(0,2) Dynamics From Four Dimensions

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in preparation

Introduction

- Field theories with four supercharges have been extensively studied over the years.
- Many of their properties have been elucidated using symmetries, anomalies, holomorphy, strong-weak coupling duality and related ideas.
- Important insights came from string theory, where many such theories can be realized as low energy theories on branes.
- Deep connections were discovered between such theories in different dimensions.

- Natural to ask how much of this progress can be extended to theories with two supercharges, which exist in $d \leq 3$.

- In three dimensions such theories have N=1 SUSY; many of the above techniques are inapplicable to them.

- In two dimensions, (1,1) or (0,2) SUSY. The former suffers from the same problems. The latter is more promising.

- In this talk we will explore the 2d – 4d connection for (0,2) supersymmetric theories.

A natural way to associate a (0,2) model to a 4d N=1 SQFT with a global U(1) symmetry:

1) Couple the U(1) current supermultiplet to an external vector superfield, which consists of a vector field, gaugino and auxiliary field:

$$(A_\mu, \lambda_\alpha, D)$$

2) Compactify the theory on a two-torus and turn on a magnetic field for the U(1) through the torus:

$$F_{12} = B$$

This breaks supersymmetry completely, but it can be restored by

3) Turning on the auxiliary field D in the U(1) vector superfield. The variation of the background gaugino is

$$\delta\lambda = (F_{\mu\nu}\sigma^{\mu\nu} + iD)\epsilon$$

or

$$\delta\lambda = i \begin{pmatrix} D - B & 0 \\ 0 & D + B \end{pmatrix} \begin{pmatrix} \epsilon_- \\ \epsilon_+ \end{pmatrix}$$

For B=D the background preserves (0,2) SUSY. The purpose of this work is to explore the resulting theories.

(0,2) SUSY in two dimensions

- (0,2) superspace:

bosonic coordinates: (x^0, x^3)

fermionic coordinates: $(\theta^+, \bar{\theta}^+)$

right-moving supercharges:

$$Q_+ = \frac{\partial}{\partial \theta^+} + i\overline{\theta}^+(\partial_0 + \partial_3)\ , \qquad \overline{Q}_+ = -\frac{\partial}{\partial \overline{\theta}^+} - i\theta^+(\partial_0 + \partial_3)$$

superspace covariant derivatives:

$$D_+ = \frac{\partial}{\partial \theta^+} - i\overline{\theta}^+(\partial_0 + \partial_3) \ , \qquad \overline{D}_+ = -\frac{\partial}{\partial \overline{\theta}^+} + i\theta^+(\partial_0 + \partial_3)$$

In the presence of a gauge field:

$$\mathcal{D}_0 + \mathcal{D}_3 = \partial_0 + \partial_3 + i(A_0 + A_3) \ ,$$

$$\mathcal{D}_+ = \frac{\partial}{\partial \theta^+} - i\overline{\theta}^+(\mathcal{D}_0 + \mathcal{D}_3) \ ,$$

$$\overline{\mathcal{D}}_+ = -\frac{\partial}{\partial \overline{\theta}^+} + i\theta^+(\mathcal{D}_0 + \mathcal{D}_3) \ ,$$

$$\mathcal{D}_0 - \mathcal{D}_3 = \partial_0 - \partial_3 + iV \ ,$$

V is a (0,2) vector superfield:

$$V = A_0 - A_3 - 2i\theta^+\overline{\lambda}_- - 2i\overline{\theta}^+\lambda_- + 2\theta^+\overline{\theta}^+ D$$

It is usually described in terms of a field strength:

$$\Upsilon = [\overline{D}_+, D_0 - D_3] = -2(\lambda_- - i\theta^+(D - iF_{03}) - i\theta^+\overline{\theta}^+(D_0 + D_3)\lambda_-),$$

which satisfies the chirality condition

$$\overline{\mathcal{D}}_+\Upsilon = 0.$$

Two types of matter superfields in a representation r of a gauge group G play a role in our discussion:

- Chiral superfield:

$$\Phi = \phi + \sqrt{2}\theta^+\psi_+ - i\theta^+\overline{\theta}^+(D_0 + D_3)\phi$$

- Fermi superfield:

$$\Lambda = \psi_- - \sqrt{2}\theta^+ F - i\theta^+\overline{\theta}^+(D_0 + D_3)\psi_- - \sqrt{2}\overline{\theta}^+ E$$

which satisfies the chirality constraint

$$\overline{\mathcal{D}}_+\Lambda = \sqrt{2}E \ , \qquad \overline{\mathcal{D}}_+E = 0$$

- Actions:

$$S_\Upsilon = \frac{1}{8g^2}\mathrm{Tr}\int d^2x d^2\theta \overline{\Upsilon}\Upsilon \ ,$$

$$S_\Phi = -\frac{i}{2}\int d^2x d^2\theta \overline{\Phi}(\mathcal{D}_0 - \mathcal{D}_3)\Phi \ ,$$

$$S_\Lambda = -\frac{1}{2}\int d^2x d^2\theta \overline{\Lambda}\Lambda \ ,$$

In components:

$$S_\Upsilon = \frac{1}{g^2}\mathrm{Tr}\int d^2x \left\{ \frac{1}{2}F_{03}^2 + i\overline{\lambda}_-(D_0 + D_3)\lambda_- + \frac{1}{2}D^2 \right\} \ ,$$

$$S_\Phi = \int d^2x \left\{ -|D_\mu\phi|^2 + i\overline{\psi}_+(D_0 - D_3)\psi_+ - i\sqrt{2}\overline{\phi}T^a\lambda_-^a\psi_+ + i\sqrt{2}\phi T^a\overline{\psi}_+\overline{\lambda}_-^a + \overline{\phi}T^a\phi D^a \right.$$

$$S_\Lambda = \int d^2x \left\{ i\overline{\psi}_-(D_0 + D_3)\psi_- + |\mathcal{F}|^2 - |E|^2 - \left(\overline{\psi}_-\frac{\partial E}{\partial \phi_i}\psi_{+i} + \overline{\psi}_{+i}\frac{\partial \overline{E}}{\partial \overline{\phi}_i}\psi_- \right) \right\} \ .$$

- FI term for a U(1):

$$S_{\rm FI} = \frac{t}{4}\int d^2x d\theta^+ \Upsilon \Big|_{\overline{\theta}^+=0} + {\rm c.c.} = \frac{it}{2}\int d^2x (D - iF_{01}) + {\rm c.c.}$$

$$t = ir + \frac{\theta}{2\pi}$$

- (0,2) superpotential:

$$S_{\mathcal{W}} = -\frac{1}{\sqrt{2}}\int d^2x d\theta^+ \Lambda_a J^a \Big|_{\overline{\theta}^+=0} + {\rm c.c.} = -\int d^2x \left\{ \mathcal{F}_a J^a(\phi_i) + \psi_{-a}\psi_{+i}\frac{\partial J^a}{\partial \phi_i} \right\} + {\rm c.c.}$$

- Reduction of (2,2) superfields under (0,2) SUSY:

Before turning on the background B and D fields, the theories we will study have (2,2) supersymmetry. The background superfields split the (2,2) multiplets into (0,2) ones.

The (2,2) twisted chiral superfield $\Sigma^{(2,2)}$ that describes the gauge multiplet decomposes as

$$\Sigma^{2,2} \to (\Sigma, \Upsilon)$$

a chiral superfield in the adjoint, and the (0,2) field strength.

The (2,2) chiral superfield splits into a (0,2) chiral superfield and a Fermi superfield

$$\Phi^{(2,2)} \to (\Phi, \Lambda)$$

The action of a (2,2) chiral superfield that transforms non-trivially under a gauge symmetry includes a non-zero E,

$$E = i\sqrt{2}\Sigma^a T^a \Phi$$

Free fields in a magnetic field

Consider a free massless scalar field ϕ of charge e under a U(1) gauge field A_μ. We turn on a background magnetic field B,

$$A_2 = Bx_1.$$

The Klein-Gordon equation for ϕ takes the form:

$$(-\partial_0^2 + \partial_3^2 + \partial_1^2 + \widetilde{D}_2^2)\phi = 0$$

where $$\widetilde{D}_2 = \partial_2 + ieBx_1$$

To study the 1+1 dimensional spectrum we separate variables

$$\phi(x^0, x^3; x^1, x^2) = \varphi(x^0, x^3)\chi(x^1, x^2) \ .$$

Taking χ to be an eigenfunction of

$$H = -(\partial_1^2 + \widetilde{D}_2^2) = p_1^2 + (p_2 + eBx_1)^2 \ , \qquad H\chi = m^2\chi,$$

gives rise to a two dimensional scalar field φ of mass m. H is the Hamiltonian of a particle in a magnetic field. The spectrum is:

$$m_n^2 = (2n+1)|eB| \ .$$

To preserve supersymmetry, we need also to turn on a D field. This shifts the spectrum to

$$m_n^2 = (2n+1)|eB| - eD \ .$$

For B=D (the (0,2) supersymmetric case), eB>0 gives a massless particle, while for eB<0 the spectrum is massive.

Can repeat the analysis for free fermions. Get

$$m_+^2 = (2n+1)|eB| - eB, \qquad m_-^2 : (2n+1)|eB| + eB$$

For right (+) and left (-) movers in 2d.

Summary:

A four dimensional free massless chiral superfield Φ with U(1) charge e reduces in a constant B=D>0 field to:

e>0: massless (0,2) chiral superfield.

e<0: massless (0,2) Fermi superfield.

Compactification:

So far we discussed the effects of the magnetic field in non - compact spacetime. We are actually interested in turning on a magnetic field on a torus. This leads to Dirac quantization:

$$eB\mathcal{A} \in 2\pi \mathrm{Z}$$

The states have degeneracy

$$n_e = |e|B\mathcal{A}/2\pi \ .$$

Below, we will normalize the charges such that the degeneracy is |e|.

Four dimensional N=1 theories on a magnetized torus

We start with a four dimensional N=1 supersymmetric gauge theory with gauge group G and matter fields Φ_i in representations r_i. We compactify on a two-torus, and turn on a magnetic field to the U(1) global symmetry under which Φ_i have charges e_i. The symmetry must be non-anomalous:

$$\sum_i e_i T(r_i) = 0 \ ,$$

$$\mathrm{Tr}_r T^a T^b = T(r)\delta^{ab}, \text{ with } a, b = 1, \cdots, \mathrm{dim}\ G$$

The two dimensional matter content is:

1) A gauge superfield Υ.

2) An adjoint chiral superfield Σ.

3) e_i (0,2) chiral superfields Φ_i in the representation r_i for fields with $e_i > 0$.

4) $|e_i|$ Fermi superfields Λ_i in the representation r_i for $e_i < 0$.

5) A chiral superfield Φ_i and a Fermi superfield Λ_i for $e_i = 0$.

Example: N=1 SQCD

Gauge group $G = U(N_c)$, N_f flavors of fundamentals $Q^i, \widetilde{Q}_i$.

Global symmetry:

$$SU(N_f)_L \times SU(N_f)_R \ .$$

The U(1) we use assigns charges e_i to Q^i and $\widetilde{e}_i$ to $\widetilde{Q}_i$.

Anomaly freedom implies:

$$\sum_i e_i = \sum_i \widetilde{e}_i = 0$$

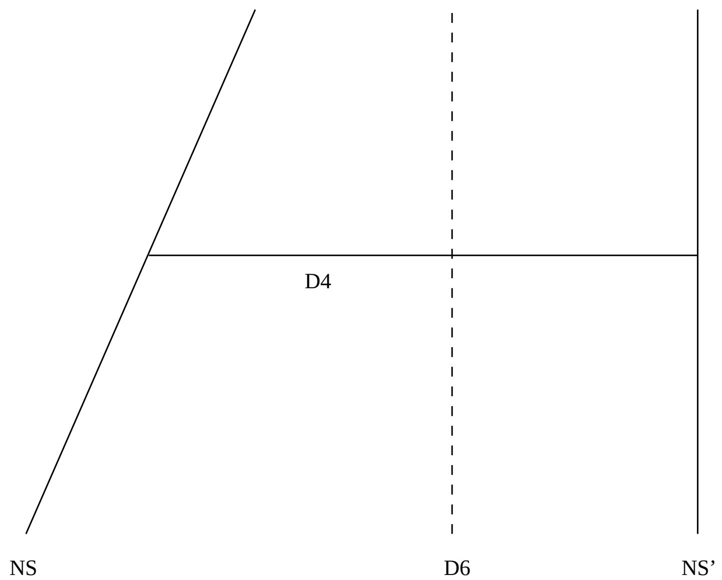
E.g. can take half of the e_i to be +1 and half to be -1; same for $\widetilde{e}_i$.

Three dimensional description

Since we are planning to compactify to two dimensions, we can view the construction by starting with a 3d theory with N=2 SUSY, obtained by compactifying N=1 SQCD on a circle, and place it on an extra circle. The magnetic field $A_2 = Bx^1$ in the 4d (0123) becomes in the three dimensional theory in (013) an expectation value of a scalar field $\phi_2 = Bx^1$. This can be thought of as a position-dependent real mass term for the chiral superfields.

This three dimensional description is useful for analyzing the dynamics.

The gauge theories in question have a useful string theory description as theories on systems of intersecting D-branes and NS5-branes. Four dimensional N=1 SQCD is described by:



$$N_c\ D4 : (01236); \quad N_f\ D6 : (0123789)$$

$$NS : \ (012345); \quad NS' : \ (012389)$$

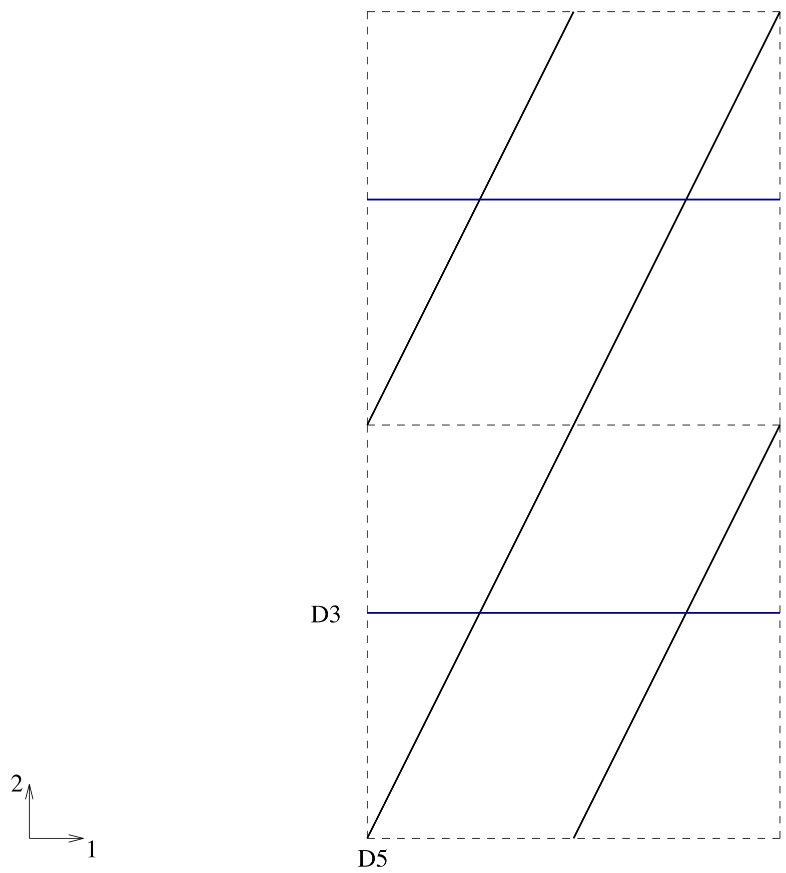
The three dimensional theory is obtained by replacing:

D4 (01236) --> D3 (0136)

D6 (0123789) --> D5 (013789)

We want to compactify x^1 on a circle and turn on the background fields B and D.

Turning on the B field corresponds in the brane system to rotating the D5-branes in the (12) plane:



- The D field corresponds to a rotation of the D5-branes in the (67) plane.

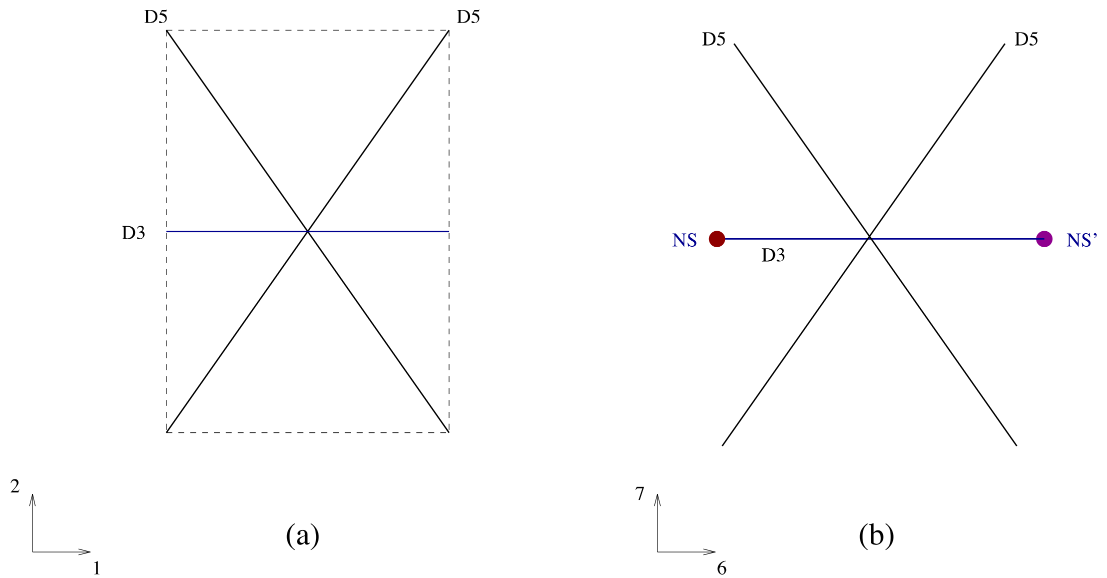
- Quantization of B, localization of the fundamentals on the torus, and the degeneracy |e| are manifest in the brane picture.

- The fact that the configuration preserves (0,2) SUSY in (03) is the familiar fact that a rotation in two complex planes preserves ½ of SUSY. Define $z_1 = x^1 + ix^2,\ z_2 = x^7 + ix^6$ and rotate:

$$z_1 \to e^{i\theta} z_1; \qquad z_2 \to e^{-i\theta} z_2$$

An example

We are now ready to discuss a full model of this sort. We will look for simplicity at the special case of U(1) gauge theory with two flavors, $Q^i,\ \tilde{Q}_i,\ i=1,2.$ The brane picture is:



(a) (b)

The two dimensional theory has gauge group U(1), two chiral superfields Q^1, $\widetilde{Q}_2$, and two Fermi superfields Λ^2, $\widetilde{\Lambda}_1$. Classically, the theory has a Coulomb branch, corresponding to the position of the D3-brane in X^2, and a Higgs branch corresponding to vanishing D-term

$$D = |Q^1|^2 - |\widetilde{Q}_2|^2 \ .$$

This Higgs branch can be parametrized by the gauge invariant meson field

$$M^1_2 = Q^1 \widetilde{Q}_2 \ .$$

- Quantum mechanically, the Coulomb branch is lifted (perturbatively).

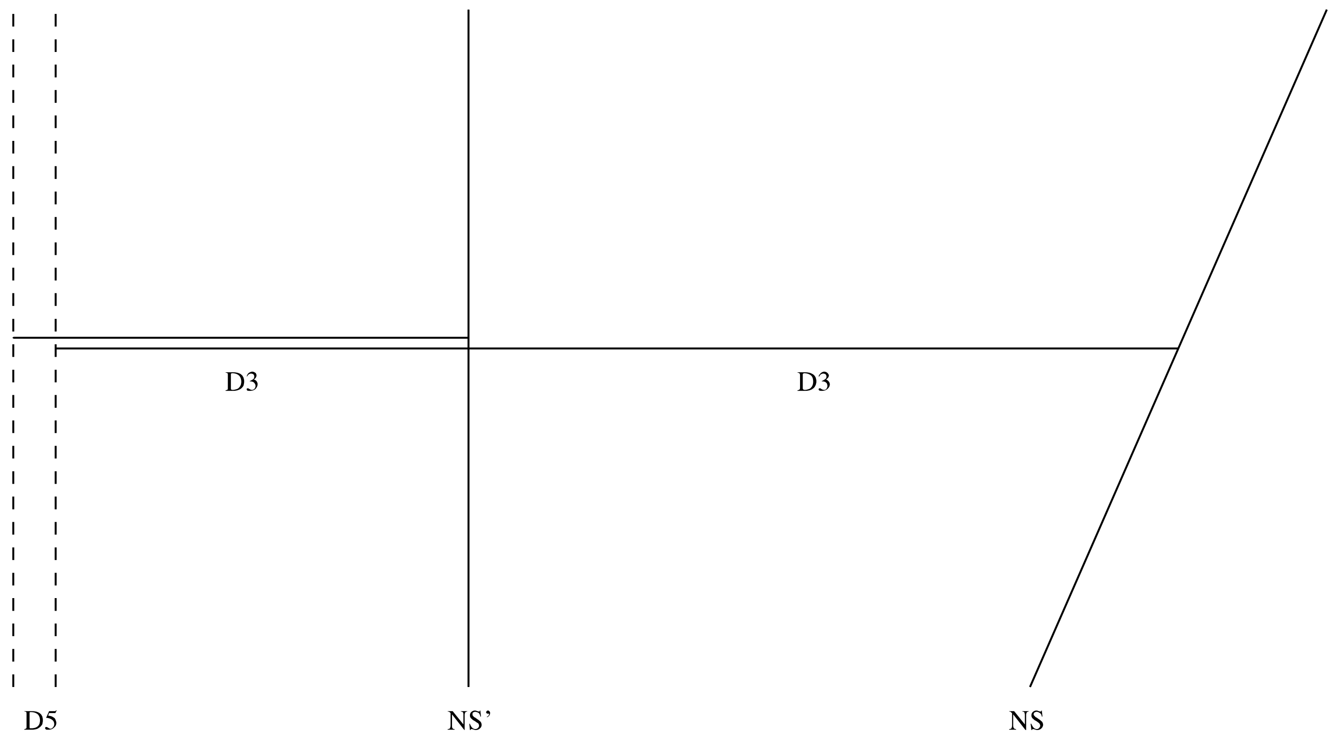
- The Higgs branch survives; it is a theory of one chiral superfield M_2^1, and a Fermi superfield Λ_1^2. The central charge of the infrared theory is

$$c_L = c_R = 3 \ .$$

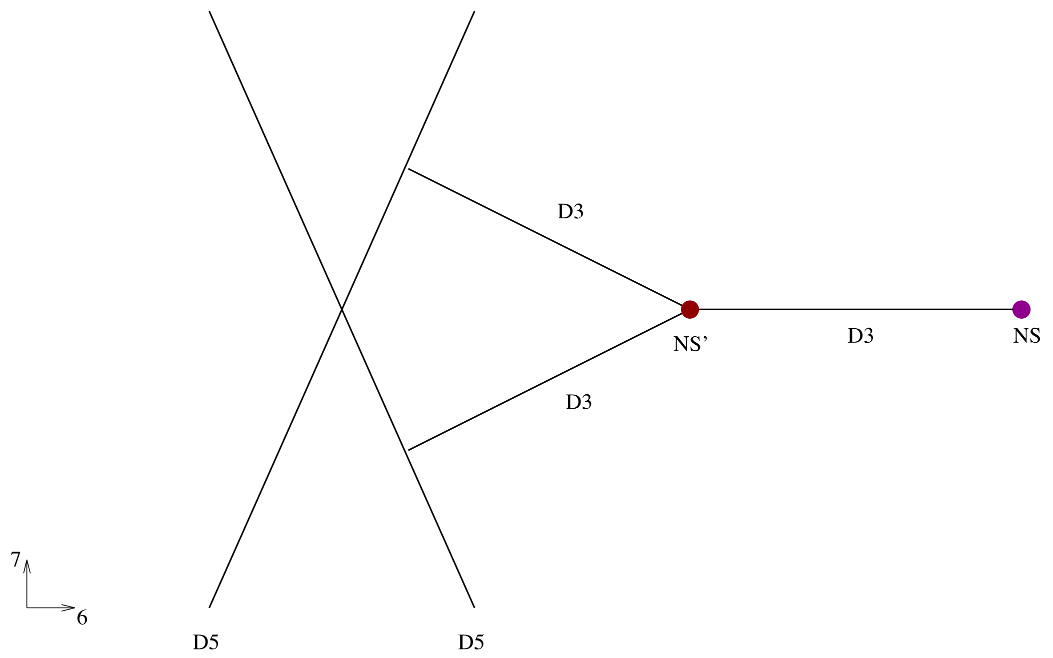
- In three dimensions, the theory we discuss has a dual (Aharony) which is another U(1) gauge theory with two flavors $q_i,\ \widetilde{q}^i,\ i = 1, 2,$ and a singlet chiral superfield M_j^i coupled to the charged fields via the superpotential

$$\mathcal{W} = M_j^i q_i \widetilde{q}^j \ .$$

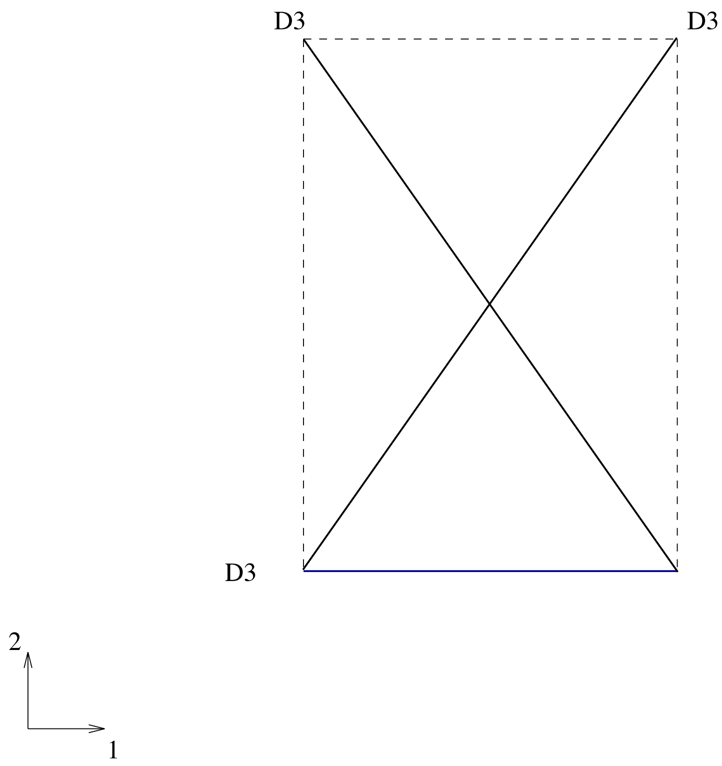
It is interesting to see what happens to this duality when we turn on the background B and D fields. It is convenient to do this using the brane construction. The brane system corresponding to 3d N=2 SQCD is:



After turning on the B and D fields we find the brane configuration:



It is useful to exhibit it in the (12) plane at the location of the NS’-brane:



In black are flavor D3-branes. Blue: color D3-brane.

- Coulomb branch is again lifted, and is replaced by two points, where the color D3-brane intersects the flavor ones at a point.

- The vacuum drawn corresponds to the electric one from before.

- Dynamics at the upper intersection is simple: at the intersection of the flavor D3's we have a chiral superfield M_2^1, and a Fermi superfield Λ_1^2 , in agreement with the electric theory.

- At the lower intersection we have a theory of the same sort as the electric one analyzed before (U1 gauge theory with two flavors $q_i, \ \widetilde{q}^i$), coupled to a magnetic meson (M_2^1, Λ_1^2).

This theory can be analyzed in two steps:

1) Turn off the coupling of M to the q's. The resulting theory is the same as the electric one analyzed above. It is a theory of a meson field $N_2^1 = q_2 \widetilde{q}^1$ and a Fermi superfield $\widehat{\Lambda}_1^2$.

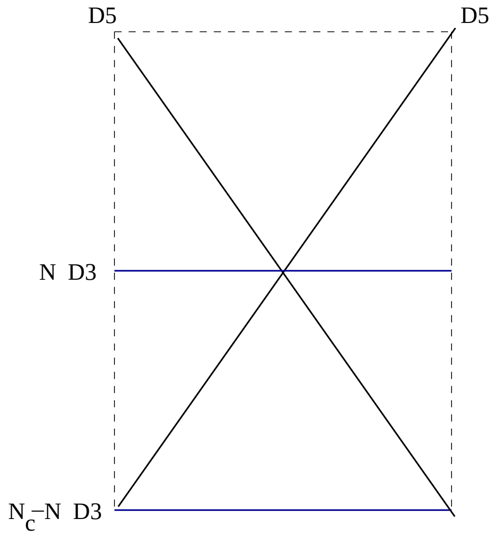
2) Now add the coupling of the magnetic meson M to the quarks. In terms of the gauge invariant fields the superpotential term is

$$\mathcal{L}_W = \int d\theta^+ \left(\Lambda_1^2 N_2^1 + \widehat{\Lambda}_1^2 M_2^1 \right) \ .$$

This is nothing but a mass term for all fields. Thus, the low energy theory at this intersection is empty, in agreement with electric one.

Comments

- Can generalize the discussion to higher $N_f, \; N_c$. The Coulomb branch again collapses to a discrete set of vacua, which are described in the brane language by



The integer N runs over the range

$$\max(0, N_c - \frac{1}{2}N_f) \leq N \leq \min(N_c, \frac{1}{2}N_f)$$

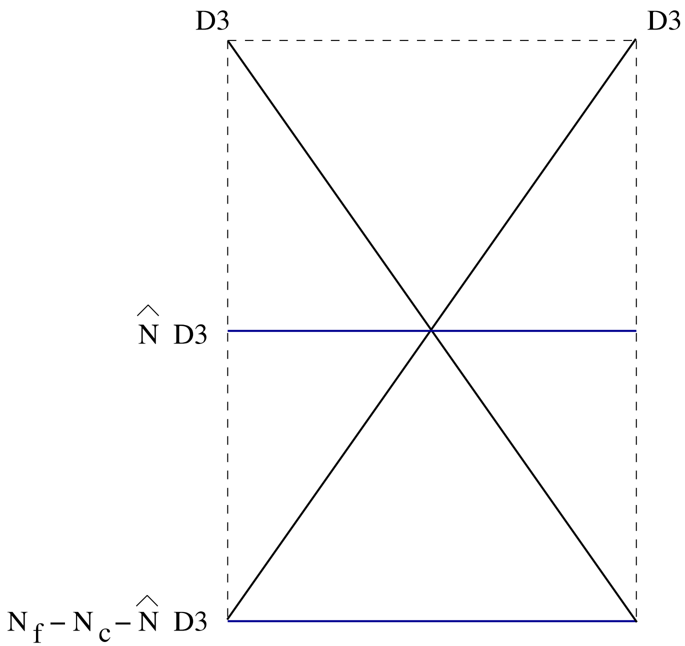
The IR theory at one of the intersections is a sigma model on the Higgs branch of a U(N) gauge theory. It has central charge

$$c_R = c_L = 3(N_f N - N^2) \ .$$

At the other intersection there is a similar theory with

$$N \to N_c - N.$$

- The magnetic theory can also be studied for all $N_f,\ N_c$. The theory has again multiple vacua described in terms of the branes by



with

$$\max(0, \frac{1}{2}N_f - N_c) \leq \widehat{N} \leq \min(N_f - N_c, \frac{1}{2}N_f)\ .$$

Electric-magnetic duality holds, with the vacuum map

$$\widehat{N} = \frac{N_f}{2} - N \ .$$

- The two dimensional theory has a global symmetry group

$$SU(N_f/2)_L \times SU(N_f/2)_L \times U(1)_L \times SU(N_f/2)_R \times SU(N_f/2)_R \times U(1)_R \ .$$

One can calculate the `t Hooft anomalies for these symmetries, and check that they match in the electric and magnetic theories.

- The right-moving N=2 supersymmetry implies the existence of a $U(1)_R$ current, whose anomaly is related to the central charge of the IR theory. It is conserved in the full theory, and can be identified in both the electric and magnetic theories.

- There are many additional properties of the two dimensional (0,2) theories obtained this way that I didn’t have time to describe.