Anomalies and hydrodynamics

Amos Yarom

(Together with K. Jensen, R. Loganayagam)

Anomalies

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In 3+1 dimensions:

$$\nabla_\mu J^\mu = 0$$

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$$\nabla_\mu J^\mu = \frac{3}{4} c_A \epsilon^{\kappa\sigma\alpha\beta} F_{\kappa\sigma} F_{\alpha\beta}$$

Anomalies

In 3+1 dimensions:

$$\nabla_\mu T^{\mu\nu} = F^{\nu\alpha} J_\alpha$$

$$\nabla_\mu J^\mu = \frac{3}{4} c_A \epsilon^{\kappa\sigma\alpha\beta} F_{\kappa\sigma} F_{\alpha\beta}$$

Anomalies

In 3+1 dimensions:

$$\nabla_\mu T^{\mu\nu} = F^{\nu\alpha} J_\alpha + \frac{1}{2} c_m \epsilon^{\kappa\sigma\alpha\beta} \nabla_\mu \left(F_{\kappa\sigma} R^{\nu\mu}{}_{\alpha\beta} \right)$$

$$\nabla_\mu J^\mu = \frac{3}{4} c_A \epsilon^{\kappa\sigma\alpha\beta} F_{\kappa\sigma} F_{\alpha\beta} + \frac{1}{4} c_m \epsilon^{\kappa\sigma\alpha\beta} R^\nu{}_{\lambda\kappa\sigma} R^\lambda{}_{\nu\alpha\beta}$$

The role of anomalies in hydrodynamics

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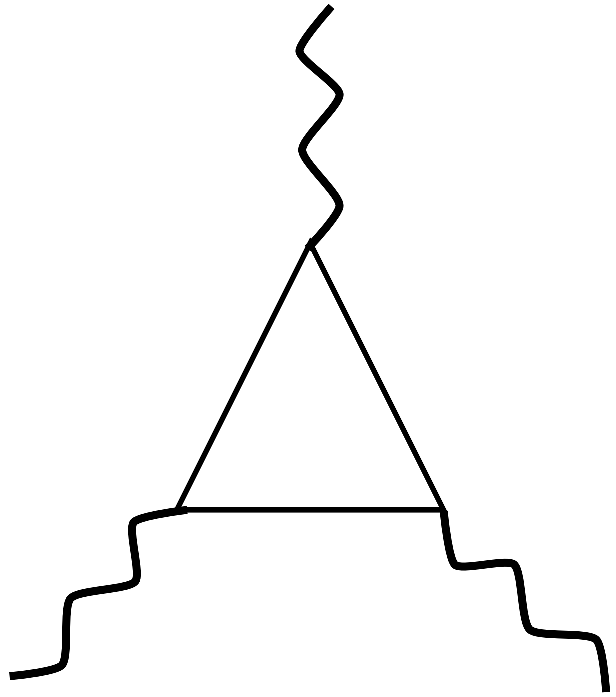

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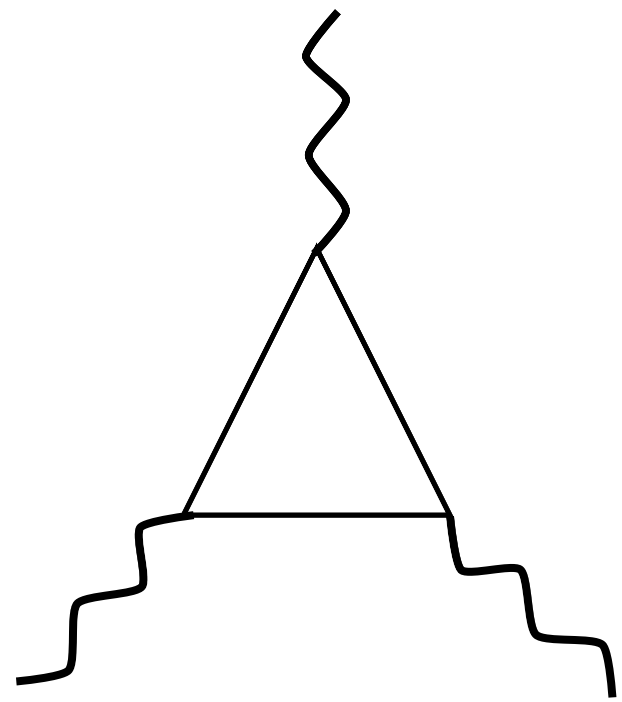

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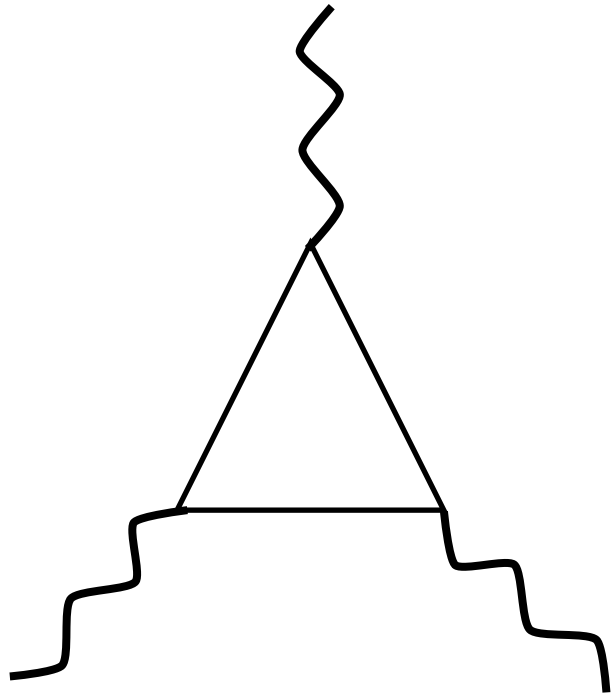
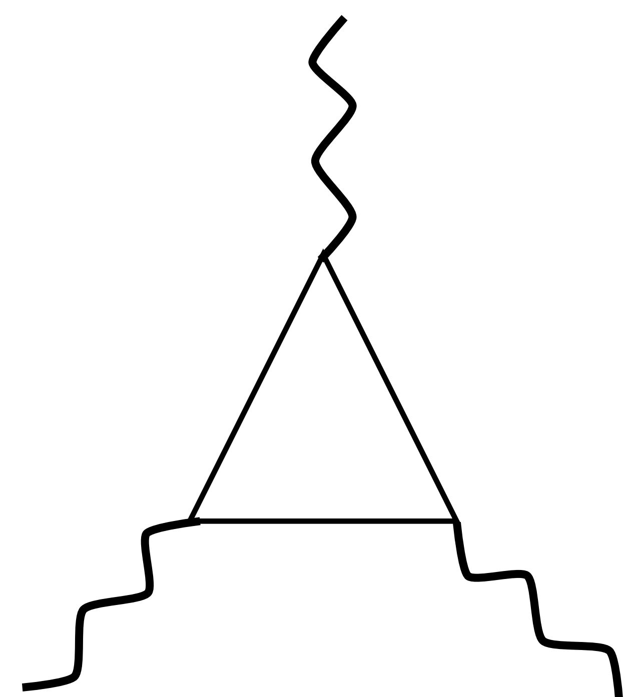


The role of anomalies in hydrodynamics

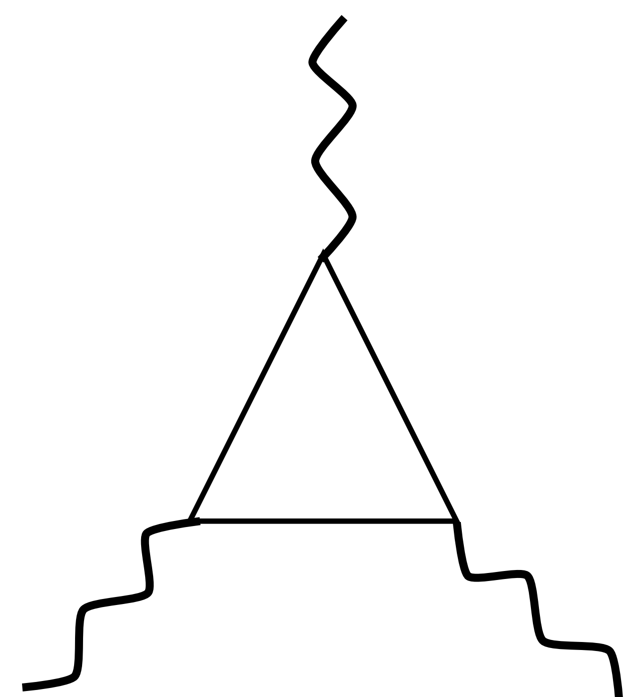
$$\vec{J} \sim \nabla \times \vec{v}$$



The role of anomalies in hydrodynamics

Vilenkin (1980)

$$\vec{J} \sim \nabla \times \vec{v}$$



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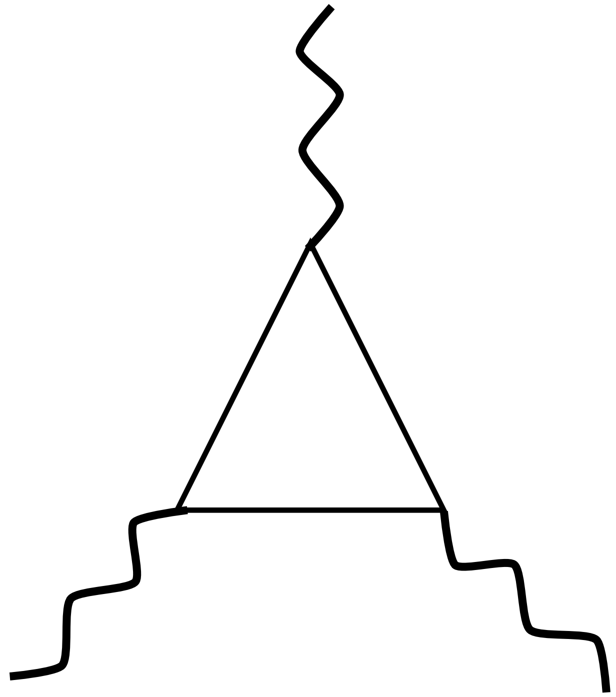

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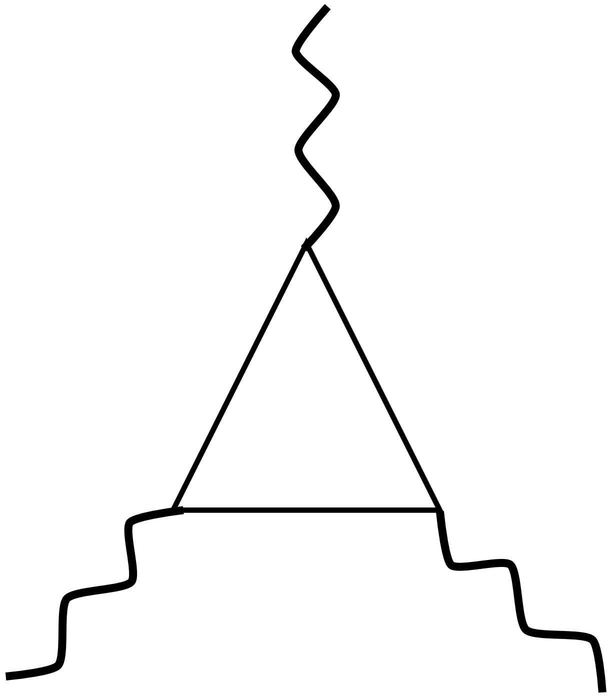

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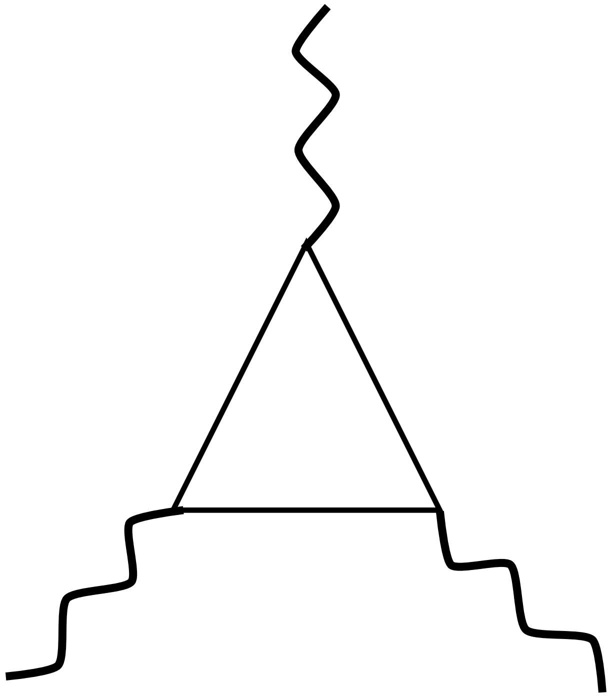

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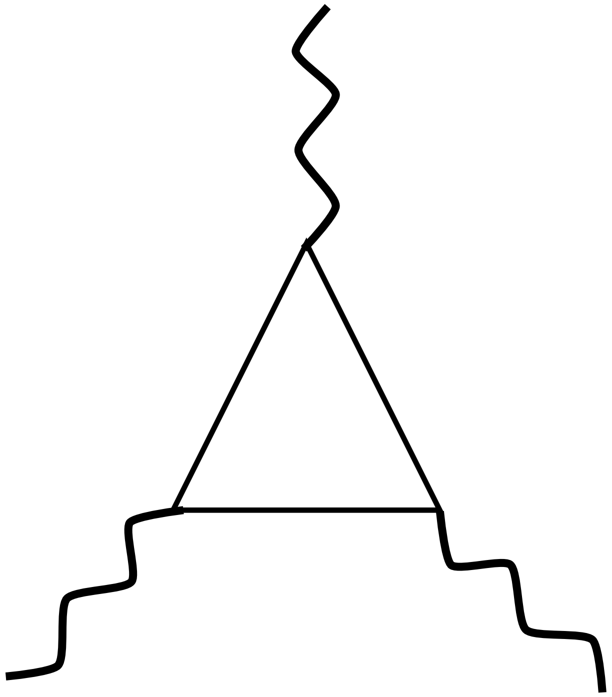

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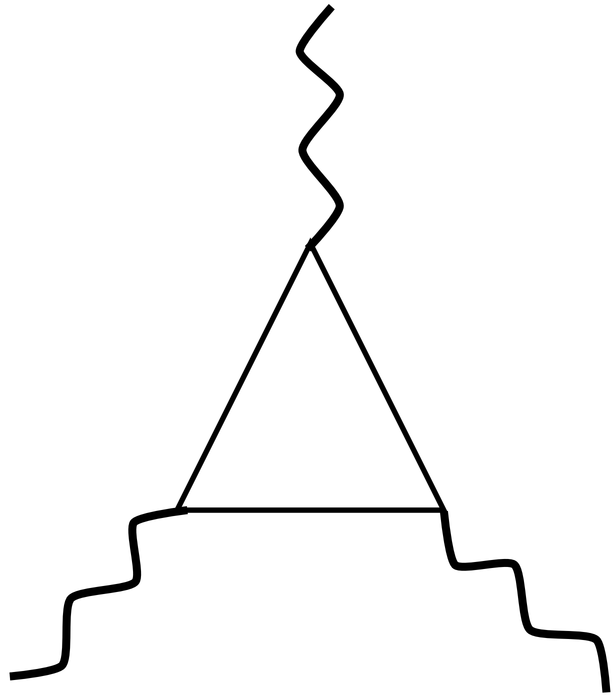

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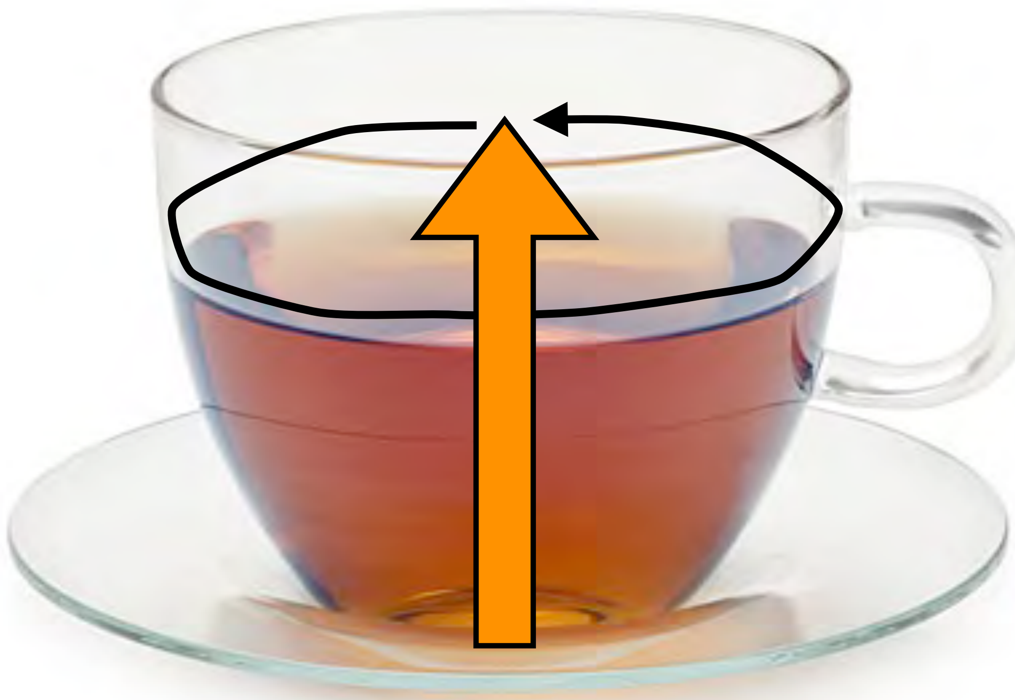

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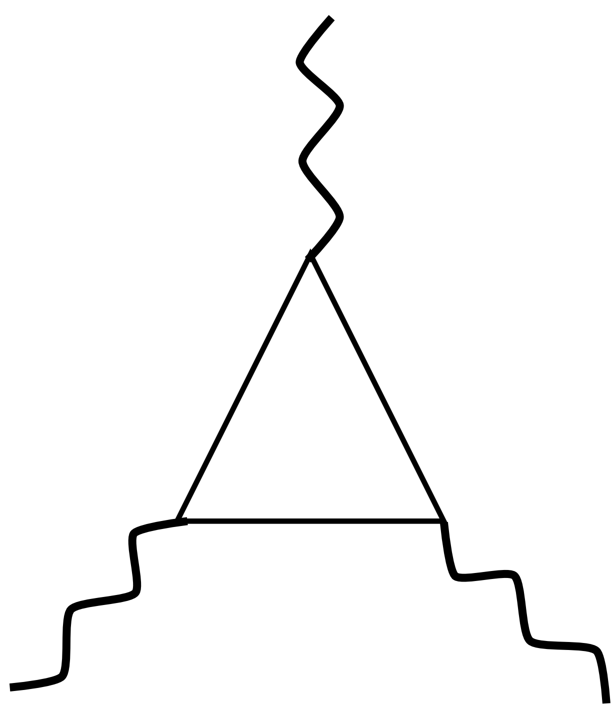

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The role of anomalies in hydrodynamics

Vilenkin (1980)

$$J^\mu = \rho u^\mu + (8\pi^2 c_m T^2 - c_A \mu^2)\epsilon^{\mu\nu\rho\sigma} u_\nu \partial_\rho u_\sigma$$

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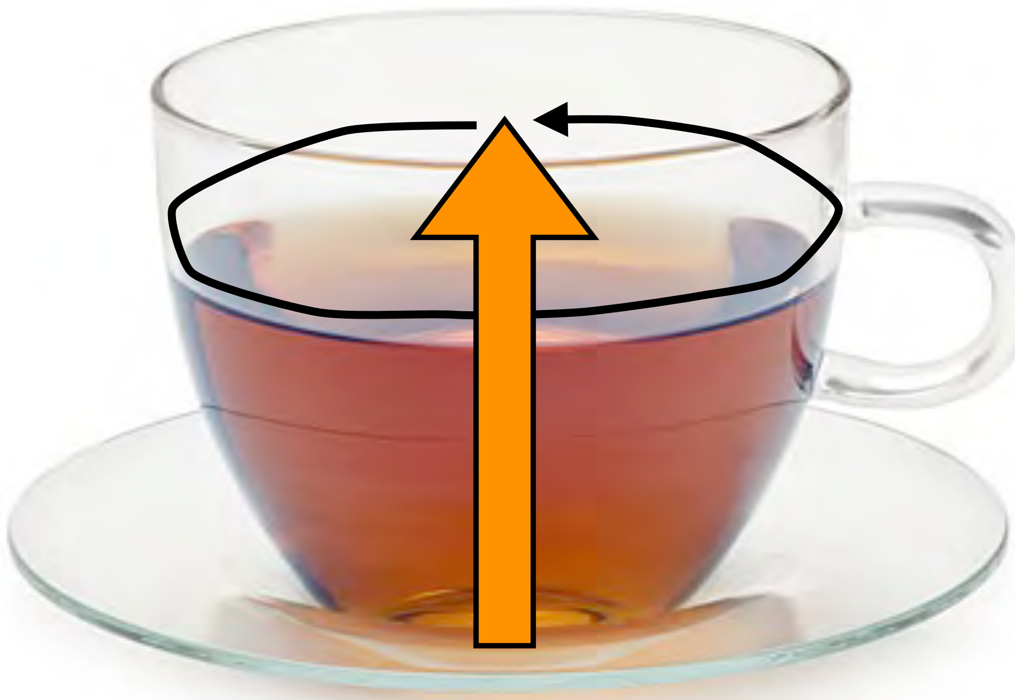

$$^*\mathbf{J} = \frac{\partial \mathbf{V}_T}{\partial \mathbf{B}}$$

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The role of anomalies in hydrodynamics

$$^*\mathbf{J} = \frac{\partial \mathbf{V}_T}{\partial \mathbf{B}} \qquad \mathbf{V}_T = \frac{\mathbf{u}}{2\mathbf{w}} \wedge \left(\mathbf{P}_T - \hat{\mathbf{P}}_T\right)$$

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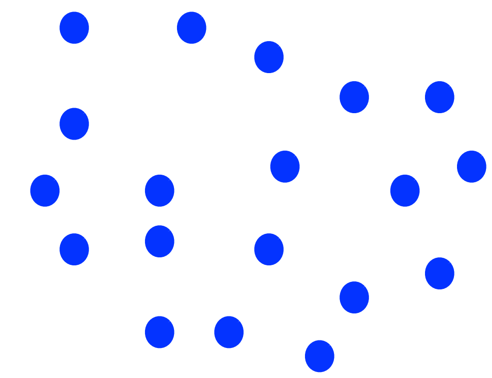
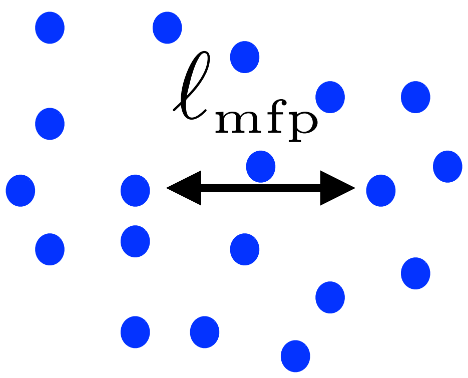
Plan

1. Hydrodynamics vs. hydrostatics.
2. A generating function for hydrostatics.
3. Components of the generating function.
4. Constructing the potential V_T.

Hydrodynamics



Hydrodynamics



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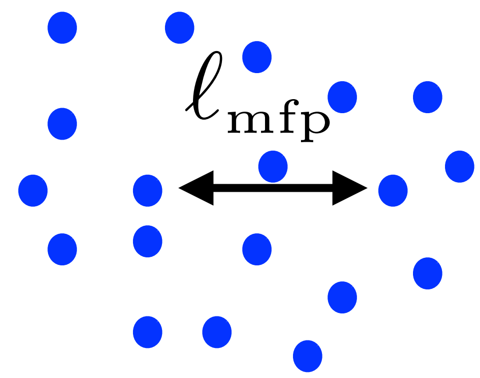
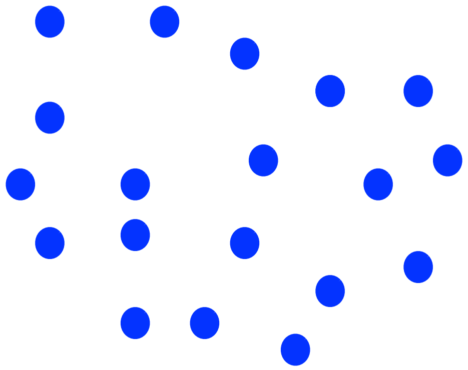


$$L \gg \ell_{\mathrm{mfp}}$$

Hydrodynamics



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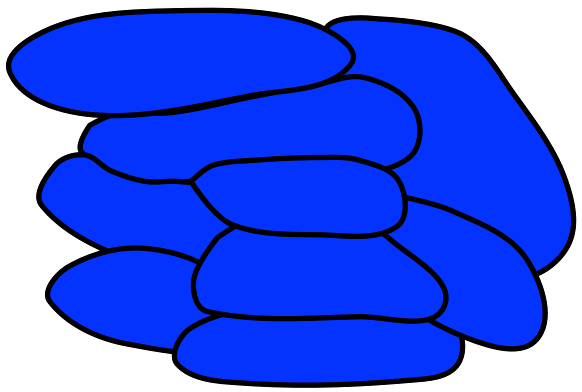

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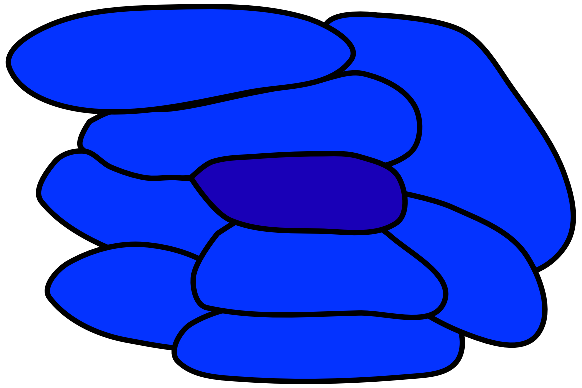
Hydrodynamics



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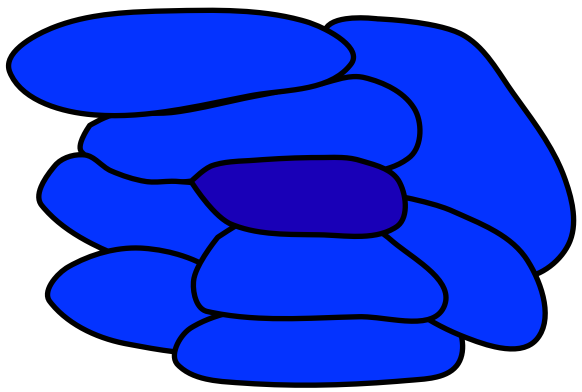
Hydrodynamics

$T(x^\alpha)$ Temperature



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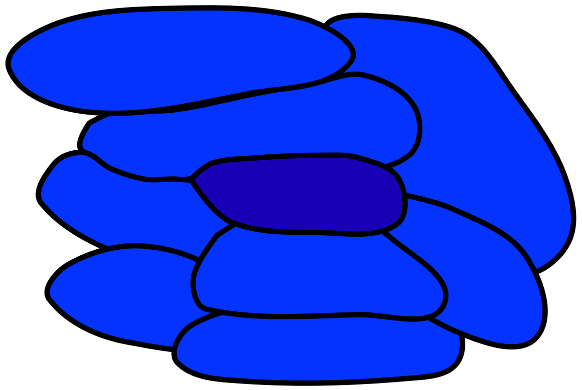
Hydrodynamics

$T(x^\alpha)$ Temperature

$\mu(x^\alpha)$ Chemical potential



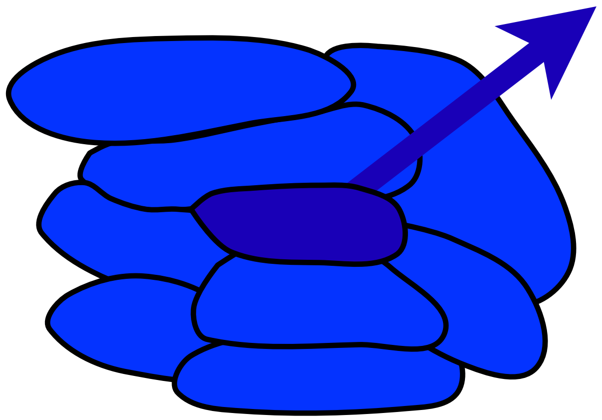
$$L \gg \ell_{\mathrm{mfp}}$$

Hydrodynamics

$T(x^\alpha)$ Temperature

$\mu(x^\alpha)$ Chemical potential

$u^\nu(x^\mu)$ Velocity field



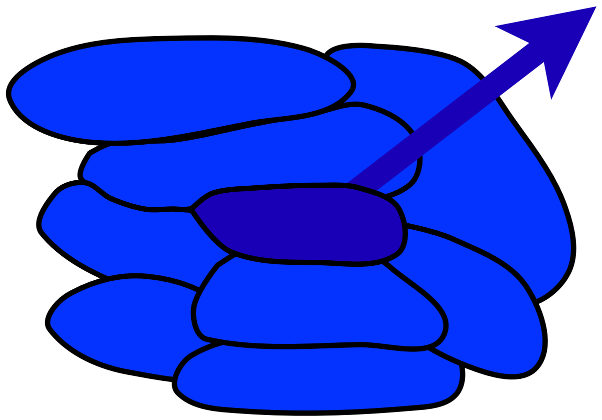
$L \gg \ell_{\rm mfp}$

Hydrodynamics

$T(x^\alpha)$ Temperature

$\mu(x^\alpha)$ Chemical potential

$u^\nu(x^\mu)$ Velocity field $(u_\mu u^\mu = -1)$



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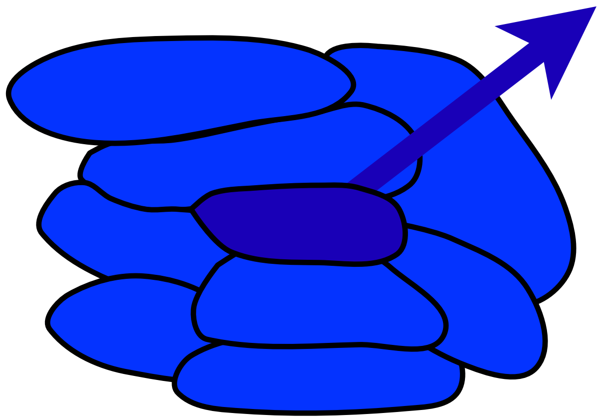

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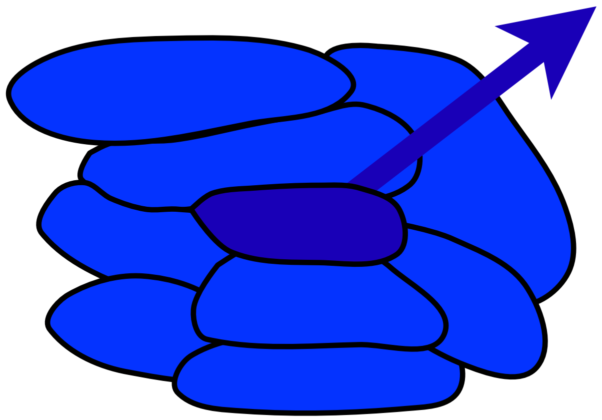
$L \gg \ell_{\mathrm{mfp}}$

$T^{\mu\nu}[u^\alpha, \mu, T]$

$J^\mu[u^\alpha, \mu, T]$

Hydrodynamics



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$L \gg \ell_{\rm mfp}$

$T^{\mu\nu}[u^\alpha, \mu, T]$

$J^\mu[u^\alpha, \mu, T]$

$$\partial_\mu T^{\mu\nu} = 0$$

$$\partial_\mu J^\mu = 0$$

Hydrodynamics

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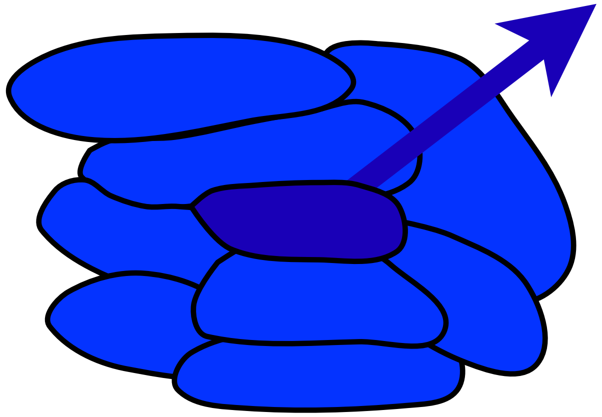


$$L \gg \ell_{\rm mfp}$$

To leading order the fields are uniform.

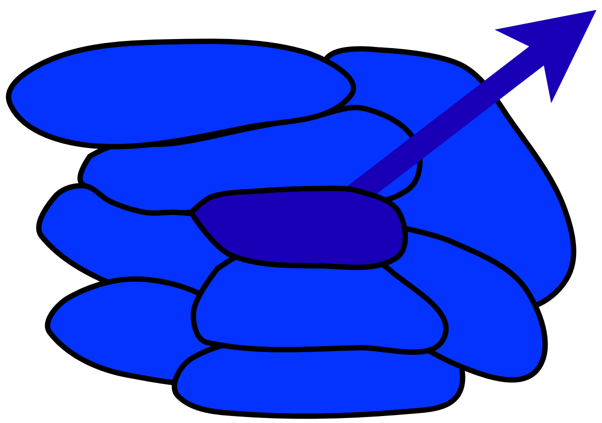
$$T^{\mu\nu}[u^\alpha, \mu, T]$$

$$J^\mu[u^\alpha, \mu, T]$$

$$\partial_\mu T^{\mu\nu} = 0$$

$$\partial_\mu J^\mu = 0$$

Hydrodynamics



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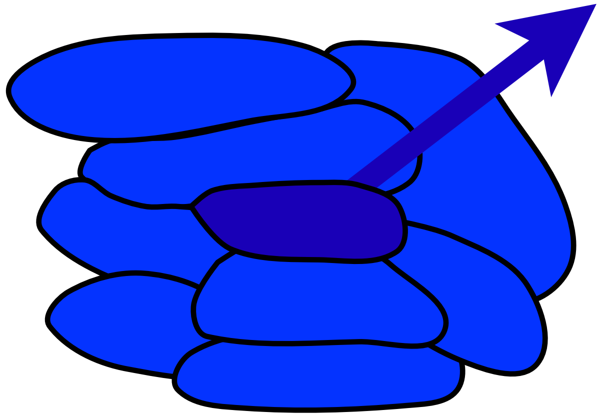

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$T^{\mu\nu}[u^\alpha, \mu, T]$

$J^\mu[u^\alpha, \mu, T]$

$\partial_\mu T^{\mu\nu} = 0$

$\partial_\mu J^\mu = 0$

We need to construct a vector out of:

$\mu,\ T,\ u^\mu$

Hydrodynamics

$T(x^\alpha)$ Temperature

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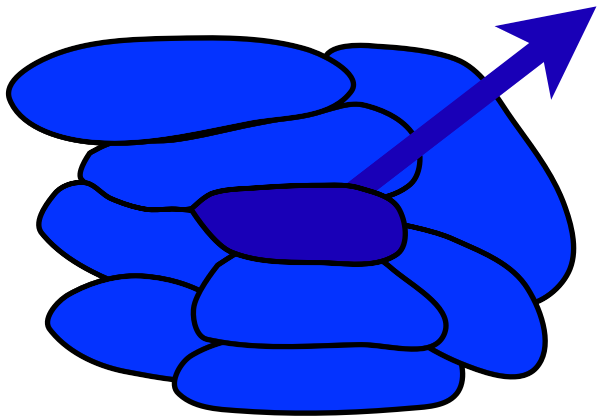
$u^\nu(x^\mu)$ Velocity field $(u_\mu u^\mu = -1)$



$L \gg \ell_{\rm mfp}$

To leading order the fields are uniform.

$T^{\mu\nu}[u^\alpha, \mu, T]$

$J^\mu[u^\alpha, \mu, T] = \rho(T,\mu) u^\mu$

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Hydrodynamics

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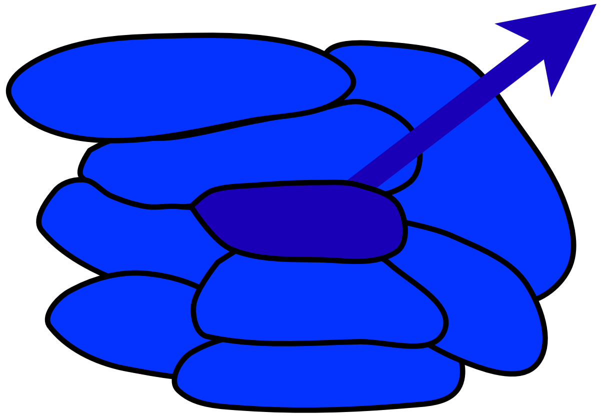

$L \gg \ell_{\rm mfp}$

To leading order the fields are uniform.

$T^{\mu\nu}[u^\alpha, \mu, T]$

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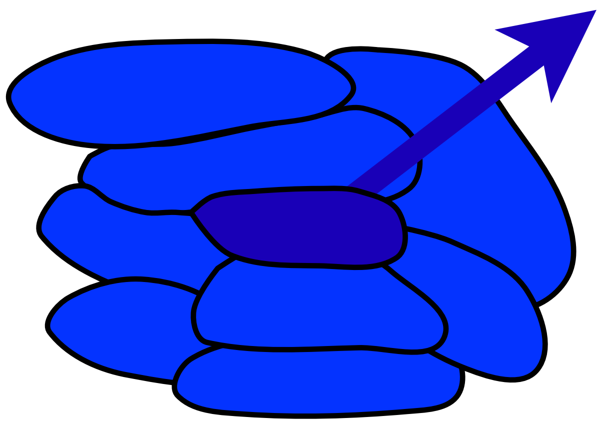

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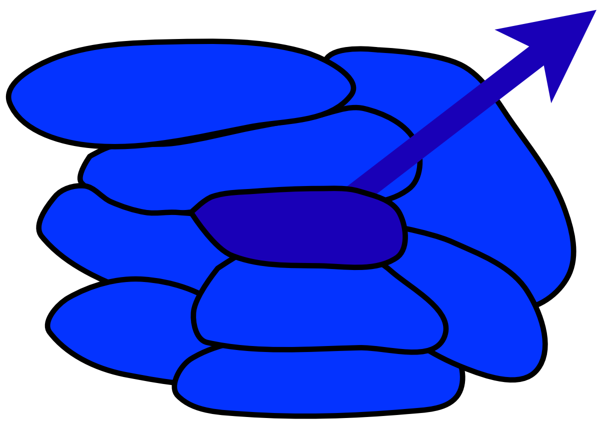

$$L \gg \ell_{\rm mfp}$$

To leading order the fields are uniform.

$$T^{\mu\nu}[u^\alpha, \mu, T] = \epsilon(T,\mu)u^\mu u^\nu + P(T,\mu)(u^\mu u^\nu + \eta^{\mu\nu}) + \mathcal{O}(\partial)$$

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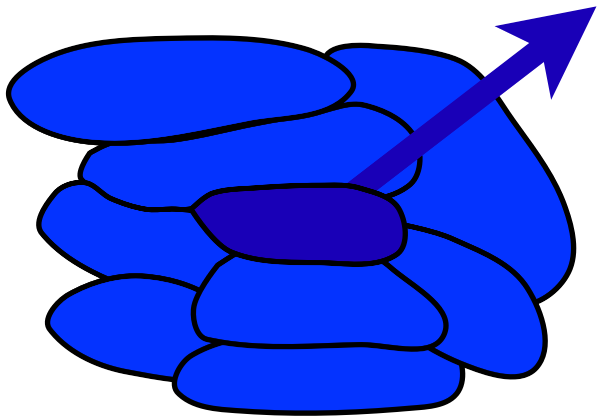

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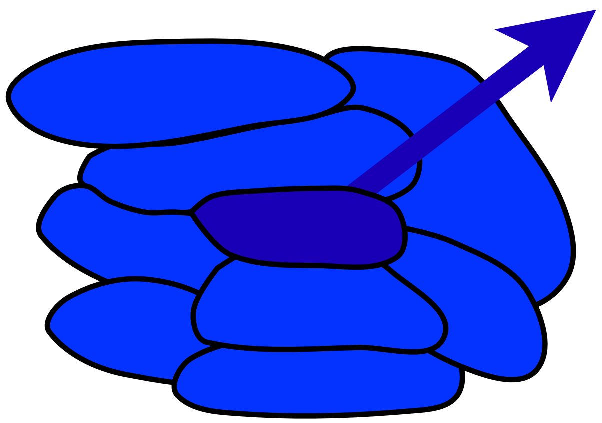

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$$J^\mu[u^\alpha, \mu, T] = \rho(T,\mu)u^\mu + \mathcal{O}(\partial)$$

$$u^\mu u^\nu + \eta^{\mu\nu} = P^{\mu\nu}$$

$$\partial_\mu T^{\mu\nu} = 0$$

$$\partial_\mu J^\mu = 0$$

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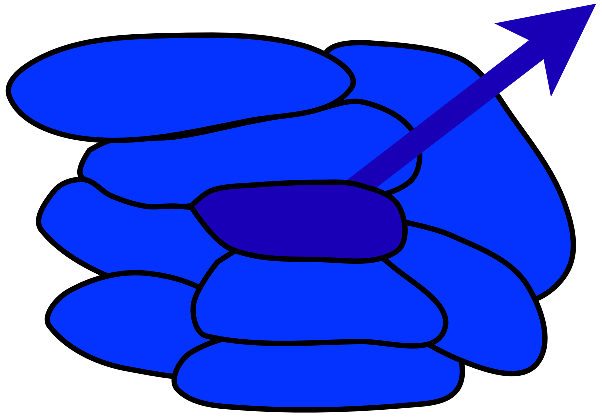

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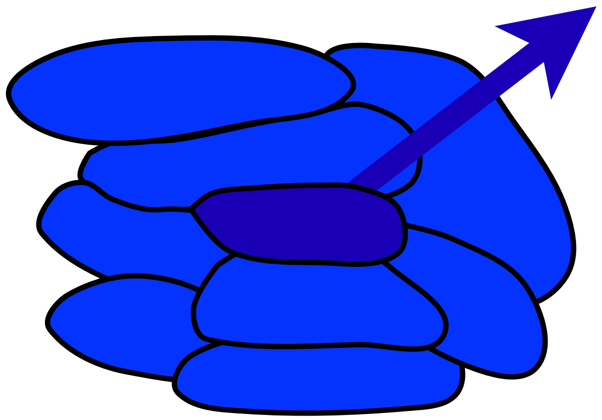

$T(x^\alpha)$ Temperature

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$u^\nu(x^\mu)$ Velocity field $(u_\mu u^\mu = -1)$

$L \gg \ell_{\rm mfp}$

At subleading order we allow slowly varying fields

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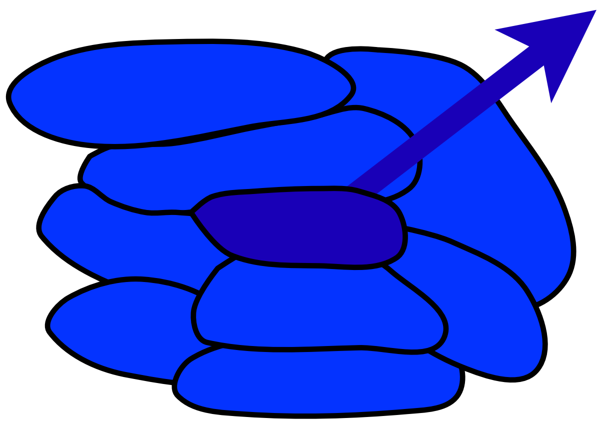
$$J^\mu[u^\alpha, \mu, T] = \rho(T,\mu)u^\mu + \mathcal{O}(\partial)$$

$$\partial_\mu T^{\mu\nu} = 0$$

$$\partial_\mu J^\mu = 0$$

Hydrodynamics



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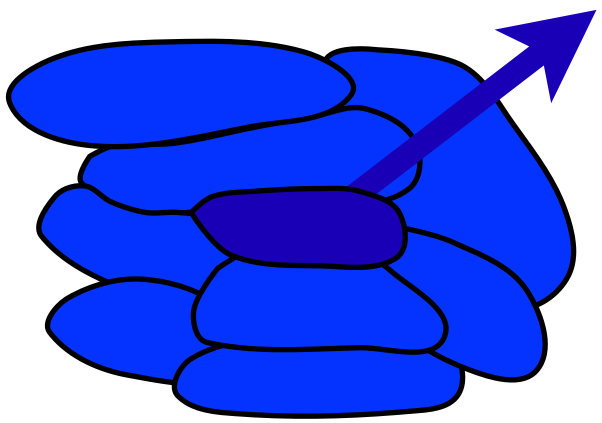
A field redefinition of the charge density fixes the longitudinal part of the current. The allowed transverse vectors at first order in derivatives are:

Hydrodynamics



$T(x^\alpha)$ Temperature

$\mu(x^\alpha)$ Chemical potential

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$L \gg \ell_{\rm mfp}$

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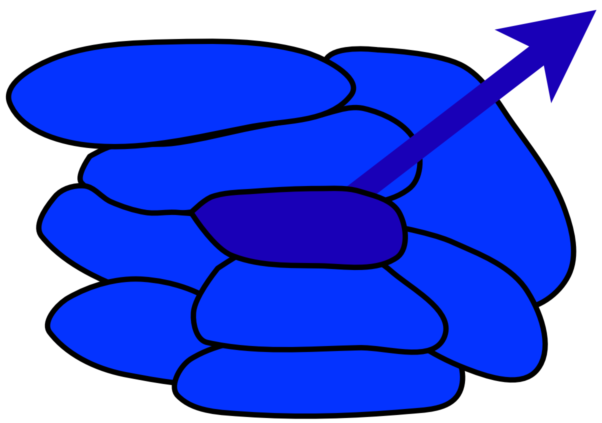

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Hydrostatic equilibrium

(Time independent fluid configurations)

Hydrostatic equilibrium

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Thermodynamic equilibrium

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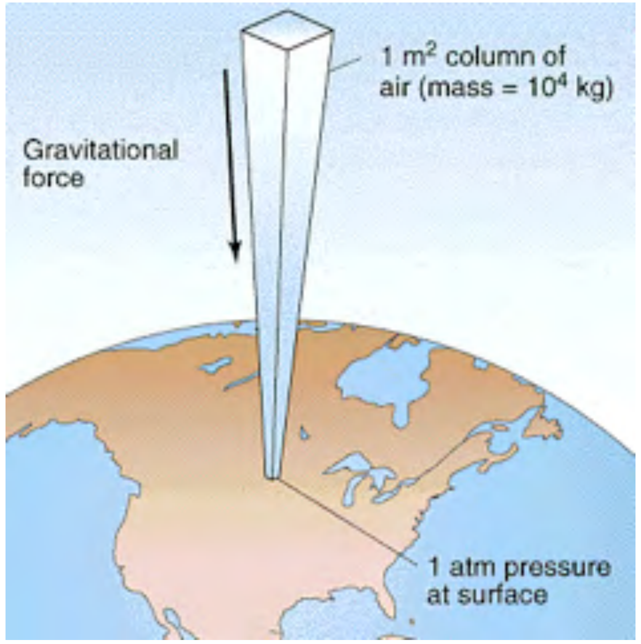
Thermodynamic equilibrium

Column of air



Hydrostatic equilibrium

(A time independent fluid configuration which is a local function of time-independent slowly varying sources)

Thermodynamic equilibrium

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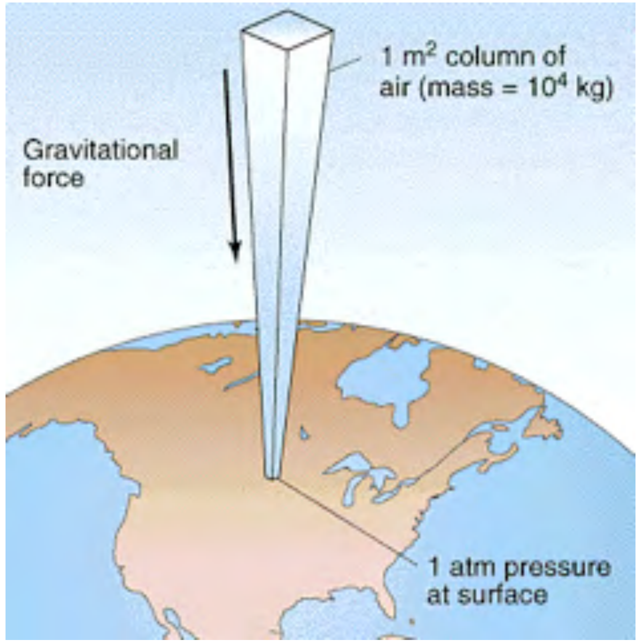

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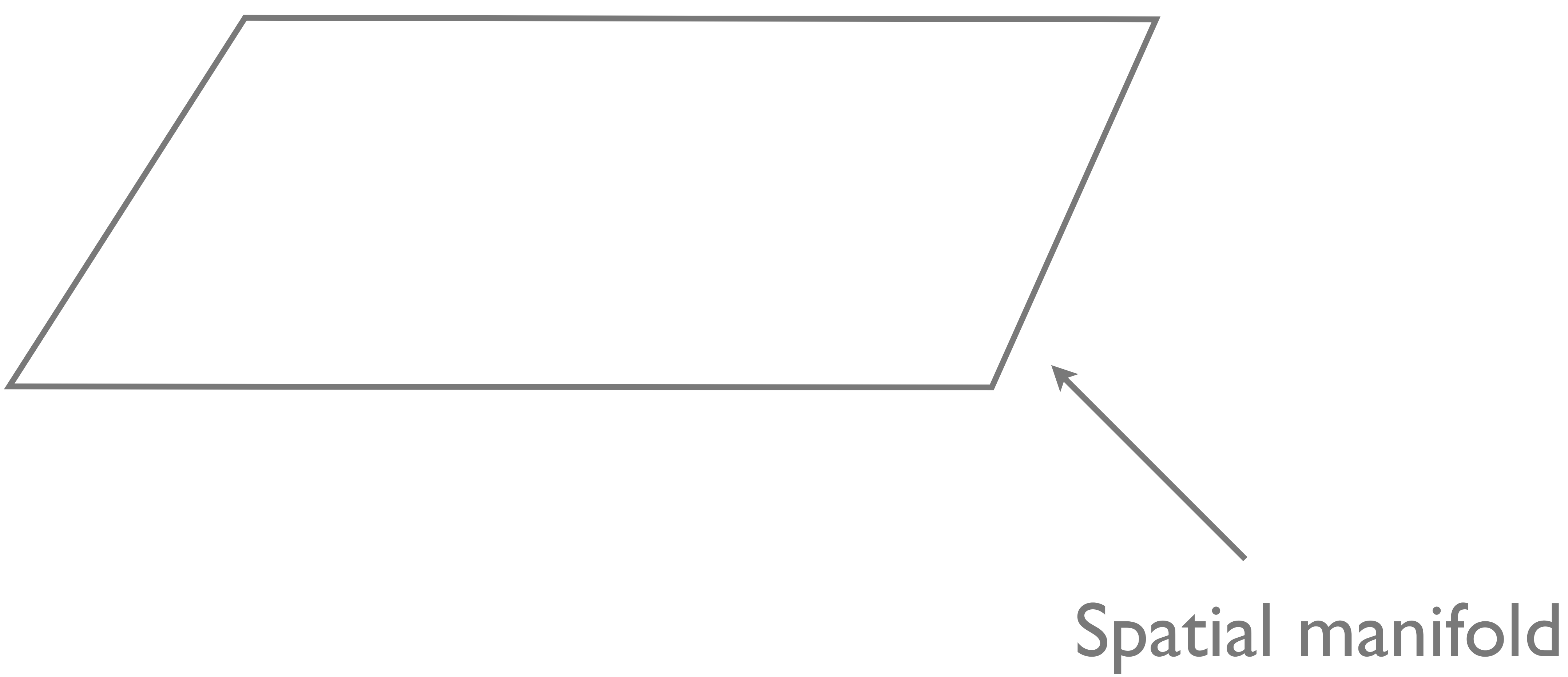

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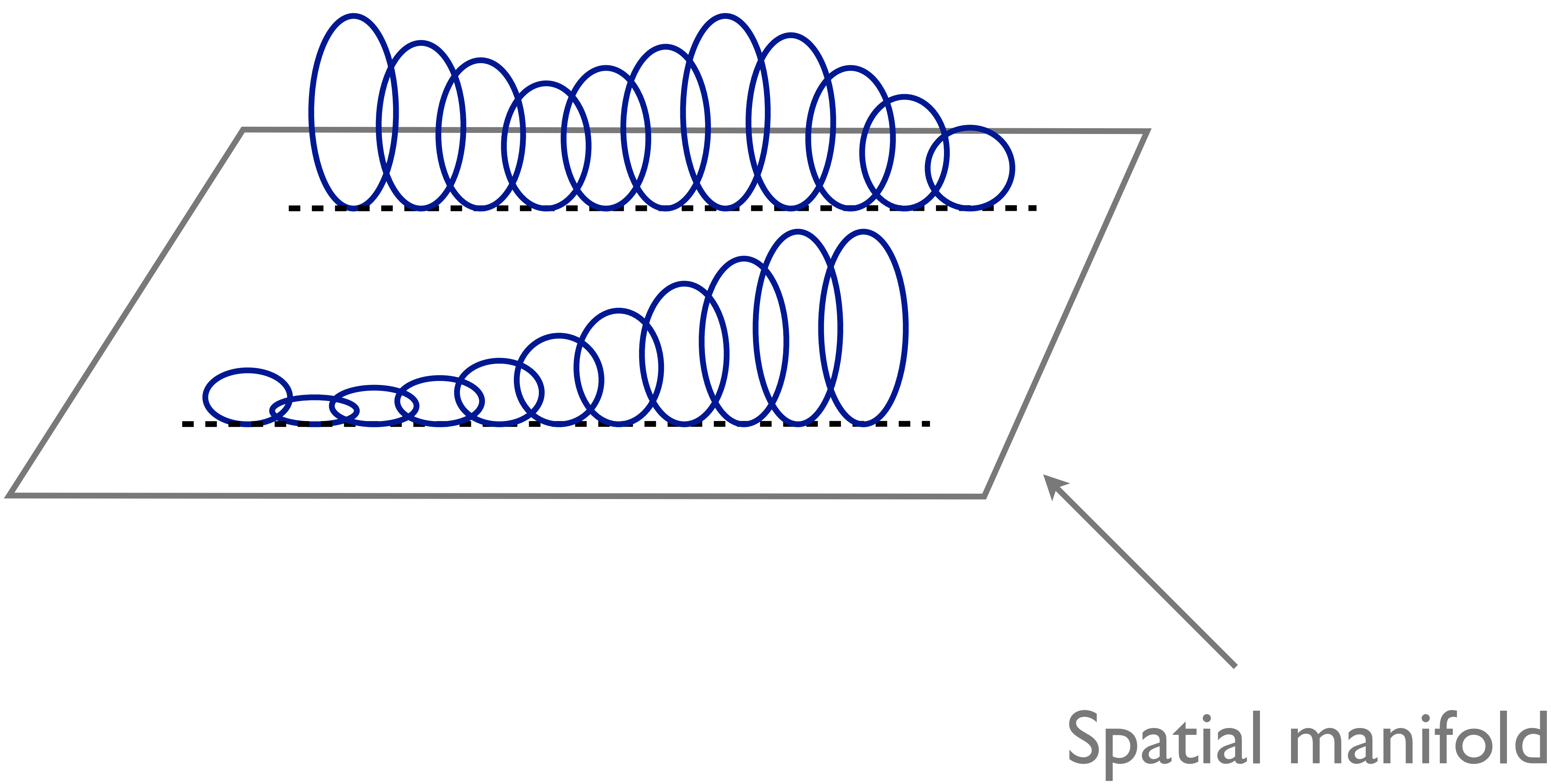

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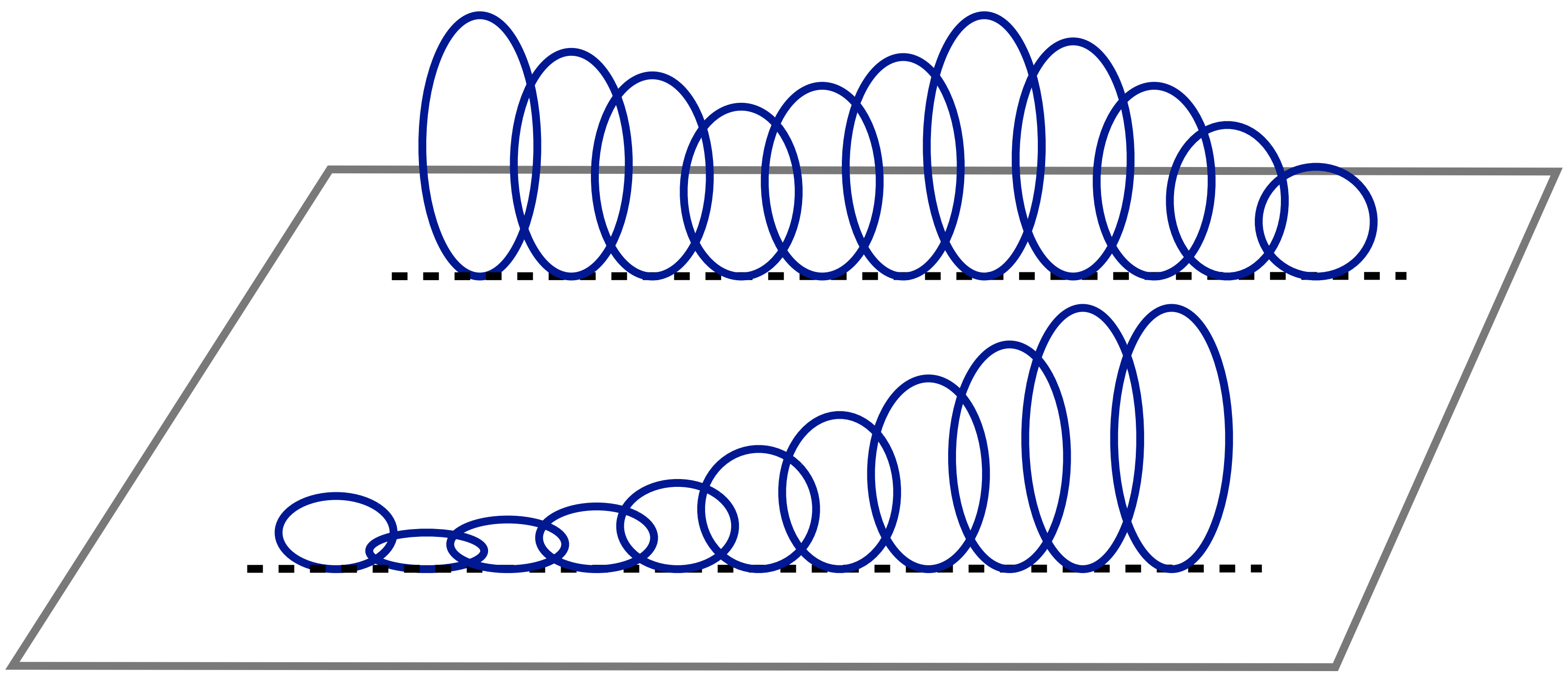

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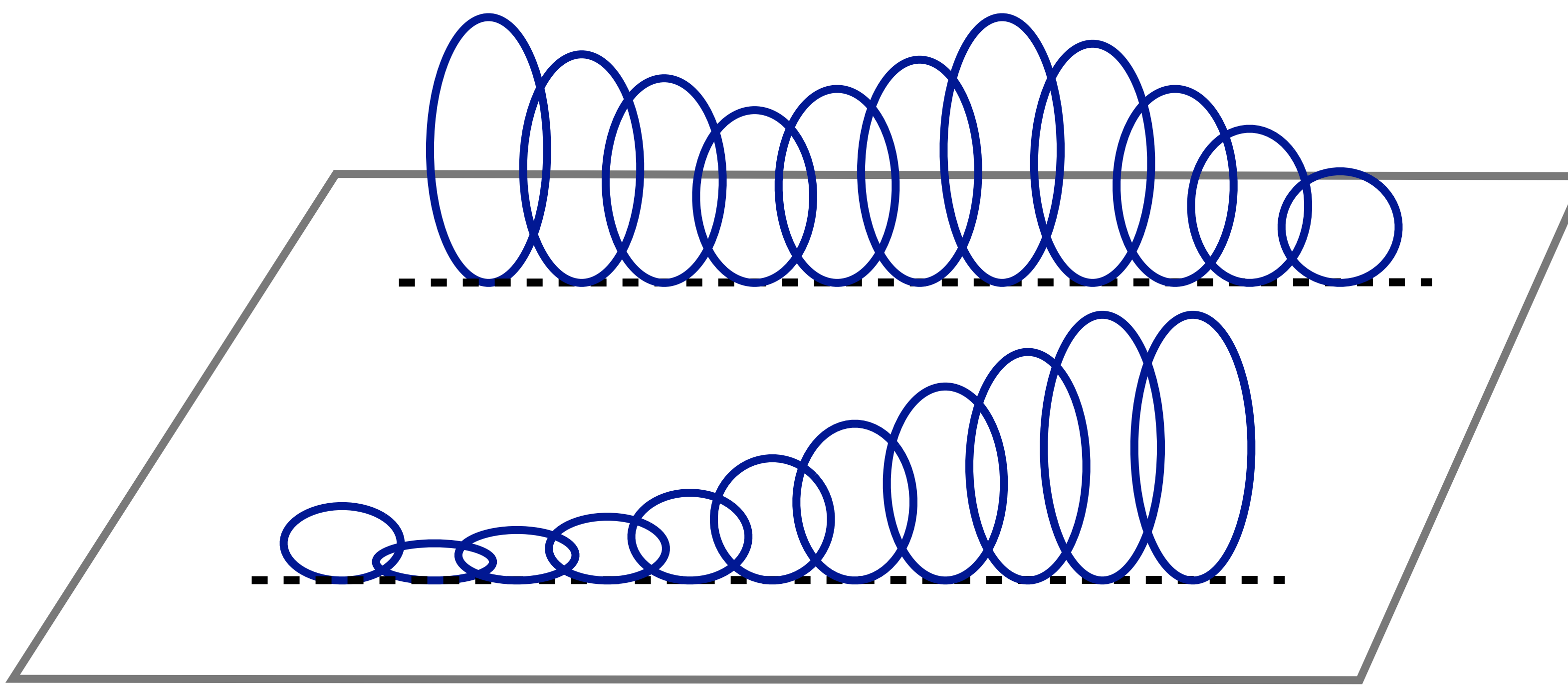

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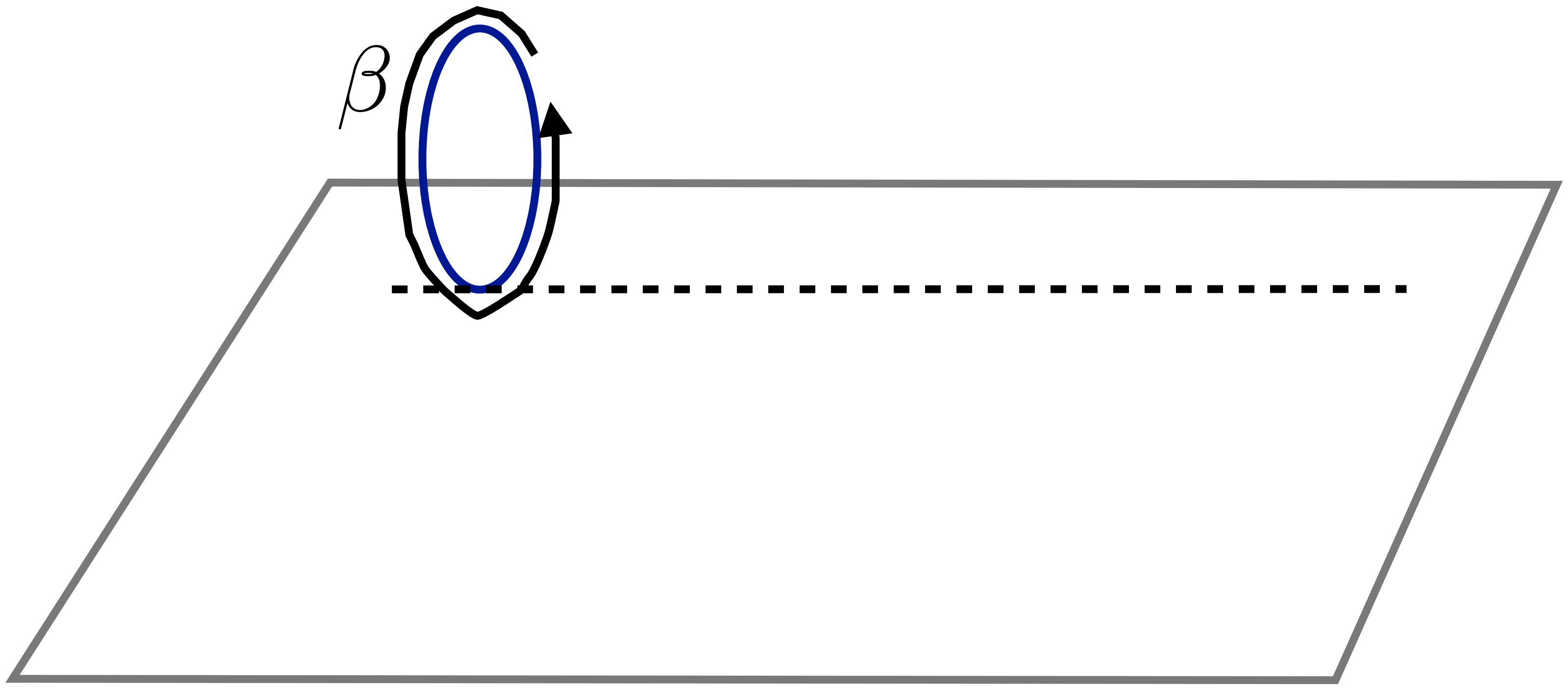

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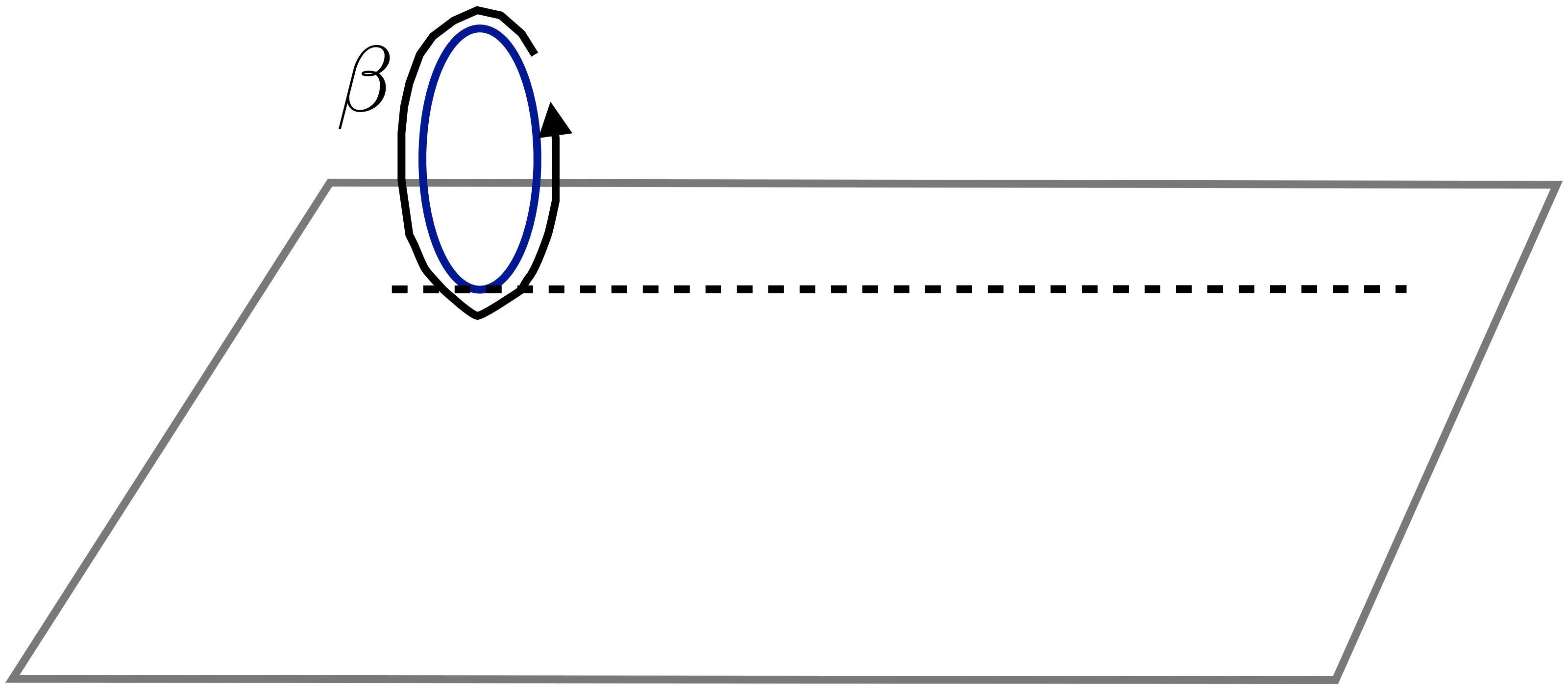

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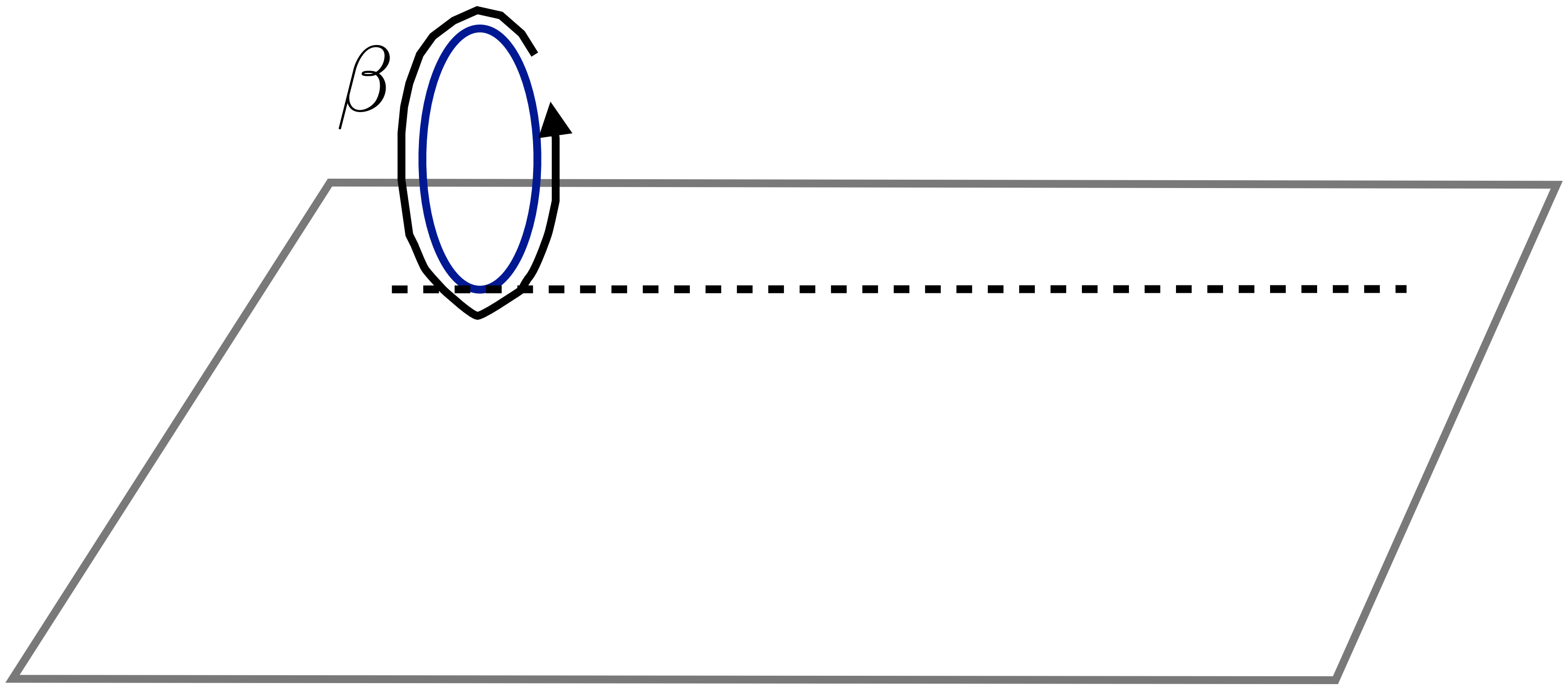

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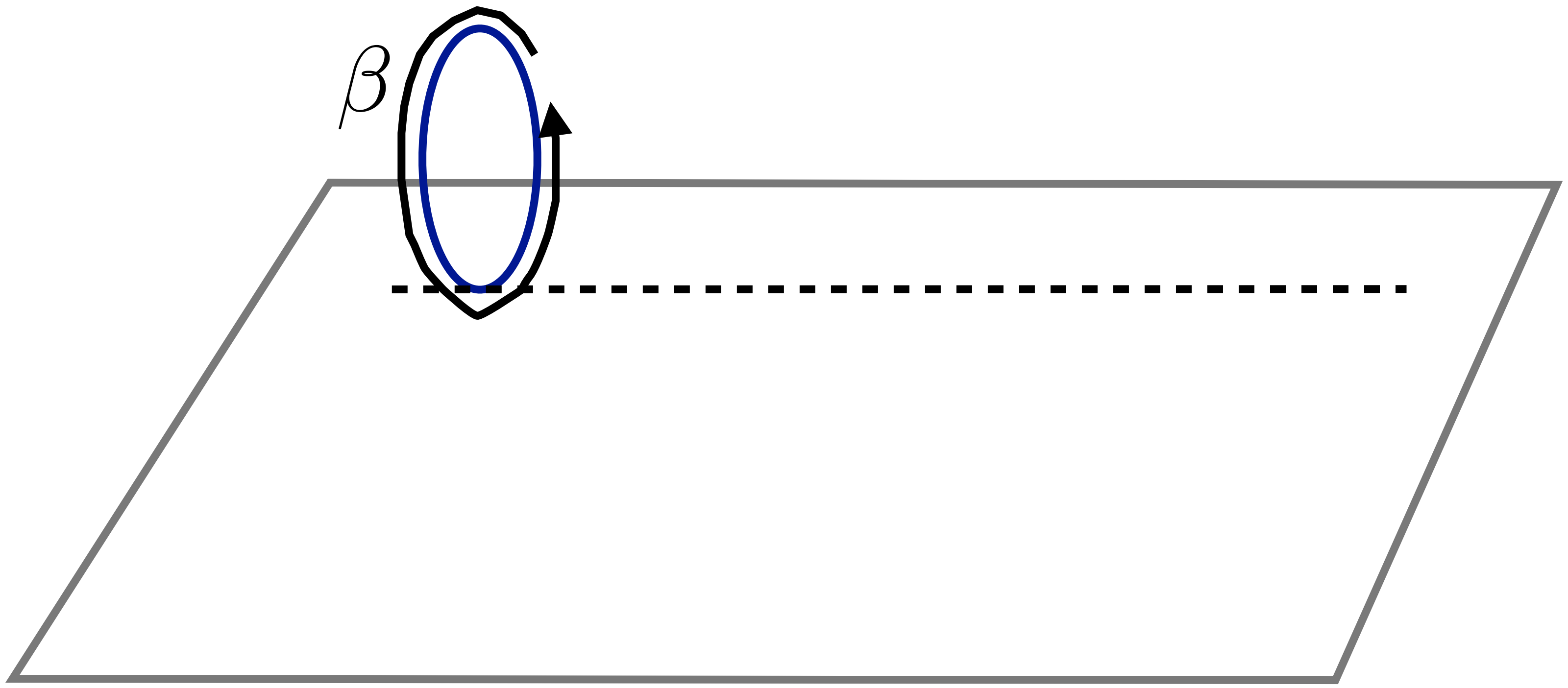

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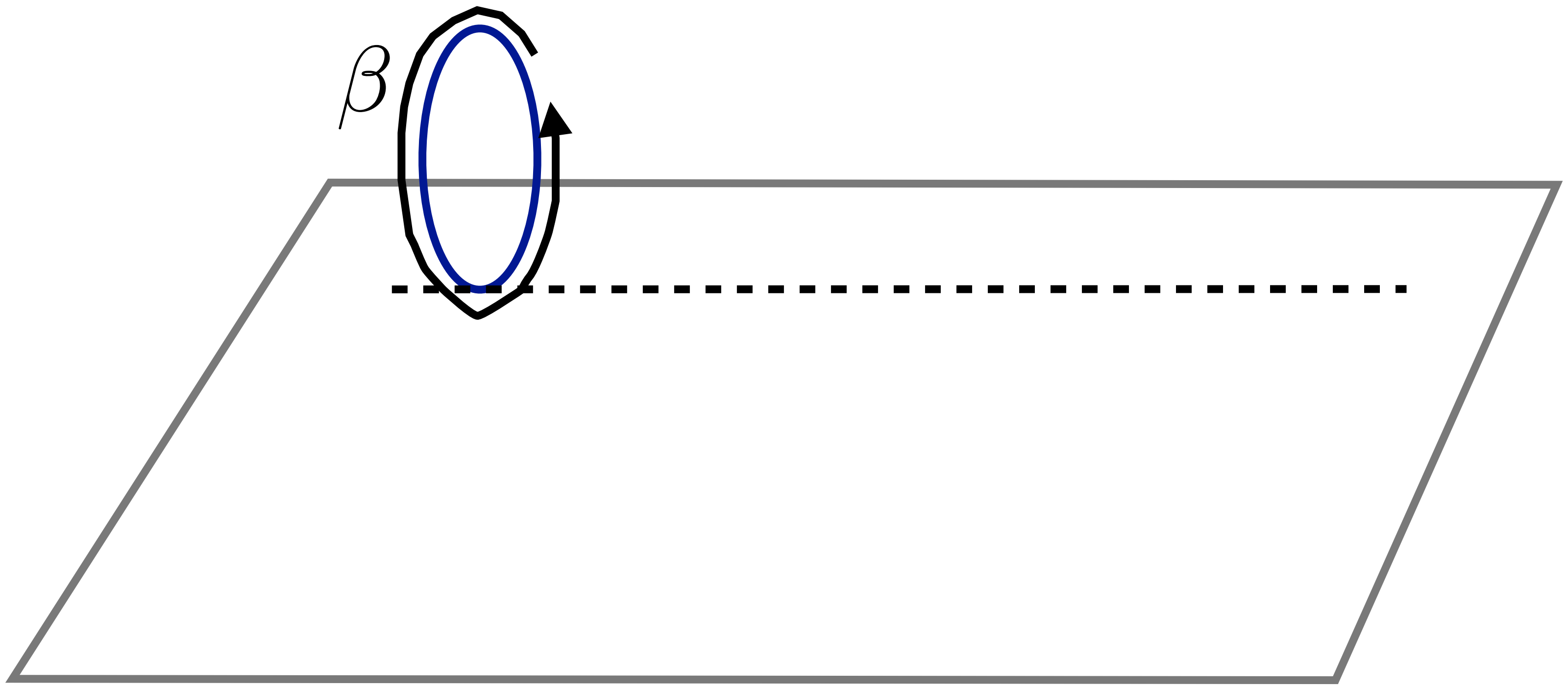

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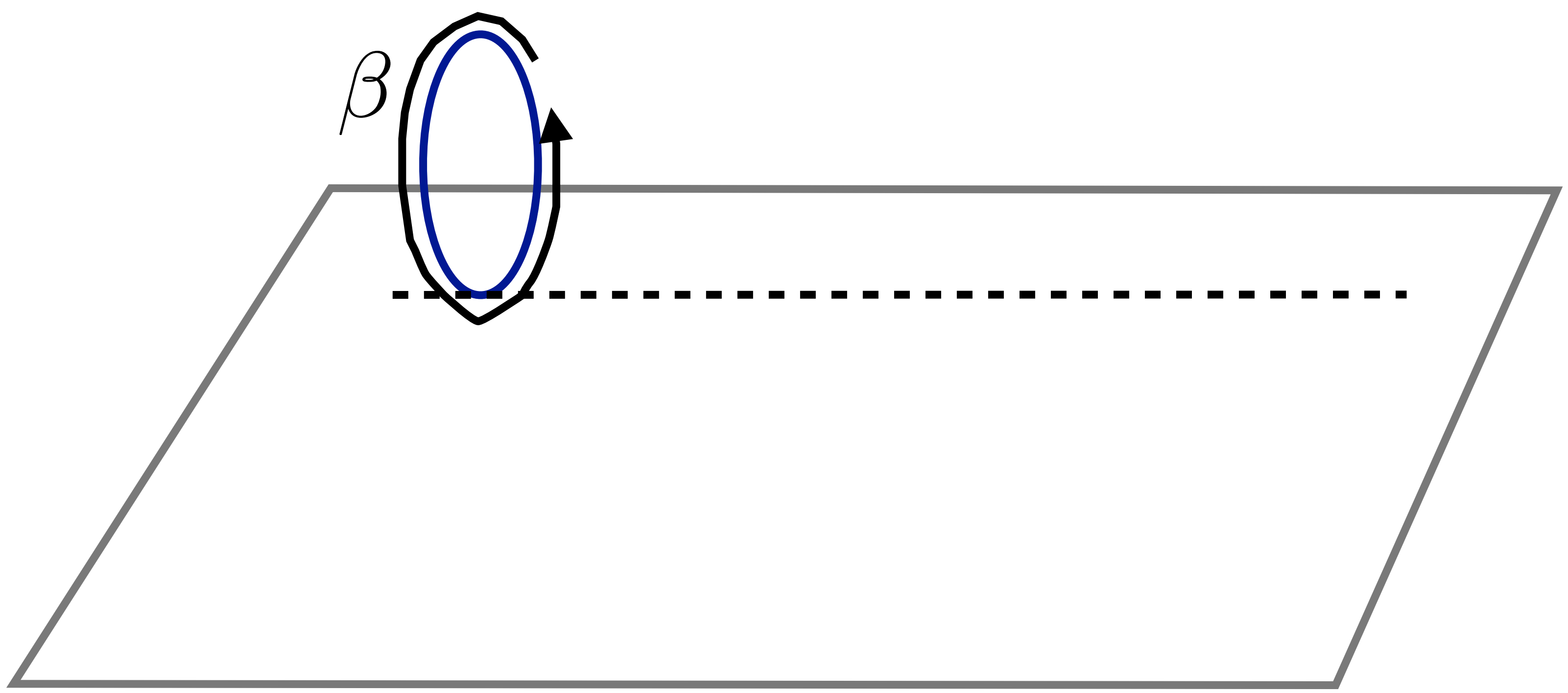

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We construct the partition function for an equilibrated theory from the Euclidian partition function:

$$W = \ln Z$$

Prescription:

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The hydrostatic current is then:

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Hence:

$$\chi = 0$$

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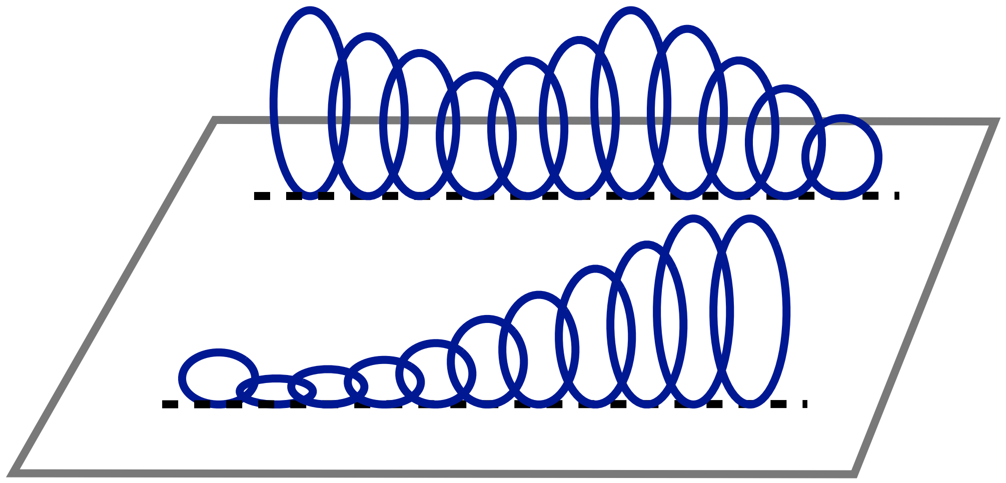

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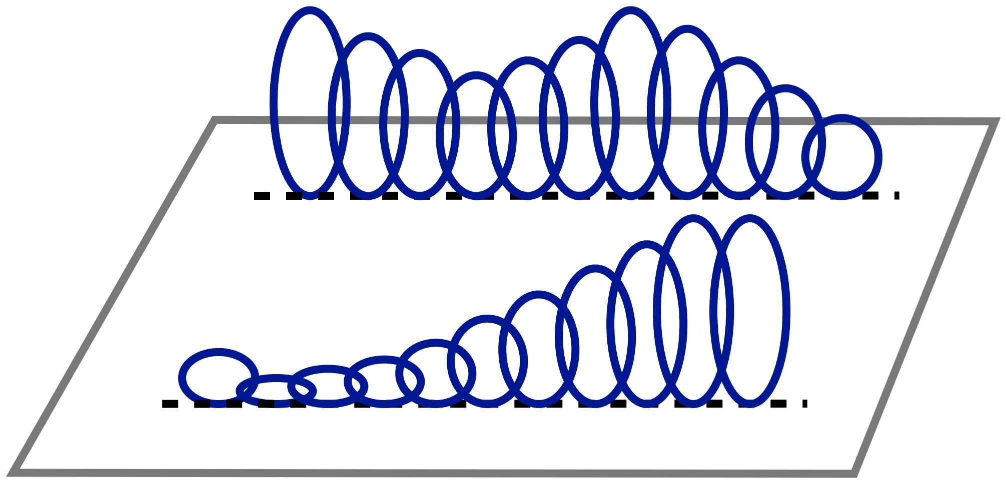

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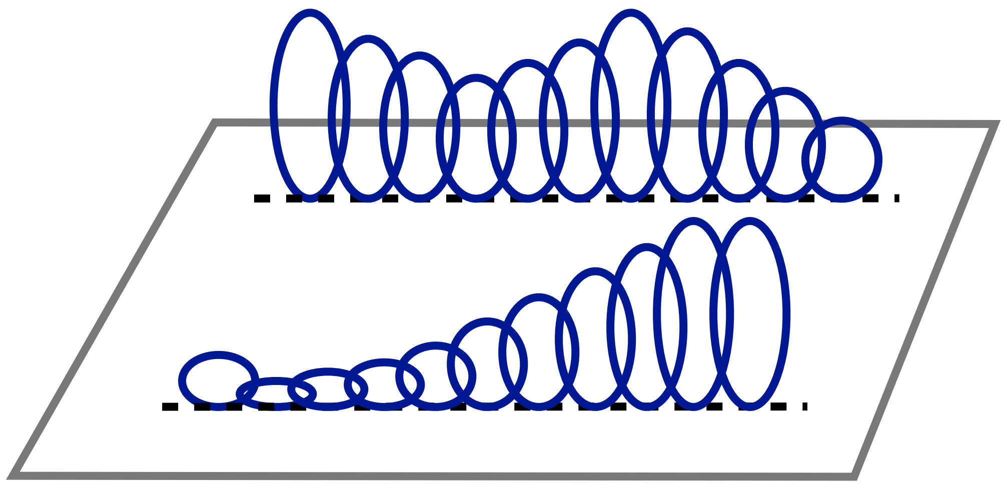

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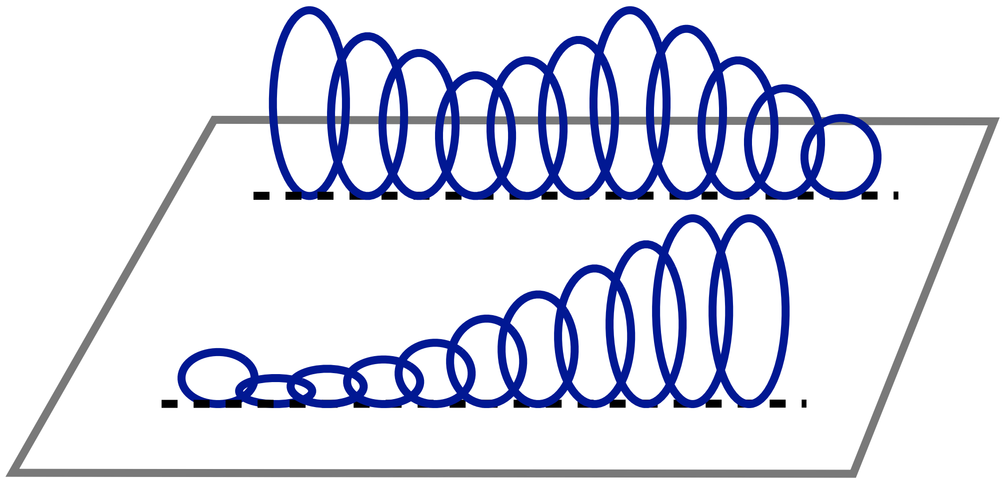

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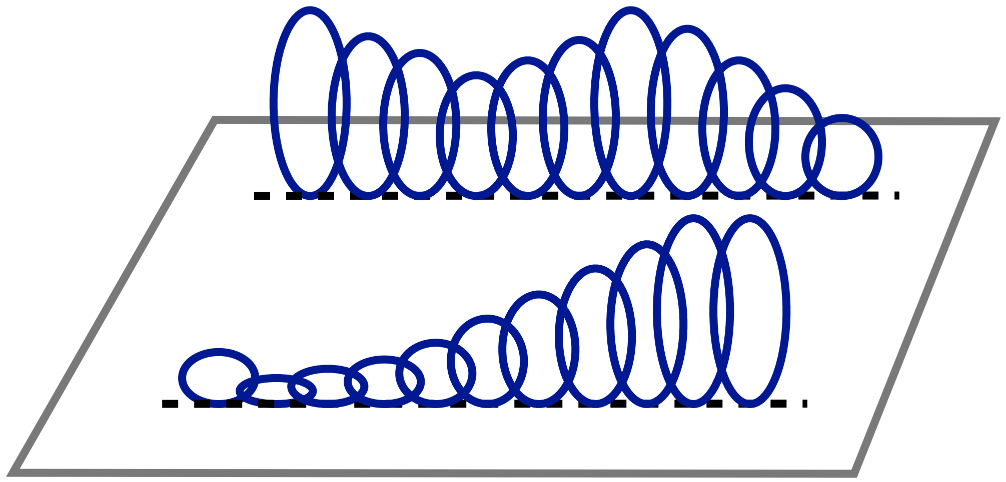

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$$W = \ln Z \;\; = W_0 + W_{trans} + W_{anom}$$

$$\delta \mathbf{I}_{CS} = d\mathbf{G} \qquad\qquad \delta W_{anom} = -\int \mathbf{G}$$

Claim:

$$W_{anom} = -\int \mathbf{W}_{CS} \qquad\qquad \mathbf{W}_{CS} = \frac{\mathbf{u}}{2\mathbf{w}}\left(\mathbf{I}_{CS} - \hat{\mathbf{I}}_{CS}\right)$$

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Hatted connections are transverse and so are their field strengths:

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Where:

$$(\mu_R)^\mu{}_\nu = \nabla_\nu \frac{u^\mu}{T}$$

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$$W = \ln Z \;\; = W_0 + W_{trans} + W_{anom}$$

$$\delta \mathbf{I}_{CS} = d\mathbf{G} \qquad\qquad \delta W_{anom} = -\int \mathbf{G}$$

$$\hat{\mathbf{I}}_{CS} = \mathbf{I}_{CS}(\hat{\mathbf{A}}, \hat{\mathbf{F}}, \hat{\mathbf{\Gamma}}, \hat{\mathbf{R}})$$

Claim:

$$W_{anom} = -\int \mathbf{W}_{CS} \qquad\qquad \mathbf{W}_{CS} = \frac{\mathbf{u}}{2\mathbf{w}}\left(\mathbf{I}_{CS} - \hat{\mathbf{I}}_{CS}\right)$$

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$$\delta \mathbf{I}_{CS} = d\mathbf{G} \qquad \delta W_{anom} = -\int \mathbf{G}$$

$$\mathbf{I}_{CS} - \hat{\mathbf{I}}_{CS} = \sum_{i=1} c_i \mathbf{w}^i$$

Claim:

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Example: $U(1)^3$ anomaly

$$\mathbf{I}_{CS} = \mathbf{A} \wedge \mathbf{F}^2$$

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Example: $U(1)^3$ anomaly

$$\mathbf{I}_{CS} = \mathbf{A} \wedge \mathbf{F}^2$$

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Anomalies

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Anomalies

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Non gauge-invariant contribution

Anomalies

$$W = \ln Z \;\; = W_0 + W_{trans} + W_{anom}$$

Chern-Simons
terms on the
base manifold

Anomalies

$$W = \ln Z \quad = W_0 + W_{trans} + W_{anom}$$

All the rest

Hydrodynamics with anomalies

$$W = \ln Z \;\; = W_0 + W_{trans} + W_{anom}$$

Hydrodynamics with anomalies

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$$J^\mu = \frac{1}{\sqrt{g}} \frac{\delta W}{\delta A_\mu}$$

Hydrodynamics with anomalies

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Hydrodynamics with anomalies

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Hydrodynamics with anomalies

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$$\delta_\Lambda \delta W = \delta_\Lambda \int \sqrt{g} \delta A_\mu J^\mu d^4 x$$

Hydrodynamics with anomalies

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$$= \int \sqrt{g} \delta A_\mu \delta_\Lambda J^\mu d^4 x$$

$$\delta \delta_\Lambda W = \delta \int \Lambda \mathbf{F}^2 d^4 x$$

Hydrodynamics with anomalies

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Define Covariant currents

Bardeen & Zumino (1984)

Hydrodynamics with anomalies

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For $U(1)^3$ anomaly in 3+1 d

$$J^\mu_{BZ} = c_A \epsilon^{\mu\nu\rho\sigma} F_{\nu\rho} A_\sigma$$

Hydrodynamics with anomalies

$$W = \ln Z \;\; = W_0 + W_{trans} + W_{anom}$$

$$W_{cov} = W + \int \mathbf{I}_{CS}$$

Define Covariant currents

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Claim:

Hydrodynamics with anomalies

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$$\mathbf{V_P} = \frac{\mathbf{u}}{2\mathbf{w}} \wedge \left(\mathbf{P} - \hat{\mathbf{P}} \right)$$

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Claim:

$$\mathbf{V_P} = \frac{\mathbf{u}}{2\mathbf{w}} \wedge \left(\mathbf{P} - \hat{\mathbf{P}}\right) \qquad \mathbf{V_P}(\mathbf{u}, \mathbf{B}_R\,, \mathbf{B}\,, \mathbf{w})$$

Hydrodynamics with anomalies

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$$^{*}\mathbf{J_P} = \frac{\partial \mathbf{V_P}}{\partial \mathbf{B}}$$

Hydrodynamics with anomalies

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$$^{*}\mathbf{J_P} = \frac{\partial \mathbf{V_P}}{\partial \mathbf{B}}$$

$$T^{\mu\nu}_{\mathbf{P}} = u^{\mu} q^{\nu}_{\mathbf{P}} + u^{\nu} q^{\mu}_{\mathbf{P}} + \nabla_{\rho}\left(L^{\mu[\nu\rho]}_{\mathbf{P}} + L^{\nu[\mu\rho]}_{\mathbf{P}} - L^{\rho(\mu\nu)}_{\mathbf{P}}\right)$$

Hydrodynamics with anomalies

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$$^*\mathbf{J_P} = \frac{\partial \mathbf{V_P}}{\partial \mathbf{B}} \qquad ^*\mathbf{q_P} = \frac{1}{2}\frac{\partial \mathbf{V_P}}{\partial \mathbf{w}} \qquad ^*\mathbf{L_P} = \frac{\partial \mathbf{V_P}}{\partial \mathbf{B}_R}$$

$$T^{\mu\nu}_{\mathbf{P}} = u^\mu q^\nu_{\mathbf{P}} + u^\nu q^\mu_{\mathbf{P}} + \nabla_\rho \left(L^{\mu[\nu\rho]}_{\mathbf{P}} + L^{\nu[\mu\rho]}_{\mathbf{P}} - L^{\rho(\mu\nu)}_{\mathbf{P}}\right)$$

Hydrodynamics with anomalies

Claim:

$$\mathbf{V}_{\mathbf{P}} = \frac{\mathbf{u}}{2\mathbf{w}} \wedge \left(\mathbf{P} - \hat{\mathbf{P}}\right) \qquad \mathbf{V}_{\mathbf{P}}(\mathbf{u}, \mathbf{B}_R\,, \mathbf{B}\,, \mathbf{w})$$

$$^*\mathbf{J}_{\mathbf{P}} = \frac{\partial \mathbf{V}_{\mathbf{P}}}{\partial \mathbf{B}} \qquad ^*\mathbf{q}_{\mathbf{P}} = \frac{1}{2}\frac{\partial \mathbf{V}_{\mathbf{P}}}{\partial \mathbf{w}} \qquad ^*\mathbf{L}_{\mathbf{P}} = \frac{\partial \mathbf{V}_{\mathbf{P}}}{\partial \mathbf{B}_R}$$

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Sketch of proof:

Hydrodynamics with anomalies

Claim:

$$\mathbf{V}_{\mathbf{P}} = \frac{\mathbf{u}}{2\mathbf{w}} \wedge \left(\mathbf{P} - \hat{\mathbf{P}}\right) \qquad \mathbf{V}_{\mathbf{P}}(\mathbf{u}, \mathbf{B}_R\,, \mathbf{B}\,, \mathbf{w})$$

$$^*\mathbf{J}_{\mathbf{P}} = \frac{\partial \mathbf{V}_{\mathbf{P}}}{\partial \mathbf{B}} \qquad ^*\mathbf{q}_{\mathbf{P}} = \frac{1}{2}\frac{\partial \mathbf{V}_{\mathbf{P}}}{\partial \mathbf{w}} \qquad ^*\mathbf{L}_{\mathbf{P}} = \frac{\partial \mathbf{V}_{\mathbf{P}}}{\partial \mathbf{B}_R}$$

$$T^{\mu\nu}_{\mathbf{P}} = u^\mu q^\nu_{\mathbf{P}} + u^\nu q^\mu_{\mathbf{P}} + \nabla_\rho \left(L^{\mu[\nu\rho]}_{\mathbf{P}} + L^{\nu[\mu\rho]}_{\mathbf{P}} - L^{\rho(\mu\nu)}_{\mathbf{P}} \right)$$

Sketch of proof:

Formally: $d\left(\frac{\mathbf{u}}{2\mathbf{w}}\right) = 1$

Hydrodynamics with anomalies

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$$W = \ln Z \quad = W_0 + W_{trans} + W_{anom}$$

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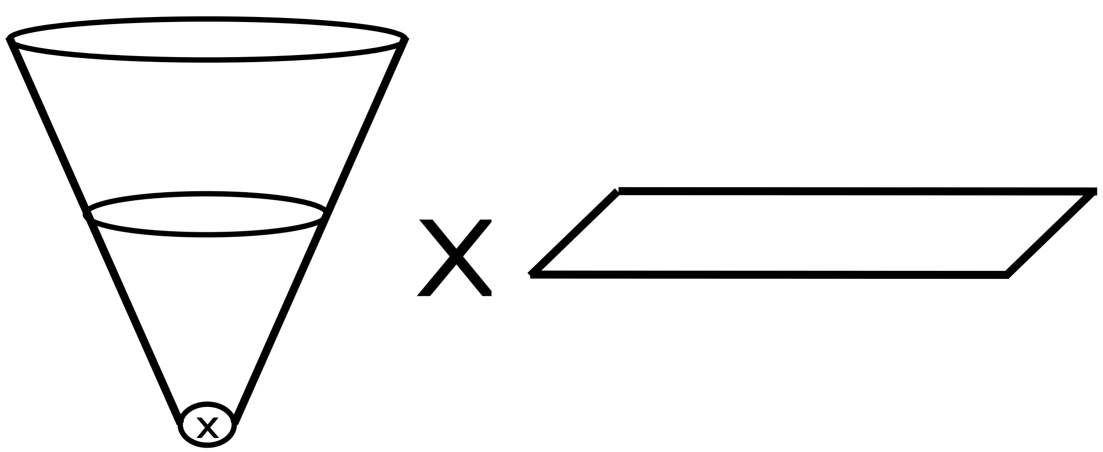

$$\phi \sim \phi + 2\pi\delta$$

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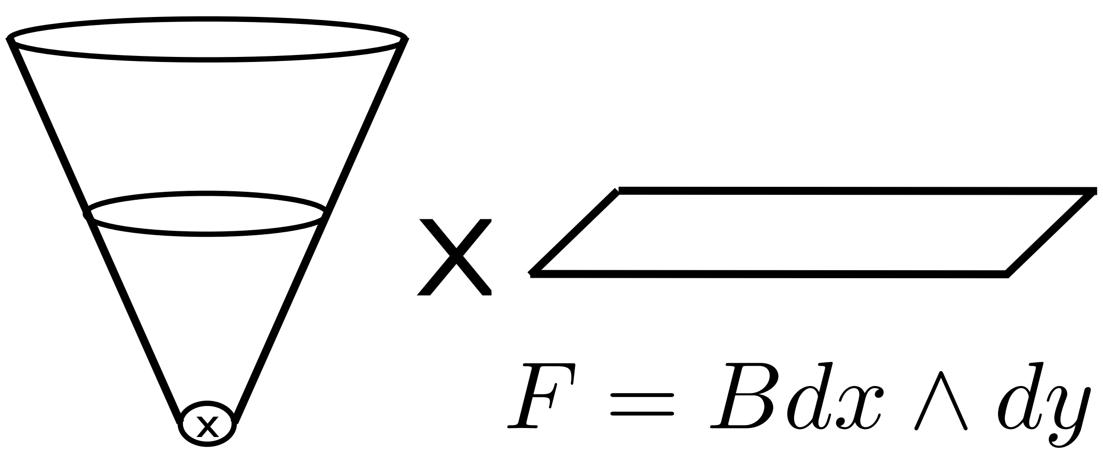

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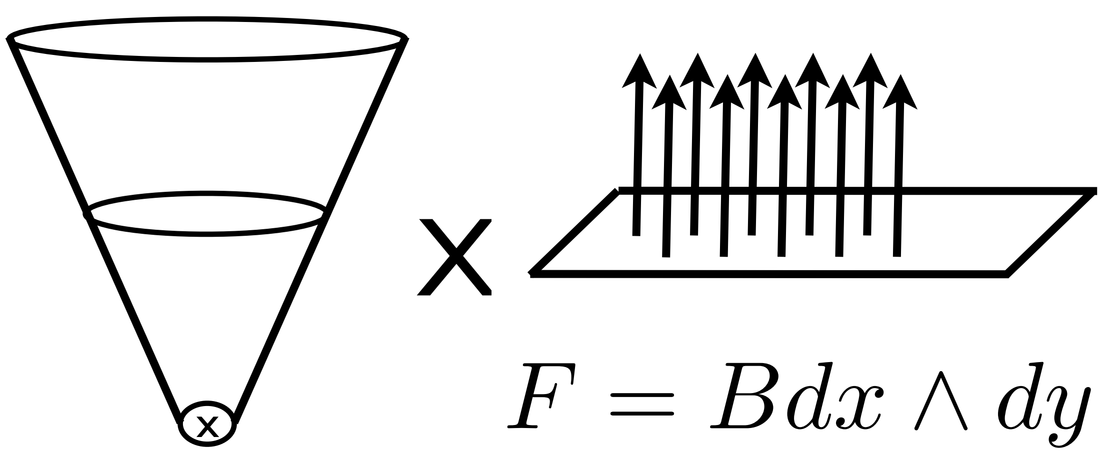

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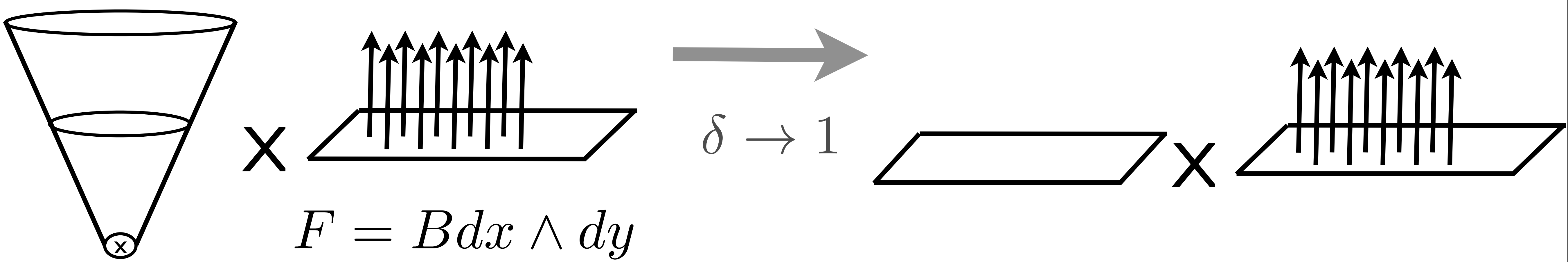

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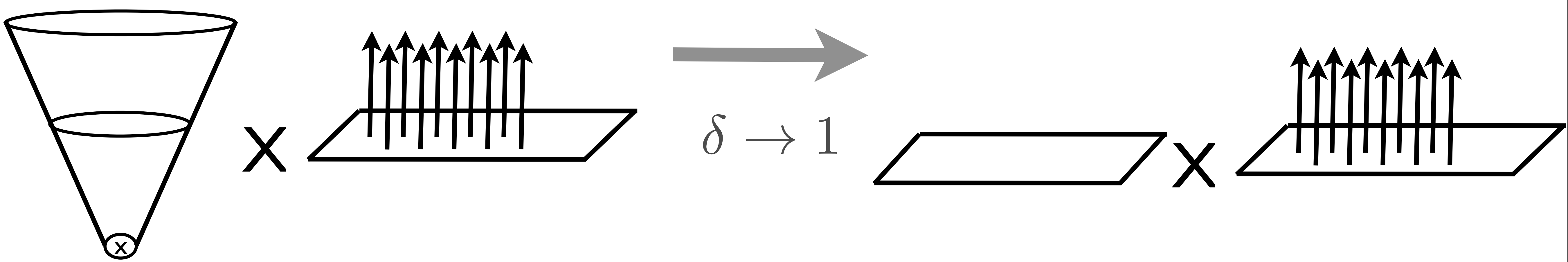

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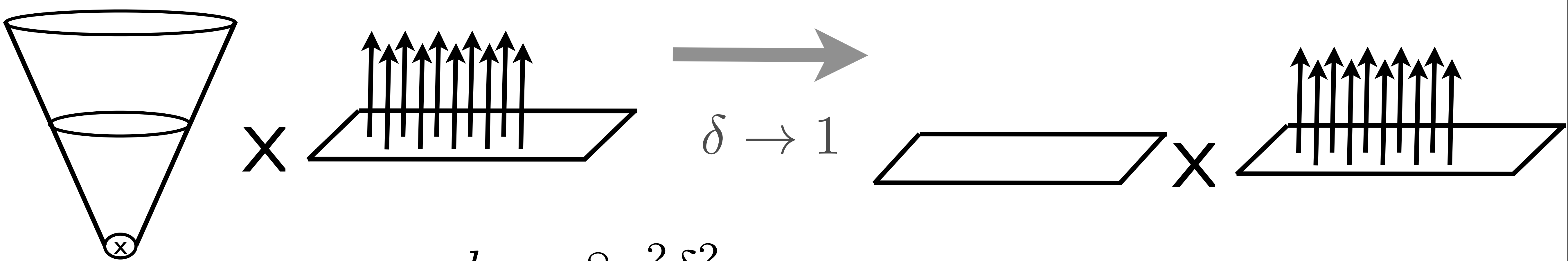

$$\mathrm{Tr}(\varrho_{cone} T^{tr}) = B \frac{k_1 - 8\pi^2 \delta^2 c_m}{4\pi^2 \delta^2 r^3}$$

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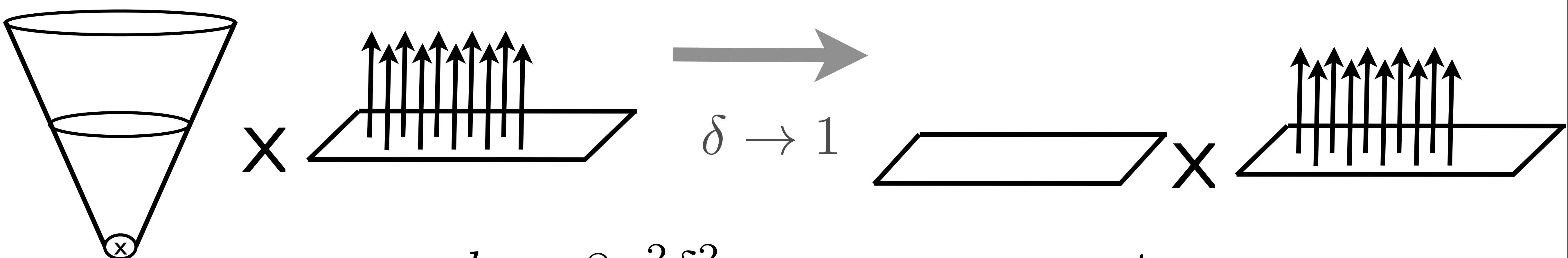

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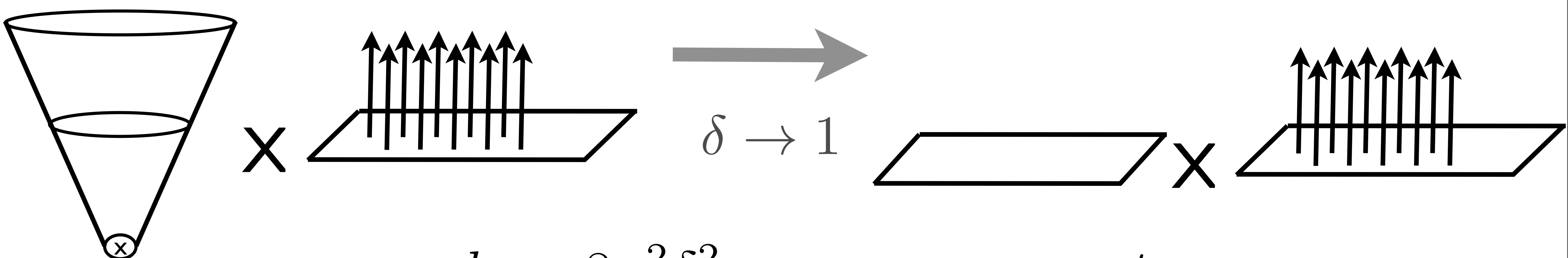

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The “cone” argument

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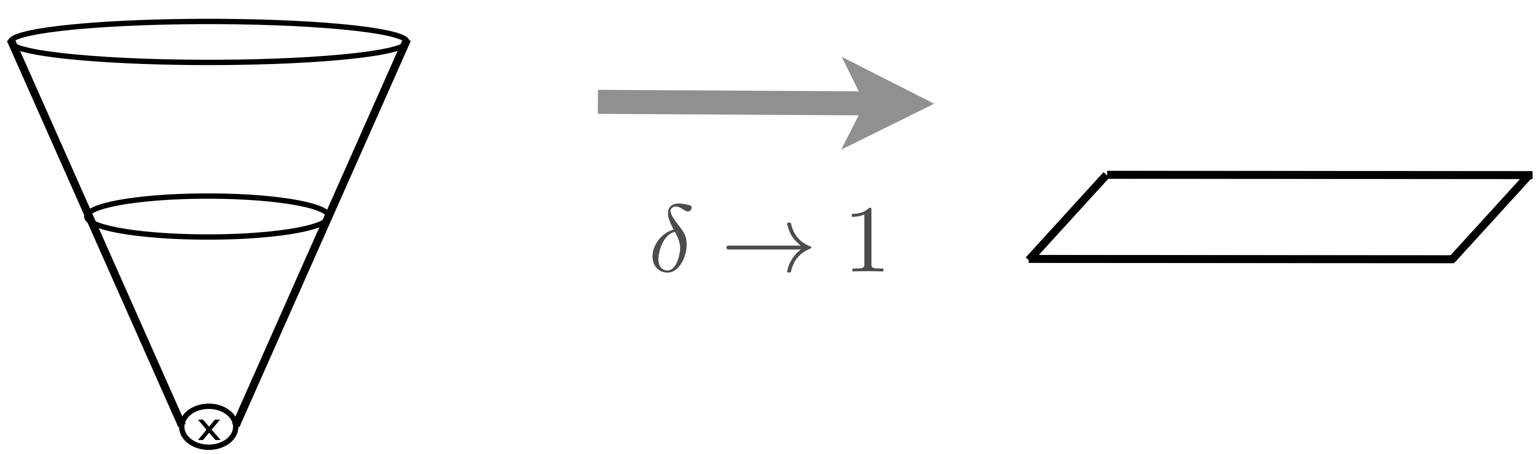
$$ds^2 = dr^2 + r^2 d\phi^2$$



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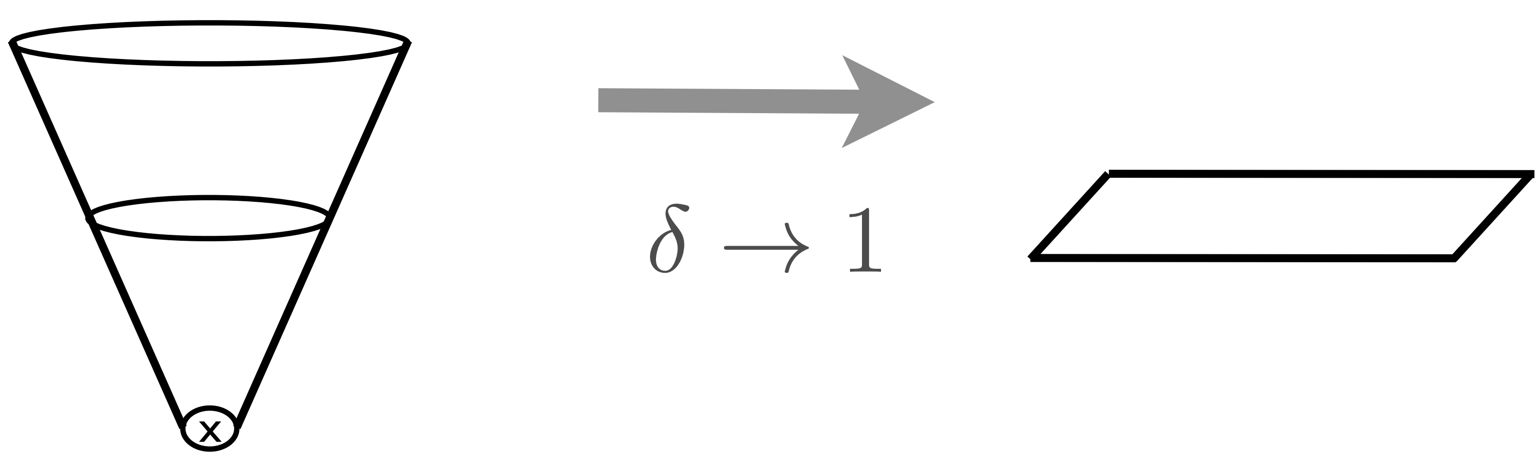

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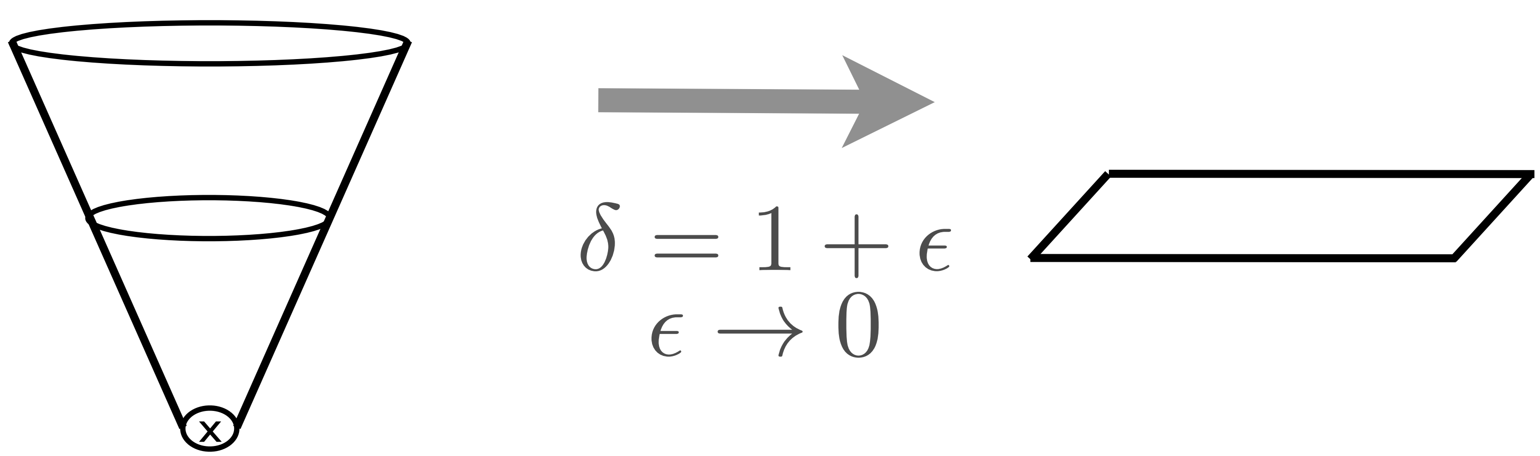

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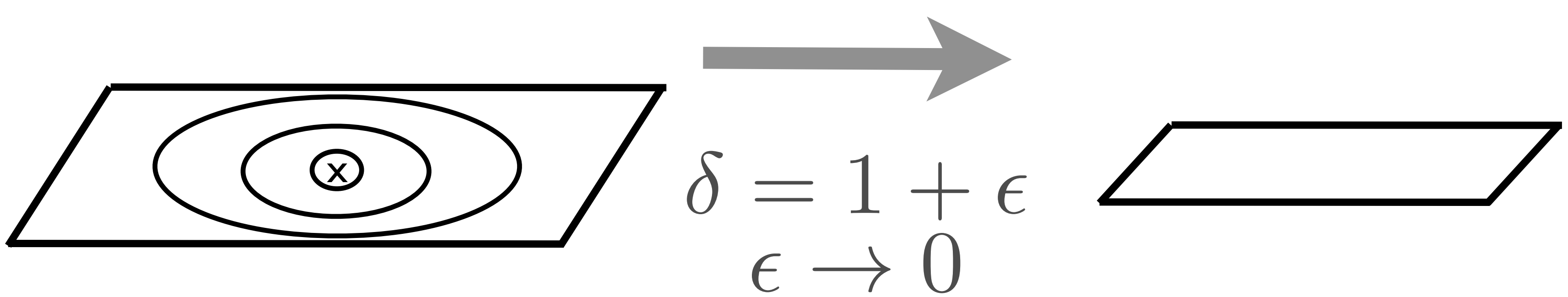

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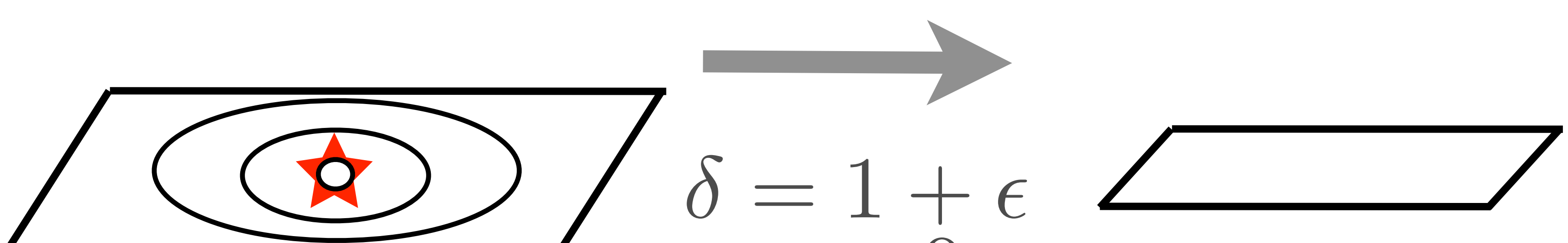

$$\delta = 1 + \epsilon$$
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The “cone” argument

Modes localized at the tip

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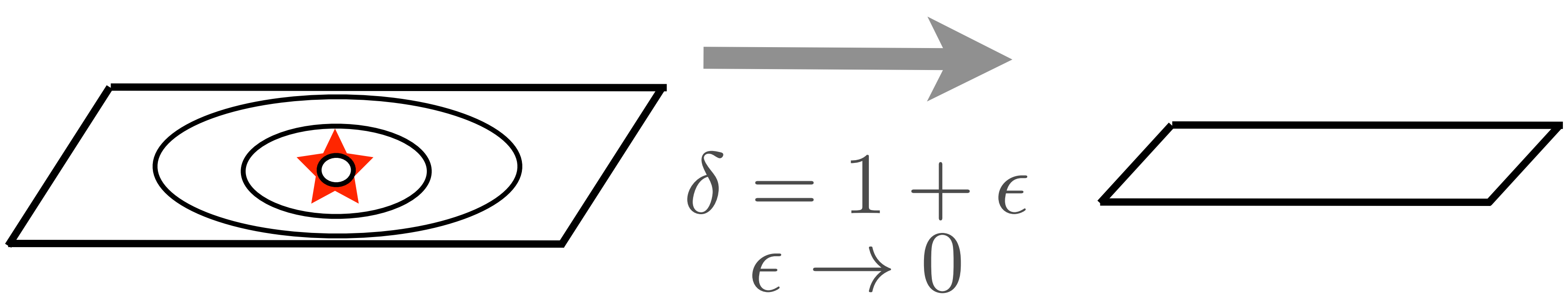

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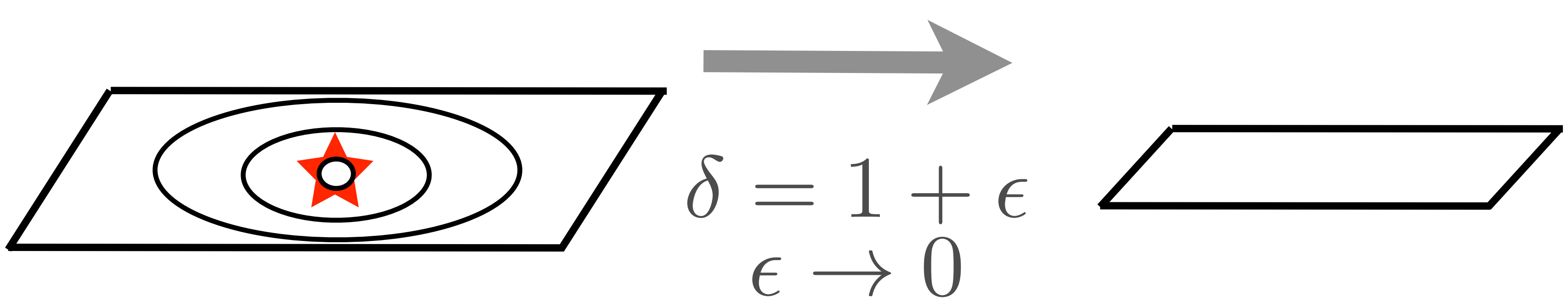

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won't affect our argument

$$T^{\mu\nu} \to T^{\mu\nu} + \mathcal{O}(\delta(r))$$

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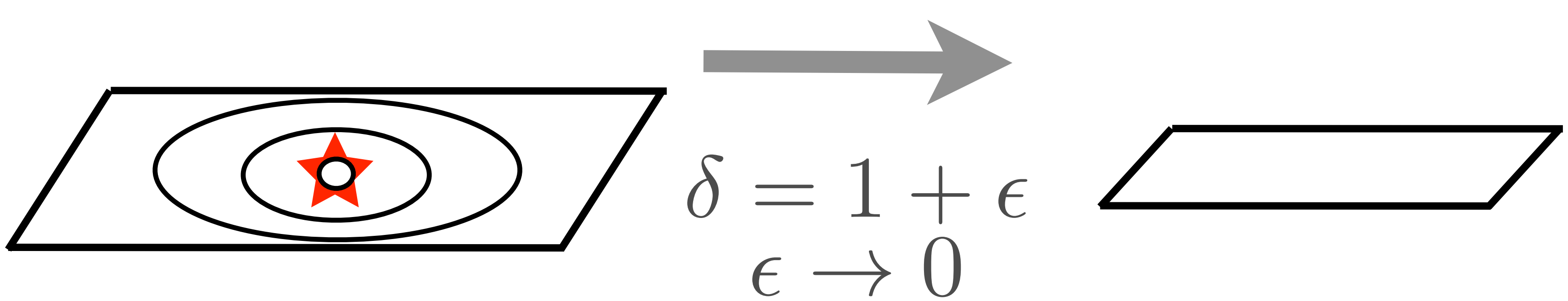

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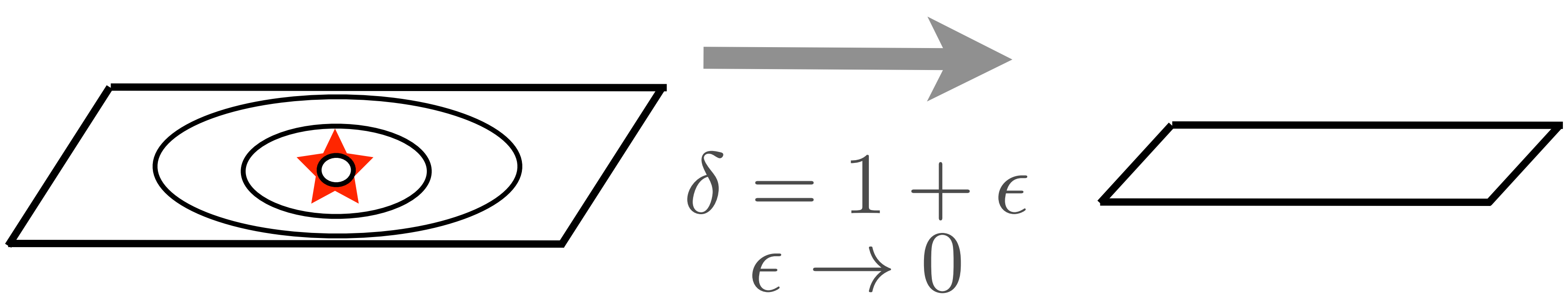

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Modes which delocalize in the flat space limit will

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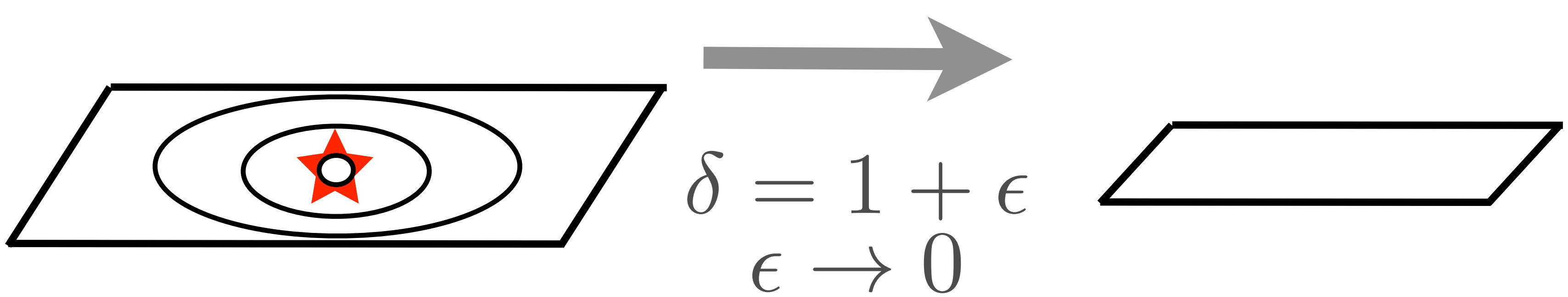

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Loganayagam (2012)

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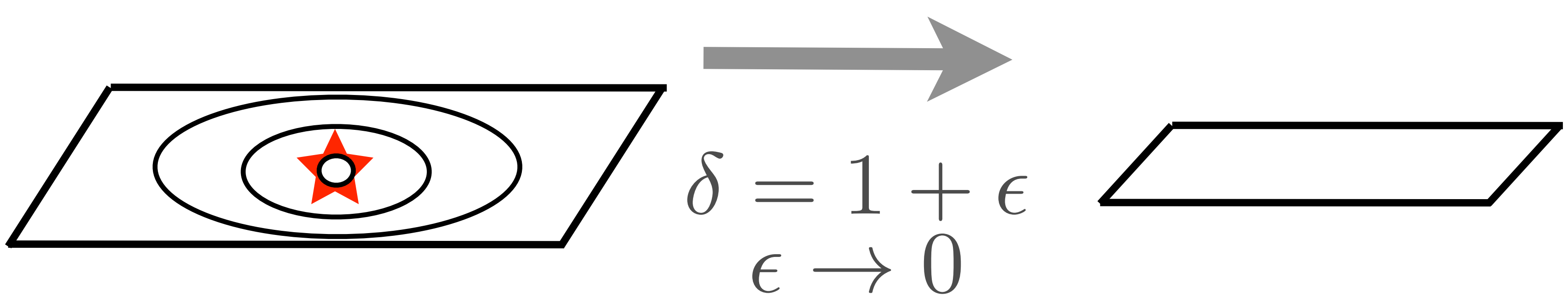

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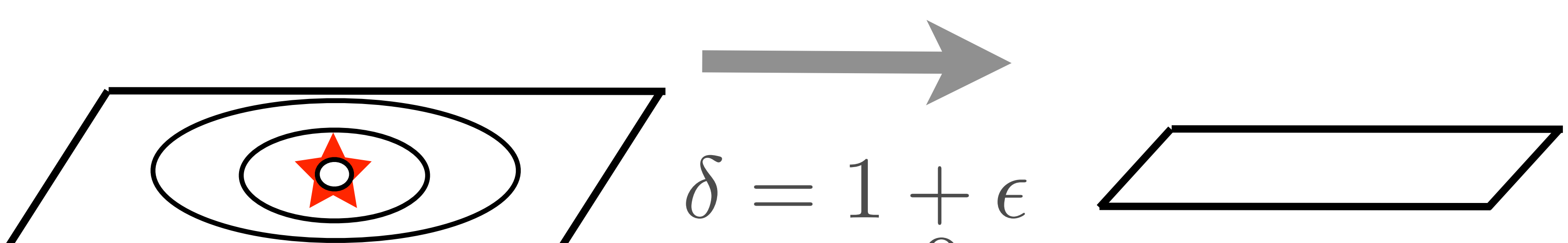

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(See also Eling, Oz, Theisen, Yankielowicz (2013))

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$$J^\mu_{cov} = \rho u^\mu + \left(k_1 T^2 - 2c_A \mu^2\right)\epsilon^{\mu\nu\rho\sigma} u_\nu \partial_\rho u_\sigma + \mathcal{O}(\partial^3)$$

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(Compare with: $\mathbf{V}_\mathbf{P} = \frac{\mathbf{u}}{2\mathbf{w}} \wedge \left(\mathbf{P} - \hat{\mathbf{P}}\right)$)

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