta} & 0 & (\bar{\alpha}\bar{\beta})^2 & 0 & \ldots \\ 0 & \beta\bar{\beta} & 0 & \bar{\alpha}\bar{\beta} & 0 & (\bar{\alpha}\bar{\beta})^2 & \ldots \\ \alpha\beta & 0 & \beta\bar{\beta} & 0 & \bar{\alpha}\bar{\beta} & 0 & \ldots \\ 0 & \alpha\beta & 0 & \beta\bar{\beta} & 0 & \bar{\alpha}\bar{\beta} & \ldots \\ \vdots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots \end{pmatrix} \\ \begin{array}{cccccccc} & \uparrow & \uparrow & \uparrow & \uparrow & \uparrow & \uparrow & \\ \mathcal{N}= & 1, & 2, & 3, & 4, & 5, & 6 & \cdots \end{array} \end{array} .$$

Horizon = high redshift surface

➔

off-diagonal corrections in mode-occupation number are exponentially suppressed

$$\alpha\beta \sim \delta\left(\omega + \omega'\right)$$

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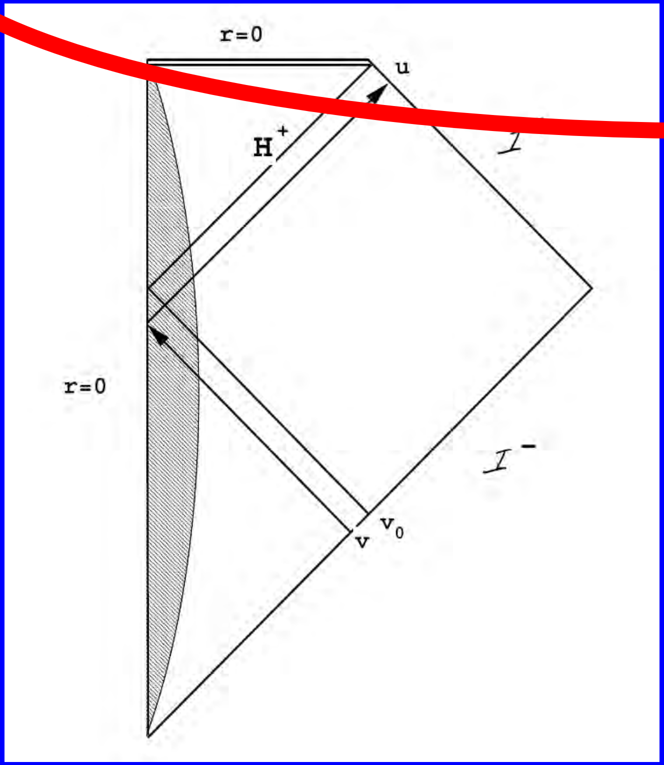
Repeat Hawking's calculation with

Classical shell←→ Semiclassical shell , wavefunction



State of the emitted radiation

The semiclassical model - Density matrix diagonal in particle number but not in frequency!

$$\hat{\rho} = \frac{1}{Z} e^{-b_i^{+} \Omega^i{}_j b^j}$$

$$\rho = \frac{1}{e^{\Omega} - 1}$$

Horizon = high redshift surface → off-diagonal corrections in mode-occupation number exponentially suppressed

$$(\rho_{SC})_{ii} = 1$$

$$(\rho_{SC})_{ij} = \sqrt{\frac{1}{S_{BH}}}\, e^{i\Theta_{ij}}$$

When the number of emitted particles is ~ S_{BH} number of large eigenvalues starts to decrease

State of the emitted radiation is highly quantum

Total number of modes $W = \omega \Delta t \sim T_H \Delta t$

$\hat{\rho}$ effectively $W \times W$ matrix

$$N \sim W$$

Total number of emitted particles $N = \Gamma \Delta t \sim T_H \Delta t$

→ Small occupation numbers

→ Diagonal entries of order unity $\rho^{jn}{}_{jn} \sim 1$

→ Very different from "normal" black body

State of the emitted radiation

model	diagonal	Off-diagonal frequency	Off-diagonal Mode-occupation
Hawking	1	$\ll 1/S_{BH}$	$\ll e^{-S_{BH}}$
SC	1	$1/\sqrt{S_{BH}}$	$\ll e^{-S_{BH}}$
Page	1	N/A	$e^{-S_{BH}/2}$

State of the emitted radiation is highly quantum – expected property for any model of unitary evolution that approximates Hawking's model

State of the interior
General considerations

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- BH "purifier" of radiation ➔ State of the interior highly quantum
- BH interior *cannot* be described by a semiclassical metric
- BH is a highly excited state, highly degenerate ➔

  density of states >> density of states of "normal" bound states in known QFT's➔ string theory ??

State of the interior : A proposal

- A gravitationally bound state
- $N \sim S_{BH}$ relativistic modes
- Energy of each mode $\sim 1/R_S$

} Similar to Dvali & Gomez, Luest talk

- Large quantum fluctuations
- Entropy bounds saturated for any region in the interior
- "Horizon" – transition region between the highly quantum interior and highly classical exterior
- Particle production near the edge
- Falling objects "burn" (not a "firewall")

State of the interior

scaling relations

$$N = S_{BH} = R_S^2/l_P^2$$

# of relativistic modes

$$\rho = p = \frac{1}{r^2 l_P^2}$$

$$E = M_{BH} = N/R_S = R_S/l_P^2$$

$$s = \sqrt{\rho}$$

$$S(E) = l_P^2 E^2$$

$$\frac{dN}{dr\,d\omega} = \frac{r^2}{l_P^2}\Theta(1 - r\omega)$$

$$s = \frac{1}{r l_P^2}$$

$$\rho = \frac{1}{r^2 l_p^2}$$

Energy of each mode

$\sim 1/R_S$

State of the interior

stability of the bound state

$$E_{ij}^{int} = \frac{G_N E_i E_j}{r} \simeq \frac{l_P^2}{R_S^3}$$

$$E_{int} = \sum_{i,j}^{N} E_{ij}^{int} \simeq N^2 \times \frac{l_P^2}{R_S^3} = \frac{N}{R_S} = M_{BH}$$

- Emission of particles with wavelength > R_S
  is suppressed ←→ geometric suppression factor
- Emission of particles with wavelength << R_S
  is suppressed ←→ gravitational barrier + suppressed production

- Stable against emission of particles of wavelength $\simeq R_S$ If a single particle of energy $1/R_S$ is emitted, the scaling relations are still valid with entropy $N-1$ and energy $M_{BH} - 1/R_S$

State of the interior

Large quantum fluctuations

$$|S\rangle \;\equiv\; \phi_1^\dagger \phi_2^\dagger \cdots \phi_N^\dagger |0\rangle$$

$$\frac{\left(\Delta\widehat{\Phi}^2\right)_S^2}{\langle\widehat{\Phi}^2\rangle_S^2} = \frac{\langle\widehat{\Phi}^4\rangle_S - \langle\widehat{\Phi}^2\rangle_S^2}{\langle\widehat{\Phi}^2\rangle_S^2} \simeq \frac{5}{4}$$

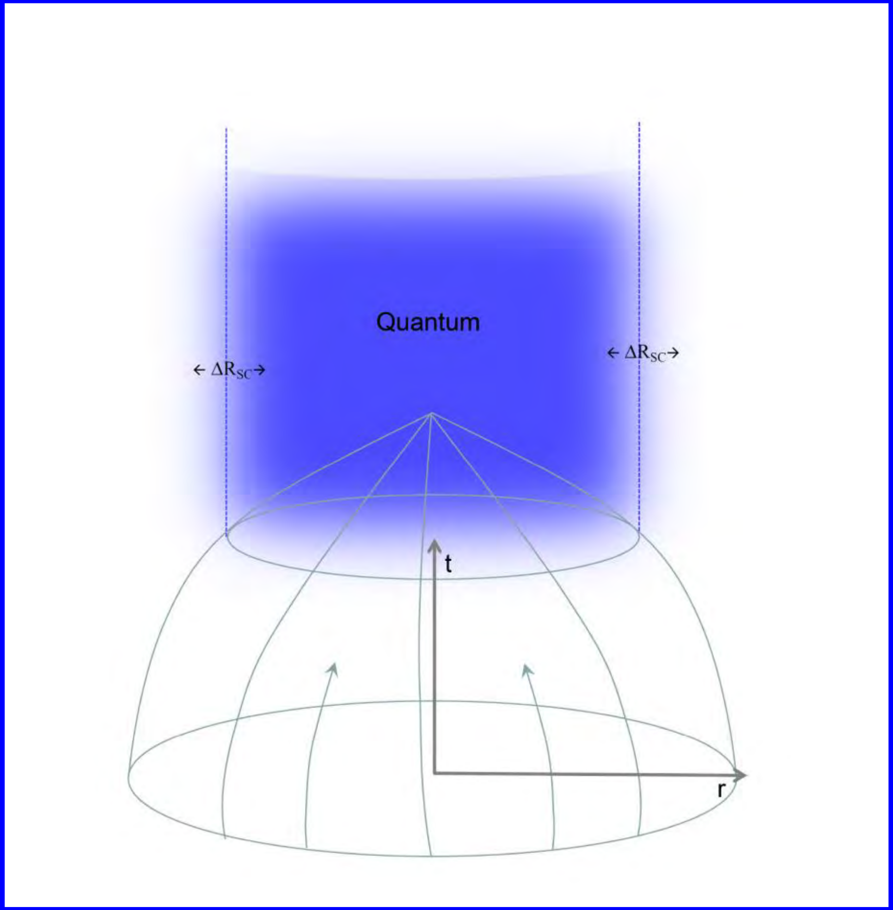
Small occupation numbers → **Large** quantum fluctuations

Large occupation numbers → **Small** quantum fluctuations

$$|\alpha\rangle \;=\; e^{-\frac{1}{2}|\alpha|^2} e^{\alpha\phi^\dagger} |0\rangle$$

$$\frac{\left(\Delta\widehat{\Phi}\right)_\alpha^2}{\langle\widehat{\Phi}\rangle_\alpha^2} \sim \frac{1}{|\alpha|^2} \sim \frac{1}{N} \ll 1$$

End of semiclassical spacetime

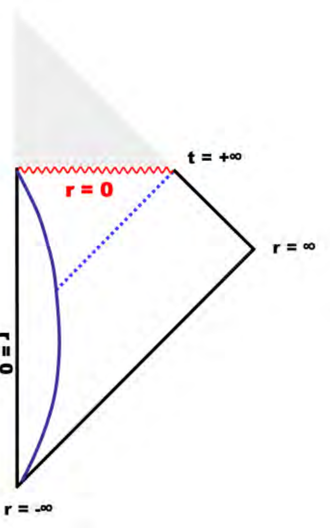


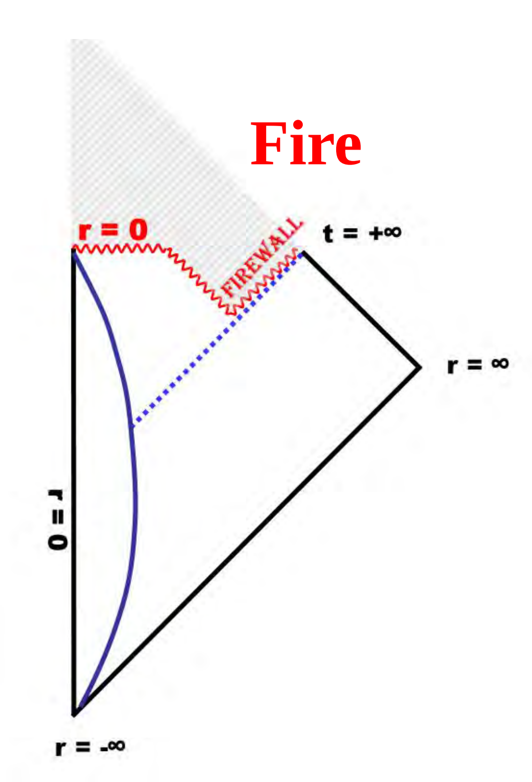
Also:

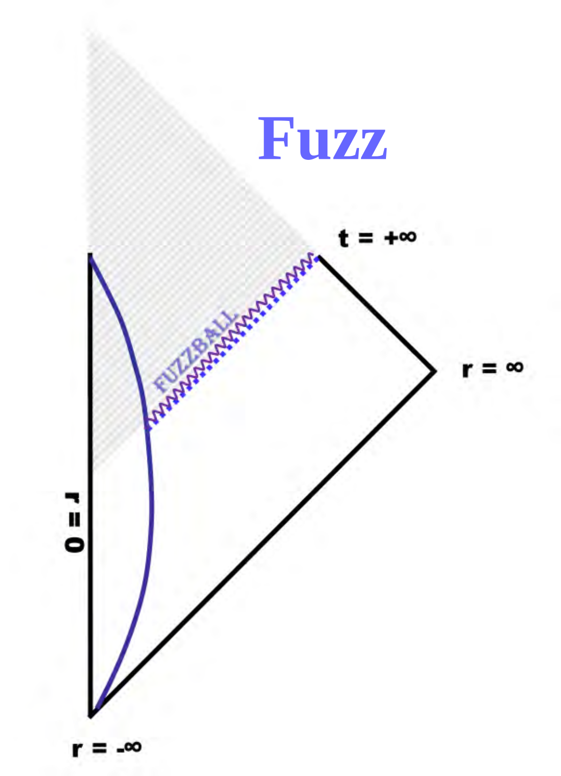
- Particle production
- Fate of falling object
- Mean free path

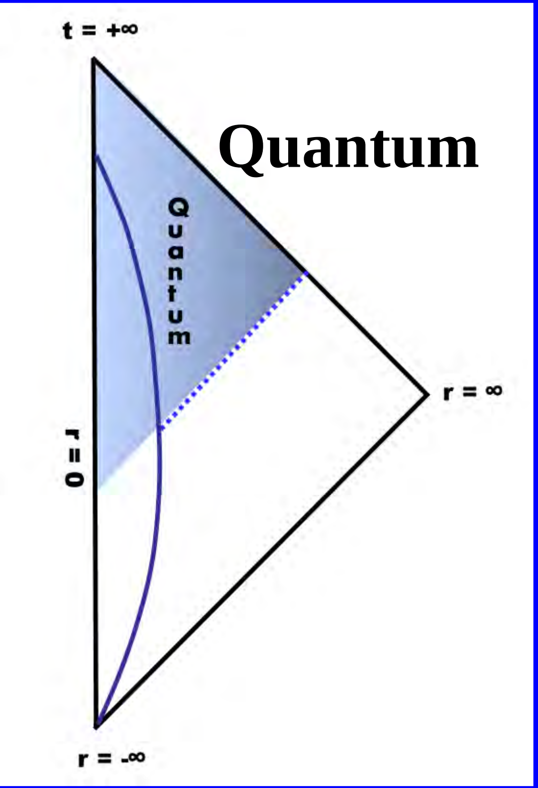
State of the interior

Comparison with other models









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