Flatspace Holography as a Limit of AdS/CFT

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Based on:
A. Bagchi, R. F. (arXiv:1203.5795),
A. Bagchi, S. Detournay, R. F. , Joan Simon (arXiv:1208.4372)

Motivation

- AdS/CFT provides a holographic picture for the gravitational theory.
- Is it possible to generalize holography for gravity in other backgrounds?
- Our universe is more like flat rather than AdS!
- Does flat space have a holographic picture as a lower dimensional field theory!
- Our answer is yes! Starting from AdS/CFT, one can study flat-space holography.

First step: Symmetries

- AdS$_d$ isometry= $SO(2, d-1)$ = symmetries of CFT$_{d-1}$.
- Dictionary of AdS/CFT $\rightarrow$ (Asymptotic symmetries of gravity=symmetries of dual theory)
- Often, ASG$\rightarrow$ isometry of the vacuum: $ASG(AdS_{d+2}) = SO(d+1, 2)$. (for $d > 1$.)
- Famous exception: $ASG(AdS_3) = Virasoro \otimes Virasoro$. [Brown, Henneaux 1986]
- Another interesting example: Near Horizon Extreme Kerr. [Bardeen, Horowitz 1999]; [Guica, Hartman, Song, Strominger 2008]

- Non-trivial ASG for asymptotically Minkowski spacetimes at null infinity in three and four dimensions.

Asymptotic Symmetries and Flat-spacetimes

- BMS_3:

$$[J_m, J_n] = (m-n)J_{m+n}, \quad [J_m, P_n] = (m-n)P_{m+n}, \quad [P_m, P_n] = 0. \tag{1}$$

J_m = Global Conformal Generators of S^1.
P_m = "Super" translations (depends on θ) $\rightarrow$ Infinite dimensional.
[Ashtekar, Bicak, Schmidt 1996]

- Further Extension: Can compute central extensions:

$$C_{JJ} = 0, \quad C_{JP} = \frac{3}{G}, \quad C_{PP} = 0 \tag{2}$$

[Barnich-Compere 2006]

Asymptotic Symmetries and Flat-spacetimes .

- BMS_4:

$$[l_m, l_n] = (m-n) l_{m+n}, \quad [\bar{l}_m, \bar{l}_n] = (m-n)\bar{l}_{m+n}, \quad [l_m, \bar{l}_n] = 0,$$
$$[l_l, T_{m,n}] = (\frac{l+1}{2} - m) T_{m+l,n}, \quad [\bar{l}_l, T_{m,n}] = (\frac{l+1}{2} - n) T_{m,n+l}.$$

$l_m, \bar{l}_m$ = Global Conformal Generators of S^2.
$T_{m,n}$ = "Super" translations → Infinite dimensional.
$(m, n = 0, \pm 1)$: Poincare group in 4d.
[Bondi, van der Burg, Metzner; Sachs 1962]

- Virasoros can have central extensions. [Barnich, Troessaert 2009]

One may expect that holographic dual of flat space has BMS symmetry.What is this theory?

Flat space limit of AdS

- AdS_3 space-time:

$$ds^2 = -(1 + \frac{r^2}{\ell^2})dt^2 + (1 + \frac{r^2}{\ell^2})^{-1}dr^2 + r^2 d\phi^2 \qquad (3)$$

- In the limit $\ell \to \infty$(zero cosmological constant), we find flat space.
- Asymptotic Symmetry is $\text{Vir} \otimes \text{Vir}$:

$$\begin{aligned} [\mathcal{L}_m, \mathcal{L}_n] &= (m-n)\mathcal{L}_{m+n} + \frac{c}{12}m(m^2-1)\delta_{m+n,0} \\ [\bar{\mathcal{L}}_m, \bar{\mathcal{L}}_n] &= (m-n)\bar{\mathcal{L}}_{m+n} + \frac{\bar{c}}{12}m(m^2-1)\delta_{m+n,0} \end{aligned}$$

where $c = \bar{c} = 3\ell/2G$.

Flat space limit of AdS.

- Define: $L_n = \mathcal{L}_n - \bar{\mathcal{L}}_{-n}, \quad M_n = (\mathcal{L}_n + \bar{\mathcal{L}}_{-n})/\ell.$
- $\ell \to \infty$ is well-defined and yields

$$
\begin{aligned}
[L_m, L_n] &= (m-n)L_{m+n} + \frac{C_1}{12} m(m^2-1)\delta_{m+n,0} \\
[L_m, M_n] &= (m-n)M_{m+n} + \frac{C_2}{12} m(m^2-1)\delta_{m+n,0}
\end{aligned}
$$

where

$$C_1 = c - \bar{c} = 0, \quad C_2 = \frac{(\bar{c}+c)}{\ell} = \frac{3}{G}$$

[Barnich-Compere 2006]

Making sense of flat space limit in the CFT side

What does flat space limit mean in the CFT side?

- Dual of AdS_3 in the global coordinate is a two dimensional CFT on the cylinder with generators:

$$\mathcal{L}_n = -e^{nw}\partial_w, \quad \bar{\mathcal{L}}_n = -e^{n\bar{w}}\partial_{\bar{w}} \quad (w = t + ix, \bar{w} = t - ix) \tag{4}$$

- Define $L_n = \mathcal{L}_n - \bar{\mathcal{L}}_{-n}, \quad M_n = \epsilon(\mathcal{L}_n + \bar{\mathcal{L}}_{-n})$.
- Use a spacetime contraction: $t \to \epsilon t, x \to x$.
- Final generators in the $\epsilon \to 0$ limit:

$$L_n = -e^{inx}(i\partial_x + nt\partial_t), \quad M_n = -e^{inx}\partial_t \tag{5}$$

- The resultant algebra in the $\epsilon \to 0$ limit is BMS_3 with central charges: $c_{LL} = C_1 = c - \bar{c}, \quad c_{LM} = C_2 = \epsilon(c + \bar{c})$ [A. Bagchi, R. F. (2012)]

Making sense of flat space limit in the CFT side.

- Proposal: Holographic dual of flat space-time is a *t-contracted* CFT.
- For the two-dimensional case, it is a non-relativistic field theory which has Galilean Conformal symmetry.
- For the higher-dimensional cases we can apply contraction of time. The resultant algebra is higher-dimensional BMS.

Correlation function

- The limit from the correlation functions of parent CFT yield interesting structures.
- Two point function of two primary operators is given by

$$G^{(2)}(\phi_1, \tau_1, \phi_2, \tau_2) = C_1 \left(1 - \cos\phi_{12}\right)^{-2h_L} e^{ih_M \tau_{12} \cot(\phi_{12}/2)}$$

where $\quad h_L = \lim_{\epsilon\to 0}(h - \bar{h}), \qquad h_M = \lim_{\epsilon\to 0} \epsilon(h + \bar{h})$

Next Step: Flat limit of BTZ

- No asymptotically flat black hole solutions in the three dimensional gravity. [D. Ida, 2000]
- The flat limit of BTZ is well-defind!
- BTZ black holes:

$$ds^2 = -\frac{(r^2-r_+^2)(r^2-r_-^2)}{r^2\ell^2}dt^2 + \frac{r^2\ell^2}{(r^2-r_+^2)(r^2-r_-^2)}dr^2 + r^2\left(d\phi - \frac{r_+r_-}{\ell r^2}dt\right)^2$$

- $\ell \to \infty$:

$$r_- \to r_0 = \sqrt{\frac{2G}{M}}|J|, \qquad r_+ \to \ell\hat{r}_+ = \ell\sqrt{8GM} \tag{6}$$

- Flat BTZ: a cosmiological solution of 3d Einstein gravity

$$ds^2_{FBTZ} = \hat{r}_+^2dt^2 - \frac{r^2dr^2}{\hat{r}_+^2(r^2-r_0^2)} + r^2d\phi^2 - 2\hat{r}_+r_0dtd\phi \tag{7}$$

Next Step: Flat limit of BTZ.

- FBTZ is a shifted- boost orbifold of $R^{1,2}$ [L. Cornalba, M. S. Costa(2002)]
- $r = r_0$ is a cosmological horizon:

$$T = \frac{\hat{r}_+^2}{2\pi r_0}, \qquad S = \frac{\pi r_0}{2G} \tag{8}$$

- Satisfy the inner horizon first law!

$$TdS = -dM + \Omega dJ \tag{9}$$

Dual field theory analysis: a Cardy-like formula

- 3d asymptotically flat spacetimes $\longrightarrow$ states of field theory with BMS symmetry.
- The states are labelled by eigenvalues of L_0 and M_0:

$$L_0|h_L, h_M\rangle = h_L|h_L, h_M\rangle, \qquad M_0|h_L, h_M\rangle = h_M|h_L, h_M\rangle,$$

where

$$h_L = \lim_{\epsilon\to 0}(h - \bar{h}) = J, \quad h_M = \lim_{\epsilon\to 0}\epsilon(h + \bar{h}) = GM + \frac{1}{8} \quad (10)$$

- Degeneracy of states with (h_L, h_M) $\longrightarrow$ Entropy of cosmological solution with $(r_0, \hat{r}_+)$

Dual field theory analysis: a Cardy-like formula

- Is there any Cardy-like formula? YES
- Partition function of t-contracted CFT:

$$Z(\eta,\rho)=\sum d(h_L,h_M)e^{2\pi(\eta h_L+\rho h_M)} \tag{11}$$

where

$$\eta=\frac{1}{2}(\tau+\bar{\tau}),\quad \rho=\frac{1}{2}\epsilon(\tau-\bar{\tau}) \tag{12}$$

- S-transformation:

$$(\tau,\bar{\tau})\to(-\frac{1}{\tau},-\frac{1}{\bar{\tau}})\Rightarrow(\eta,\rho)\to(-\frac{1}{\eta},\frac{\rho}{\eta^2}) \tag{13}$$

Dual field theory analysis: a Cardy-like formula .

- Modular invariance of

$$Z^0(\eta,\rho) = e^{-\pi i \rho C_M} Z(\eta,\rho) \qquad (14)$$

results in

$$S = \log d(h_L, h_M) = 2\pi h_L \sqrt{\frac{C_M}{2h_M}} \qquad (15)$$

[A. Bagchi, S. Detournay, R. F. , Joan Simon (2012)]

- In agreement with the entropy of cosmological solution!
- At sufficiently high temperature hot flat space tunnels into a universe described by flat space cosmology.[A. Bagchi, S. Detournay, D. Grumiller, J. Simon]

Current Directions

Extension of BMS algebra to higher spin case [Work in progress with Hamid Afshar, Daniel Grumiller and Jan Rosseel]

- $sl(3,\mathbb{R})$ algebra:

$$[\mathcal{L}_m,\mathcal{L}_n]=(m-n)\mathcal{L}_{m+n}$$
$$[\mathcal{L}_m,\mathcal{W}_n]=(2m-n)\mathcal{W}_{m+n}$$
$$[\mathcal{W}_m,\mathcal{W}_n]=(m-n)(2m^2+2n^2-mn-8)\mathcal{L}_{m+n} \tag{16}$$

  where m,n are $0,\pm1$ in $\mathcal{L}_m$ and $0,\pm1,\pm2$ in $\mathcal{W}_m$.

- New generators:

$$L_m=\mathcal{L}_m-\bar{\mathcal{L}}_{-m},\qquad M_m=\epsilon(\mathcal{L}_m+\bar{\mathcal{L}}_{-m})$$
$$U_m=(\mathcal{W}_m-\bar{\mathcal{W}}_{-m}),\qquad V_m=\epsilon(\mathcal{W}_m+\bar{\mathcal{W}}_{-m}) \tag{17}$$

- $\epsilon\to0$ is well-defined:

Current Directions

$$
\begin{aligned}
[L_m, L_n] &= (m-n)L_{m+n} \qquad & [L_m, M_n] &= (m-n)M_{m+n} \\
[L_m, U_n] &= (2m-n)U_{m+n} \qquad & [L_m, V_n] &= (2m-n)V_{m+n}
\end{aligned}
$$

$$
\begin{aligned}
[M_m, U_n] &= (2m-n)V_{m+n} \\
[U_m, U_n] &= (m-n)(2m^2+2n^2-mn-8)L_{m+n} \\
[U_m, V_n] &= (m-n)(2m^2+2n^2-mn-8)M_{m+n} \qquad (18)
\end{aligned}
$$

- Extension of BMS algebra $\longrightarrow$ WBMS algebra!

Current Directions

- Question: Is WBMS a symmetry of higher spin gravity in the flat spacetimes?
- Spin 2 case: Chern-Simons formulation

$$A_{flat} = A_M^n M_n + A_L^n L_n, \qquad n = 0, \pm 1 \tag{19}$$

L_n and M_n are generators of BMS and

$$\begin{aligned} A_M &= e^1, & A_M^1 &= \frac{1}{2}(e^0 + e^2), & A_M^{-1} &= \frac{1}{2}(e^0 - e^2), \\ A_L^0 &= \omega^1, & A_L^1 &= \frac{1}{2}(\omega^0 + \omega^2), & A_L^{-1} &= \frac{1}{2}(\omega^0 - \omega^2). \end{aligned} \tag{20}$$

Current Directions

- Our observation:

$$L_n = \lim_{\ell\to\infty}(\mathcal{L}_n - \bar{\mathcal{L}}_{-n}), \qquad M_n = \lim_{\ell\to\infty}\frac{1}{\ell}(\mathcal{L}_n + \bar{\mathcal{L}}_{-n}),$$
$$A^n_L = \lim_{\ell\to\infty}\frac{1}{2}(A^n + \bar{A}^n), \qquad A^n_M = \lim_{\ell\to\infty}\frac{\ell}{2}(A^n - \bar{A}^n). \quad (21)$$

- A proposal: 3d spin three flat gravity with WBMS symmetry

$$A_{flat} = A^n_M M_n + A^n_L L_n + A^m_U U_m + A^m_V V_m, \quad (22)$$
$$n = 0, \pm 1, \qquad m = 0, \pm 1, \pm 2$$

where

$$U_m = \lim_{\ell\to\infty}(\mathcal{W}_m - \bar{\mathcal{W}}_{-m}), \qquad V_m = \lim_{\ell\to\infty}\frac{1}{\ell}(\mathcal{W}_m + \bar{\mathcal{W}}_{-m})$$
$$A^m_U = \lim_{\ell\to\infty}\frac{1}{2}(A^m_W + \bar{A}^m_W), \qquad A^m_V = \lim_{\ell\to\infty}\frac{\ell}{2}(A^m_W - \bar{A}^m_W).$$

- Poisson-brackets of boundary preserving gauge transformations results in WBMS algebra.

Summary

- Flat space limit in the bulk side is equivalent to t-contraction in the boundary theory.
- Equivalence of symmetries is an evidence.
- Correlation functions are well-defined.
- Counting of states supports this proposal.
- One can talk about higher-spin gravity in the 3d flat spacetime.

Future Direction

Entanglement Entropy of BMS field theory

- $\ell \to \infty$ limit of holographic entanglement entropy in the global AdS_3 is well-defined:

$$S = \frac{\Lambda}{2G} \sin\left(\frac{\Delta\phi}{2}\right) = \frac{1}{6} c_M \Lambda \sin\left(\frac{\Delta\phi}{2}\right) \tag{23}$$

- Is it possible to derive it from BMS field theory?

Black Hole Entropy

- Possible existence of BMS algebra in the NH geometries of non-extremal Black holes (Branes).
- Can we reproduce entropy of non-extremal black holes through a microscopic description in terms of contracted CFT?

Thank you