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Holographic models for QCD in the Veneziano limit

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1. Introduction and motivation
2. Holographic V-QCD models

[MJ, Kiritsis arXiv:1112.1261]

3. Turning on finite temperature

[Alho, MJ, Kajantie, Kiritsis, Tuominen arXiv:1210.4516]

4. Spectrum and correlators

[Arean, Iatrakis, MJ, Kiritsis, arXiv:1211.6125, arXiv:1306.xxxx]

1. Introduction

Motivation

QCD: $SU(N_c)$ gauge theory with N_f quark flavors (fundamental)

- ▶ Often useful: “quenched” or “probe” approximation, $N_f \ll N_c$
- ▶ Here **Veneziano limit**: large N_f, N_c with $x = N_f/N_c$ fixed \Rightarrow backreaction

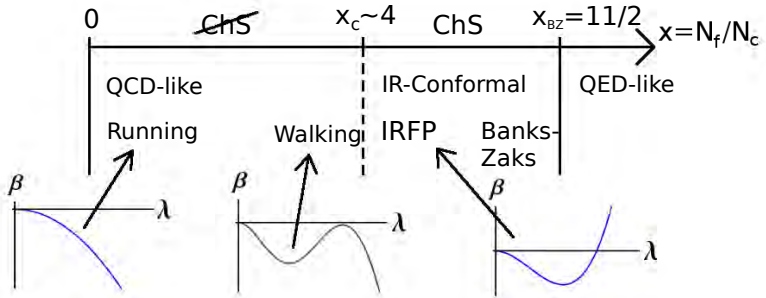
Important new features can be captured in the Veneziano limit:

- ▶ Phase diagram of QCD (at zero temperature, baryon density, and quark mass), varying $x = N_f/N_c$
- ▶ The QCD thermodynamics as a function of x
- ▶ Phase diagram as a function of baryon density

QCD phases in the Veneziano limit

Expected structure at zero T , μ , and quark mass; finite $x = N_f/N_c$

- ▶ Phases:
 - ▶ $0 < x < x_c$: QCD-like IR, chiral symmetry broken
 - ▶ $x_c \leq x < 11/2$: Conformal window, chirally symmetric
- ▶ **Conformal transition** at $x = x_c$
- ▶ RG flow of the coupling: running, walking, or fixed point



2. V-QCD

Holographic V-QCD: the fusion

The fusion:

1. IHQCD: model for glue by using dilaton gravity

[Gursoy, Kiritsis, Nitti; Gubser, Nellore]

2. Adding flavor and chiral symmetry breaking via tachyon
brane actions

[Klebanov, Maldacena; Bigazzi, Casero, Cotrone, Iatrakis, Kiritsis, Paredes]

Consider 1 + 2 in the Veneziano limit with **full backreaction**
⇒ V-QCD models

[MJ, Kiritsis arXiv:1112.1261]

Defining V-QCD

Degrees of freedom ($T = \tau\mathbb{I}$):

- ▶ The tachyon $\tau \leftrightarrow \bar{q}q$, and the dilaton $\lambda \leftrightarrow \text{Tr}F^2$
- ▶ $\lambda = e^\phi$ is identified as the 't Hooft coupling $g^2 N_c$

$$\mathcal{S}_{\text{V-QCD}} = N_c^2 M^3 \int d^5x \sqrt{g} \left[R - \frac{4}{3} \frac{(\partial\lambda)^2}{\lambda^2} + V_g(\lambda) \right] \\ - N_f N_c M^3 \int d^5x V_f(\lambda, \tau) \sqrt{-\det(g_{ab} + \kappa(\lambda) \partial_a \tau \partial_b \tau)}$$

$$V_f(\lambda, \tau) = V_{f0}(\lambda) \exp(-a(\lambda)\tau^2); \quad ds^2 = e^{2A(r)}(dr^2 + \eta_{\mu\nu}x^\mu x^\nu)$$

- ▶ Need to choose V_g , V_{f0} , a , and $\kappa \dots$
- ▶ The simplest and most reasonable choices do the job!
- ▶ We have, however, also explored generic choices

Matching to QCD

In the UV ($\lambda \rightarrow 0$):

- ▶ UV expansions of potentials matched with perturbative QCD beta functions \Rightarrow

$$\lambda(r) \simeq -\frac{\beta_0}{\log r} \quad \tau(r) \simeq m(-\log r)^{-\gamma_0/\beta_0} r + \sigma(-\log r)^{\gamma_0/\beta_0} r^3$$

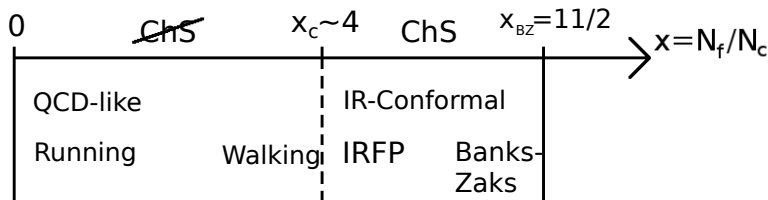
with $r \sim 1/\mu \rightarrow 0$

In the IR ($\lambda \rightarrow \infty$):

- ▶ $V_g(\lambda)$ chosen as for Yang-Mills, so that a “good” IR singularity exists
- ▶ $V_{f_0}(\lambda)$, $a(\lambda)$, and $\kappa(\lambda)$ chosen to produce tachyon divergence: several possibilities (\rightarrow Potentials I and II)
- ▶ Extra constraints from the asymptotics of the meson spectra

Phase diagram

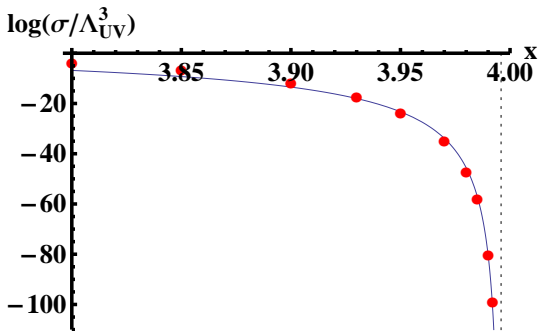
Fixing the potentials reasonably, at zero quark mass, after some analysis:



- ▶ Meets standard expectations from QCD!
- ▶ Conformal transition

[Kaplan,Son,Stephanov;Kutasov,Lin,Parnachev]

Other important features



$$\langle \bar{q}q \rangle \sim \sigma \sim \exp\left(-\frac{\kappa}{\sqrt{x_c - x}}\right)$$

1. Miransky/BKT scaling as $x \rightarrow x_c$ from below
 - ▶ E.g., The chiral condensate $\langle \bar{q}q \rangle \propto \sigma$
(From tachyon UV $\tau(r) \sim m_q(\log r)r + \sigma(\log r)r^3$)
2. Unstable Efimov vacua observed for $x < x_c$
3. Turning on the quark mass possible

3. V-QCD at finite T

Finite temperature – definitions

Lagrangian as before

$$\mathcal{S}_{V\text{-QCD}} = N_c^2 M^3 \int d^5x \sqrt{g} \left[R - \frac{4}{3} \frac{(\partial\lambda)^2}{\lambda^2} + V_g(\lambda) \right] \\ - N_f N_c M^3 \int d^5x V_f(\lambda, \tau) \sqrt{-\det(g_{ab} + \kappa(\lambda) \partial_a \tau \partial_b \tau)}$$

A more general metric, A and f solved from EoMs

$$ds^2 = e^{2A(r)} \left(\frac{dr^2}{f(r)} - f(r) dt^2 + dx^2 \right)$$

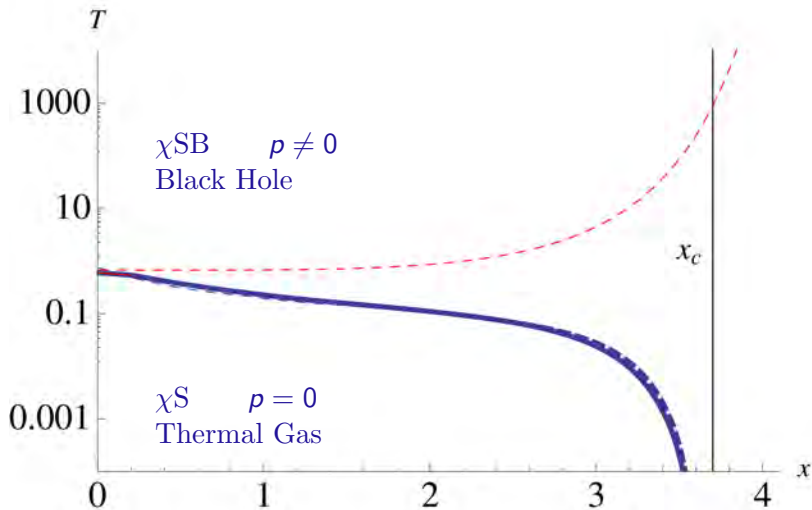
Black hole thermodynamics:

$$f(r) = 4\pi T (r_h - r) + \mathcal{O}((r - r_h)^2) ; \quad s = 4\pi M^3 N_c^2 e^{3A(r_h)}$$

Also: Thermal gas solutions ($f \equiv 1$, \sim zero T solutions)

Phase diagram: example

Phases on the (x, T) -plane (PotII)



4. Spectrum of V-QCD

Fluctuation analysis

Study at qualitative level:

1. Meson spectra (at zero temperature and quark mass)
 - ▶ Add gauge fields in $\mathcal{S}_{V\text{-QCD}}$
 - ▶ Four towers: scalars, pseudoscalars, vectors, and axial vectors
 - ▶ Flavor singlet and nonsinglet ($SU(N_f)$) states
2. The S-parameter

$$S \sim \frac{d}{dq^2} q^2 [\Pi_V(q^2) - \Pi_A(q^2)]_{q^2=0}$$

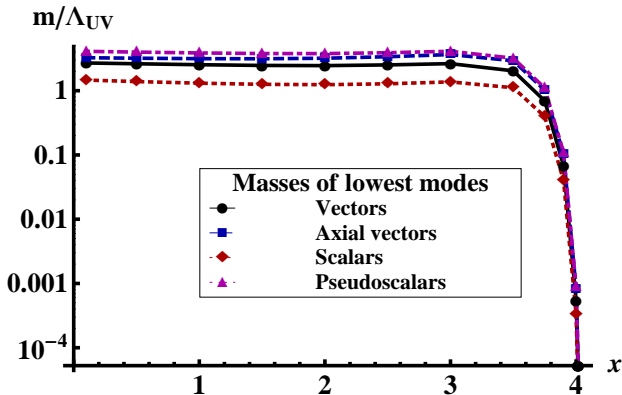
Open questions in the region relevant for “walking” technicolor ($x \rightarrow x_c$ from below):

- ▶ The S-parameter might be reduced
- ▶ Possibly a light “dilaton” (flavor singlet scalar): Goldstone mode due to almost unbroken conformal symmetry.

The 125 GeV state seen at the LHC?

Meson masses

Flavor nonsinglet masses (Example: PotI)

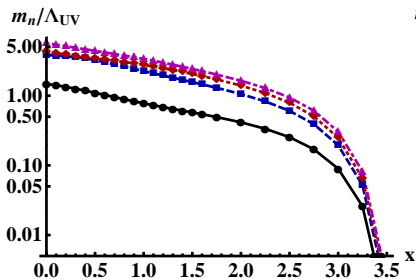


- ▶ All masses show Miransky scaling as $x \rightarrow x_c$
- ▶ Roughly $m_n^2 \sim n$ or $m_n^2 \sim n^2$ depending on potentials

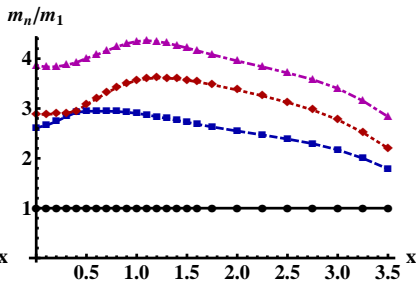
Scalar singlet masses

Scalar singlet spectrum (PotII):

In log scale



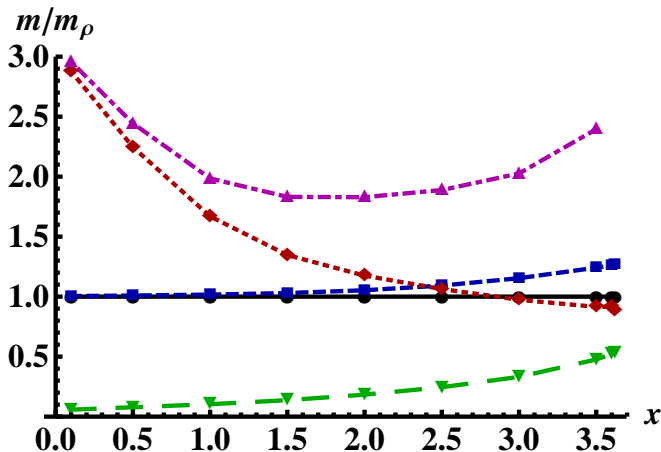
Normalized to the lowest state



No light dilaton?

Meson mass ratios

Mass ratios (PotII): Lowest states normalized to ρ

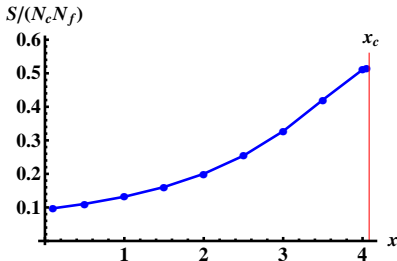


All ratios tend to constants as $x \rightarrow x_c$: indeed **no dilaton**

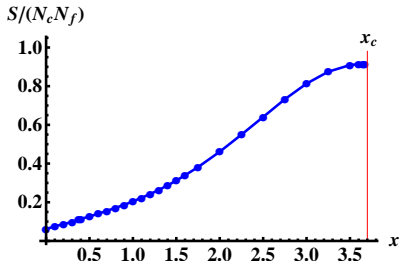
S-parameter

$$S \sim \frac{d}{dq^2} q^2 [\Pi_V(q^2) - \Pi_A(q^2)]_{q^2=0}$$

PotI



PotII



The S-parameter **increases** with x : **expected suppression absent**

Jumps discontinuously to zero at $x = x_c$

Summary

- ▶ We explored bottom up models for QCD in the Veneziano limit
- ▶ A class of models, V-QCD, was obtained by a fusion of IHQCD with tachyonic brane action
- ▶ V-QCD models meet expectations from QCD at qualitative level
- ▶ Future work:
finite μ and quantitative fits to QCD

Extra slides

Matching to QCD

Similar strategy as in IHQCD

Matching in the UV ($\lambda \rightarrow 0$):

- ▶ Take analytic potentials at $\lambda = 0$
 \Rightarrow RG flow consistent with QCD (when $A \leftrightarrow \log \mu$)
- ▶ Require correct (naive) operator dimensions in the deep UV
- ▶ Match expansions of potentials with perturbative QCD beta functions
 - ▶ $V_g(\lambda)$ with (two-loop) Yang-Mills beta function
 - ▶ $V_g(\lambda) - xV_{f0}(\lambda)$ with QCD beta function
 - ▶ $a(\lambda)/\kappa(\lambda)$ with the anomalous dimension of the quark mass/chiral condensate (\Rightarrow properly running quark mass!)
- ▶ After this, a single undetermined parameter in the UV: W_0

$$V_{f0}(\lambda) = W_0 + W_1\lambda + \mathcal{O}(\lambda^2)$$

In the IR ($\lambda \rightarrow \infty$), there must be a solution where the tachyon action $\propto e^{-a(\lambda)\tau^2} \rightarrow 0$

- ▶ $V_g(\lambda)$ chosen as for Yang-Mills, so that a “good” IR singularity exists
- ▶ $V_{f0}(\lambda)$, $a(\lambda)$, and $\kappa(\lambda)$ chosen to produce tachyon divergence: several possibilities (\rightarrow Potentials I and II)
- ▶ Extra constraints from the asymptotics of the meson spectra

Working potentials often string-inspired power-laws, multiplied by logarithmic corrections (!)

Background analysis: zero temperature

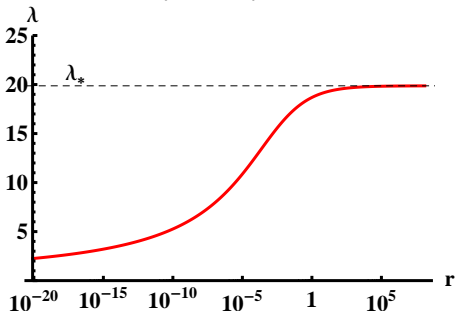
Analysis of the backgrounds (r -dependent solutions of EoMs) at zero temperature

- ▶ Expect two kinds of solutions, with
 1. Nontrivial tachyon profile (chirally broken)
 2. Identically vanishing tachyon (chirally symmetric)
- ▶ Identify the dominant vacua
- ▶ Fully backreacted system \Rightarrow rich dynamics, complicated numerical analysis ...

Backgrounds at zero quark mass

Sketch of behavior in the conformal window ($x > x_c$):

- ▶ Tachyon vanishes (no ChSB)
- ▶ Similar to IHQCD, different potential
⇒ IR fixed point
- ▶ Dilaton flows between UV/IR fixed points

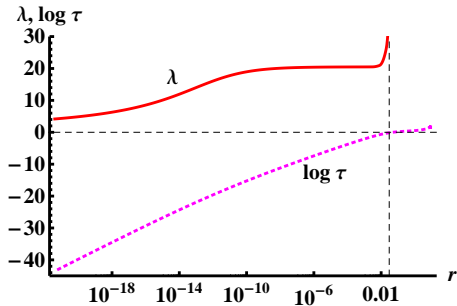


Here UV: $r \rightarrow 0$, IR: $r \rightarrow \infty$

As x goes below x_c , this solution becomes unstable (tachyon BF bound)

Right below the conformal window ($x < x_c$; $|x - x_c| \ll 1$)

- ▶ Dilaton flows very close to the IR fixed point
- ▶ “Small” nonzero tachyon induces an IR singularity



Result: “walking”

Actual solutions

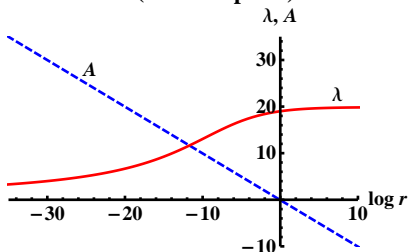
UV: $r = 0$

IR: $r = \infty$

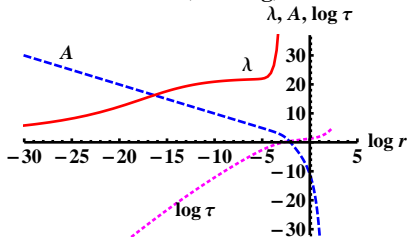
$A \sim \log \mu \sim -\log r$

$x_c \simeq 3.9959$

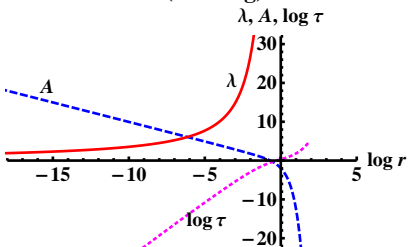
$x = 4$ (IR fixed point)



$x = 3.9$ (walking)



$x = 2$ (running)



The BF bound and x_c

At an fixed point

$$\tau(r) \sim C_1 r^\Delta + C_2 r^{4-\Delta}$$

with

$$-m^2 \ell^2 = \Delta(4 - \Delta)$$

Requiring real Δ gives the Breitenlohner-Freedman bound for the tachyon (Starinets' lectures)

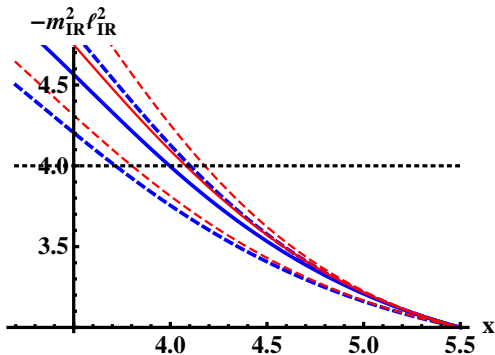
$$-m^2 \ell^2 = \Delta(4 - \Delta) \leq 4$$

- ▶ Saturated for $\Delta = 2$, then $\tau(r) \sim C_1 r^2 + C_2 r^2 \log r$
- ▶ Violation of BF bound \Rightarrow instability

Analysis of this instability of the tachyon $\Rightarrow x_c$

Dependence on the UV parameter W_0 and IR choices for the potentials

Resulting variation of the edge of conformal window
 $x_c = 3.7 \dots 4.2$



Agrees with most of the other estimates

Potentials I

$$\begin{aligned}V_g(\lambda) &= 12 + \frac{44}{9\pi^2}\lambda + \frac{4619}{3888\pi^4} \frac{\lambda^2}{(1 + \lambda/(8\pi^2))^{2/3}} \sqrt{1 + \log(1 + \lambda/(8\pi^2))} \\V_f(\lambda, \tau) &= V_{f0}(\lambda)e^{-a(\lambda)\tau^2} \\V_{f0}(\lambda) &= \frac{12}{11} + \frac{4(33 - 2x)}{99\pi^2}\lambda + \frac{23473 - 2726x + 92x^2}{42768\pi^4}\lambda^2 \\a(\lambda) &= \frac{3}{22}(11 - x) \\\kappa(\lambda) &= \frac{1}{\left(1 + \frac{115 - 16x}{288\pi^2}\lambda\right)^{4/3}}\end{aligned}$$

In this case the tachyon diverges exponentially:

$$\tau(r) \sim \tau_0 \exp \left[\frac{81 \cdot 3^{5/6} (115 - 16x)^{4/3} (11 - x)}{812944 \cdot 2^{1/6}} \frac{r}{R} \right]$$

Potentials II

$$\begin{aligned}V_g(\lambda) &= 12 + \frac{44}{9\pi^2}\lambda + \frac{4619}{3888\pi^4} \frac{\lambda^2}{(1 + \lambda/(8\pi^2))^{2/3}} \sqrt{1 + \log(1 + \lambda/(8\pi^2))} \\V_f(\lambda, \tau) &= V_{f0}(\lambda)e^{-a(\lambda)\tau^2} \\V_{f0}(\lambda) &= \frac{12}{11} + \frac{4(33 - 2x)}{99\pi^2}\lambda + \frac{23473 - 2726x + 92x^2}{42768\pi^4}\lambda^2 \\a(\lambda) &= \frac{3}{22}(11 - x) \frac{1 + \frac{115 - 16x}{216\pi^2}\lambda + \lambda^2/(8\pi^2)^2}{(1 + \lambda/(8\pi^2))^{4/3}} \\\kappa(\lambda) &= \frac{1}{(1 + \lambda/(8\pi^2))^{4/3}}\end{aligned}$$

In this case the tachyon diverges as

$$\tau(r) \sim \frac{27 \cdot 2^{3/4} 3^{1/4}}{\sqrt{4619}} \sqrt{\frac{r - r_1}{R}}$$

Effective potential

For solutions with $\tau = \tau_* = \text{const}$

$$\mathcal{S} = M^3 N_c^2 \int d^5x \sqrt{g} \left[R - \frac{4}{3} \frac{(\partial\lambda)^2}{\lambda^2} + V_g(\lambda) - xV_f(\lambda, \tau_*) \right]$$

IHQCD with an **effective potential**

$$V_{\text{eff}}(\lambda) = V_g(\lambda) - xV_f(\lambda, \tau_*) = V_g(\lambda) - xV_{f0}(\lambda) \exp(-a(\lambda)\tau_*^2)$$

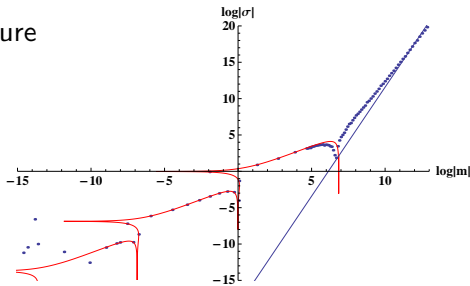
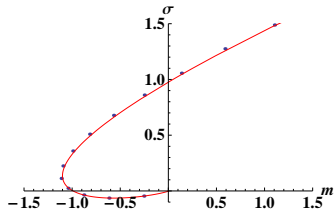
Minimizing for τ_* we obtain $\tau_* = 0$ and $\tau_* = \infty$

- ▶ $\tau_* = 0$: $V_{\text{eff}}(\lambda) = V_g(\lambda) - xV_{f0}(\lambda)$;
fixed point with $V'_{\text{eff}}(\lambda_*) = 0$
- ▶ $\tau_* \rightarrow \infty$: $V_{\text{eff}}(\lambda) = V_g(\lambda)$ (like YM, no fixed points)

Efimov spiral

Ongoing work: the dependence $\sigma(m)$ of the chiral condensate on the quark mass

- ▶ For $x < x_c$ spiral structure

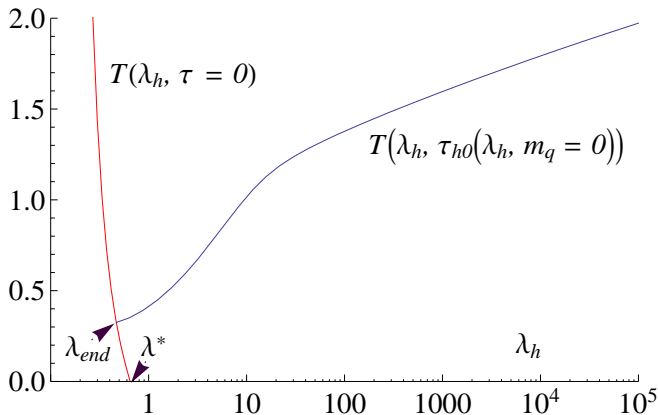


- ▶ Dots: numerical data
- ▶ Continuous line: (semi-)analytic prediction

Allows to study the effect of double-trace deformations

Black hole branches

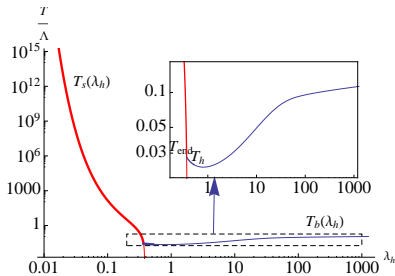
Example: PotII at $x = 3$, $W_0 = 12/11$



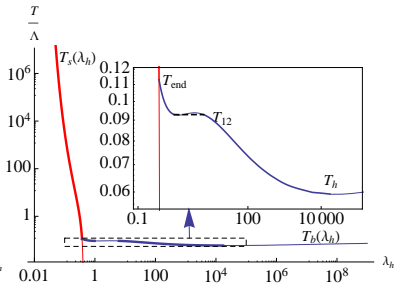
Simple phase structure: 1st order transition at $T = T_h$ from thermal gas to (chirally symmetric) BH

More complicated cases:

PotII at $x = 3$, W_0 SB



PotI at $x = 3.5$, $W_0 = 12/11$

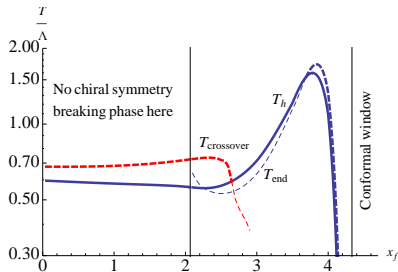


- ▶ Left: chiral symmetry restored at 2nd order transition with $T = T_{\text{end}} > T_h$
- ▶ Right: Additional first order transition between BH phases with broken chiral symmetry

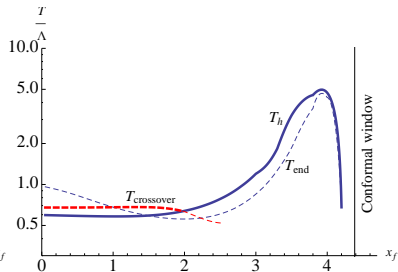
Also other cases ...

Phase diagrams on the (x, T) -plane

PotI* W_0 SB

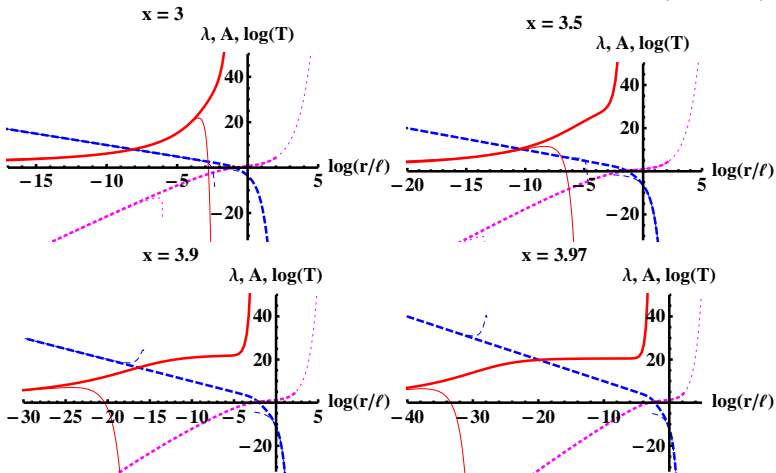


PotII* W_0 SB

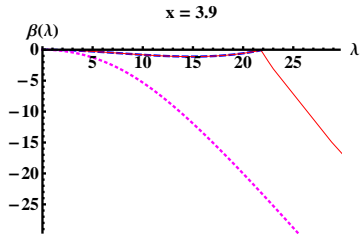
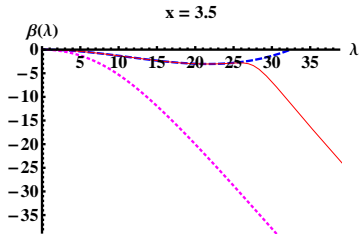
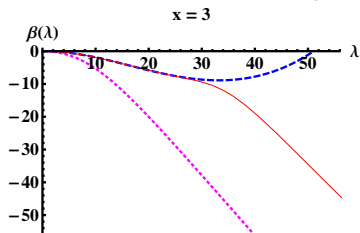
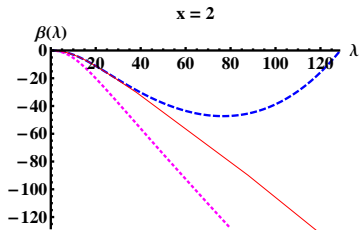


Backgrounds in the walking region

Backgrounds with zero quark mass, $x < x_c \simeq 3.9959$ (λ , A , τ)



Beta functions **along the RG flow** (evaluated on the background),
 zero tachyon, YM $x_c \simeq 3.9959$



Holographic beta functions

Generalization of the holographic RG flow of IHQCD

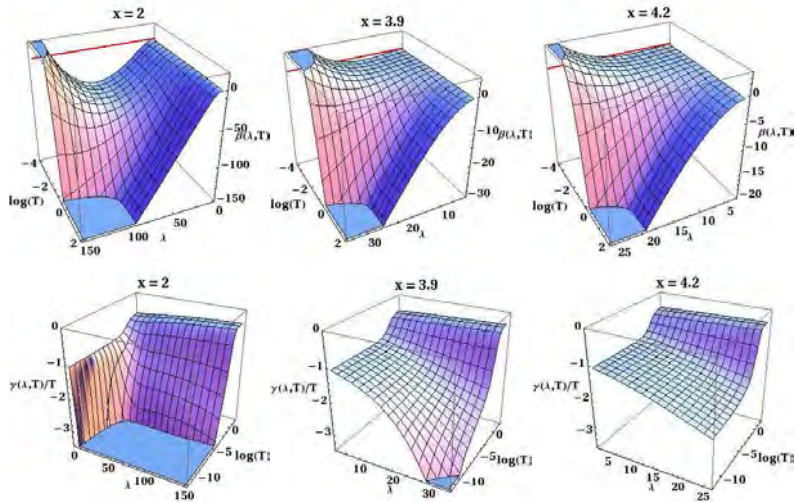
$$\beta(\lambda, \tau) \equiv \frac{d\lambda}{dA} ; \quad \gamma(\lambda, \tau) \equiv \frac{d\tau}{dA}$$

linked to

$$\frac{dg_{\text{QCD}}}{d \log \mu} ; \quad \frac{dm}{d \log \mu}$$

The **full** equations of motion boil down to two first order partial non-linear differential equations for β and γ

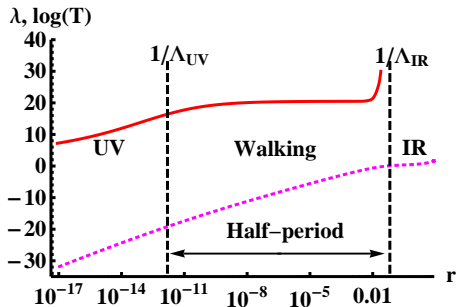
“Good” solutions numerically (unique)



Miransky/BKT scaling

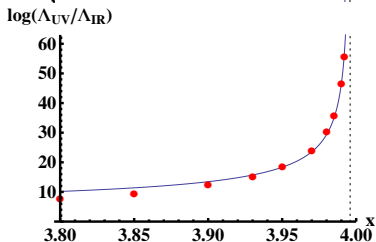
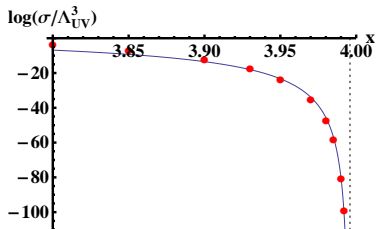
As $x \rightarrow x_c$ from below: walking, dominant solution

- ▶ BF-bound for the tachyon violated at the IRFP
- ▶ x_c fixed by the BF bound:
 $\Delta = 2$ & $\gamma_* = 1$
at the edge of the conformal window



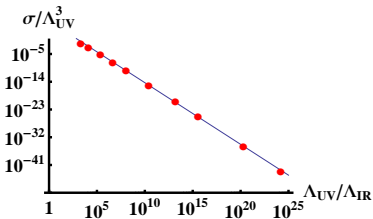
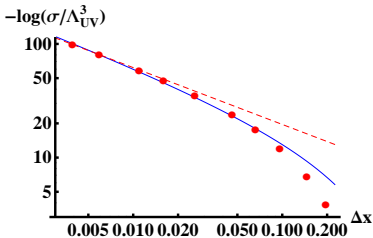
- ▶ $\tau(r) \sim r^2 \sin(\kappa \sqrt{x_c - x} \log r + \phi)$ in the walking region
- ▶ “0.5 oscillations” \Rightarrow Miransky/BKT scaling,
amount of walking $\Lambda_{UV}/\Lambda_{IR} \sim \exp(\pi/(\kappa \sqrt{x_c - x}))$

As $x \rightarrow x_c$
with known κ



$$\langle \bar{q}q \rangle \sim \sigma \sim \exp(-2\pi/(\kappa\sqrt{x_c - x}))$$

$$\Lambda_{UV}/\Lambda_{IR} \sim \exp(\pi/(\kappa\sqrt{x_c - x}))$$



γ_* in the conformal window

Comparison to other guesses

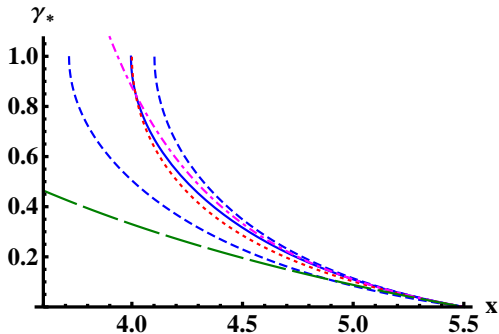
V-QCD (dashed: variation
due to W_0)

Dyson-Schwinger

2-loop PQCD

All-orders β

[Pica, Sannino arXiv:1011.3832]



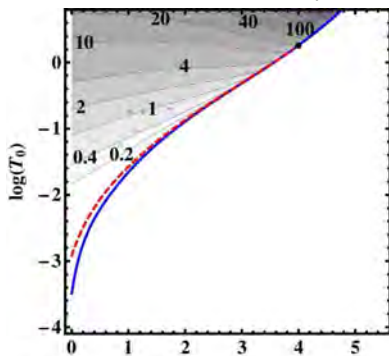
Parameters

Understanding the solutions for generic quark masses requires discussing parameters

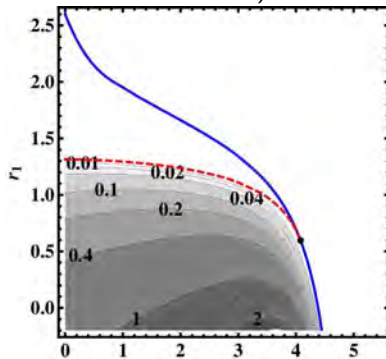
- ▶ YM or QCD with massless quarks: **no parameters**
- ▶ QCD with flavor-independent mass m : a **single** (dimensionless) parameter m/Λ_{QCD}
- ▶ In this model, after rescalings, this parameter can be mapped to a parameter (τ_0 or r_1) that controls the diverging tachyon in the IR
- ▶ x has become continuous in the Veneziano limit

Map of all solutions

All “good” solutions ($\tau \neq 0$) obtained varying x and τ_0 or r_1
Contouring: quark mass (zero mass is the red contour)

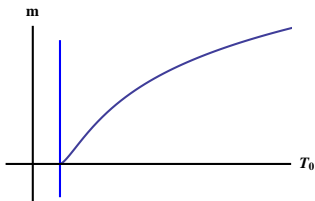


“Potentials I” $\leftrightarrow T_0$



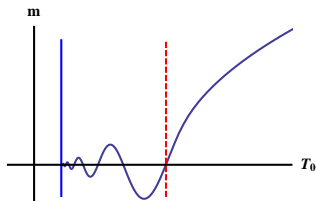
“Potentials II” $\leftrightarrow r_1$

Mass dependence and Efimov vacua



Conformal window ($x > x_c$)

- ▶ For $m = 0$, unique solution with $\tau \equiv 0$
- ▶ For $m > 0$, unique “standard” solution with $\tau \neq 0$

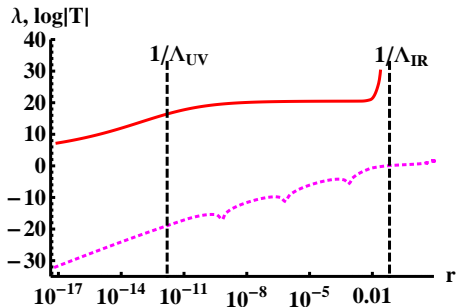


Low $0 < x < x_c$: Efimov vacua

- ▶ Unstable solution with $\tau \equiv 0$ and $m = 0$
- ▶ “Standard” stable solution, with $\tau \neq 0$, for all $m \geq 0$
- ▶ Tower of unstable Efimov vacua (small $|m|$)

Efimov solutions

- ▶ Tachyon oscillates over the walking regime
- ▶ $\Lambda_{UV}/\Lambda_{IR}$ increased wrt. “standard” solution



Effective potential: zero tachyon

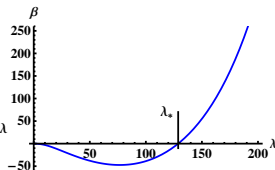
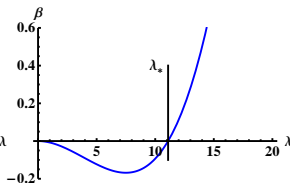
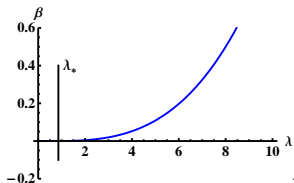
Start from Banks-Zaks region, $\tau_* = 0$, chiral symmetry conserved
($\tau \leftrightarrow \bar{q}q$), $V_{\text{eff}}(\lambda) = V_g(\lambda) - xV_{f0}(\lambda)$

- ▶ V_{eff} defines a β -function as in IHQCD – Fixed point guaranteed in the BZ region, moves to higher λ with decreasing x
- ▶ Fixed point λ_* runs to ∞ either at finite $x(<x_c)$ or as $x \rightarrow 0$

Banks-Zaks
 $x \rightarrow 11/2$

Conformal Window
 $x > x_c$

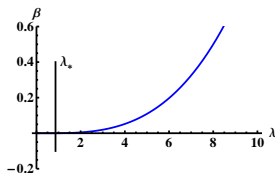
$x < x_c$??



Effective potential: what actually happens

Banks-Zaks

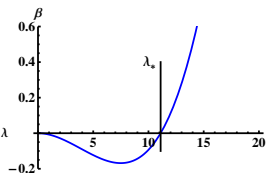
$$x \rightarrow 11/2$$



$$\tau \equiv 0$$

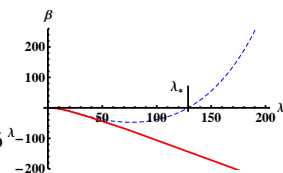
Conformal Window

$$x > x_c$$



$$\tau \equiv 0$$

$$x < x_c$$



$$\tau \neq 0$$

- ▶ For $x < x_c$ vacuum has nonzero tachyon (checked by calculating free energies)
- ▶ The tachyon **screens the fixed point**
- ▶ In the deep IR τ diverges, $V_{\text{eff}} \rightarrow V_g \Rightarrow$ dynamics is YM-like

Where is x_c ?

How is the edge of the conformal window stabilized?

Tachyon IR mass at $\lambda = \lambda_*$ \leftrightarrow quark mass dimension

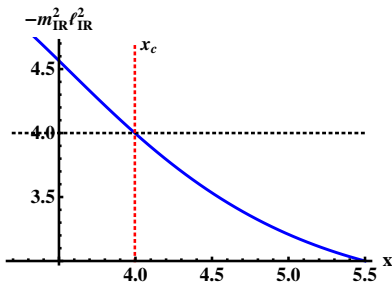
$$-m_{\text{IR}}^2 \ell_{\text{IR}}^2 = \Delta_{\text{IR}}(4 - \Delta_{\text{IR}}) = \frac{24a(\lambda_*)}{\kappa(\lambda_*)(V_g(\lambda_*) - xV_0(\lambda_*))}$$

$$\gamma_* = \Delta_{\text{IR}} - 1$$

Breitenlohner-Freedman
(BF) bound (horizontal line)

$$-m_{\text{IR}}^2 \ell_{\text{IR}}^2 = 4 \Leftrightarrow \gamma_* = 1$$

defines x_c



Why $\gamma_* = 1$ at $x = x_c$?

No time to go into details ... the question boils down to the linearized tachyon solution at the fixed point

- ▶ For $\Delta_{\text{IR}}(4 - \Delta_{\text{IR}}) < 4$ ($x > x_c$):

$$\tau(r) \sim m_q r^{\Delta_{\text{IR}}} + \sigma r^{4 - \Delta_{\text{IR}}}$$

- ▶ For $\Delta_{\text{IR}}(4 - \Delta_{\text{IR}}) > 4$ ($x < x_c$):

$$\tau(r) \sim Cr^2 \sin[(\text{Im}\Delta_{\text{IR}}) \log r + \phi]$$

Rough analogy:

Tachyon EoM \leftrightarrow Gap equation in Dyson-Schwinger approach

Similar observations have been made in other holographic frameworks

[Kutasov, Lin, Parnachev arXiv:1107.2324, 1201.4123]

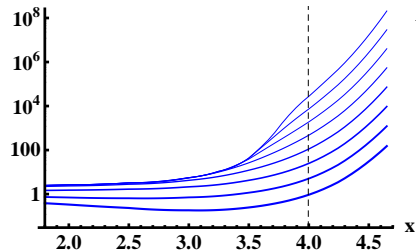
Mass dependence

For $m > 0$ the conformal transition disappears

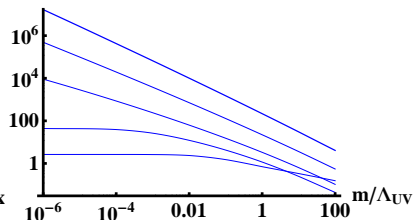
The ratio of typical UV/IR scales $\Lambda_{UV}/\Lambda_{IR}$ varies in a natural way

$m/\Lambda_{UV} = 10^{-6}, 10^{-5}, \dots, 10$ $x = 2, 3.5, 3.9, 4.25, 4.5$

$\Lambda_{UV}/\Lambda_{IR}$



$\Lambda_{UV}/\Lambda_{IR}$



sQCD phases

The case of $\mathcal{N} = 1$ $SU(N_c)$ superQCD with N_f quark multiplets is known and provides an interesting (and more complex) example for the nonsupersymmetric case. From Seiberg we have learned that:

- ▶ $x = 0$ the theory has confinement, a mass gap and N_c distinct vacua associated with a spontaneous breaking of the leftover R symmetry Z_{N_c} .
- ▶ At $0 < x < 1$, the theory has a runaway ground state.
- ▶ At $x = 1$, the theory has a quantum moduli space with no singularity. This reflects confinement with ChSB.
- ▶ At $x = 1 + 1/N_c$, the moduli space is classical (and singular). The theory confines, but there is no ChSB.
- ▶ At $1 + 2/N_c < x < 3/2$ the theory is in the non-abelian magnetic IR-free phase, with the magnetic gauge group $SU(N_f - N_c)$ IR free.
- ▶ At $3/2 < x < 3$, the theory flows to a CFT in the IR. Near $x = 3$ this is the Banks-Zaks region where the original theory has an IR fixed point at weak coupling. Moving to lower values, the coupling of the IR $SU(N_c)$ gauge theory grows. However near $x = 3/2$ the dual magnetic $SU(N_f - N_c)$ is in its Banks-Zaks region, and provides a weakly coupled description of the IR fixed point theory.
- ▶ At $x > 3$, the theory is IR free.

Saturating the BF bound (sketch)

Why is the BF bound saturated at the phase transition (massless quarks)??

$$\Delta_{\text{IR}}(4 - \Delta_{\text{IR}}) = \frac{24a(\lambda_*)}{\kappa(\lambda_*)(V_g(\lambda_*) - xV_0(\lambda_*))}$$

- ▶ For $\Delta_{\text{IR}}(4 - \Delta_{\text{IR}}) < 4$:
 $\tau(r) \sim m_q r^{4 - \Delta_{\text{IR}}} + \sigma r^{\Delta_{\text{IR}}}$
- ▶ For $\Delta_{\text{IR}}(4 - \Delta_{\text{IR}}) > 4$:
 $\tau(r) \sim Cr^2 \sin[(\text{Im}\Delta_{\text{IR}}) \log r + \phi]$
- ▶ Saturating the BF bound, the tachyon solutions will entangle
→ required to satisfy boundary conditions
- ▶ Nodes in the solution appear through UV → massless solution

Saturating the BF bound (sketch)

Does the nontrivial (ChSB) massless tachyon solution exist?

Two possibilities:

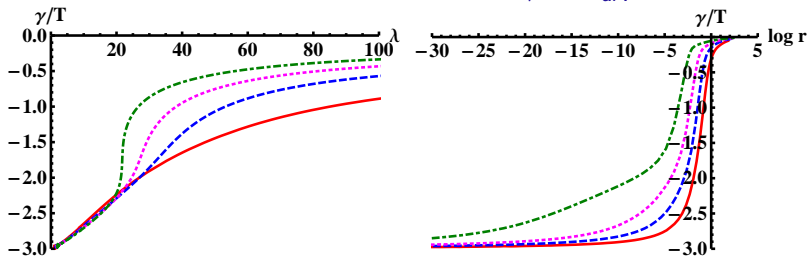
- ▶ $x > x_c$: BF bound satisfied at the fixed point \Rightarrow only trivial massless solution ($\tau \equiv 0$, ChS intact, fixed point hit)
- ▶ $x < x_c$: BF bound violated at the fixed point \Rightarrow a nontrivial massless solution exist, which drives the system away from the fixed point

Conclusion: **phase transition** at $x = x_c$

As $x \rightarrow x_c$ from below, need to approach the fixed point to satisfy the boundary conditions \Rightarrow nearly conformal, **“walking” dynamics**

Gamma functions

Massless backgrounds: gamma functions $\frac{\gamma}{\tau} = \frac{d \log \tau}{dA}$



$x = 2, 3, 3.5, 3.9$