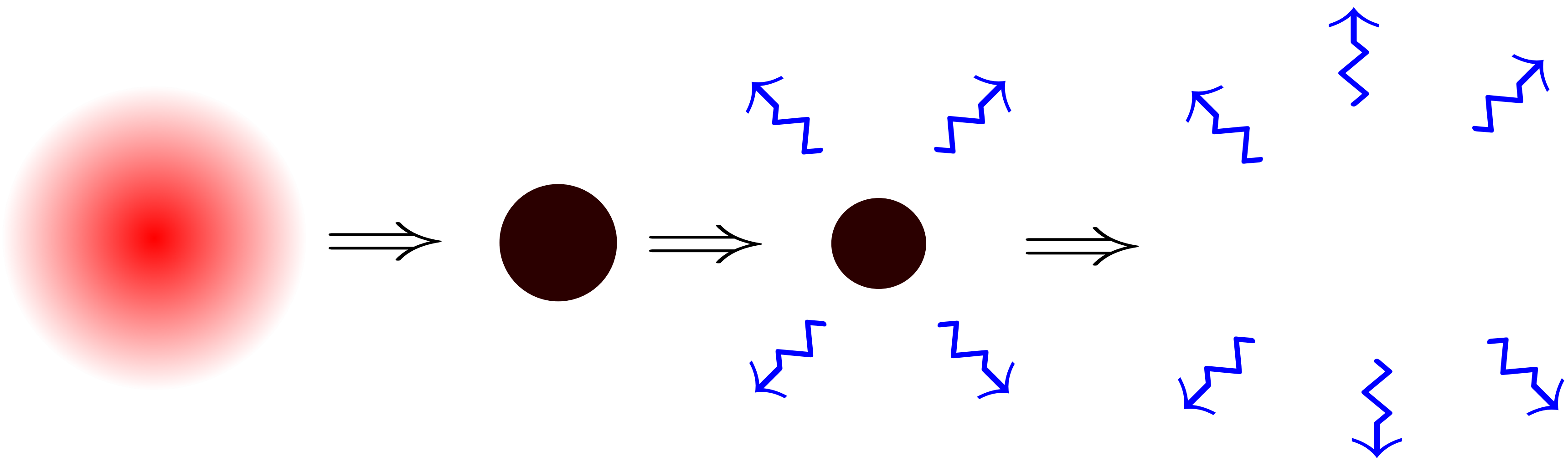
The infalling observer in AdS/CFT and the black hole information paradox

Kyriakos Papadodimas
University of Groningen

20 June 2013
7th Regional String Meeting, Kolymbari

based on arXiv:1211.6767, K.P and Suvrat Raju
+ work in progress

Hawking Radiation vs Unitarity



gas cloud
in pure state

Hawking radiation

$$|\Psi_0\rangle \quad \Longrightarrow \quad \cdots \quad \Longrightarrow \quad \rho_{\text{thermal}}$$

INCONSISTENT WITH UNITARY EVOLUTION

Horizon is smooth

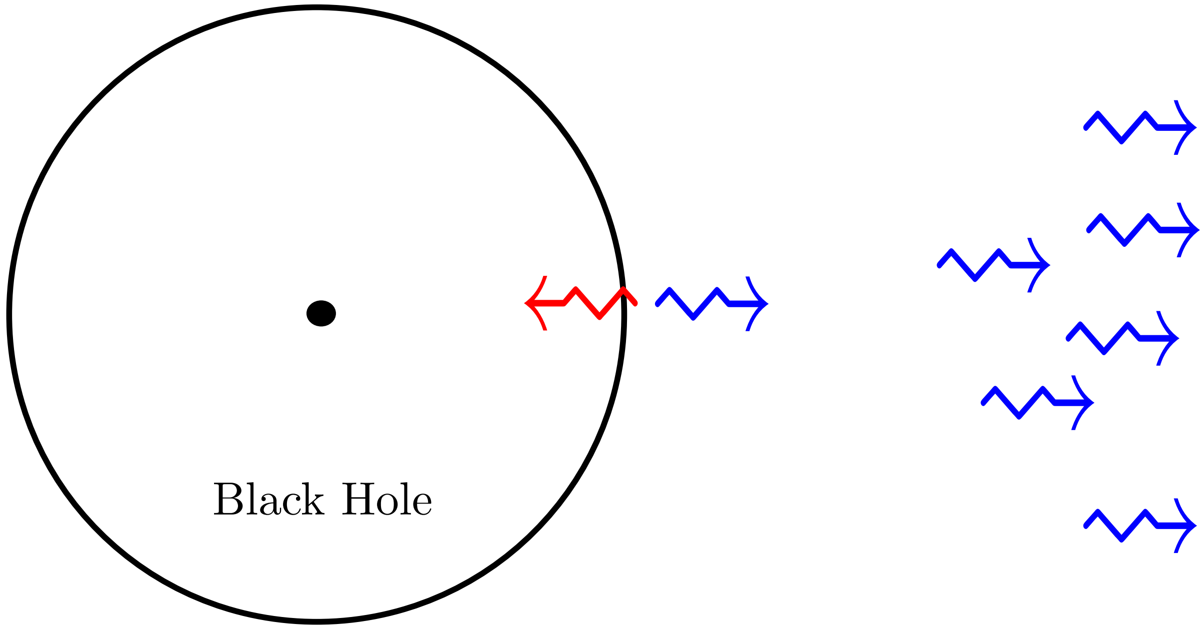
- Classically, the black hole horizon is a smooth region of spacetime

$$R^2_{\mu\nu\rho\sigma} \sim \frac{1}{M^4}$$

- A freely falling observer does not feel anything when crossing the horizon of a big black hole

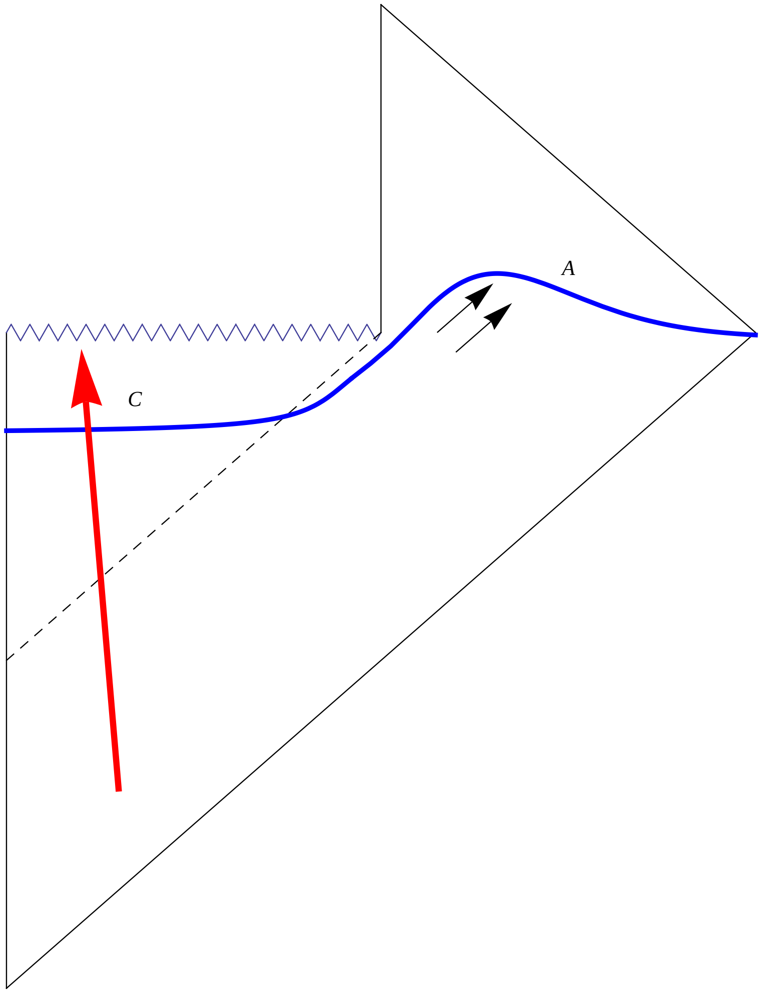
- Horizon is featureless (looks like “empty space”)

Information Paradox



- For unitarity: final state must carry information of initial state
- (In some sense) Hawking quanta are created near the horizon
- **If horizon is featureless and we have locality, how is information transferred to outgoing radiation?**

Quantum Cloning on the nice slices



We have tension between

- Unitarity
- Locality
- Equivalence Principle (smooth horizon)

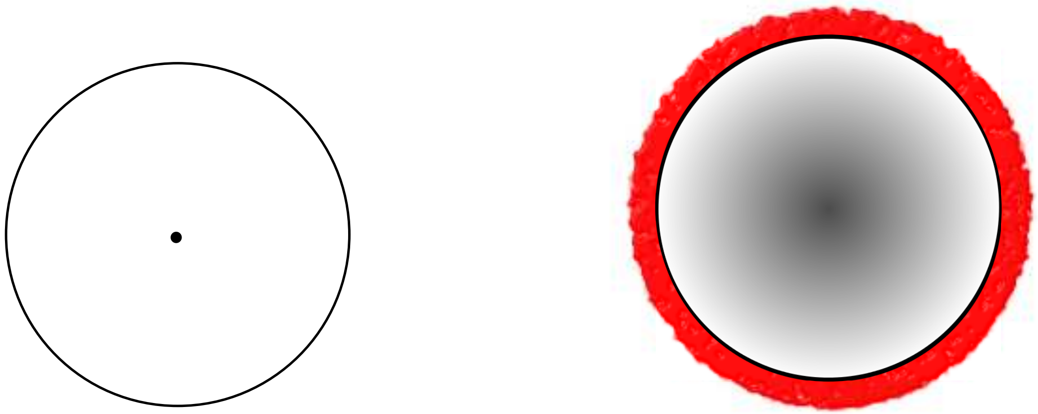
CAN SMALL CORRECTIONS RESOLVE THE PARADOX?

Will try to argue that the answer is **YES**

"Small" amount of non-locality is sufficient to restore unitarity and at the same time preserve the smoothness of the horizon

Modification of black hole geometry?

- Proposals to modify interior of black hole:
  -Fuzzballs (Mathur, ...)
  -Firewalls (Almheiri, Marolf, Polchinski, Sully – "AMPS")
  -Rami's talk ?....
  -....



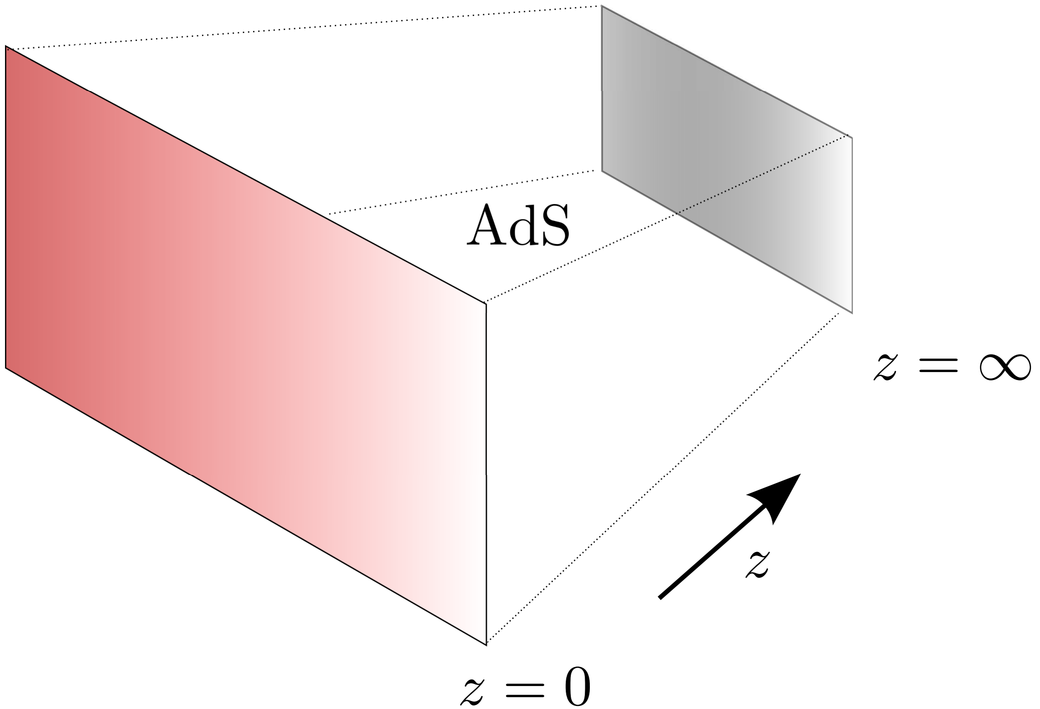
-interior black hole geometry $\neq$ Schwarzschild solution

-infalling observer feels deviations from GR/burns-up when crossing the horizon

Free infall or not?

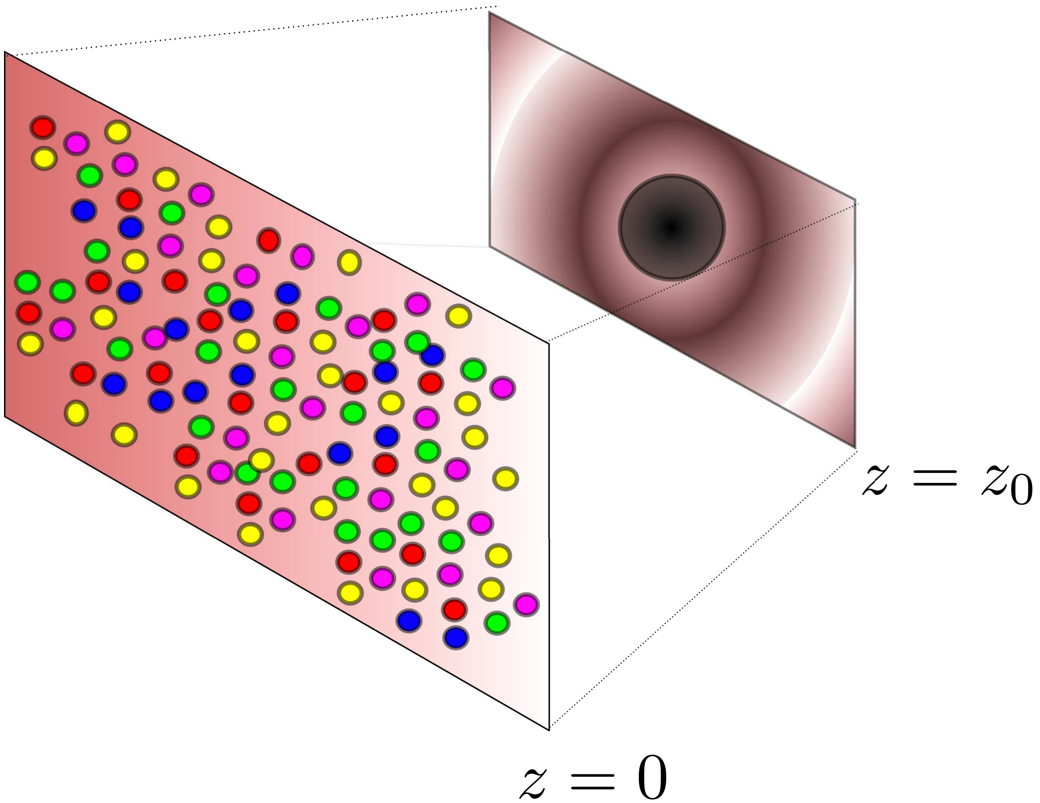
Does an infalling observer notice something when crossing the horizon or not?

AdS/CFT correspondence



Quantum Gravity in AdS $\quad \Leftrightarrow \quad$ large N gauge theory in lower dimensions

AdS/CFT correspondence



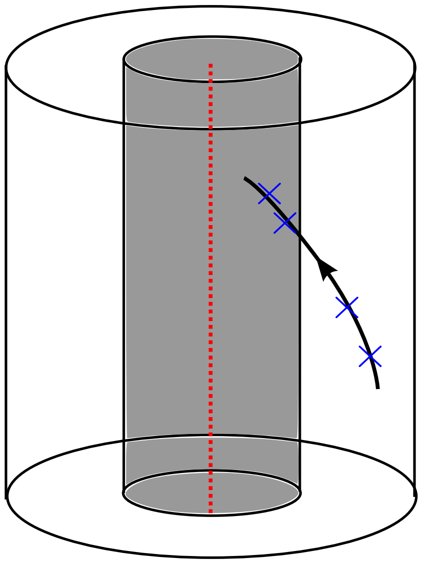
Black Hole in AdS $\Leftrightarrow$ Quark-Gluon Plasma (QGP) in gauge theory

Black Holes in AdS/CFT

Main goals:

- Is the region behind the horizon encoded in the boundary CFT?
- Understand what happens to an observer falling into a black hole
- Address the information paradox

An infalling observer in AdS



- Consider a big black hole in AdS and an observer freely falling towards it
- The observer performs local experiments
- We will reconstruct these experiments from the boundary gauge theory
- We will argue that the results of these experiments are the same as those of semi-classical GR

Reconstructing local observables in empty AdS

In AdS/CFT we know that

“S-matrix elements” in AdS $\Leftrightarrow$ Correlation functions in CFT

Local bulk correlators in AdS $\Leftrightarrow$?

Our first goal:

Construct local bulk observables from CFT

(based on earlier works: Banks, Douglas, Horowitz, Martinec, Bena, Balasubramanian, Giddings, Lawrence, Kraus, Trivedi, Susskind, Freivogel Hamilton, Kabat, Lifschytz, Lowe, Heemskerk, Marolf, Polchinski, Sully...)

Reconstructing local observables in empty AdS

- Large N CFTs contain in their spectrum **generalized free fields** i.e. (composite) local operators $\mathcal{O}(x)$ whose correlators factorize

$$\langle\mathcal{O}(x_1)...\mathcal{O}(x_{2n})\rangle = \langle\mathcal{O}(x_1)\mathcal{O}(x_2)\rangle\ ...\ \langle\mathcal{O}(x_{2n-1})\mathcal{O}(x_{2n})\rangle + \ldots$$

- Factorization ≈ "superposition principle". However, the operators $\mathcal{O}$ **do not satisfy any linear equation of motion in the CFT**.

- Hence, they are not **free fields**, but rather **generalized free fields**

- Excitations created by $\mathcal{O}$ behave like **ordinary free particles** in a higher dimensional AdS spacetime

Reconstructing local observables in empty AdS

- First we define the Fourier modes of $\mathcal{O}_{\omega,\vec{k}}$ by

$$\mathcal{O}(t,\vec{x}) = \int dt d\vec{x} \left(\mathcal{O}_{\omega,\vec{k}}\, e^{-i\omega t + i\vec{k}\vec{x}} + \mathrm{h.c.}\right)$$

- Conformal invariance fixes the 2-point function to be

$$\langle \mathcal{O}(t,\vec{x})\mathcal{O}(0,\vec{0})\rangle = \left(\frac{-1}{t^2 - \vec{x}^2 - i\epsilon}\right)^{\Delta}$$

- From this we find

$$\mathcal{O}_{\omega,\vec{k}}|0\rangle = 0, \qquad \omega > 0$$

and (**using large N factorization**)

$$[\mathcal{O}_{\omega,\vec{k}}, \mathcal{O}^{\dagger}_{\omega',\vec{k}'}] = \mathcal{N}\theta(\omega^2 - \vec{k}^2)(\omega^2 - \vec{k}^2)^{\Delta - d/2}\delta(\omega - \omega')\delta(\vec{k} - \vec{k}')$$

- From this commutation relation we see that the modes $\mathcal{O}_{\omega,\vec{k}}$ create a **freely generated Fock space** of excitations.

- For an ordinary free field we have dispersion relation $\omega^2 = \vec{k}^2 + m^2$.

- For the generalized free fields, excitations labeled by the **independent** parameters ω and $\vec{k}$.

- $\Rightarrow$ excitations behave like ordinary particles in higher dimensional AdS space

Reconstructing local observables in empty AdS

- Consider AdS in Poincare patch

$$ds^2 = \frac{-dt^2 + d\vec{x}^2 + dz^2}{z^2}$$

- and a scalar field satisfying $\Box\phi = m^2\phi$.

- We take m^2 to be related to the conformal dimension Δ of $\mathcal{O}$ by

$$\Delta = \frac{d}{2} + \sqrt{m^2 + d^2/4}$$

- For each value of $\omega, \vec{k}$ we find a solution of the Klein-Gordon equation of the form

$$f_{\omega,\vec{k}}(t,\vec{x},z) = e^{-i\omega t + i\vec{k}\vec{x}} z^{d/2} J_{\Delta-d/2}(\sqrt{\omega^2 - \vec{k}^2} z)$$

Reconstructing local observables in empty AdS

- We construct non-local CFT operators as

$$\phi_{\rm CFT}(t,\vec{x},z) = \int_{\omega>0} d\omega\, d\vec{k}\, \Big(\mathcal{O}_{\omega,\vec{k}}\; f_{\omega,\vec{k}}(t,\vec{x},z) + \text{h.c.}\Big)$$

  Notice that while these are labeled by the coordinate z, they are really operators in the CFT. They are smeared, nonlocal operators.

- Using the previous results we can show that they satisfy

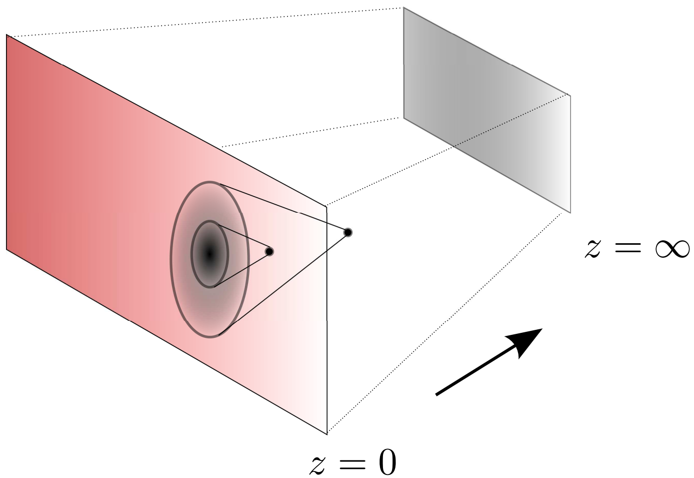
$$\Box_{\rm AdS}\,\phi_{\rm CFT} = m^2\,\phi_{\rm CFT}$$

  and

$$[\phi_{\rm CFT}(t,\vec{x},z)\,,\,\phi_{\rm CFT}(t',\vec{x}',z')] = 0$$

  for points $(t,\vec{x},z)$ and $(t',\vec{x}',z')$ spacelike **with respect to the AdS metric**.

Reconstructing local observables in empty AdS



- From the point of view of the CFT, coordinate z is an "auxiliary" parameter, which controls the smearing of the operators

- We can explicitly see how AdS space emerges from the lower dimensional CFT, as the combination of the coordinates $t, \vec{x}$ together with the extra parameter z

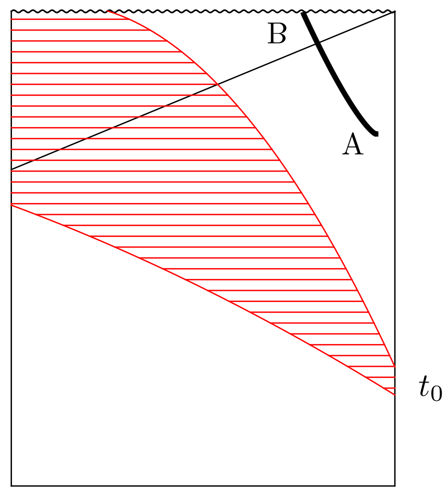
Reconstructing local observables in empty AdS

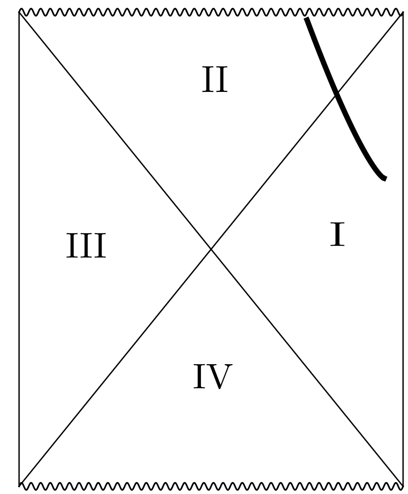
We can also interchange the order of the Fourier transforms to write

$$\phi_{\mathrm{CFT}}(t,\vec{x},z) = \int dt' d\vec{x}' \; K(t,\vec{x},z\;;\;t',\vec{x}')\mathcal{O}(t',\vec{x}')$$

where K is some kernel — sometimes called the *transfer function*.

Subtleties: 1/N expansion, gauge invariance....





BH formed by collapse $\approx$ Typical (QGP) pure state $|\Psi\rangle$

Eternal Black Hole in AdS $\approx$ Thermal ensemble in gauge theory

CFT Correlators at finite temperature

We use the notation

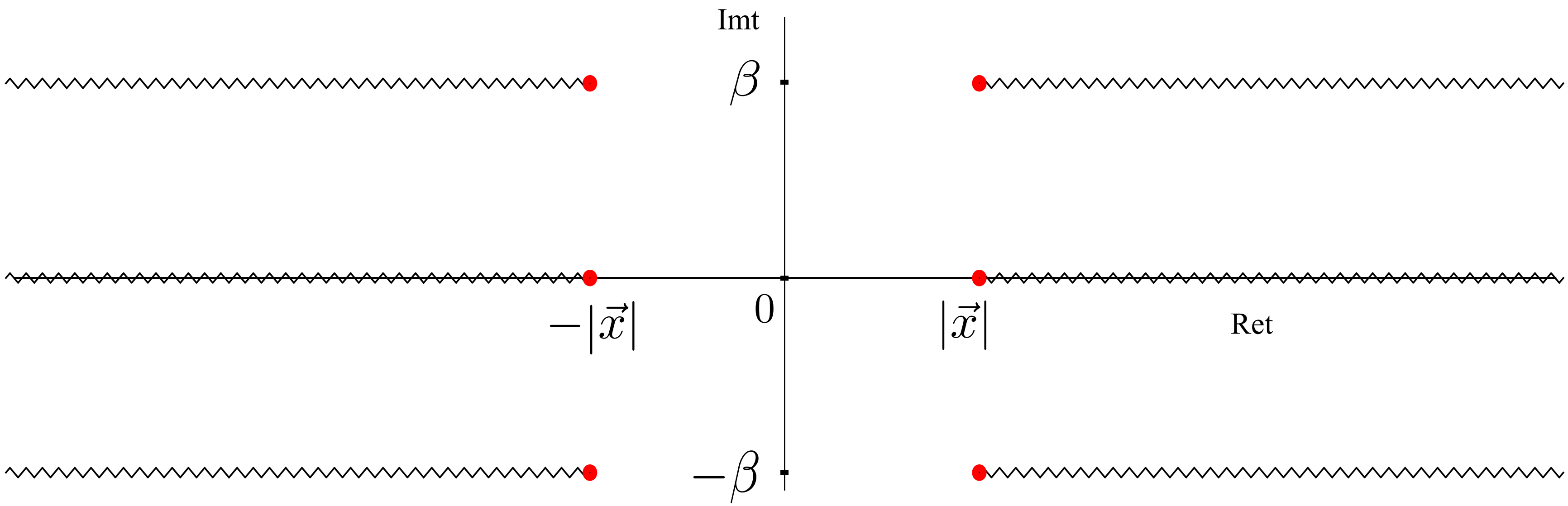
$$\langle A_1 ... A_n \rangle_\beta = \frac{1}{Z(\beta)} \mathrm{Tr} \left(e^{-\beta H} A_1 ... A_n \right)$$

- At large N thermal correlators factorize

$$\langle \mathcal{O}(x_1) ... \mathcal{O}(x_{2n}) \rangle_\beta = \langle \mathcal{O}(x_1) \mathcal{O}(x_2) \rangle_\beta \dots \langle \mathcal{O}(x_{2n-1}) \mathcal{O}(x_{2n}) \rangle_\beta + \dots$$

- Of course $\langle \mathcal{O}(x_1) \mathcal{O}(x_2) \rangle_\beta \neq \langle 0 | \mathcal{O}(x_1) \mathcal{O}(x_2) | 0 \rangle$
- Factorization can fail if we scale the parameters of the correlator with N (for example: number of insertions, distances x_i, dimension of operators etc.)

CFT Correlators at finite temperature



- Consider the 2-point function $G_\beta(t,\vec{x}) = \langle \mathcal{O}(t,\vec{x})\mathcal{O}(0,\vec{0})\rangle_\beta$
- Satisfies the **KMS condition**

$$G_\beta(t - i\beta, \vec{x}) = G_\beta(-t, -\vec{x})$$

- In Fourier space

$$G_\beta(-\omega, \vec{k}) = e^{-\beta\omega} G_\beta(\omega, \vec{k})$$

CFT Correlators at finite temperature

- If we again define the Fourier modes $\mathcal{O}_{\omega,\vec{k}}$ by

$$\mathcal{O}(t,\vec{x}) = \int dt d^{d-1}x \left(\mathcal{O}_{\omega,\vec{k}}\, e^{-i\omega t + i\vec{k}\vec{x}} + \mathrm{h.c.} \right)$$

- we find that they satisfy an oscillator algebra

$$[\mathcal{O}_{\omega,\vec{k}}\,,\, \mathcal{O}^{\dagger}_{\omega',\vec{k}'}] = \left(G_{\beta}(\omega,\vec{k}) - G_{\beta}(-\omega,\vec{k}) \right) \delta(\omega-\omega')\delta(\vec{k}-\vec{k}')$$

- but now the (canonically normalized) oscillators are thermally populated

$$\langle \hat{\mathcal{O}}^{\dagger}_{\omega,\vec{k}}\, \hat{\mathcal{O}}_{\omega,\vec{k}} \rangle_{\beta} = \frac{1}{e^{\beta\omega}-1}$$

(this is the CFT analogue of the "thermal atmosphere" of the black hole)

Reconstructing the region outside the black hole

- Consider a black hole in AdS given by the metric

$$ds^2 = \frac{-h(z)dt^2 + dx^2 + h^{-1}(z)dz^2}{z^2} \qquad , \qquad h(z) = 1 - \frac{z^d}{z_0^d}$$

- Look for solutions of the Klein-Gordon equation of the form

$$f_{\omega,\vec{k}}(t,\vec{x},z) = e^{-i\omega t + i\vec{k}\vec{x}}\psi_{\omega,\vec{k}}(z)$$

- For every $(\omega, \vec{k})$ there is a unique solution, normalizable at the boundary $z = 0$.
- These are the usual "Schwarzschild modes" that we get when we quantize a scalar field near a black hole. We relate

$$f_{\omega\vec{k}}(t,\vec{x},z) \qquad \Leftrightarrow \qquad \mathcal{O}_{\omega,\vec{k}}$$

Reconstructing the region outside the black hole

- As before, we can write nonlocal CFT operators

$$\phi_{\rm CFT}(t,\vec{x},z) = \int_{\omega>0} d\omega d\vec{k} \left(\mathcal{O}_{\omega,\vec{k}}\, f_{\omega,\vec{k}}(t,\vec{x},z) + \text{h.c.} \right)$$

- which behave like local fields around a black hole

$$(\Box - m^2)\phi_{\rm CFT} = 0$$

$$[\phi_{\rm CFT}(t,\vec{x},z)\,,\,\phi_{\rm CFT}(t',\vec{x}',z')] = 0 \qquad , \quad \text{for spacelike points}$$

- and more generally

$$\langle \phi_{\rm CFT}(P_1)\ldots\phi_{\rm CFT}(P_n)\rangle_\beta = \langle \phi_{gravity}(P_1)\ldots\phi_{gravity}(P_n)\rangle_{\text{Hartle Hawking}}$$

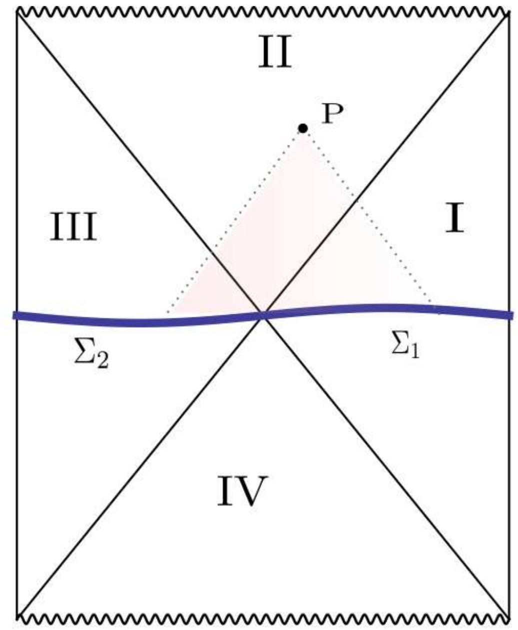
Reconstructing the region outside the black hole

- We have understood how to reconstruct the region outside the black hole from the point of view of the gauge theory

- We can write local observables in gravity as non-local operators in the gauge theory

Falling behind the horizon

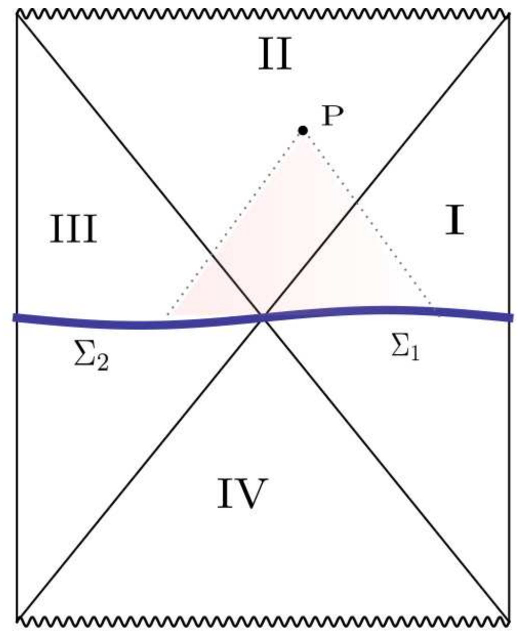
- Penrose diagram of (eternal) AdS black hole



- Cauchy slice for points in II is $\Sigma_1 \oplus \Sigma_2$
- To reconstruct local operator at P we need **both** modes on Σ_1 and Σ_2

$$\begin{array}{ccc} \text{Modes on } \Sigma_1 & \Leftrightarrow & \mathcal{O}_{\omega,\vec{k}} \\ \text{Modes on } \Sigma_2 & \Leftrightarrow & ? \end{array}$$

Falling behind the horizon



- Maldacena: eternal black hole = 2 copies of CFT in entangled state
- In this formalism, modes on Σ_2 are the operators $\widetilde{\mathcal{O}}_{\omega,\vec{k}}$ in the second copy of the CFT
- Do we really need the two entangled copies?
- If we work **with a single CFT**, what is the meaning of the operators $\widetilde{\mathcal{O}}_{\omega,\vec{k}}$?

Pure states and thermalization

In eternal AdS black hole:

2nd CFT $\sim$ "heat bath"

A pure state $|\Psi\rangle$ in an **isolated** system may "look thermal", without the need for an external heat bath

The large N $\mathcal{N}=4$ SYM "self-thermalizes"

Coarse-graining and doubling of operators

- Consider complicated (ergodic) system in pure state $|\Psi\rangle$. Intuitive expectation $\Rightarrow$ system "thermalizes"

- For some observables $\{\mathcal{O}_i\}$- called **coarse-grained observables**, their correlators on $|\Psi\rangle$ come close to thermal correlators

$$\langle\Psi|\mathcal{O}_1..\mathcal{O}_n|\Psi\rangle \approx \mathrm{Tr}\left(e^{-\beta H}\mathcal{O}_1...\mathcal{O}_n\right)$$

(example: single trace operators in large N gauge theory)

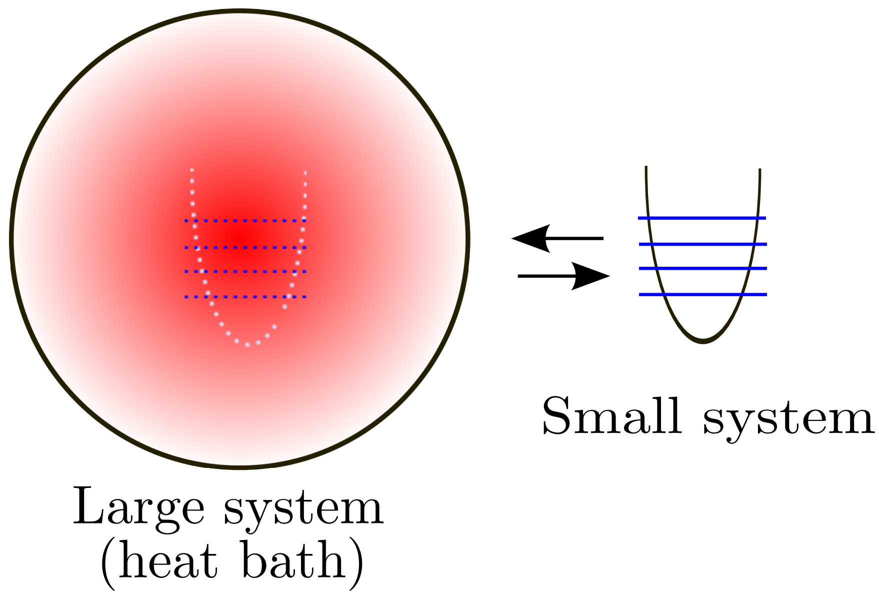
- This is not true for all observables, there are also **fine grained observables** which do not thermalize

- To simplify the language let us assume that the Hilbert space (expanded around $|\Psi\rangle$) has the form

$$\mathcal{H} = \mathcal{H}_{\mathrm{coarse}} \otimes \mathcal{H}_{\mathrm{fine}}$$

- $\mathcal{H}_{\mathrm{fine}}$ plays the role of a **heat bath** for $\mathcal{H}_{\mathrm{coarse}}$

Coarse graining and doubling of operators



SMALL SUBSYSTEM IS MIRRORED IN HEAT BATH

- For us the Quark-Gluon-Pasma is the heat bath
- The glueball operators $\mathcal{O}_i$ are the coarse-grained observables
- They are mirrored in the QGP, which leads to new operators $\widetilde{\mathcal{O}}_i$ (special "collective excitations" of the QGP)

Coarse graining and doubling of operators

- Every state $|\Psi\rangle$ can be written as

$$|\Psi\rangle = \sum_{ij} c_{ij} |\Psi_i^c\rangle \otimes |\Psi_j^f\rangle$$

where $|\Psi_i^c\rangle$, $|\Psi_j^f\rangle$ are orthonormal basis of $\mathcal{H}_{\mathrm{coarse}}$ and $\mathcal{H}_{\mathrm{fine}}$ respectively

- We can choose Schmidt basis

$$|\Psi\rangle = \sum_{i} a_i |\hat{\Psi}_i^c\rangle \otimes |\hat{\Psi}_i^f\rangle$$

- If $\mathcal{H}_{coarse}$ thermalizes, it means that the reduced density matrix

$$\rho_{\mathrm{coarse}} = Z_c^{-1} e^{-\beta H_{\mathrm{coarse}}}$$

- which means we can redefine our orthonormal basis such that

$$|\Psi\rangle = \sum_{i} \frac{e^{-\frac{\beta E_i^c}{2}}}{\sqrt{Z_c}} |\hat{\Psi}_i^c\rangle \otimes |\hat{\Psi}_i^f\rangle$$

Coarse graining and doubling of operators

- The state $|\Psi\rangle$ can be written as

$$|\Psi\rangle = \sum_i \frac{e^{-\frac{\beta E_i^c}{2}}}{\sqrt{Z_c}} |\hat{\Psi}_i^c\rangle \otimes |\hat{\Psi}_i^f\rangle$$

- Consider a **coarse-grained** operator acting on $\mathcal{H}_{\text{coarse}}$ as

$$\mathcal{O} = \sum_{ij} a_{ij} |\hat{\Psi}_i^c\rangle \otimes \langle\hat{\Psi}_j^c|$$

- Then we **define** a new operator

$$\widetilde{\mathcal{O}} = \sum_{ij} a_{ij}^* |\hat{\Psi}_i^f\rangle \otimes \langle\hat{\Psi}_j^f|$$

acting on the fine-grained Hilbert space.

Coarse graining and doubling of operators

- We started with a set of coarse-grained operators $\mathcal{O}_i$

- The operators $\widetilde{\mathcal{O}}_i$ constructed as above, have the properties

1. The operator algebra $\widetilde{\mathcal{O}}_i$ is isomorphic to that of $\mathcal{O}_i$

2. Operators $\mathcal{O}_i$ commute with operators $\widetilde{\mathcal{O}}_i$

Coarse graining and doubling of operators

- At large N, correlation functions of the mirrored operators $\widetilde{\mathcal{O}}$ on a pure state, agree with those of analytically continued operators

$$\mathcal{O}(t + i\beta/2)$$

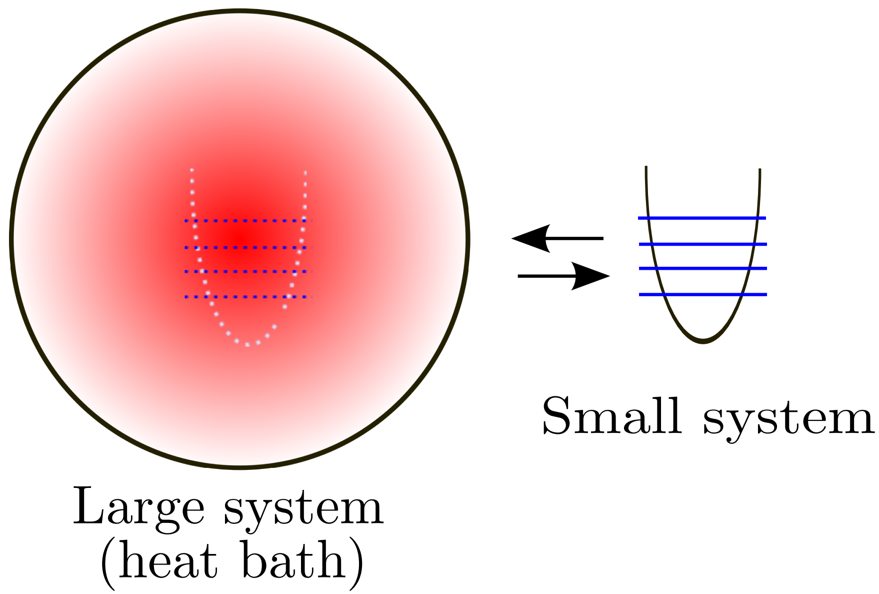
  and in particular correlators computed with the "thermofield doubling"

- However the $\widetilde{\mathcal{O}}$, **as operators acting on pure states**, were defined via the coarse/fine-grained decomposition

Fine-grained observables

- The "tilde operators" are very special: they are **fine-grained** observables
- They are state-dependent operators, will not "click correctly" with different microstate $|\Psi'\rangle$ (which may be a good thing....)
- Among all possible fine-grained operators, the "tilde operators" are selected/protected via their entanglement with the coarse-grained ones
- They are "very sparse operators"

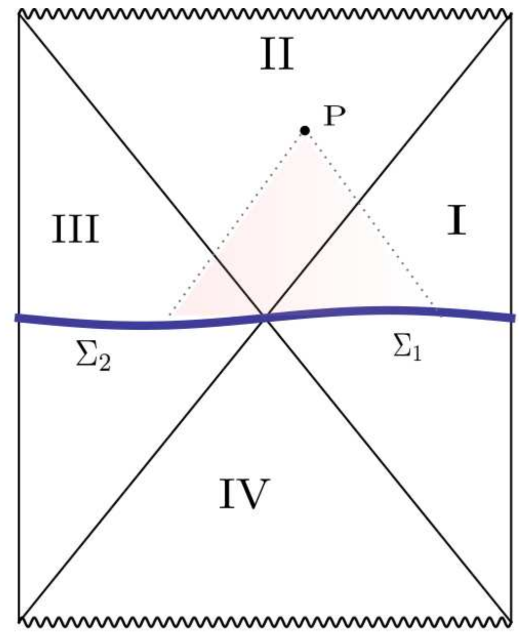
Coarse graining and doubling of operators



SMALL SUBSYSTEM IS MIRRORED IN HEAT BATH

- For us the Quark-Gluon-Pasma is the heat bath
- The glueball operators $\mathcal{O}_i$ are the coarse-grained observables
- They are mirrored in the QGP, which leads to new operators $\widetilde{\mathcal{O}}_i$
- This mirroring involves the fine-degrees of freedom

Falling behind the horizon



$$\text{Modes on } \Sigma_1 \quad \Leftrightarrow \quad \mathcal{O}_{\omega,\vec{k}}$$

$$\text{Modes on } \Sigma_2 \quad \Leftrightarrow \quad \widetilde{\mathcal{O}}_{\omega,\vec{k}}$$

where $\widetilde{\mathcal{O}}_{\omega,\vec{k}}$ are the Fourier transforms of the mirrored operators $\widetilde{\mathcal{O}}$

Local operators behind the horizon

Using both $\mathcal{O}_{\omega,\vec{k}}$ and $\widetilde{\mathcal{O}}_{\omega,\vec{k}}$ we can write local observables behind the horizon of the black hole.

$$\phi_{\rm CFT}(t,\vec{x},z) = \int_{\omega>0} d\omega d\vec{k} \left[\mathcal{O}_{\omega,\vec{k}}\ g^{(1)}_{\omega,\vec{k}}(t,\vec{x},z) + \widetilde{\mathcal{O}}_{\omega,\vec{k}}\ g^{(2)}_{\omega,\vec{k}}(t,\vec{x},z) + \text{h.c.}\right]$$

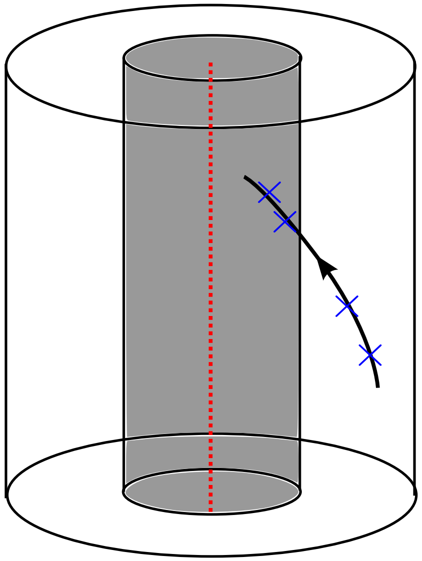
here $g^{(1),(2)}$ are solutions of the Klein-Gordon equation in region II

In the large N limit, correlators of $\phi_{\rm CFT}(t,\vec{x},z)$ on a typical pure state $|\Psi\rangle$ (corresponding to a black hole microstate) agree with those computed in semiclassical gravity.

We have reconstructed both the exterior and the interior of the black hole from the dual gauge theory

Fate of the infalling observer

Using the operators ϕ_{CFT} we can reconstruct the experiments of the infalling observer



MAIN CONCLUSION: For a big AdS black hole, an infalling semi-classical observer does not notice anything special when crossing the horizon

In contradiction to (fuzzball) and firewall proposals.

Various subtleties

- Sensitivity to pure state $|\Psi\rangle$?
- Including $1/N$ corrections?
- Spread of transfer function as we approach the horizon?
- Sensitivity to late times - Poincare recurrences?

If large N expansion at finite temperature holds, we can argue that they are under control.

The information paradox

- We argued that horizon is smooth.

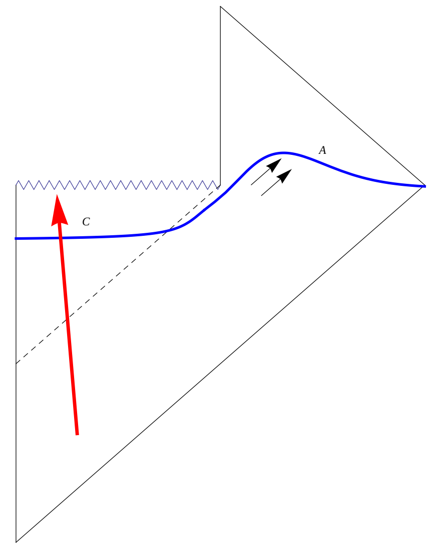
- Mathur and AMPS argue that free infall cannot be compatible with unitary evaporation

- Sharpened version of information paradox (strong subadditivity argument)

What does our construction teach us about the information paradox?

Black Hole Complementarity

Black Hole interior is not independent Hilbert space, but highly scrambled version of exterior



In our construction:

- Exterior of black hole $\Rightarrow$ operators $\mathcal{O}$ (single trace operators)
- Interior of black hole $\Rightarrow$ operators $\widetilde{\mathcal{O}}$ (special, QGP operators)
- In low-point correlators $\mathcal{O}$, $\widetilde{\mathcal{O}}$ seem to be independent
- If we act with too many (order N^2) of $\mathcal{O}$'s we can "reconstruct" the $\widetilde{O}$'s

THANK YOU

“Asymptotic” version of the information paradox

- Hawking's computation $\Rightarrow$ mixed (thermal) state ρ_{Hawking}
- Starting from pure state $|\Psi\rangle$ we end up with mixed state ρ_{Hawking}
- Inconsistent with unitary evolution in Quantum Mechanics

"Asymptotic" version of the information paradox

However, consider what happens when a normal object burns (say a piece of coal)

- Outgoing photons seem to be thermal to a very good approximation.
- How is unitarity preserved? Where is the information of the original piece of coal stored in the outgoing radiation?
- ANSWER: It is encoded in very small correlations (entanglement) between the outgoing photons.
- While final state **looks like** a thermal density matrix ρ_{thermal} in reality it is a pure state.

SMALL CORRECTIONS TO LEADING THERMAL APPROXIMATION CAN RESTORE UNITARITY

“Asymptotic” version of the information paradox

However, consider what happens when a normal object burns (say a piece of coal)

- Outgoing photons seem to be thermal to a very good approximation.
- How is unitarity preserved? Where is the information of the original piece of coal stored in the outgoing radiation?
- ANSWER: It is encoded in very small correlations (entanglement) between the outgoing photons.
- While final state **looks like** a thermal density matrix $\rho_{\rm thermal}$ in reality it is a pure state.

SMALL CORRECTIONS TO LEADING THERMAL APPROXIMATION CAN RESTORE UNITARITY

“Asymptotic” version of the information paradox

- Imagine that outgoing photons can be in 2 states. For N photons we have

$$2^N \qquad \text{states}$$

- Density matrix of outgoing radiation is of size $2^N \times 2^N$.
- Consider

$$\rho_{\text{exact}} = \rho_{\text{thermal}} + 2^{-N}\rho_{\text{correction}}$$

  where $\rho_{\text{correction}} \sim \mathcal{O}(1)$.
- Can easily check that ρ_{exact} of this form, can correspond to a **pure state**

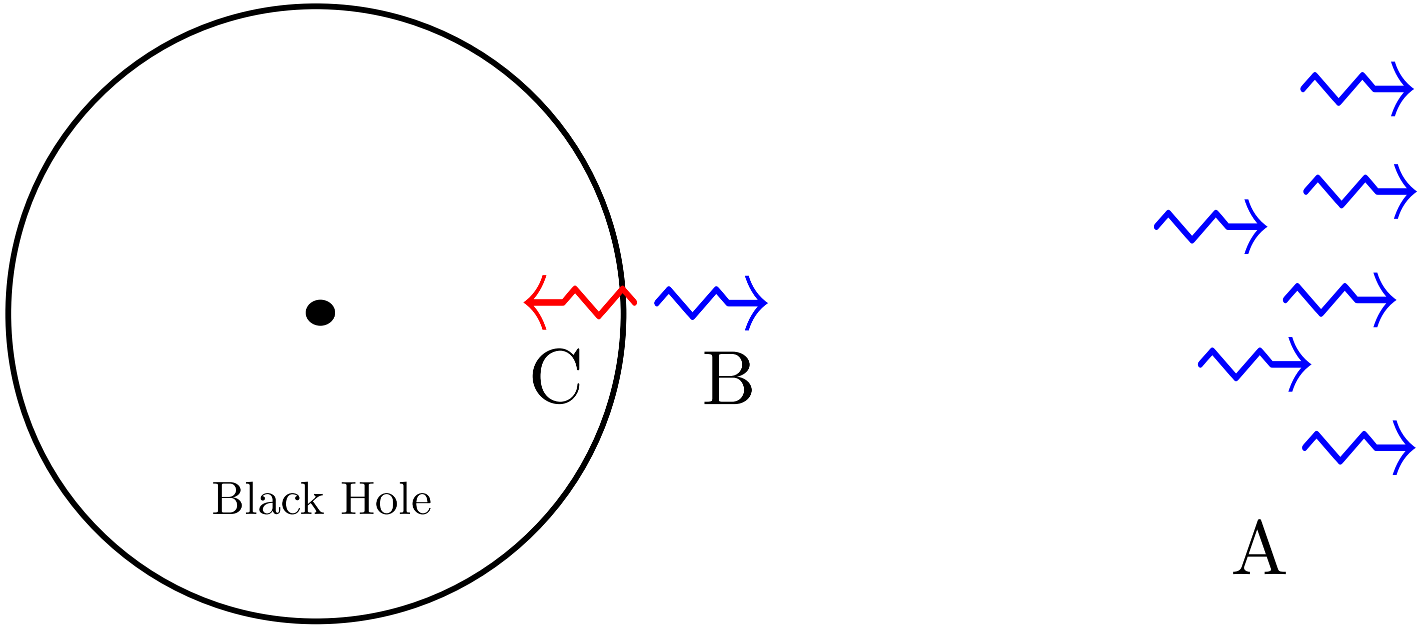
EXPONENTIALLY SMALL CORRECTIONS CAR RESTORE UNITARITY

Resolution of (asymptotic version of) the information paradox

- Hawking's computation is only **the leading order** result
- We certainly expect corrections to the leading order computation, from quantum gravity effects (saddle points etc.)
- Even extremely small corrections are able to restore unitarity due to the large number of particles

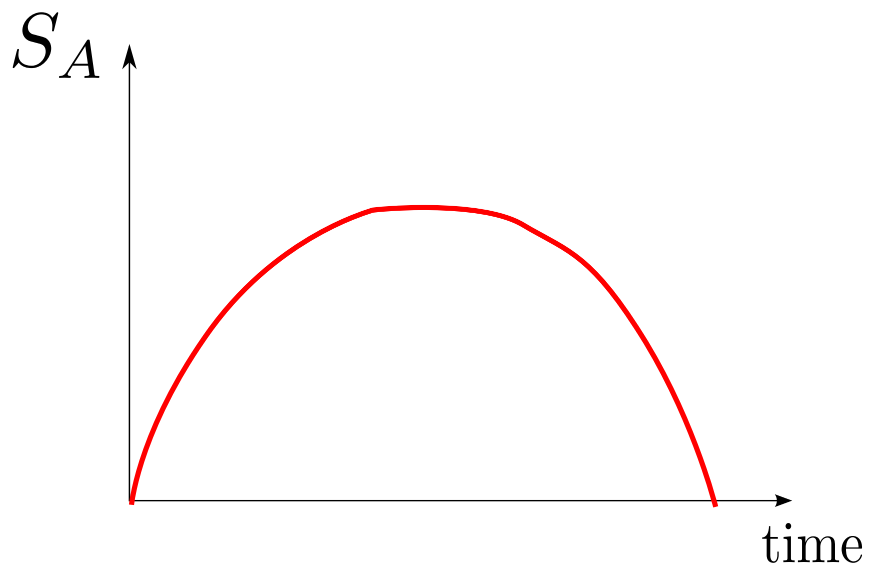
Sharpened version of the information paradox (Mathur, AMPS)

Consider the process of Hawking radiation



- A: old radiation, far from black hole
- B: newly created Hawking particle, outgoing
- C: ingoing partner of B

Sharpened version of the information paradox (Mathur, AMPS)

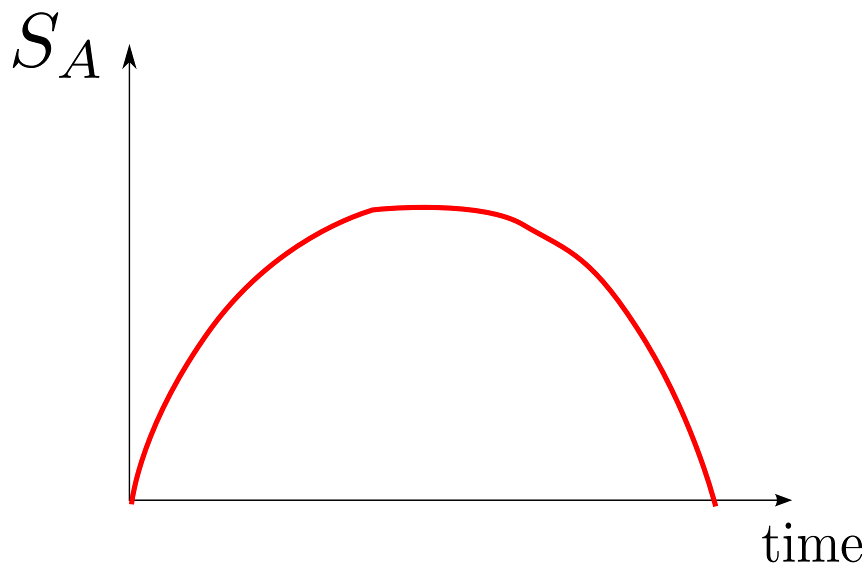


Consider the entropy of radiation A as a function of time

$$S_A = -\mathrm{Tr}\left(\rho_A \log \rho_A\right)$$

If initial state is pure then S_A must go to zero after complete evaporation of the black hole

Sharpened version of the information paradox (Mathur, AMPS)



In the beginning adding a B to A increases the entropy i.e. we expect

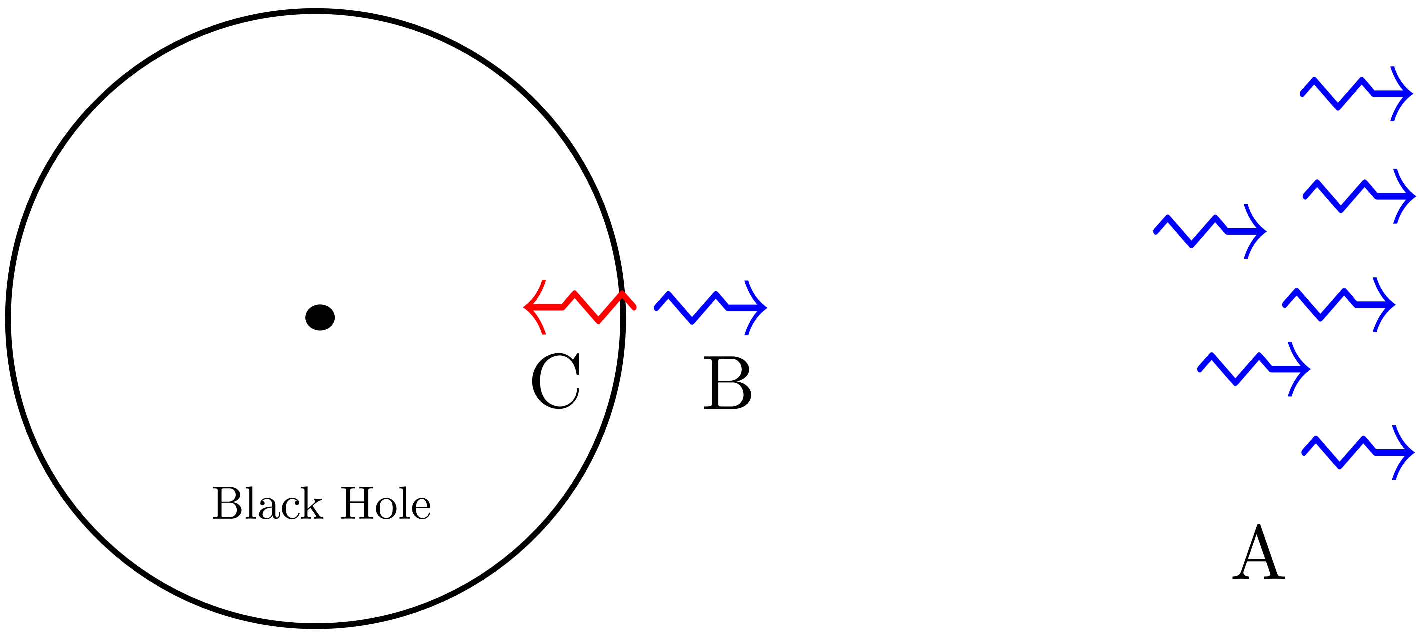
$$S_{AB} > S_A$$

but eventually this must turn around and for an **old black hole** we expect

$$S_{AB} < S_A$$

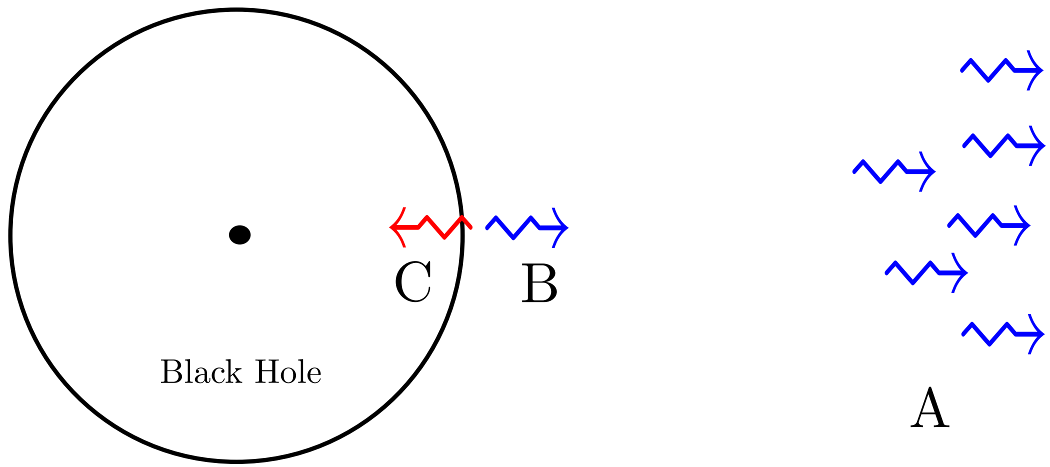
Sharpened version of the information paradox (Mathur, AMPS)

Consider the process of Hawking radiation



- For **information recovery**: B must be entangled to A
- For **free infall**: B must be entangled to C
- Are these two statements compatible?

Sharpened version of the information paradox (Mathur, AMPS)



Strong subadditivity theorem: for 3 **independent** systems A,B,C we have

$$S_{AB} + S_{BC} \geq S_A + S_C$$

For the Hawking pair production we have $S_{BC} \approx 0$ and $S_C > 0$ which would imply

$$S_{AB} > S_A$$

For unitarity: after Page time we need $S_{AB} < S_A$ $\Rightarrow$ PARADOX

Proposed resolution

- In our language the C's are fine-grained operators defined via their entanglement with coarse grained operators
- After Page time the early radiation A plays the role of the heat bath
- Hence C's are "highly scrambled" combinations of A's

  Systems A,B,C are not really independent

$$\Downarrow$$

Strong subadditivity theorem cannot be applied to A,B,C

Consistent with locality

- System C is not independent, but rather a highly scrambled version of (part of) A
- Locality for **simple measurements** is preserved. For P_1 inside horizon and P_2 outside

$$[\phi(P_1)\,,\,\phi(P_2)] \approx 0$$

  up to very small corrections.
- If $\mathcal{W}$ is a complicated operator acting outside the horizon which measures many of the As then we allow

$$[\phi(P_1)\,,\,\mathcal{W}] \neq 0$$

Coarse-grained vs fine-grained observables

- Fine-grained are "sparse" (very few nonzero eigenvalues)
- Coarse-grained are "not-sparse"

The fact that C's are fine-grained (state-dependent) makes it easier to simultaneously satisfy

$$[\phi(A), \phi(C)] \approx 0$$

while at the same time $C \subset A$.

(spin chain toy-model $+$ scrambling)

On complementarity

How we understand complementarity

- There is a large Hilbert space describing **both** the interior and the exterior of the black hole
- We can construct operators acting on this Hilbert space and describing observables outside the black hole
- We can construct operators acting on the same Hilbert space and describing observables inside the black hole
- For few, light observables, they approximately commute.
- But not for too accurate measurements, or measurements involving too many insertions

Summary

- **Local bulk physics from CFT**: local observables both outside and inside the black hole
- **Infalling observer:** does not notice anything special
- **Interior of black hole:** coarse-grained observables effectively doubled in fine grained degrees of freedom. Black hole interior is a combination of both.
- **Information paradox:** small corrections can restore unitarity.
- **Strong subadditivity argument (Mathur, AMPS)**: A,B,C are not independent systems. C is a highly scrambled rewriting of A
- **Non-locality:** The amount of non-locality required is small...

Future directions

- Dynamics, "stability" of fine-grained operators
- Thermalization
- Meaning of singularity
- $1/N$ corrections etc.
- Bounds on non-locality, toy model
- ...

- Consider (sub)-algebra of "coarse observables" $\mathcal{A}$. Select Cartan basis and corresponding projection operators $\mathcal{P}_a, \quad a = 1, ..n$.
- A typical pure state $|\Psi\rangle$ can be expanded as

$$|\Psi\rangle = \sum_{a=1}^{n} c_a |a\rangle_\Psi, \qquad |a\rangle_\Psi = \frac{P_a|\Psi\rangle}{||P_a|\Psi\rangle||}$$

  For typical states $c_a \approx 1/\sqrt{n}$.
- By acting with "coarse observables" in $\mathcal{A}$, we can map states with eigenalues $\{a\}$ into eigenvalues $\{b\}$, without acting on the "fine degrees of freedom". This defines "transition operators" $T_{a\to b}$ (made out of the $\mathcal{A}$'s) (example: spin-flip)
- In this way we get a set of n^2 states

$$|b; a\rangle_\Psi \equiv T_{a\to b}|a\rangle_\Psi$$

- For given typical pure state $|\Psi\rangle$ we associate an n^2 dimensional Hilbert space $\mathcal{H}_\Psi$ (subset of the full Hilbert space), spanned by the (almost orthogonal)

$$|b;a\rangle_\Psi$$

- Original state

$$|\Psi\rangle = \sum c_a |a;a\rangle_\Psi$$

- In this notation, coarse grained operators $\mathcal{O} \in \mathcal{A}$ act as

$$\mathcal{O} = \sum_{bb',a} \mathcal{O}_{bb'} |b;a\rangle_\Psi \, {}_\Psi\langle b';a|$$

$$\widetilde{\mathcal{O}} = \sum_{aa'} (\mathcal{O}_{aa',b})^* |b;a\rangle_\Psi \, {}_\Psi\langle b;a'|$$