Attraction and Subtraction

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Crete. 18 June, 2013

Work with Avik Chakraborty (1212.1875, 1212.6919)
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Consider an action of the form

$$S = \int d^4x \sqrt{-g} \Big(R - 2(\partial\phi)^2 - f_{ab}(\phi) F^a_{\mu\nu} F^{b\ \mu\nu} \Big) \tag{1}$$

A very natural action to consider. Contains all the low spin bosons that have long range (classical) effects.

(a) scalar is uncharged, (b) it is a modulus, (c) coupling happens because of the scalar dependence of the gauge coupling.

Eg: Ungauged supergravity, coming from torus reductions of string.

This theory admits black hole solutions. We will focus on extremal black holes - these have charge=mass (roughly) $\implies$ Double-zero horizon

$$ds^2 = -dt^2\left(1-\frac{r_H}{r}\right)^2 + \frac{dr^2}{\left(1-\frac{r_H}{r}\right)^2} + r^2 d\Omega^2. \tag{2}$$

Attractor Mechanism (eg., Goldstein et al): The scalar $\phi(r)$ can have a non-trivial radial profile, so that $\phi(r_H)$ is fixed but $\phi(r=\infty)$ can vary. The metric functions change accordingly, but it still has a double-zero at the horizon.

$$ds^2 = -a(r)^2 dt^2 + \frac{dr^2}{a(r)^2} + b(r)^2 d\Omega^2. \tag{3}$$

Why care?

1. Violation of no-hair theorem in flat space.

2. $\phi(r = \infty)$ has a coupling constant interpretation, so we might hope to tune it to weak coupling at the boundary (Dabholkar-Sen-Trivedi).

3. Charges are determined by the near-horizon scalar values, and charges determine thermodynamics. Attractor mechanism is the statement that only the near-horizon geometry really matters for microscopics/thermodynamics etc.

4. Our goal, much more modest: scan the attraction basin as a solution space of supergravity and see that the recently introduced subtracted geometry arises as the boundary of the attraction basin.

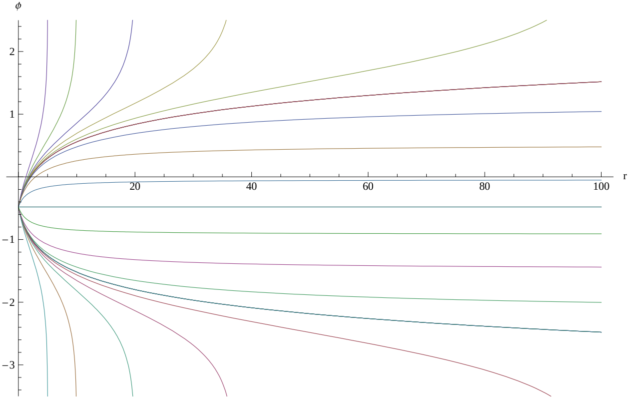


Figure : (Part of our) Result: Attraction Basin

What happens when the scalar moves outside of the attraction basin? How do we characterize the full attraction basin and its boundary? Nature of the flow? etc. etc.

One purpose of our work is to answer this question. But we stumbled upon this via another direction, so we will take a digression to discuss that.

Hidden Conformal Symmetry (Castro-Maloney-Strominger)

A piece of folk lore is that generic black holes (not just extremal ones) might be dual to conformal field theories. Crude way to motivate it: redshift at the horizon = IR fixed point.

More practically: For various (generic) black holes in string theory, the formulas for the thermodynamic quantities have a form that **begs** for a CFT interpretation. Entropy has a Cardy-like structure even away from extremality and supersymmetry (Cvetic-Larsen). Note: 2-D conformal symmetry.

But direct attempts at a holographic implementation of idea haven't gone far.

One recent way to look for the conformal symmetry makes the following observation. Consider a fairly generic black hole metric:

$$ds^2 = \frac{G(r,\theta)}{\sqrt{\Delta(r,\theta)}}(dt + A(r,\theta)d\phi)^2 + \\ +\sqrt{\Delta(r,\theta)}\Big(\frac{dr^2}{X(r)} + d\theta^2 + \frac{X(r)}{G(r,\theta)}\sin^2\theta d\phi^2\Big) \quad (4)$$

The functions are known explicitly in various reductions of string theory, but we won't write them down. Often they are very complicated.

It turns out that the wave equation in the geometry in the IR limit looks like an AdS_3 wave equation. This is interpreted as due to a hidden conformal symmetry (CFT2). (Castro et al., CK, Chen, ...)

Subtracted geometry

It turns out that the AdS_3 structure of the wave equation can be manifested at the level of the geometry, by changing ONLY the warp factor in the metric.

$$\Delta(r,\theta) \to \Delta_0(r,\theta) \tag{5}$$

where there is a specific algorithm for producing Δ_0 once we know Δ.

The resulting geometry is called the Subtracted Geometry of the original black hole (Cvetic-Larsen). The asymptotic structure changes: the black hole is now in a conical box, not asymptotically flat.

$$\Delta(r,\theta) \sim r^4 \to \Delta_0(r,\theta) \sim r \tag{6}$$

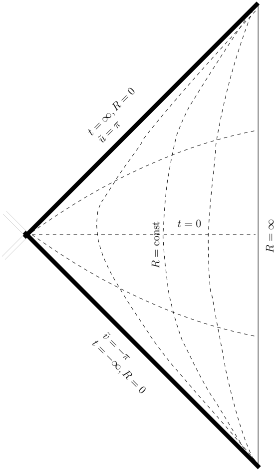
Solution of the same theory, but with matter **necessarily** excited: even in the cases where the unsubtracted BH is a vacuum solution.

The scalar diverges logarithmically at $r \to \infty$.

Temperature and Entropy do not depend on Δ, so they don't change. Proposal(Cvetic-Larsen): Can the thermodynamics of flat space black holes be thought of as captured by this boxed black hole? And the rest of the geometry is "ambience"?

Without the black hole, the metric is a conical box:

$$ds^2 = -\frac{R^6}{p^2}dt^2 + 16dR^2 + R^2 d\Omega_2^2. \tag{7}$$


$t = \infty, R = 0$
$\tilde{u} = \pi$
$t = 0$
$R = \mathrm{const}$
$R = \infty$
$\tilde{v} = -\pi$
$t = -\infty, R = 0$

For the purpose of this talk, our goal will be different and narrow: it is to connect the subtracted geometry to the attractor mechanism in flat space by looking at extremal versions of the subtracted geometry.

A usual slogan: double zero (extremality) should make the horizon attractive. But what happens to the solution as we integrate the perturbation radially outward?

Perturbation theory at the horizon + Mathematica experiments suggest: when you perturb the scalar at the horizon, the geometry flows to an asymptoticallty flat spacetime, for one choice for the sign of the perturbation. Solution blows up for the other choice of sign. Strange.

Can we find exact solutions and make sense of this?

Surprisingly (to us), it turns out that exact solutions can be found in certain cases by connecting the system to a Toda system adapting some old work by Gibbons-Maeda, Pope, ...

Exact solutions (toy example)

Consider, with $(\alpha_1, \alpha_2) = (2\sqrt{3}, -2/\sqrt{3})$,

$$S = \int d^4x\sqrt{-g}\Big(R - 2(\partial\phi)^2 - e^{\alpha_1\phi}{F_2}^2 - e^{\alpha_2\phi}{F_1}^2\Big) \quad (8)$$

Look for static solutions in the “attractor" ansatz:

$$ds^2 = -a(r)^2dt^2 + \frac{dr^2}{a(r)^2} + b(r)^2d\Omega^2,$$
$$F_{a=1,2} = Q_{a=1,2}\sin\theta d\theta \wedge d\phi, \quad \phi = \phi(r) \quad (9)$$

Solution (toy example)

Exploiting the Toda-connection, once the dust settles, the solution can be written as

$$a^2 = \frac{r^2}{\Xi\, Q_1^{1/2} Q_2^{3/2}} \frac{1}{\sqrt{(1+d_1 r)(1+d_2 r)^3}},$$
$$e^{\frac{4\phi}{\sqrt{3}}} = \frac{Q_2}{\sqrt{3} Q_1}\Big(\frac{1+d_2\, r}{1+d_1\, r}\Big), \qquad b^2 = \frac{r^2}{a^2}. \tag{10}$$

Ξ is a complicated numerical factor, whose form doesn't matter here.

Regularity for all $r \implies d_1, d_2 > 0$. These are the asymptotically flat solutions, and capture the attraction basin.

So the boundaries of the basin correspond to $d_1 = 0$ and $d_2 = 0$. It is possible to check that $d_2 = 0$ boundary is the relevant subtracted geometry for the system.

Solutions with d_1 or $d_2 < 0$ diverge at finite radius. So if we start on the subttractor and perturb the wrong way, we will see blow-ups. Otherwise, we end up in flat space. Which is what we set out to understand.

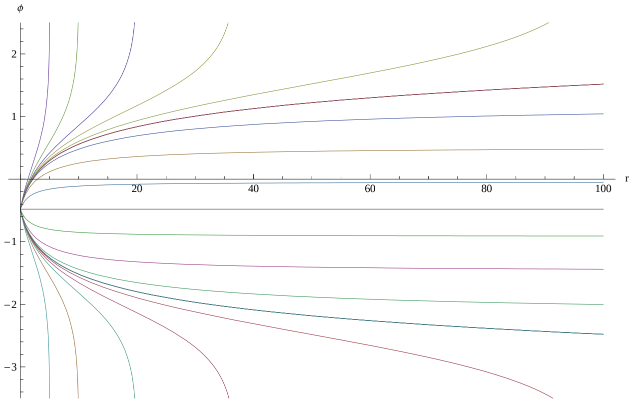


Figure : Once Again...

Further Questions

1. Similar exact (static) solutions can actually be found in more interesting theories like the STU model, and the attraction basin charted (more general d_i). Other boundaries of the basin result in more general subtractions: generalized subtracted geometry. The warp factor $\Delta_0 \sim r, r^2, r^3, r^4$. (de Boer et al, Chakraborty&CK).

2. Adding rotation is more difficult, but can be done. (Guica et al., Jana&CK)

3. Attraction basin in the d_i space can have more complex structure than the simple $(d_1 > 0, d_2 > 0)$ in our toy example (Jana&CK).

4. Subtracted geometry from solution generating techniques (Virmani, Guica et al.). SG as a scaling limit (Cvetic-Gibbons). Separability of wave equations in SG (Larsen-Keeler).

5. Now that we have an understanding of the subtracted geometry in the solution space of SUGRA, what can we say about its physics? Is it a viable box for the flat space black hole? Physics with conical box asymptotics? How to define charges? What is the appropriate vacuum - the extremal subtracted geometry? Empty conical box (unlike empty AdS) is pathological because of singularities, and so QFT cannot be defined there? etc?

Thank you!