Dynamical phase transitions and Thermalization in low dimensional field theories and AdS

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+ related

Why study thermalization?

Most equilibrium phenomena we see around involve approximate equilibrium, relative to some time scale (e.g. glass). To be able to describe the systems, we can

(a) determine the relaxation time scale τ, and for $t \gg \tau$ apply equilibrium thermodynamics,

(b) address non-equilibrium physics (are there universal laws, is there a non-equilibrium entropy etc.)?

Deeper questions: pure state ---> mixed state (in what sense)? How much mixed: how much memory of the initial state? (more later).

Holographic viewpoint

AdS/CFT translates some of the above questions beautifully:

Equilibrium <---> black holes

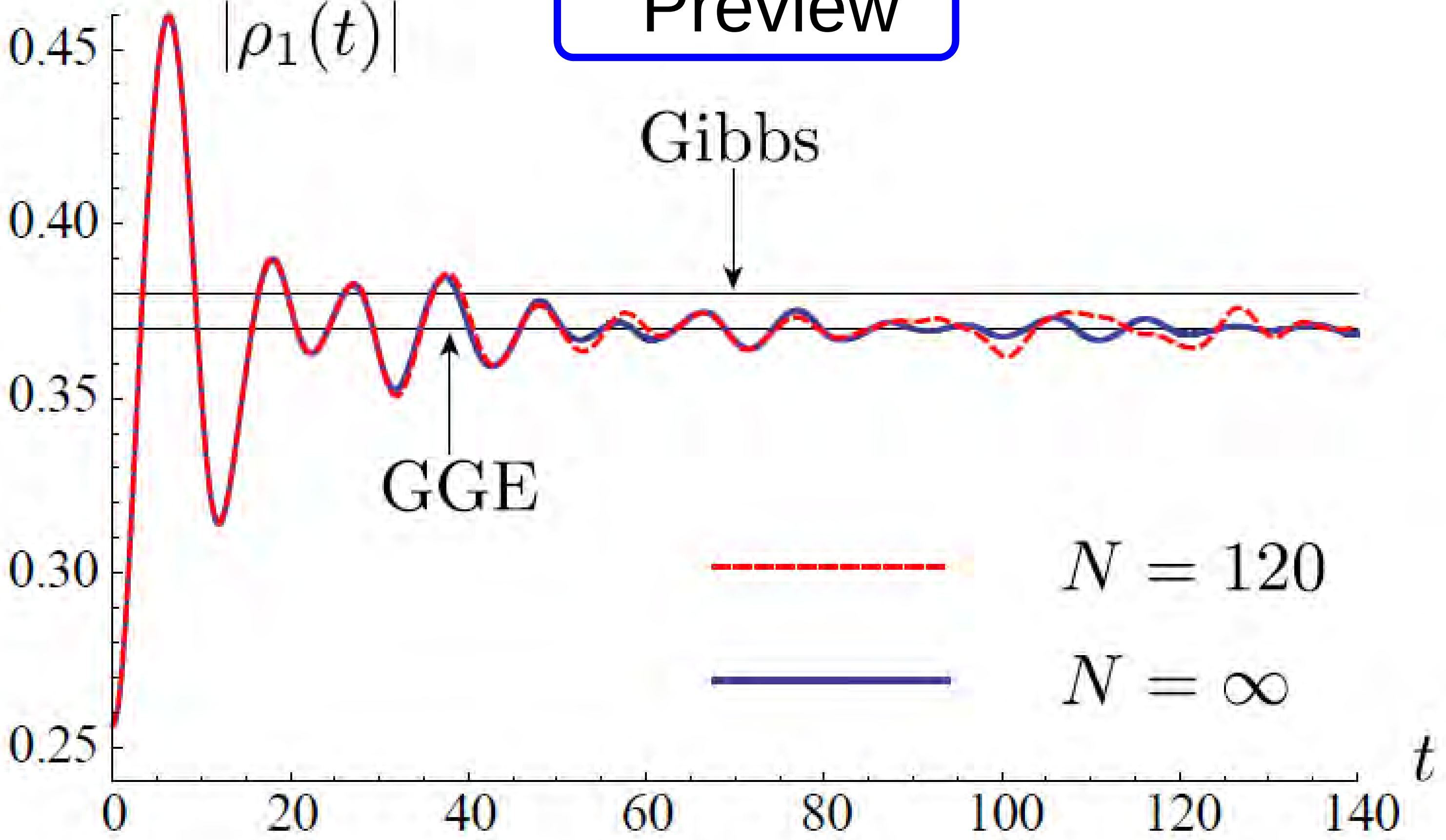
Relaxation time <---> time for black hole formation (see Najafabadi's talk)

Non-equilibrium entropy <---> area of time-evolving black hole (e.g. works for slow evolution, like in AdS/hydrodynamis, Bhattacharya et al 2008)

Memory loss <------> no hair

``Pure state to Mixed state'' <---> information loss

Preview



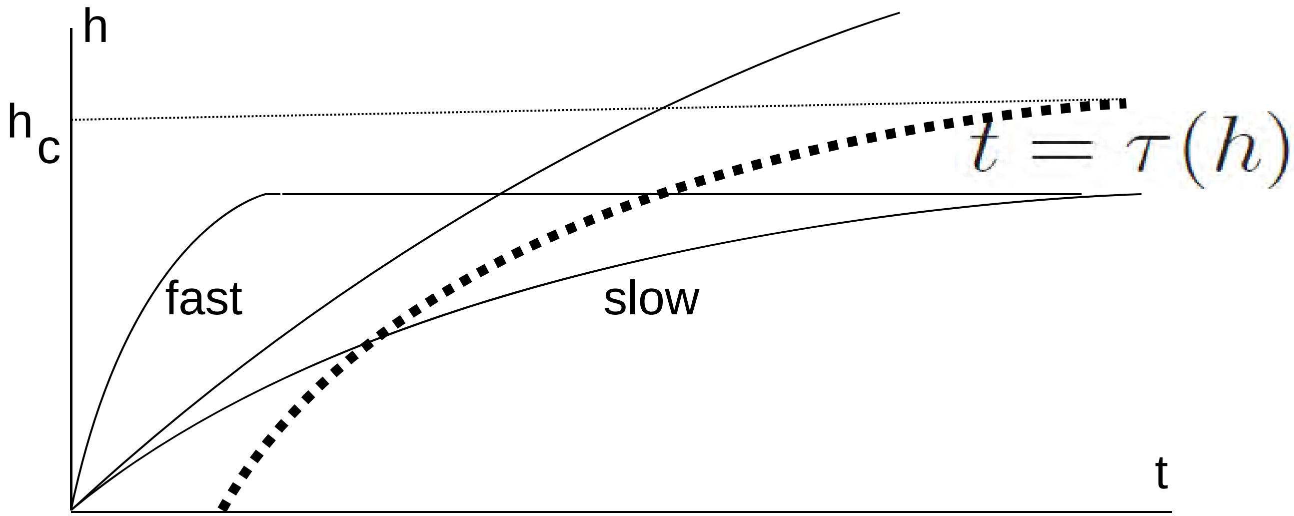
$$\langle \psi(t)|O|\psi(t)\rangle \xrightarrow{t\to\infty} \mathrm{Tr}\left(\frac{1}{\mathcal{Z}(\mu_i)}\exp[-\sum_i \mu_i Q_i]O\right)$$

Thermalizable of integrable systems. Known for TFI, Hard-core bosonic chain, and Matrix quantum mechanics [GM + Morita (2013)]; Infinite amount of "hair": hs BH?

Dynamic criticality

Another important aspect of non-equilibrium physics is dynamical phase transitions. (see Aliakbari's talk)

For very slow changes, dynamical variations due to a time-dependent hamiltonian H(h(t)) = thermodynamic variations described by exp[- H(h)/T] (adiabatic principle)



$$H = \sum_{i,j} \sigma_i^1 . \sigma_j^1 + h(t) \sum_i \sigma_i^3$$

Polkovnikov et al review 2011, M. Rigol et al 2005-2013, Calabrese et al 2005-2013, Kibble 1980-Zurek 1996 , Damsky 2004,.....

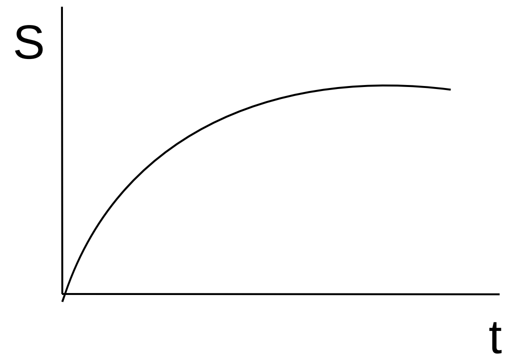
Part I: Non-equilibrium entropy

Consider the time evolution of a pure state of the hamiltonian H, which is not an eigenstate (obtained from a quantum quench, or by some other means). Under appropriate conditions, we expect the `state of the system' to approach equilibrium (see later).

Is there a notion of a non-equilibrum entropy, S(t), which, for instance satisfies

(i) dS/dt > 0?

(ii) S ---> thermal entropy?



von Neumann entropy is clearly no good, since under unitary evolution (closed system, no damping), it remains zero.

$$\mathrm{Tr}\rho(t)\ln\rho(t) = \mathrm{Tr}\rho(0)\ln\rho(0) = 0$$

Concrete examples: 1+1 dimensional field theories

Consider a finite temperature 1+1 dimensional Euclidean QFT defined on R x S(1). Split space into an (`entangling') interval A and its complement B.

Define the reduced density matrix (see Sinha, Alishahiha's talks)

$$\rho_A(t) = -\mathrm{Tr}_B |\psi(t)\rangle\langle\psi(t)|$$

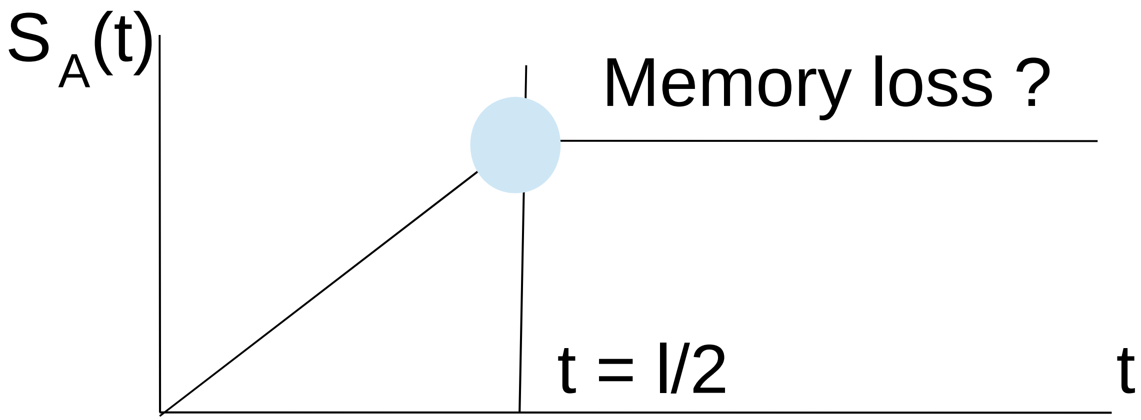
The dynamical entanglement entropy (EE) of interval A is

$$S_A(t) = -\mathrm{Tr}\rho_A(t) \ln \rho_A(t)$$

Proposal: for a sufficiently large interval A, the dynamical EE $S_A(t)$ **is a good candidate for a non-equilibrium entropy S(t) characterizing the quantum state(irrespective of choice of A).**

Calabrese-Cardy 2005, Balasubramaniam et al 2011, Hubeny-Rangamani-Takayanagi 2007, Hartman-Maldacena 2013, HongLiu- Suh 2013, Caputa-GM-Sinha 1306.xxxx

How good is the proposal?



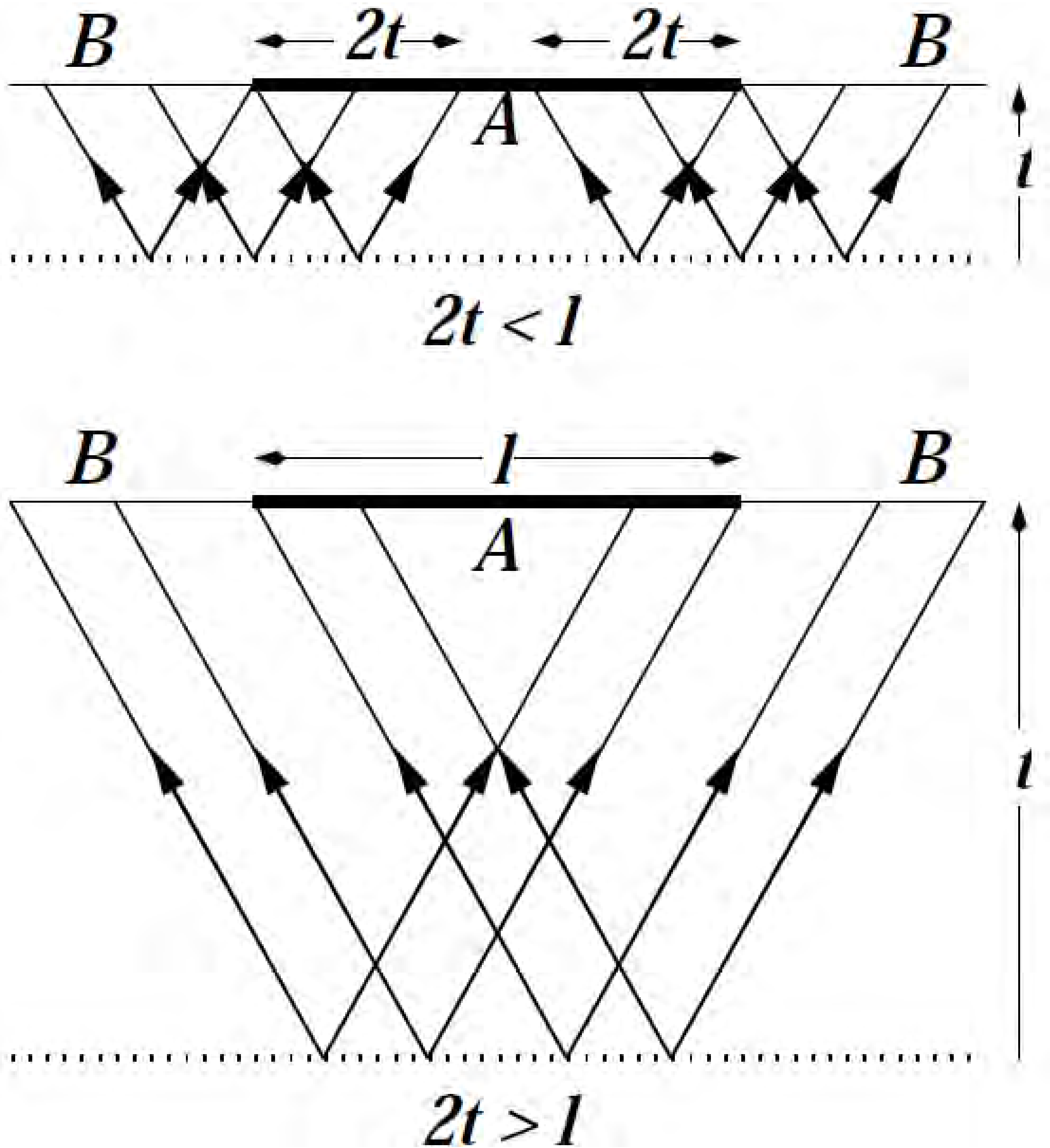
The entropy grows linearly. Does the final value remember about the initial state?

Explicit calculation: $S_A(t>l/2)=l\times c\pi/(3\beta)=\sqrt{2\pi cE/3}$ for $l\gg\beta$
(remembers only the energy of the initial state).

This is also verified by holographic calculation.

Other examples: Vaidya metric with quenched mass function.

Intuition about the linear growth (Calabrese-Cardy 2005)



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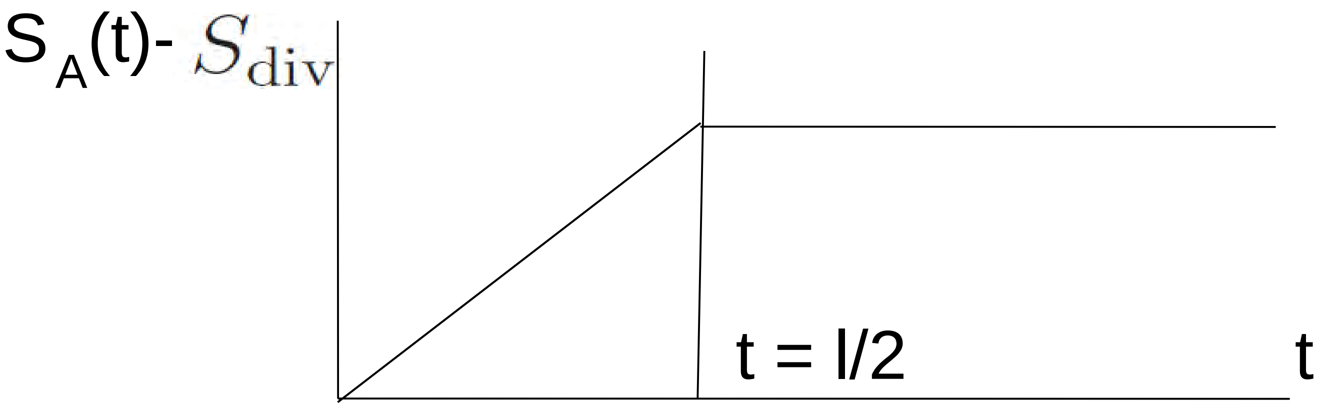
For multiple intervals, the issue of monotoniticiy is subtle.

Why is there a memory loss, and how perfect is it?

(a) Add charges (Caupta-GM-Sihna 1306.xxxx)

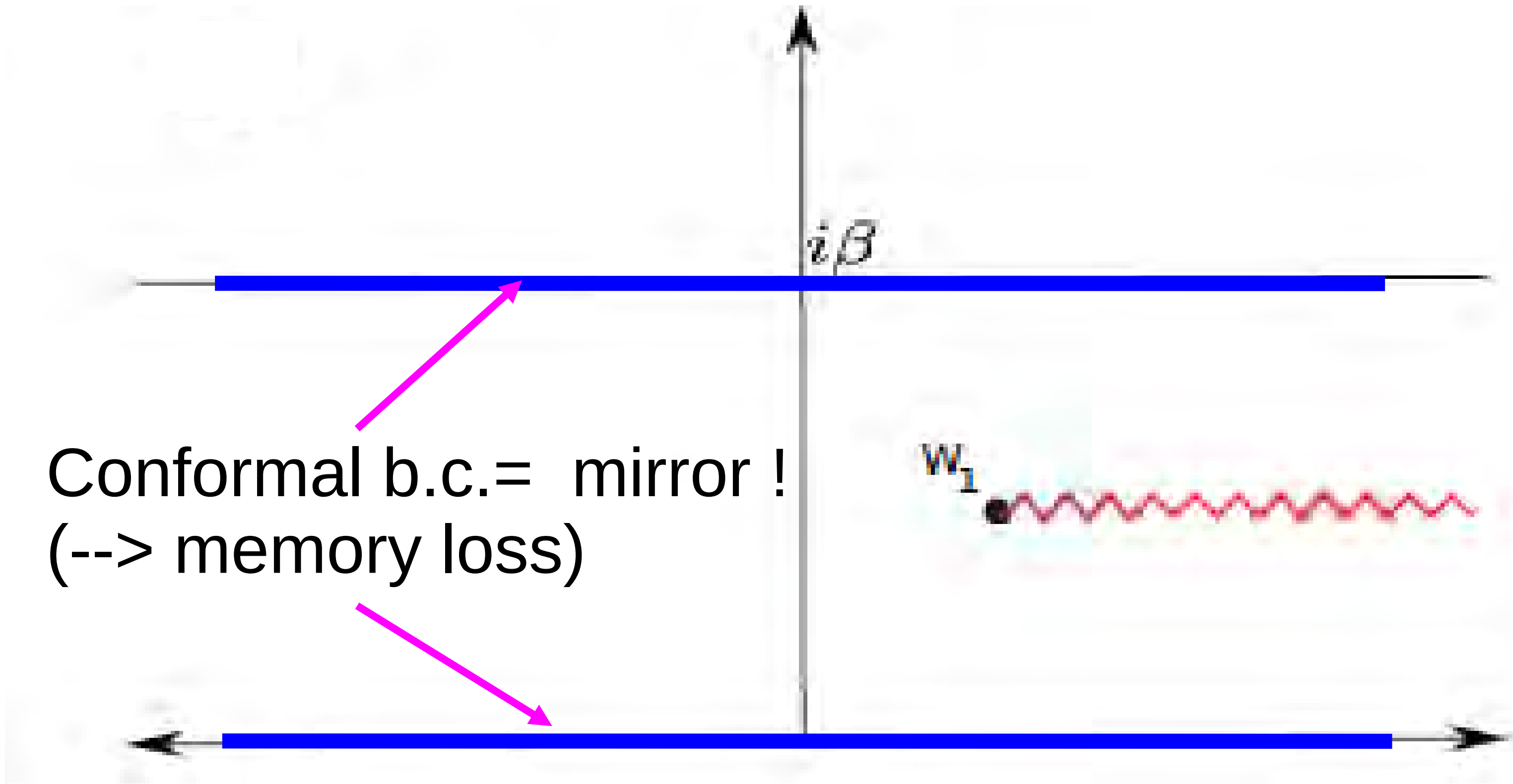


For $l, t \gg \beta$

$$S_A(t) = \begin{cases} 2t\, s_{\rm eqm} + S_{\rm div} & t \le l/2 \\ l\, s_{\rm eqm} + S_{\rm div} & t \ge l/2 \end{cases}$$

$$s_{\rm eqm} = \sqrt{\frac{\pi c}{6}\left(E + J - \frac{\pi}{2k} Q^2\right)} + \sqrt{\frac{\pi c}{6}\left(E - J - \frac{\pi}{2k} Q^2\right)}$$

Some important details

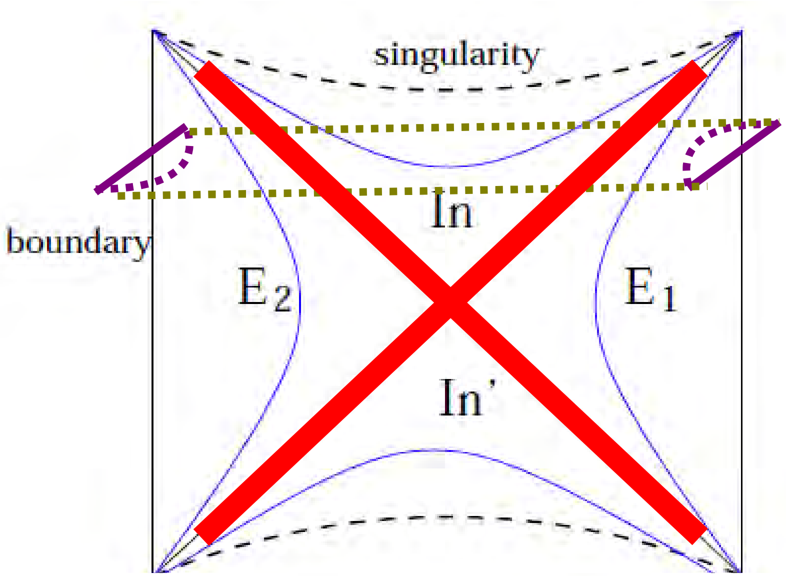


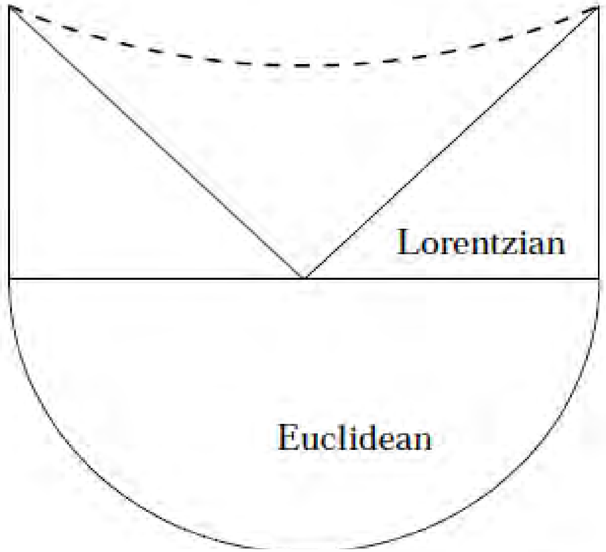
Renyi Entropy = correlation function of ``twist fields'' on the strip

$$S^{(n)} = \frac{1}{1-n}\log\langle\phi^{+}(z_1,\bar{z}_1)\phi^{-}(z_2,\bar{z}_2)\rangle = \frac{c}{6}\left(n+\frac{1}{n}\right)\log\left(\frac{\beta_+\beta_-}{\pi^2\epsilon^2}\cosh\frac{2\pi t}{\beta_+}\cosh\frac{2\pi t}{\beta_-}\right)$$

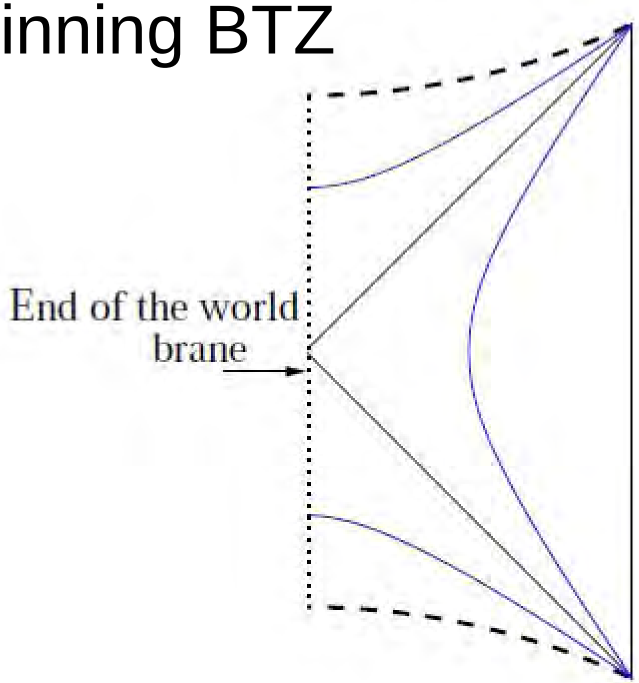
$$S_A(t) = \mathrm{limit}_{n\to 1} S^{(n)}(t)$$

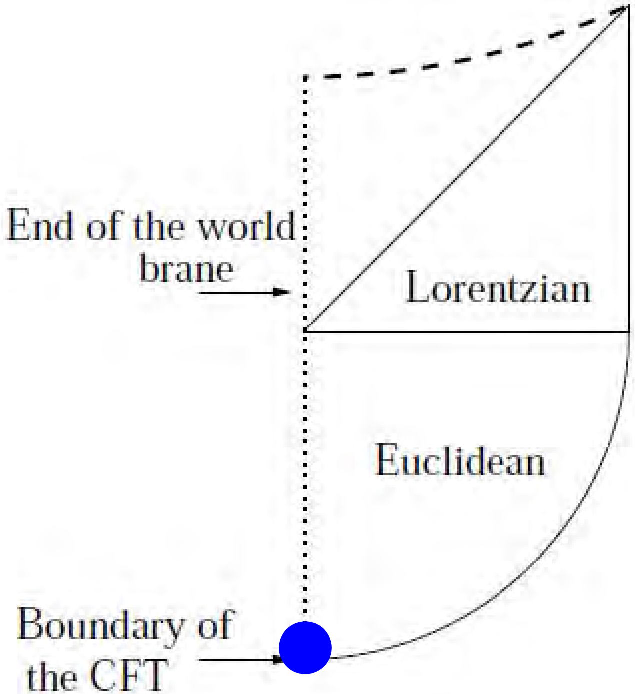
Holographic calculation

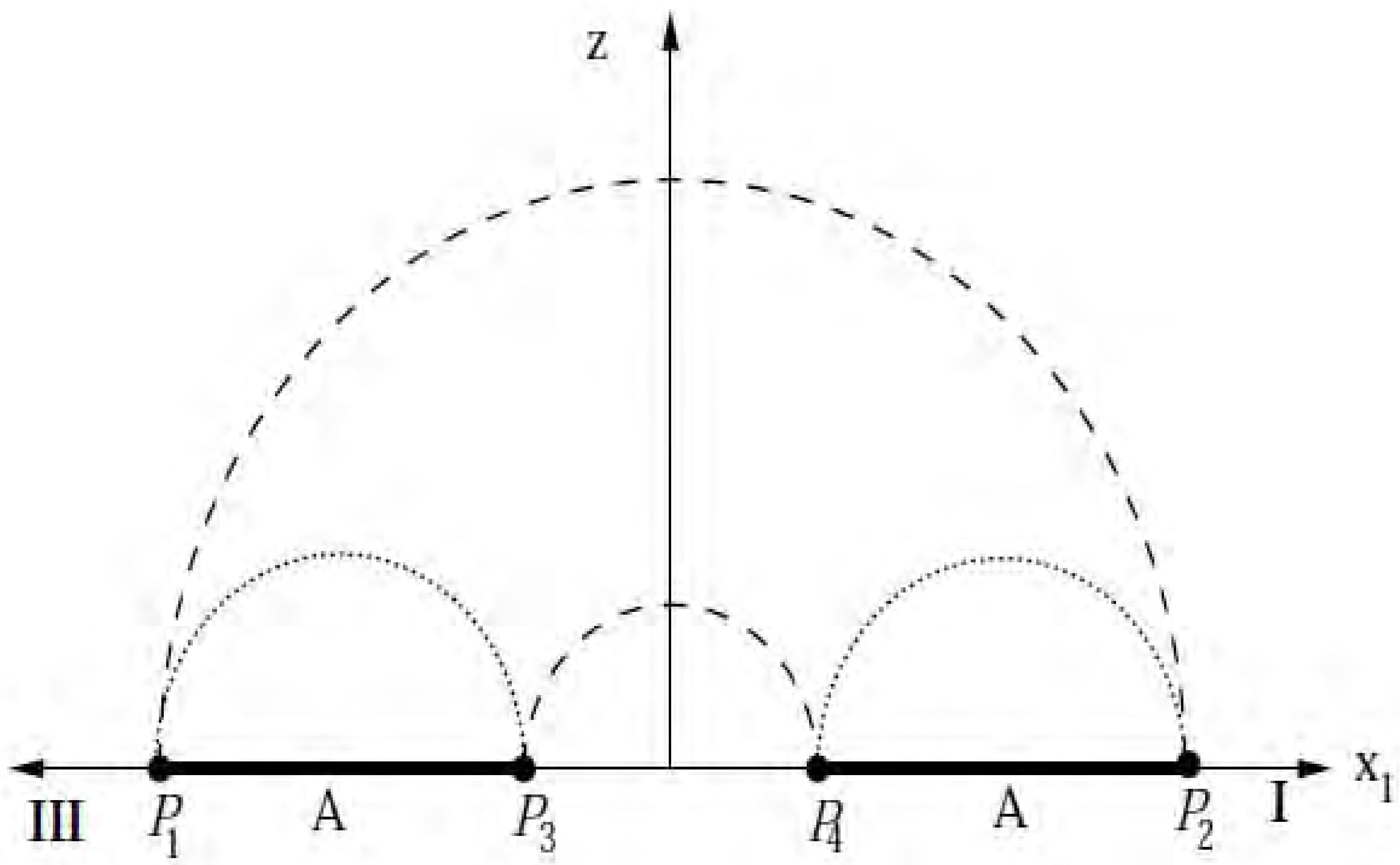




Spinning BTZ







.....,Hartman-Maldacena 2013, ...,Caputa-GM-Sinha 1306.xxxx

A puzzle with Holography

The AdS(3) dual for the CFT(2) with conserved angular momentum and global U(1) charge is described by Einstein gravity (with cosm. Term) + U(1) CS term

$$S=\int d^3x\sqrt{g}\left(R+1/L^2\right)+\int d^3x AdA+boundary\ terms$$

with the classical solution

$$ds^2=ds^2_{BTZ}(E,J),\quad A_z=A_{\bar{z}}=Q$$

The Ryu-Takayanagi prescription for HEE, in the AdS(3) case, becomes a calculation of geodesic length, which is entirely determined by the metric and is, hence, independent of the charge. Hence

$$S_A(t)\xrightarrow{t>l/2} l\left(\sqrt{\pi\frac{c}{6}(E+J)}+\sqrt{\pi\frac{c}{6}(E-J)}\right)$$

A puzzle with Holography (contd.)

Thus, the AdS calculation is independent of the U(1) charge. How does it match with the CFT expression which depends on Q?

Trouble for Ryu-Takayanagi prescription?

Not quite! Note that there are boundary terms in AdS(3) coming from the CS action:

$$S_{boundary,gauge} = \int d^2x \sqrt{h}\, h^{\mu\nu} A_\mu A_\nu$$

which implies $E_{boundary} = E_{bulk} + \pi Q^2/(2k)$ Spectral flow

Hence, $S_{A,bulk}(E_{bulk}, J) = S_{A,CFT}(E_{CFT}, J, Q)$

Alternatively, in terms of canonical quantities:

$$S_{A,bulk}(\beta,\Omega) = S_{A,CFT}(\beta,\Omega)$$

Why does the EE lose memory?

We have just seen that even in the presence of concerved charges, the EE loses memory excepting the conserved charges.

(b) We will now show (Caputa-GM-Sinha 1306.xxxx) that this is a consequence of quantum ergodicity. One statement of quantum ergodicity is that for ``macroscopic observables''

$$\langle\psi(t)|O|\psi(t)\rangle \xrightarrow{t\to\infty} \mathrm{Tr}\left(\rho_{\mathrm{mc}}(E)O\right)$$

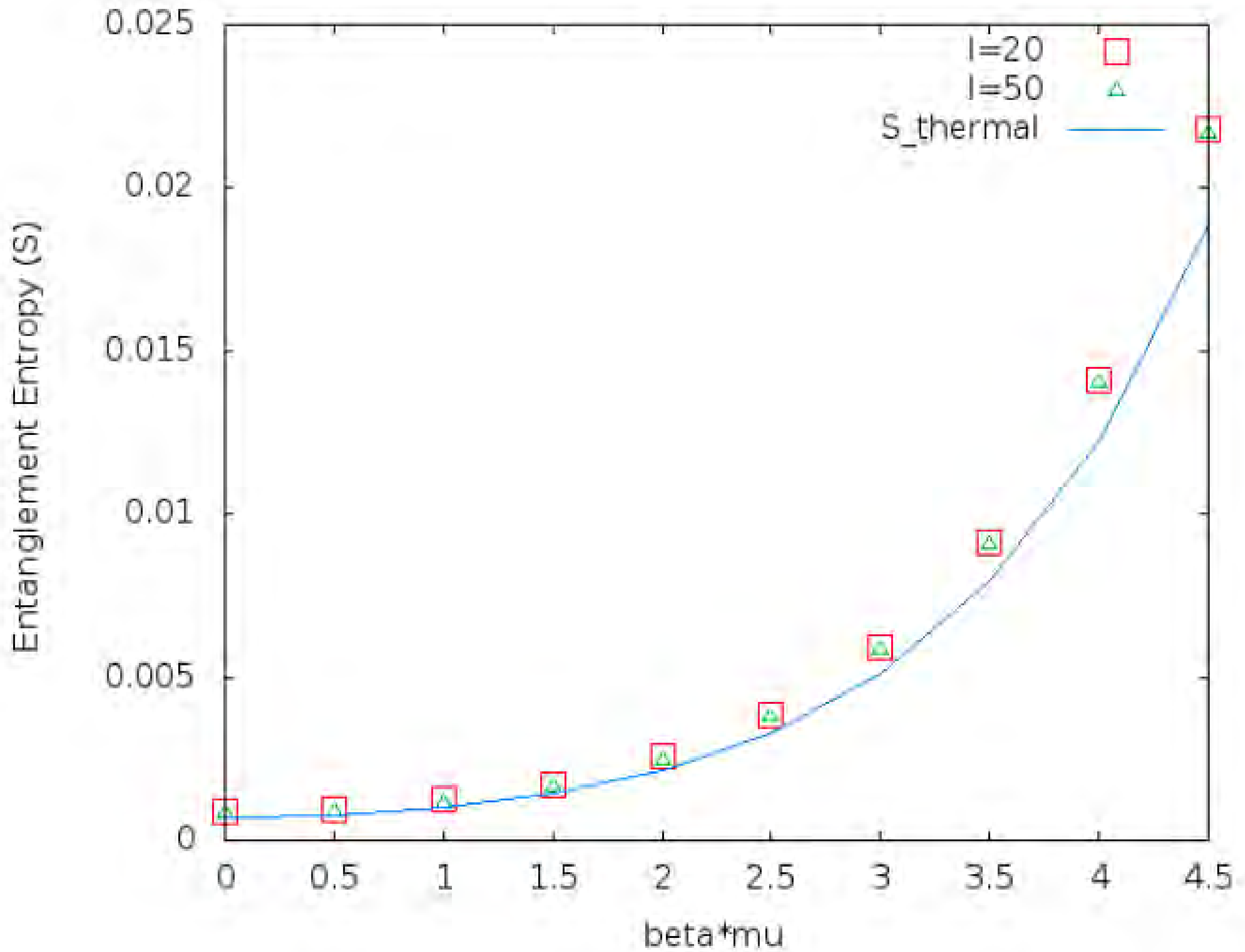
Let us choose $O = P_B|i_A\rangle\langle j_A|$

Then we get $\rho_A \xrightarrow{t\to\infty} \rho_{A,mc} \equiv \mathrm{Tr}(P_B\rho_{mc})$

Hence $S_A \xrightarrow{t\to\infty} -\mathrm{Tr}(\rho_{A,mc}\ln\rho_{A,mc}) = l \times s_{mc} = l \times s_{thermal}$

We can prove the first equality; the second equality follows from ensemble equivalence. (Need $l \gg \beta$)

The above argument is quite robust, including the last two steps. We have independently verified these for extremely non-relativistic systems as well



Combined with the Calebrese-Cardy causality argument, we expect dynamical EE to serve as a good candidate for non-equilibrium entropy in non-CFT's as well (HEE: double trace flow)

Broken Ergodicity (phases)

In case ergodicity does not hold, the final EE can depend on more than just the energy of the final state. It can, rather depend on which "part" of the constant energy subspace one starts from.

$$\langle \psi^{(\alpha)}(t)|O|\psi^{(\alpha)}(t)\rangle \xrightarrow{t\to\infty} \mathrm{Tr}\left(\rho^{(\alpha)}_{\mathrm{part\ mc}}(E)O\right)$$

By repeating the previous arguments, one can now see that the dynamical EE may saturate to values which retain memory of which "part" the initial state belonged to. (Hong Liu-Shu 2013)

The first law

An independent justification for dynamical EE as a non-equilibrium entropy is provided by the ``first law'' (see Sinha, Alishahiha's talks)

$$\Delta E_A = T_{ent}\, \Delta S_A$$

In our case, for $l \ll \beta$ the RHS becomes

$$\Delta S_l = \frac{c}{6}\log\left(\frac{\beta_+\beta_-}{\pi^2\epsilon^2}\sinh\frac{\pi l}{\beta_+}\,\sinh\frac{\pi l}{\beta_-}\right) - \frac{c}{3}\log\frac{l}{\epsilon} \sim \frac{c\pi^2(1+\Omega^2)T^2\, l^2}{18(1-\Omega^2)^2}$$

The LHS is just the mass of the BTZ black hole

$$\Delta E_A = \int_{A=l} dx\, T_{tt} = \frac{MRl}{16\pi G_N} \qquad M = (2\pi T)^2\frac{1+\Omega^2}{(1-\Omega^2)^2}$$

Hence we get

$$\Delta E_l = \frac{3}{\pi l}\Delta S_l$$

Part II: Matrix quantum mechanics (thermalization, dynamical ph.tr.)

Single trace Unitary Matrix QM (GM, T. Morita 2013)

$$Z = \int DU(t)\, e^{iNS}, \; S = \int dt \left[\frac{1}{2}\mathrm{Tr}\left(|\partial_t U|^2\right) - V(U)\right], \; V(U) = \frac{a}{2}\left(\mathrm{Tr}U + \mathrm{Tr}U^\dagger\right)$$

This model is equivalent to N free fermions on a circle.

$$H = \int d\theta\, \psi^\dagger(\theta,t)\left[-\frac{1}{2N^2}\partial_\theta^2 + V(\theta)\right]\psi(\theta,t), \qquad V(\theta) = a\cos\theta,$$

$$\rho(\theta,t) = \frac{1}{N}\psi^\dagger(\theta,t)\psi(\theta,t), \qquad \int d\theta\, \rho(\theta,t) = 1.$$

Here the fermion field has a mode expansion in terms of single-particle eigenfunctions:

$$\psi(\theta) = \sum_{m=1}^{\infty} c_m \varphi_m(\theta), \quad h\varphi_m(\theta) = \epsilon_m \varphi_m,$$

Relation to adjoint scalar QCD in 1+1

$$S = \int dt \int_0^L dx\, \mathrm{Tr}\left(\frac{1}{2g^2} F_{tx}^2 + \sum_{I=1}^{D} \frac{1}{2} \left(D_\mu Y^I\right)^2 + \sum_{I,J} \frac{g'^2}{4} [Y^I, Y^J][Y^I, Y^J] \right).$$

↓

$$S/DN^2 = \int dt \left[\frac{1}{2N} \mathrm{Tr}\left(|\partial_t U|^2\right) - \frac{\Delta}{\pi \tilde{\lambda} L} \sum_{n=1}^{\infty} \frac{K_1(n\Delta L)}{n} \left| \frac{1}{N} \mathrm{Tr} U^n \right|^2 \right]$$

$$U(t) = P \exp[i \oint A_1(t) dx]$$

AdS dual= D2 branes with Scherk-Shwarz compactification

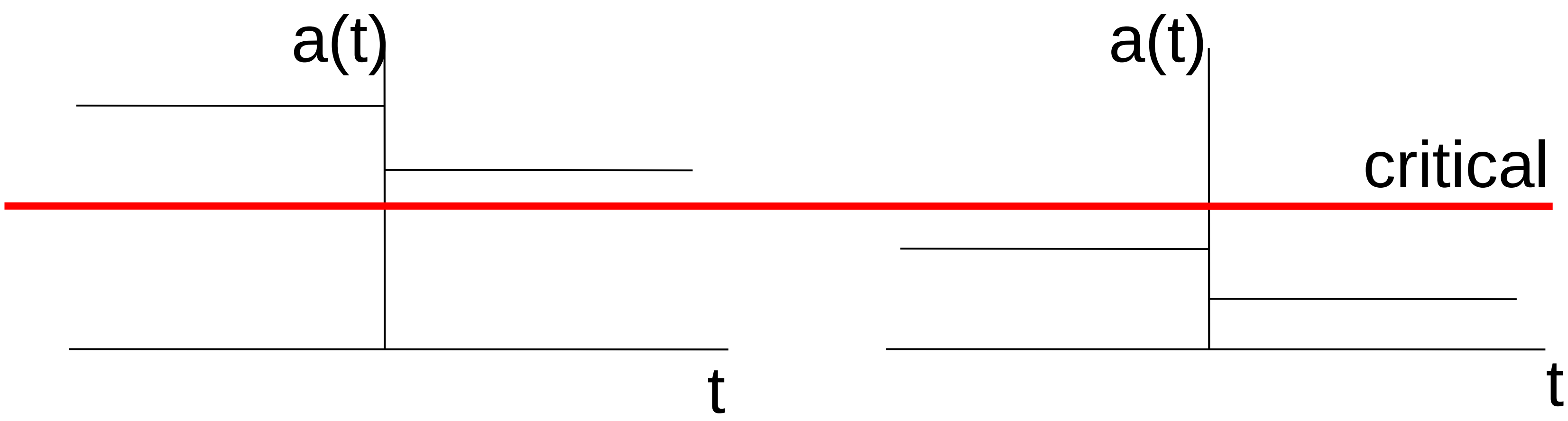
GM-Morita 2011, Mahato-GM-Morita 2009

Integrable system

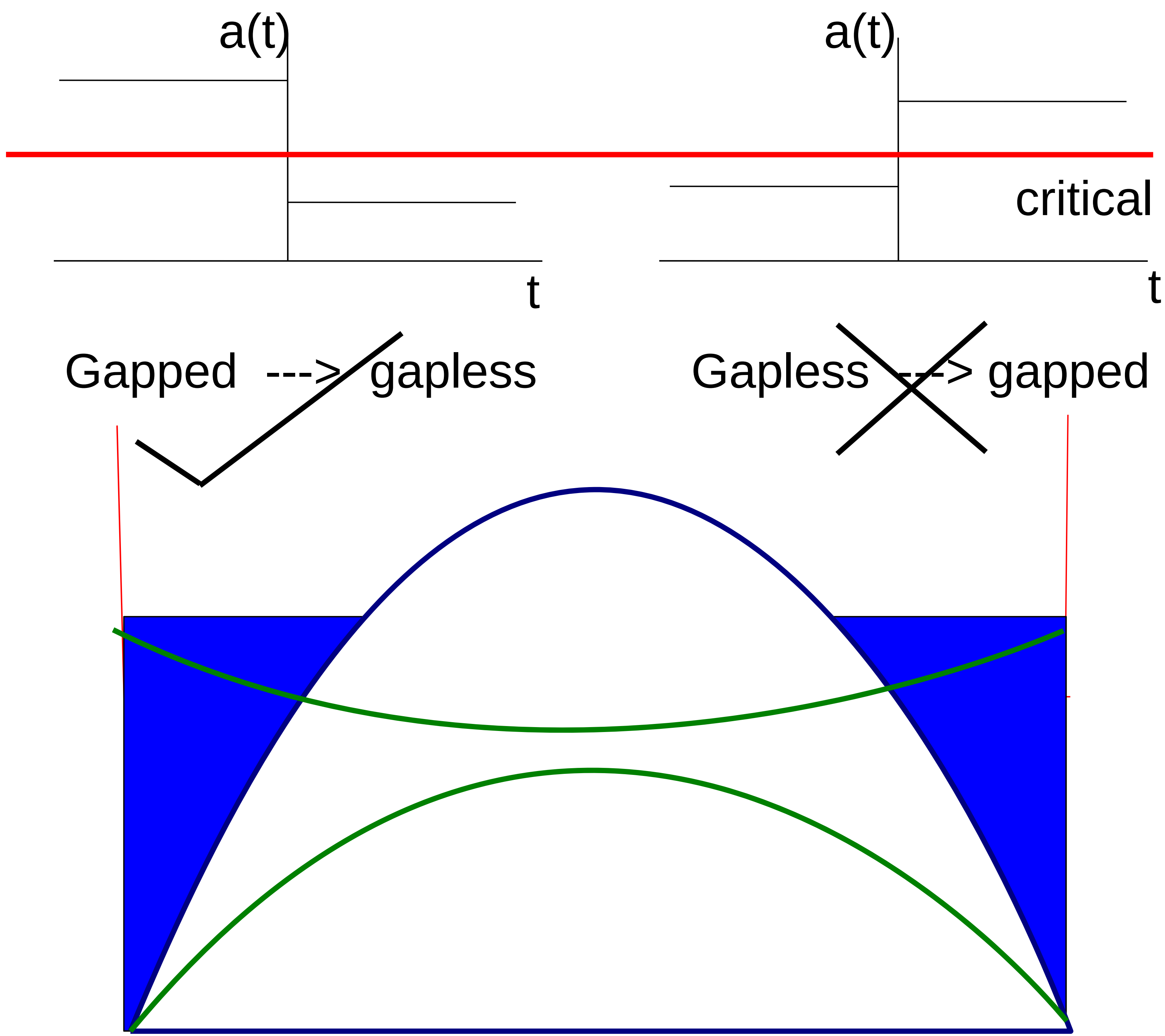
The system of free fermions clearly has an infinite number of conserved charges

$$N_m = c_m^\dagger c_m, \quad (m = 1, \cdots)$$

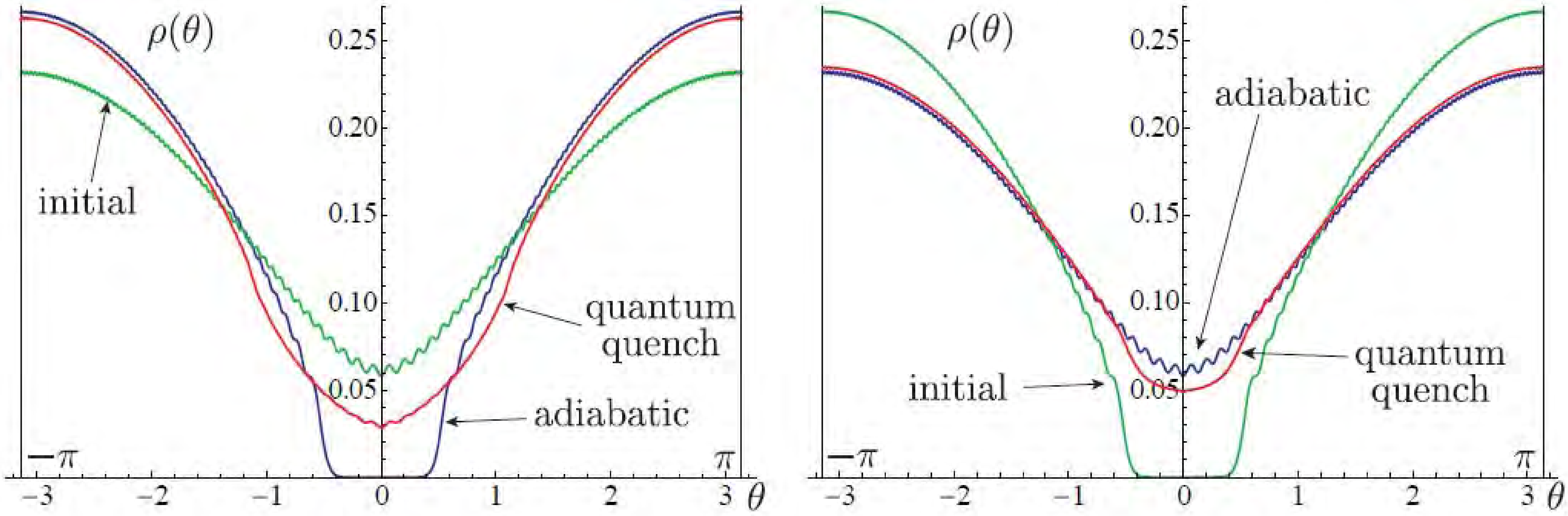
Consider a quantum quench in this system. Make a -> a(t) in the potential, e.g.



Gapped ---> gapped Gapless ---> gapless

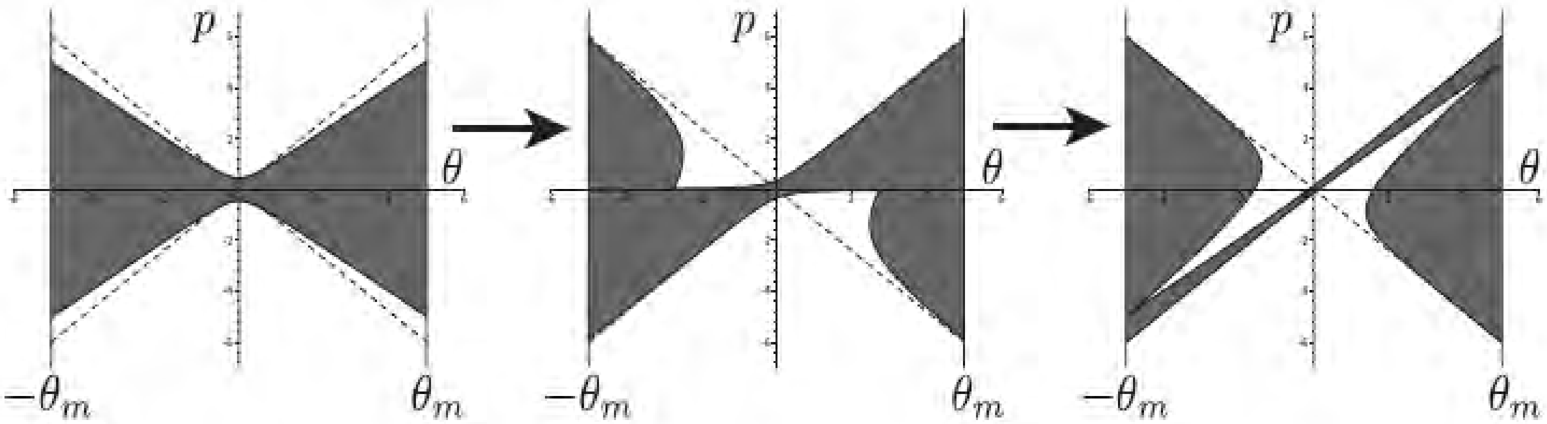

a(t)
t
a(t)
critical
t
Gapped ---> gapless
Gapless ---> gapped

Irreversibility of dynamical phase transitions under quantum quench

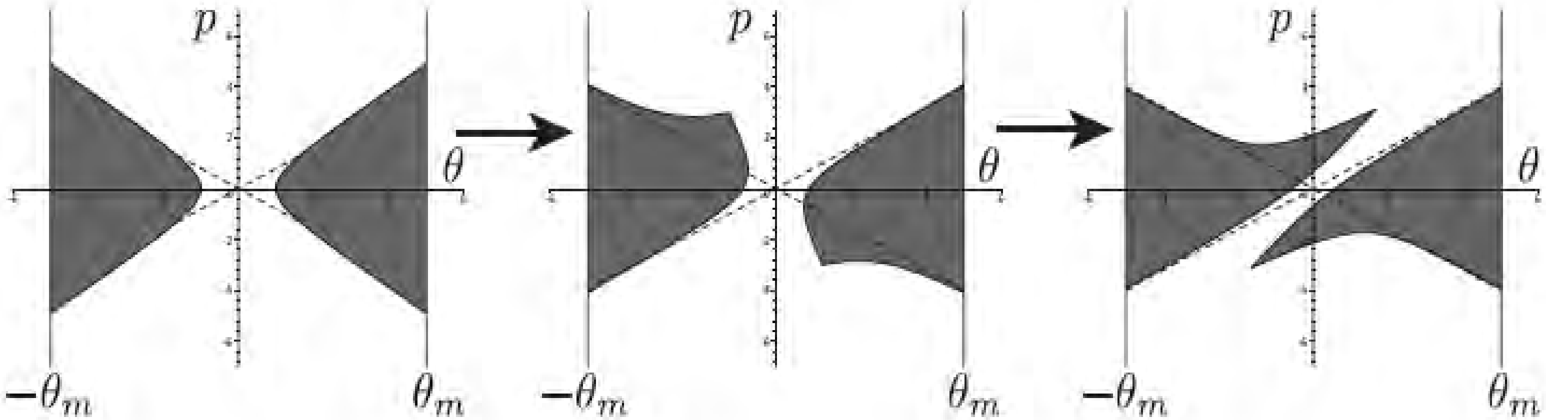


(a) $a_i < a_c < a_f$ (b) $a_i > a_c > a_f$

Intuition



(I) $a_i < a_c$, $a_f > a_c$.



(II) $a_i > a_c$, $a_f < a_c$.

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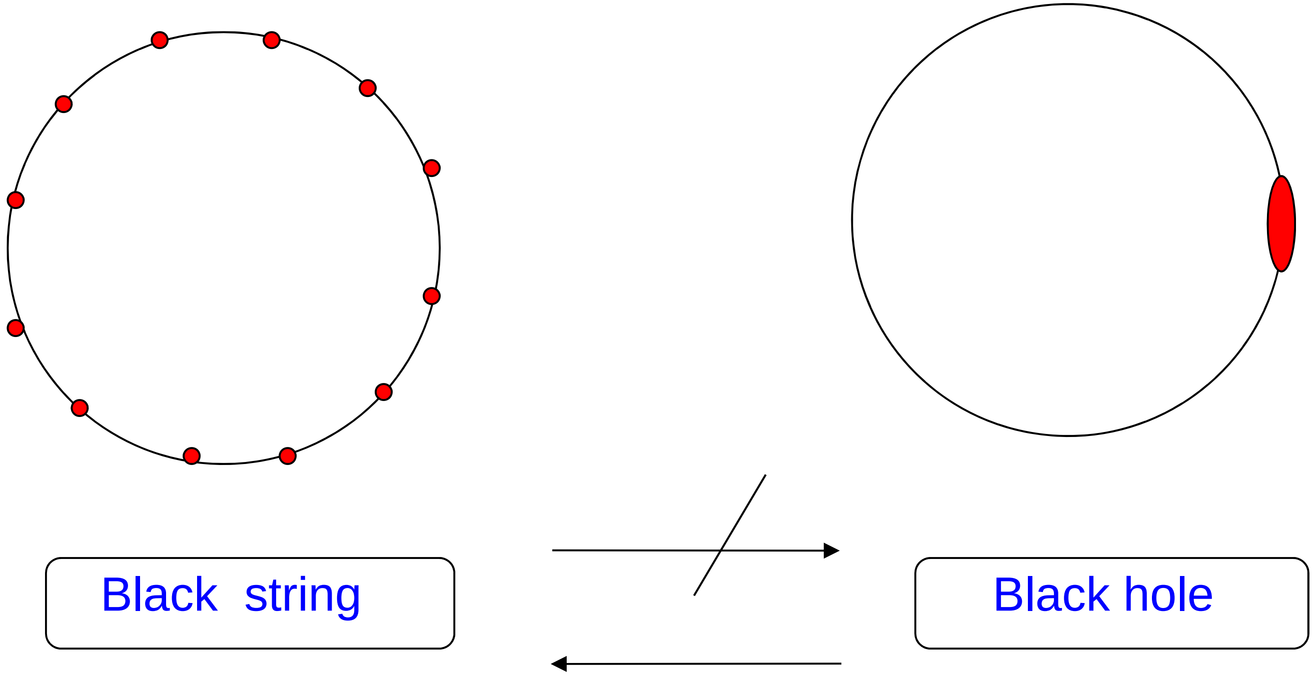
Gravity dual statement

Uniform eigenvalues of Wilson line=centre symmetry=fractional periodicity

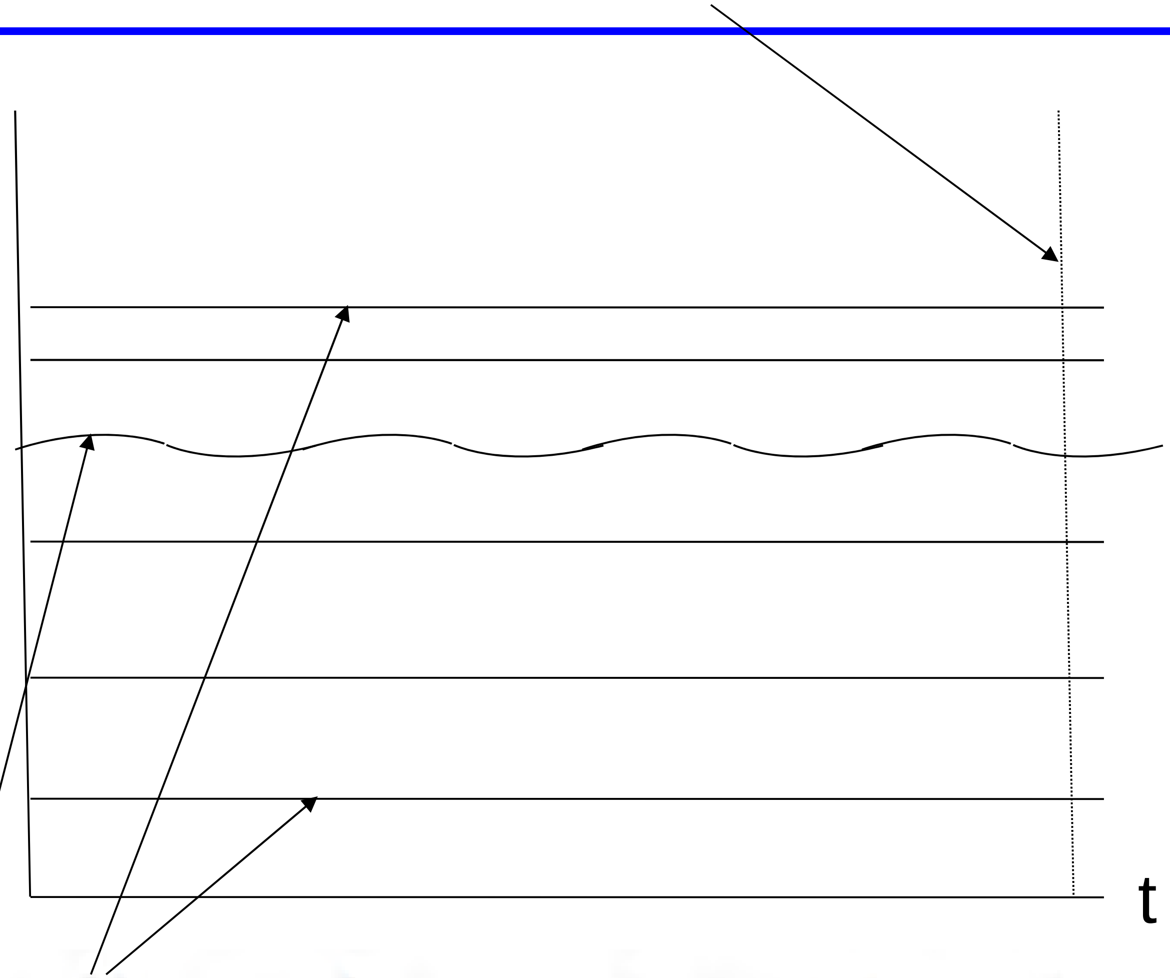
Non-uniform eigenvalues=broken centre symmetry=integral periodicity



Dynamical Gregory Laflamme transformation

No Naked singularity appears (however, see Choptuik et al)

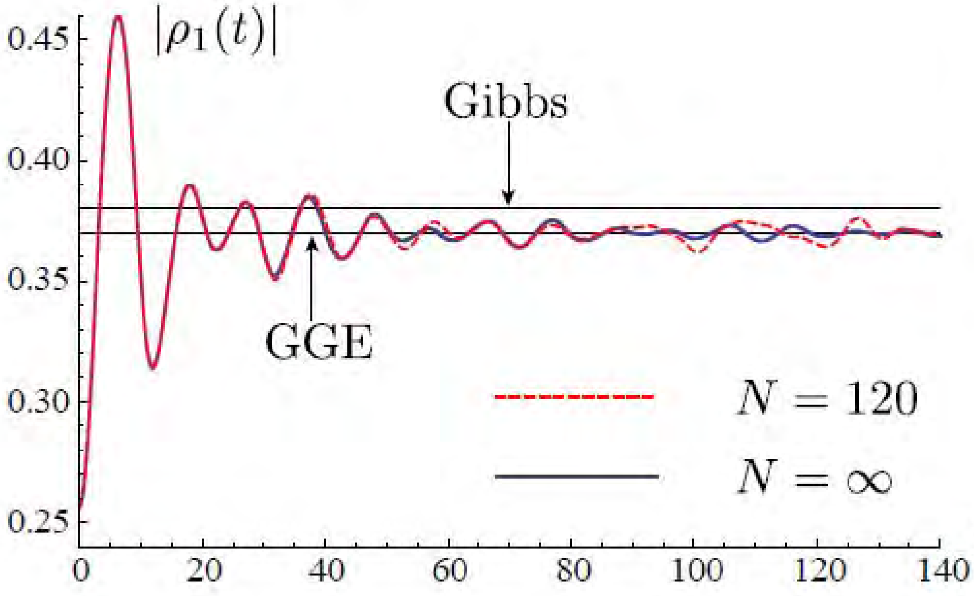
Retention of memory



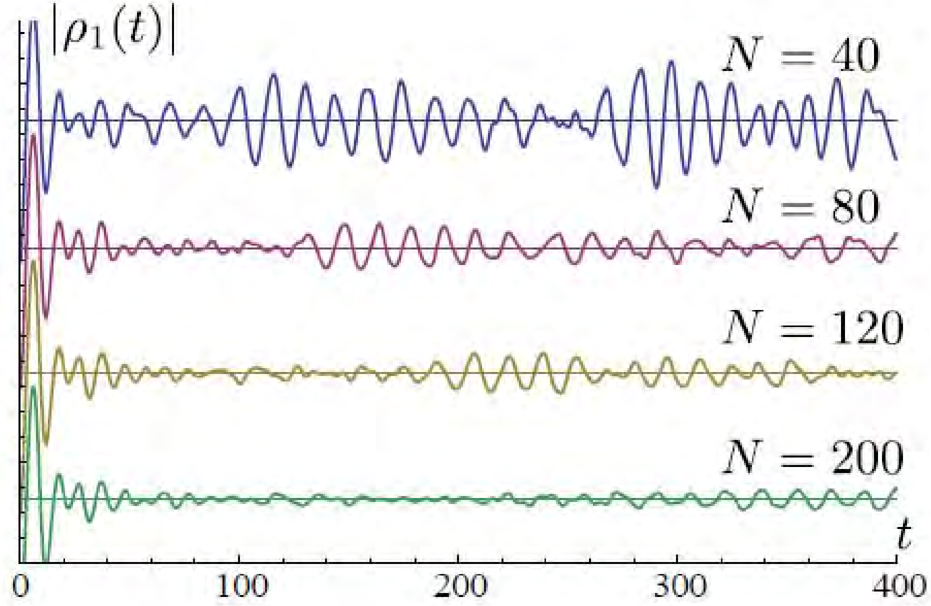
$$N_m = c_m^\dagger c_m, \quad (m = 1, \cdots)$$

$$\hat{O}_{m,n}(t) = c_m^\dagger c_n \exp[-i(\epsilon_m - \epsilon_n)t/\hbar], \quad m \neq n.$$

Thermalization of density in spite of integrability



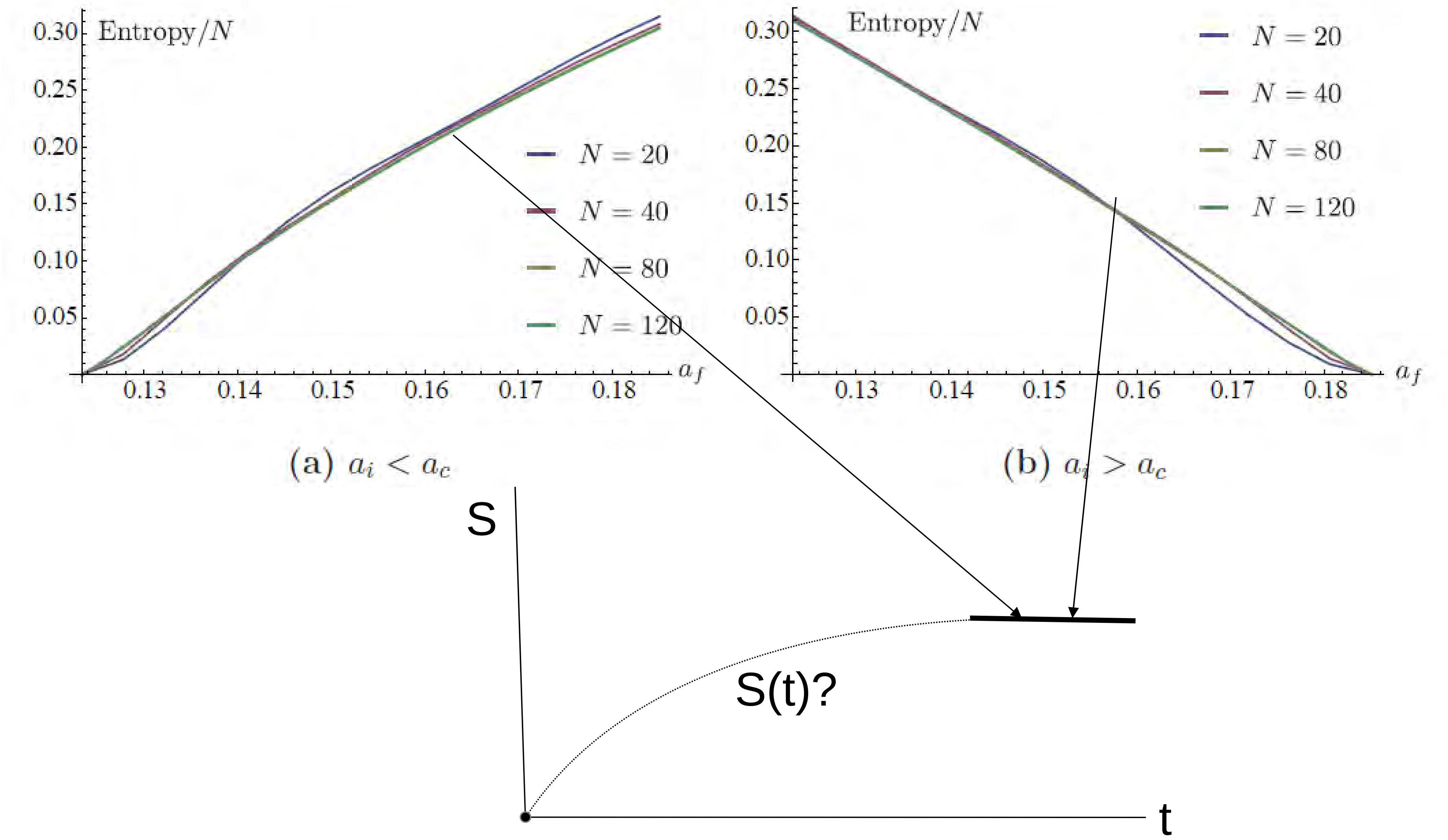
(a) $\rho_1(t)$ and GGE vs. Gibbs ensemble



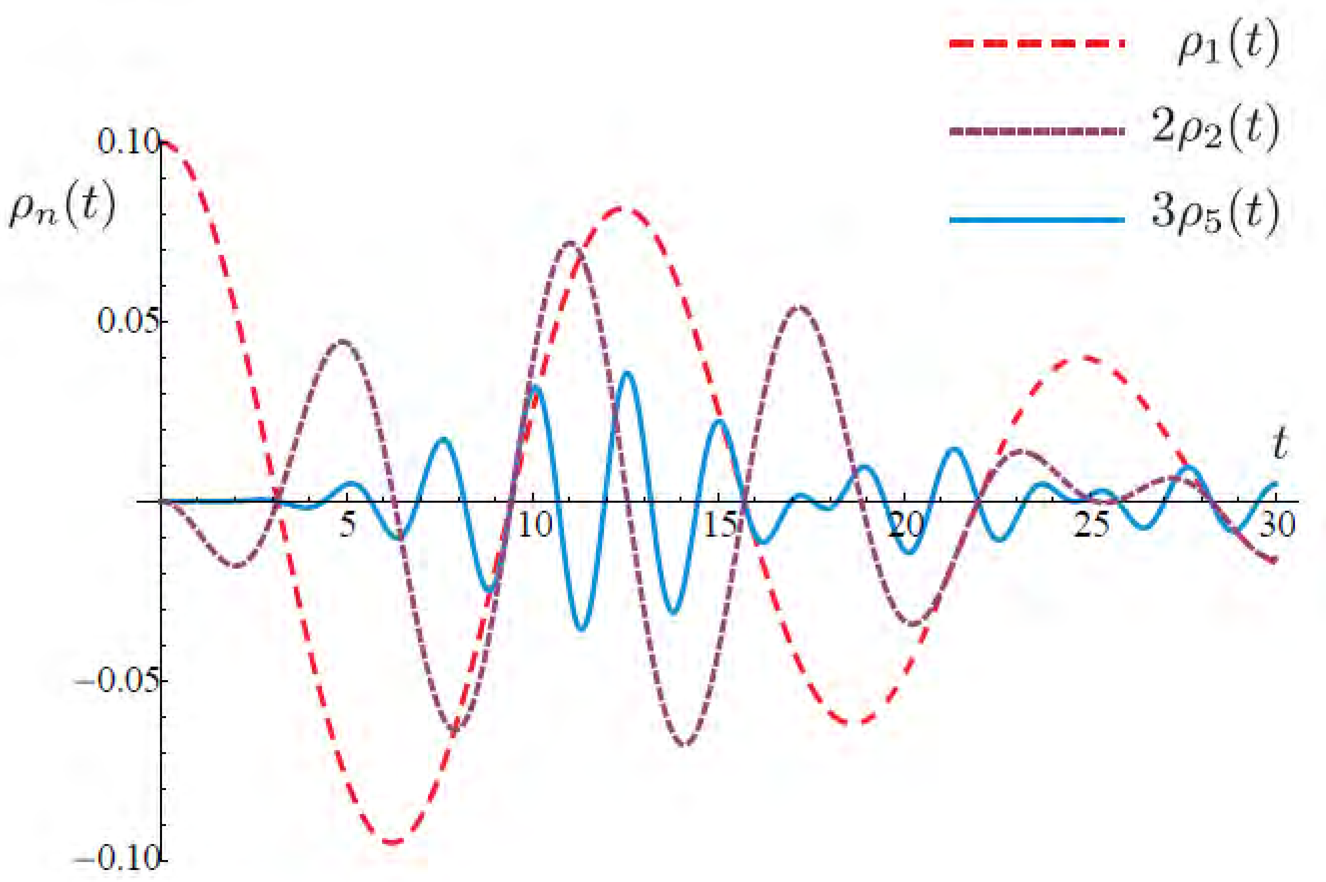
(b) Poincaré recurrence

$$\langle \rho_n(t) \rangle = \int d\theta \langle \rho(\theta, t) \rangle \cos(n\theta).$$

Entropy Production



Turbulence-like Cascades



All modes die: what happend to conservation of energy? They do not die at the same time. Long wavelength modes die off first, then shorter wavelength modes take off, after they die, even shorter modes take off, etc.

Conclusions

1. Dynamical EE promises to be a good candidate for non-equilibrium entropy. We studied this for 1+1 dimensional FT's and connected to quantum ergodicity.

2. In case of 1+1 dimensional free fermion systems (= 2d adjoint scalar QCD= Maitrx QM), dynamical phase transitions show apparent irreversibiity under quantum quench.

3. In these systems, thermalization occurs in spite of integrability. This adds a new model to the existing studies, in addition to TFI and hard-core bosonic chain. (Cold atoms)

4. Dissipation can be understood in terms of energy cascades in terms of transfer of energy from long to short wavelengths (cf. turbulence).