Blackhole Paradoxes: Origin and proposed resolution

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RB, 1209.2686
+ J. Medved, 1305.3139
+ J. Medved, 1302.6086
==============
+ J. Kupferman, 1010.4157
+ M. Hadad, 1202.5273
==============
+ M. Hadad, to appear
+ J. Medved, to appear

- BH paradoxes: Origin
- BH information paradox: Emission of thermal radiation in classical collapse
- Radiation density matrix in semiclassical collapse
- Information released

Breakdown of predictability in gravitational collapse

Hawking PRD 1976
paid up bet too soon

- Calculation: A collapsing shell of matter emits thermal radiation
- Matter in vacuum + collapsing shell →→ no shell + matter in a thermal state
- Quantum mechanics forbids this type of time evolution
- Interpretation: another layer of unpredictability in the presence of BH's =

BH INFORMATION PARADOX

Additional "Paradoxes"

- More recently, "Firewall paradox", etc. BH information paradox recast in Quantum Information terms
- No-Hair theorem, absence of global symmetries in presence of BH's
- Trans-Planckian problem
- Divergence of entanglement entropy

My Proposal

- No need to introduce another layer of unpredictability, new principles, or abandon old ones
- What has to change: description of BH

  classical geometry → Semiclassical state, wavefunction

Supremacy of QM over Geometry

How does a test particle (matter) move in a gravitational field of a spherical mass in a superposition of different locations?

Page & Geilker 1981

Quantum mechanically: **wavefunction** $|\Psi_{M,g}>$ for the geometry and matter

Classically – **geodesic motion** of the particle in the **geometry** created by the mass distribution

Matter – test particle

Geometry

$$\langle \Psi_{M,g} | \mathcal{O}_M | \Psi_{M,g} \rangle^*$$

* Expect: true quantum gravity observables are boundary observables. Here, we do expect approximate local observables.

Classically – **geodesic motion** of the particle in the **geometry** created by the mass distribution

- Classically – geodesic motion ⇔ geometry
- Quantum mechanically $\langle \Psi_{M,g} | \mathcal{O}_M | \Psi_{M,g} \rangle$
- When the mass has a localized wavefunction & the particle is far enough from the mass

g_C – Classical Schwarzschild geometry

$$\langle \Psi_{M,g} | O_M | \Psi_{M,g} \rangle \sim \langle \Psi_M(g_c) | O_M | \Psi_M(g_c) \rangle$$

Wavefunction for matter is a function of the classical geometry

In general,

$$\langle \Psi_{M,g} | O_M | \Psi_{M,g} \rangle \not\sim \langle \Psi_M(g_c) | O_M | \Psi_M(g_c) \rangle$$

Semiclassical BH vs.
Quantum fields in a fixed curved spacetime

Need to calculate:

$$\langle \mathcal{O}_M \rangle = \langle \Psi_{M,BH} | \mathcal{O}_M | \Psi_{M,BH} \rangle$$

Instead:
Quantum fields in curved space

$$\langle \mathcal{O}_M \rangle_C = \langle \Psi_M(g_c) | \mathcal{O}_M | \Psi_M(g_c) \rangle$$

Semiclassical BH vs.
Quantum fields in a fixed curved spacetime

Classical limit

$$\langle \mathcal{O}_M \rangle = \langle \Psi_{M,BH} | \mathcal{O}_M | \Psi_{M,BH} \rangle$$

$$\langle \mathcal{O}_M \rangle_C = \langle \Psi_M(g_c) | \mathcal{O}_M | \Psi_M(g_c) \rangle$$

Compton wavelength

$$\lambda_{BH} = \hbar / M_{BH}$$

Schwarzschild radius

$$R_S = 2GM_{BH}$$

"Classicality parameter"

$$C_{BH} = \frac{1}{2\pi} \frac{\lambda_{BH}}{R_S} = 1 / S_{BH}$$

Dvali & Gomez "1/N"

$$\langle O_M \rangle_C = \lim_{C_{BH} \to 0} \langle O_M \rangle$$

Semiclassical BH vs. Quantum fields in a fixed curved spacetime

Semiclassical approximation

$$\langle O_M \rangle_{SC} = \lim_{C_{BH}\to 0} \langle O_M \rangle + \text{leading } \neq 0 \text{ correction for } C_{BH}$$

Most operators – small correction ~ $C_{BH} = 1/S_{BH}$

~ $e^{-1/C_{BH}}$

Interesting cases

$$\lim_{C_{BH}\to 0} \langle O_M \rangle = 0 + O(C_{BH})$$

$$\lim_{C_{BH}\to 0} \langle O_M \rangle = \infty + O(1/C_{BH})$$

Origin of the BH information paradox

RB, 1209.2686

Need to calculate: $$\langle \mathcal{O}_M \rangle = \langle \Psi_{M,BH} | \mathcal{O}_M | \Psi_{M,BH} \rangle$$

Instead:
Quantum fields in curved space $$\langle \mathcal{O}_M \rangle_C = \langle \Psi_M(g_c) | \mathcal{O}_M | \Psi_M(g_c) \rangle$$

$$\left\langle \Psi_{M,BH} \,|\, O_M \,|\, \Psi_{M,BH} \right\rangle \neq \left\langle \tilde{\Psi}_M(\tilde{g}_c) \,|\, O_M \,|\, \tilde{\Psi}_M(\tilde{g}_c) \right\rangle$$

Restoring predictability in gravitational collapse

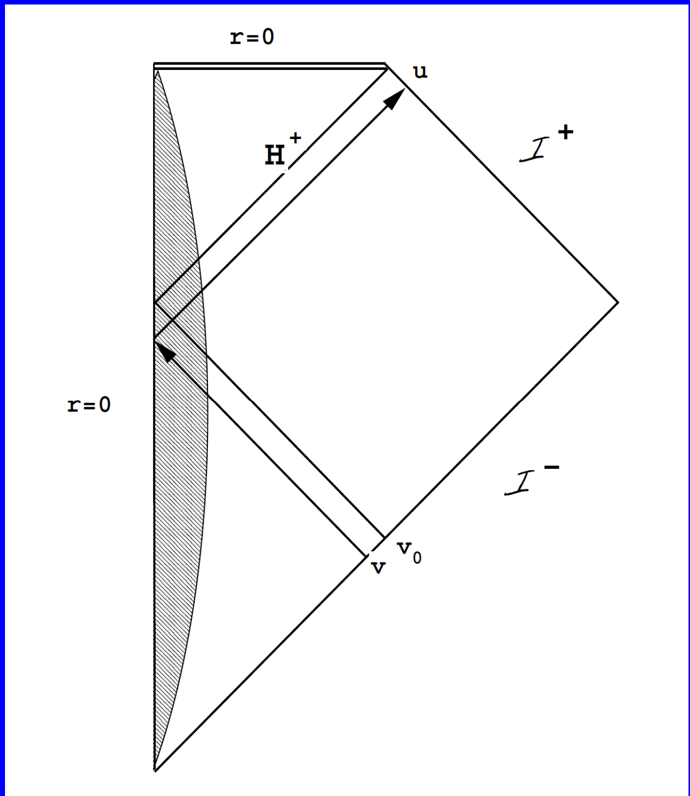
RB+J. Medved, 1305.3139

Repeat Hawking's calculation with

Classical shell ←→ Semiclassical shell , wavefunction

Thermal Radiation in classical collapse

Hawking PRD 1976



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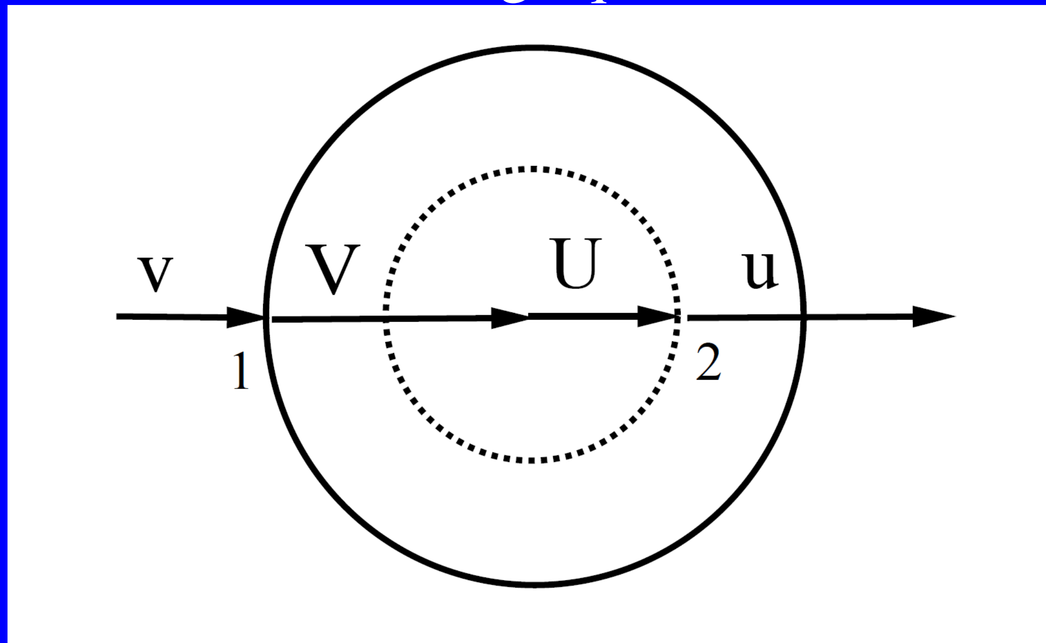
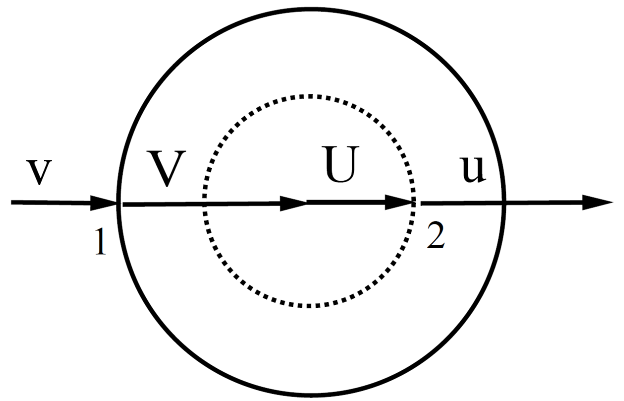


Ray tracing

Ford gr-qc/9707062

$$r^* = r + 2M\,\ln\left(\frac{r-2M}{2M}\right)$$

$$v = t + r^* \qquad u = t - r^*$$



$$R_{shell}(T) \;=\; R_S + A\,(T_0 - T) + \mathcal{O}\left[(T_0 - T)^2\right]$$

$$V = T + r$$

$$U = T - r$$

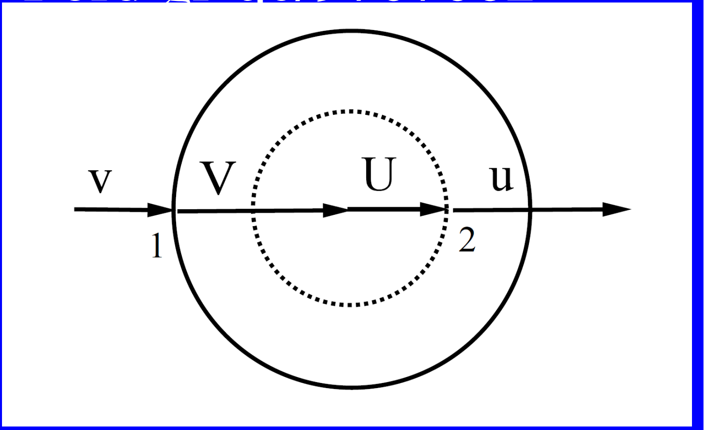
$$\left(\frac{dt}{dT}\right)^2 \approx \left(\frac{R-2M}{2M}\right)^{-2}\left(\frac{dR}{dT}\right)^2 \approx \frac{(2M)^2}{(T-T_0)^2}$$

$$t \;=\; -R_S \ln\left[T_0 - T\right] + \cdots \;,$$

$$r^* \;=\; +R_S \ln\left[R_{shell} - R_S\right] + \cdots \;,$$

Ray tracing

Ford gr-qc/9707062



$$t = -R_S \ln [T_0 - T] + \cdots ,$$

$$r^* = +R_S \ln [R_{shell} - R_S] + \cdots ,$$

$$u = t - r^* \sim -4M \ln\left(\frac{T_0 - T}{B'}\right)$$

$$V(v) = av + b$$

$$V = T + r$$

$$U = T - r$$

Phase shift

$$u = g(v) = -4M \ln\left(\frac{v_0 - v}{C}\right)$$

$$r = 0$$

$$U(V) = V.$$

$$R_{shell}(T) = R_S + A(T_0 - T) + \mathcal{O}\left[(T_0 - T)^2\right]$$

$$U = T - r = T - R(T) \sim (1 + A)T - 2M - AT_0$$

Radiation density matrix in classical collapse

$$F_\omega = \int_0^\infty d\omega' \left(\alpha_{\omega'\omega} f_{\omega'} + \beta_{\omega'\omega} f^*_{\omega'} \right)$$

$$f_{\omega'} = \frac{1}{\sqrt{2\pi}} e^{i\omega' v}$$

$$f_{\omega\ell m} \sim \frac{Y_{\ell m}(\theta,\phi)}{\sqrt{4\pi\omega}\, r} \times \begin{cases} e^{-i\omega v}, & \text{on } \mathcal{I}^- \\ e^{-i\omega G(u)}, & \text{on } \mathcal{I}^+ \end{cases}$$

$$F_{\omega\ell m} \sim \frac{Y_{\ell m}(\theta,\phi)}{\sqrt{4\pi\omega}\, r} \times \begin{cases} e^{-i\omega u}, & \text{on } \mathcal{I}^+ \\ e^{-i\omega g(v)}, & \text{on } \mathcal{I}^- \end{cases}$$

$$\rho_H(\omega,\tilde{\omega}) = \int_{-\infty}^{v_0} dv \int_0^\infty d\omega' \int_0^\infty d\omega''\, \beta^*_{\omega'\omega} \beta_{\omega''\tilde{\omega}} \frac{e^{iv(\omega'-\omega'')}}{2\pi}$$

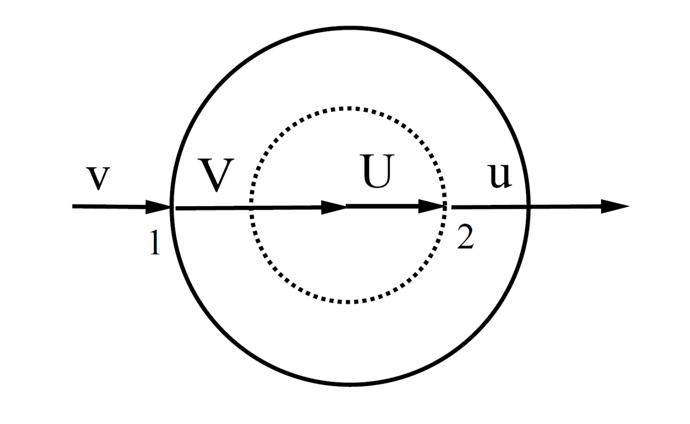
Radiation density matrix in classical collapse

$$u = g(v) = -4M \ln\left(\frac{v_0 - v}{C}\right)$$

$$F_\omega = \begin{cases} e^{4Mi\omega \ln\left(\frac{v_0 - v}{C}\right)} & , v < v_0 \\ 0 & , v > v_0 \end{cases}$$

$$\beta_{\omega'\omega} \propto \frac{1}{2\pi}\int_{-\infty}^{v_0} dv\, e^{i\omega' v}\, e^{-i2\omega R_S \ln(v_0 - v)}$$

$$= \Gamma\left(1 - i2\omega R_S\right)\left(i\omega'\right)^{-1+i2\omega R_S}\frac{1}{2\pi}e^{iv_0(\omega'-\omega)}$$



Thermal Radiation in classical collapse

$$\kappa = \frac{2\pi}{\hbar T_H}$$

$$\beta^*_{\omega'\omega}\beta_{\omega''\widetilde{\omega}} = \frac{t^*_\omega t_{\widetilde{\omega}}}{(2\pi)^2}\frac{1}{(\omega\widetilde{\omega})^{1/2}}\Gamma\left(1+i\frac{\omega}{\kappa}\right)\Gamma\left(1-i\frac{\widetilde{\omega}}{\kappa}\right)(\omega')^{-1/2-i\frac{\omega}{\kappa}}(\omega'')^{-1/2+i\frac{\widetilde{\omega}}{\kappa}}$$
$$\times\; e^{-\frac{\pi}{\kappa}\frac{\omega+\widetilde{\omega}}{2}}\; e^{iv_0(\omega''-\widetilde{\omega}-\omega'+\omega)}\,.$$

$$\rho_H(\omega,\tilde{\omega}) = \int_{-\infty}^{v_0} dv \int_0^{\infty} d\omega' \int_0^{\infty} d\omega''\, \beta^*_{\omega'\omega}\beta_{\omega''\widetilde{\omega}}\frac{e^{iv(\omega'-\omega'')}}{2\pi}$$

$$\beta^*_{\omega'\omega}\beta_{\omega''\widetilde{\omega}} = \frac{t^*_\omega t_{\widetilde{\omega}}}{(2\pi)^2}\frac{1}{(\omega\widetilde{\omega})^{1/2}}\Gamma\left(1+i\frac{\omega}{\kappa}\right)\Gamma\left(1-i\frac{\widetilde{\omega}}{\kappa}\right)(\omega')^{-1/2-i\frac{\omega}{\kappa}}(\omega'')^{-1/2+i\frac{\widetilde{\omega}}{\kappa}} \times e^{-\frac{\pi}{\kappa}\frac{\omega+\widetilde{\omega}}{2}}\, e^{iv_0(\omega''-\widetilde{\omega}-\omega'+\omega)} .$$

$$\rho_H(\omega,\tilde{\omega}) = \int_{-\infty}^{v_0} dv \int_0^\infty d\omega' \int_0^\infty d\omega''\, \beta^*_{\omega'\omega}\beta_{\omega''\widetilde{\omega}}\frac{e^{iv(\omega'-\omega'')}}{2\pi}$$

$\int_{-\infty}^{v_0} d$ ... $(\widetilde{\omega}-\omega)$

$$\rho_H(\omega,\widetilde{\omega}) \propto \frac{1}{e^{\frac{\hbar\omega}{T_H}}-1}\delta(\omega-\widetilde{\omega})$$

$\int_{-\infty}$

$$= \frac{1}{2R_S}\delta(\widetilde{\omega}-\omega) .$$

Breakdown of predictability
in gravitational collapse

RB, J. Medved, 1305.3139

Semiclassical collapse

Wavefunction of shell

$$\Psi_{shell} = \Psi_{shell}(R_{shell})$$

instead of classical geometry

Simplest and sufficient choice: One more parameter, σ^2 – "quantum width" of the shell

$$\Psi_{shell}(R_{shell}) = N^{-1/2}\, e^{-\frac{(R_{shell}-R_C)^2}{4\sigma^2}}$$

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Semiclassical collapse

Wavefunction of shell

$$\Psi_{shell} = \Psi_{shell}(R_{shell})$$

instead of classical geometry

$$\Psi_{shell} \to \Psi_{BH}$$

$$R_{shell} \to R_S$$

$$C_{BH} = \frac{\hbar G}{\pi R_S^2} = \frac{1}{S_{BH}}$$

$$\mathcal{N} = 4\pi R_S^2 \sqrt{\pi C_{BH} R_S^2}$$

$$\Psi_{BH}(R) = \mathcal{N}^{-1/2} e^{-\frac{(R-R_S)^2}{2C_{BH}R_S^2}}$$

$$\langle \widehat{O}(R_{shell}) \rangle = \frac{4\pi}{\mathcal{N}} \int_0^\infty dR_{shell}\, R_{shell}^2\, e^{-\frac{(R_{shell}-R_S)^2}{2\sigma^2}} O(R_{shell})$$

Wavefunction of shell

$$\Psi_{shell} = \Psi_{shell}(R_{shell})$$

$$\Psi_{BH}(R) = \mathcal{N}^{-1/2} e^{-\frac{(R-R_S)^2}{2C_{BH}R_S^2}}$$

$$R_{shell}(T) = R_S + A\,(T_0 - T) + \mathcal{O}\left[(T_0 - T)^2\right]$$

$$R_{shell} \sim R_S$$

$$R_{shell} - R_S \simeq v_0 - v$$

$$\langle \widehat{O}(R_{shell}) \rangle = \frac{4\pi}{\mathcal{N}} \int_0^{\infty} dR_{shell}\; R_{shell}^2\; e^{-\frac{(R_{shell}-R_S)^2}{2\sigma^2}} O(R_{shell})$$

$$\simeq \frac{4\pi}{\mathcal{N}} \int_{-\infty}^{\infty} dv_{shell} \left[R_S^2 + 2R_S(v_0 - v_{shell}) + (v_0 - v_{shell})^2\right] e^{-\frac{(v_0 - v_{shell})^2}{2\sigma^2}} O(v_{shell})$$

Radiation density matrix in semiclassical collapse

$$\rho_{SC}(\omega,\tilde{\omega}) = \int_{-\infty}^{v_0} dv \int_0^{\infty} d\omega' \int_0^{\infty} d\omega'' \left\langle \Psi_{BH} \middle| \beta^*_{SC}(\omega',\omega)\beta_{SC}(\omega'',\tilde{\omega}) \middle| \Psi_{BH} \right\rangle \frac{e^{iv(\omega'-\omega'')}}{2\pi}$$

$$\left|\Psi_{\phi}\right\rangle = \left|0_{in}\right\rangle$$

$$\left|\Psi_{shell,\phi}\right\rangle = \left|\Psi_{shell}\right\rangle\left|\Psi_{\phi}\right\rangle$$

$\beta_C \rightarrow \beta_{SC}$

Classical collapse

$$F_{\omega} = \int_0^{\infty} d\omega' \left(\alpha_{\omega'\omega} f_{\omega'} + \beta_{\omega'\omega} f^*_{\omega'}\right)$$

$$f_{\omega'} = \frac{1}{\sqrt{2\pi}} e^{i\omega' v}$$

$$\rho_H(\omega,\tilde{\omega}) = \int_{-\infty}^{v_0} dv \int_0^{\infty} d\omega' \int_0^{\infty} d\omega'' \, \beta^*_{\omega'\omega}\beta_{\omega''\tilde{\omega}} \frac{e^{iv(\omega'-\omega'')}}{2\pi}$$

Radiation density matrix in semiclassical collapse

$\beta_C \rightarrow \beta_{SC}$

$$\beta_{\omega'\omega} \propto \frac{1}{2\pi}\int_{-\infty}^{v_0} dv\ e^{i\omega' v}\ e^{-i2\omega R_S \ln(v_0 - v)}$$

$$\beta_{\omega'\omega,\ SC} \propto \frac{1}{2\pi}\int_{-\infty}^{v_{shell}} dv\ e^{i\omega' v}\ e^{-i2\omega R_S \ln(v_{shell} - v)}$$

$$= \Gamma\left(1 - i2\omega R_S\right)\left(i\omega'\right)^{-1+i2\omega R_S} \frac{1}{2\pi} e^{i v_{shell}(\omega' - \omega)} + \cdots$$

$\beta_{CL} \rightarrow \beta_{SC}$

Radiation density matrix in semiclassical collapse

$$\rho_H(\omega,\tilde{\omega}) = \int_{-\infty}^{v_0} dv \int_0^{\infty} d\omega' \int_0^{\infty} d\omega'' \, \beta^*_{\omega'\omega}\beta_{\omega''\tilde{\omega}} \frac{e^{iv(\omega'-\omega'')}}{2\pi}$$

$$\beta^*_{\omega'\omega}\beta_{\omega''\widetilde{\omega}} = \frac{t^*_\omega t_{\widetilde{\omega}}}{(2\pi)^2}\frac{1}{(\omega\widetilde{\omega})^{1/2}}\Gamma\left(1+i\frac{\omega}{\kappa}\right)\Gamma\left(1-i\frac{\widetilde{\omega}}{\kappa}\right)(\omega')^{-1/2-i\frac{\omega}{\kappa}}(\omega'')^{-1/2+i\frac{\widetilde{\omega}}{\kappa}}$$
$$\times \, e^{-\frac{\pi}{\kappa}\frac{\omega+\widetilde{\omega}}{2}} \, e^{iv_0(\omega''-\widetilde{\omega}-\omega'+\omega)} \, .$$

$$\beta_{\omega'\omega,\ SC} = \Gamma(1-i2\omega R_S)(i\omega')^{-1+i2\omega R_S}\frac{1}{2\pi}e^{iv_{shell}(\omega'-\omega)} + \cdots$$

$$I_C = \frac{1}{2\pi}\int_{-\infty}^{v_0} dv\, e^{i(v-v_0)(\omega'-\omega'')} = \frac{1}{2\pi}\int_{-\infty}^{0} dv'\, e^{iv'(\omega'-\omega'')} = \delta(\omega'-\omega'')$$

$$I_{SC}(\omega'-\omega''; v_{shell}) = \frac{1}{2\pi}\int_{-\infty}^{v_0} dv\, e^{i(v-v_{shell})(\omega'-\omega'')}$$

$$I_{SC}(\omega'-\omega''; v_{shell}) = \frac{1}{2\pi}\int_{-\infty}^{v_0-v_{shell}} dv'\, e^{iv'(\omega'-\omega'')}$$

Not a δ-function!

$$\langle \widehat{I}_{SC}(\omega' - \omega''; v_{shell}) \rangle = \left\langle \frac{1}{2\pi} \int_{-\infty}^{0} dv' \; e^{iv'(\omega' - \omega'')} + \frac{1}{2\pi} \int_{0}^{\widetilde{v}} dv' \; e^{iv'(\omega' - \omega'')} \right\rangle$$

$$= \delta(\omega' - \omega'') + \left\langle \frac{1}{2\pi} \int_{0}^{\widetilde{v}} dv' \; e^{iv'(\omega' - \omega'')} \right\rangle$$

$$\widetilde{v} = v_0 - v_{shell}$$

$$\Delta I_{SC}(\omega' - \omega'') = \frac{4\pi}{\mathcal{N}} \int_{-\infty}^{\infty} d\widetilde{v} \left[R_c^2 + 2R_c \widetilde{v} + \widetilde{v}^2 \right] e^{-\frac{\widetilde{v}^2}{2\sigma^2}} \frac{1}{2\pi} \int_{0}^{\widetilde{v}} dv' \; e^{iv'(\omega' - \omega'')}$$

$$\Delta I_{SC}(\omega' - \omega'') = \frac{4\pi}{\mathcal{N}} \int_{-\infty}^{\infty} d\widetilde{v} \left[R_S^2 + 2R_S \widetilde{v} + \widetilde{v}^2 \right] e^{-\frac{\widetilde{v}^2}{2\sigma^2}} \frac{1}{2\pi} \left[\frac{e^{i\widetilde{v}(\omega' - \omega'')} - 1}{i(\omega' - \omega'')} \right]$$

$$= \frac{4\pi}{\mathcal{N}} \int_{-\infty}^{\infty} d\widetilde{v} \left[R_S^2 + 2R_S \widetilde{v} + \widetilde{v}^2 \right] e^{-\frac{\widetilde{v}^2}{2\sigma^2}} \frac{1}{2\pi} e^{i\frac{\widetilde{v}}{2}(\omega' - \omega'')} \frac{\sin\left[\frac{\widetilde{v}}{2}(\omega' - \omega'')\right]}{\left(\frac{\omega' - \omega''}{2}\right)}$$

$$\Delta I_{SC}(\omega' - \omega'') = \frac{4\pi}{\mathcal{N}} \int_{-\infty}^{\infty} d\widetilde{v} \left[R_S^2 + 2R_S\widetilde{v} + \widetilde{v}^2\right] e^{-\frac{\widetilde{v}^2}{2\sigma^2}} \frac{1}{2\pi} \left[\frac{e^{i\widetilde{v}(\omega'-\omega'')} - 1}{i(\omega' - \omega'')}\right]$$

$$= \frac{4\pi}{\mathcal{N}} \int_{-\infty}^{\infty} d\widetilde{v} \left[R_S^2 + 2R_S\widetilde{v} + \widetilde{v}^2\right] e^{-\frac{\widetilde{v}^2}{2\sigma^2}} \frac{1}{2\pi} e^{i\frac{\widetilde{v}}{2}(\omega'-\omega'')} \frac{\sin\left[\frac{\widetilde{v}}{2}(\omega' - \omega'')\right]}{\left(\frac{\omega'-\omega''}{2}\right)}$$

$$\Delta I_{sc}(\omega' - \omega'') = \frac{1}{2\pi} R_S C_{BH}\, e^{-\frac{(\omega'-\omega'')^2}{4} R_S^2 C_{BH}}$$

SC correction to radiation density matrix

$$\Delta\rho_{SC}(\omega,\widetilde{\omega}) = \frac{t^*_\omega t_{\widetilde{\omega}}}{(2\pi)^2}\frac{1}{(\omega\widetilde{\omega})^{1/2}}\Gamma\left(1+i\frac{\omega}{\kappa}\right)\Gamma(1-i\frac{\widetilde{\omega}}{\kappa})e^{-\frac{\pi}{\kappa}\frac{\omega+\widetilde{\omega}}{2}}$$

$$\times \int_0^\infty d\omega'' \int_0^\infty d\omega' \,\frac{R_S}{2\pi}C_{BH}\; e^{-\frac{(\omega'-\omega'')^2}{4}R_S^2 C_{BH}}\;(\omega')^{-1/2-i\frac{\omega}{\kappa}}(\omega'')^{-1/2+i\frac{\widetilde{\omega}}{\kappa}}$$

$$\mathcal{I} = \int_0^\infty d\omega'' \int_0^\infty d\omega' \,\frac{R_S}{2\pi}C_{BH}\; e^{-\frac{(\omega'-\omega'')^2}{4}R_S^2 C_{BH}}\;(\omega')^{-1/2-i\frac{\omega}{\kappa}}(\omega'')^{-1/2+i\frac{\omega}{\kappa}}$$

$$\omega' = \omega'' \;,\quad \mathcal{I} \propto \int_0^\infty d\omega' \;(\omega')^{-1}$$

Correction of order $(C_{BH})^{1/2}$ to the diagonal

SC correction to radiation density matrix

$$\Delta\rho_{SC}(\omega,\widetilde{\omega}\,;C_{BH}) = \frac{t^*_\omega t_{\widetilde{\omega}}}{(2\pi)^3} 2C_{BH}^{1/2}\left(R_S^2 C_{BH}/4\right)^{+i\frac{\omega-\widetilde{\omega}}{\kappa}}$$

$$\times \quad \frac{1}{(\omega\widetilde{\omega})^{1/2}}\Gamma\left(1+i\frac{\omega}{\kappa}\right)\Gamma(1-i\frac{\widetilde{\omega}}{\kappa})\; e^{-\frac{\pi}{\kappa}\frac{\omega+\widetilde{\omega}}{2}}\;\Gamma\left(\frac{1}{2}-\frac{i}{2}\frac{\omega-\widetilde{\omega}}{\kappa}\right)$$

$$\times \quad \left\{\Gamma\left(i\frac{\omega-\widetilde{\omega}}{\kappa}\right)\left[\frac{\Gamma\left(\frac{1}{2}+i\frac{\widetilde{\omega}}{\kappa}\right)}{\Gamma\left(\frac{1}{2}+i\frac{\omega}{\kappa}\right)}+\frac{\Gamma\left(\frac{1}{2}-i\frac{\omega}{\kappa}\right)}{\Gamma\left(\frac{1}{2}-i\frac{\widetilde{\omega}}{\kappa}\right)}\right]+2\frac{i}{\frac{\omega-\widetilde{\omega}}{\kappa}}\right\}.$$

$$\Delta\rho_{SC}(\omega,\widetilde{\omega}) = \frac{t^*_\omega t_{\widetilde{\omega}}}{(2\pi)^2}\frac{1}{(\omega\widetilde{\omega})^{1/2}}\Gamma\left(1+i\frac{\omega}{\kappa}\right)\Gamma(1-i\frac{\widetilde{\omega}}{\kappa})e^{-\frac{\pi}{\kappa}\frac{\omega+\widetilde{\omega}}{2}}$$

$$\times \int_0^\infty d\omega'' \int_0^\infty d\omega' \;\frac{R_S}{2\pi}C_{BH}\; e^{-\frac{(\omega'-\omega'')^2}{4}R_S^2 C_{BH}}\;(\omega')^{-1/2-i\frac{\omega}{\kappa}}(\omega'')^{-1/2+i\frac{\widetilde{\omega}}{\kappa}}$$

Radiation density matrix in semiclassical collapse

$$\rho^{(N)}_{IJ}(\omega,\widetilde{\omega}) \;=\; \frac{1}{N}\rho_H(\omega,\widetilde{\omega})\mathbb{I}_{N\times N} \;+\; \frac{1}{N}C^{1/2}_{BH}\Delta\rho_{OD}(\omega,\widetilde{\omega})\not{\mathbb{I}}_{N\times N}$$

$$\not{\mathbb{I}}_{N\times N} = \begin{bmatrix} 0 & 1 & 1 & \cdots & 1 \\ 1 & 0 & 1 & \cdots & 1 \\ 1 & 1 & 0 & & 1 \\ \vdots & \vdots & & \ddots & \vdots \\ 1 & 1 & 1 & \cdots & 0 \end{bmatrix}_{N\times N}$$

$$\rho^{(N)}_{II} = \rho_H(\omega,\widetilde{\omega})$$

$$\rho_H \;\rightarrow\; \frac{1}{\mathrm{Tr}\;\rho_H}\rho_H$$

$$\rho^{(N)}_{I\neq J} = C^{1/2}_{BH}\Delta\rho_{OD}(\omega,\widetilde{\omega})$$

phase $e^{i\Theta_{IJ}}$

Information released

Information $I = S_H - S$

Actual Entropy $\frac{S}{N} = -\mathrm{Tr}\left[\rho^{(N)} \ln \rho^{(N)}\right]$

Thermal (Hawking) Entropy

$$\frac{S_H}{N} = -\frac{1}{N}\mathrm{Tr}\left[\mathbb{I}_{N\times N}\ \rho_H(\omega,\widetilde{\omega}) \ln \rho_H(\omega,\widetilde{\omega})\right] = -\mathrm{Tr}\left[\rho_H(\omega,\widetilde{\omega}) \ln \rho_H(\omega,\widetilde{\omega})\right]$$

Information released

$$\frac{S}{N} = -\frac{1}{N}\mathrm{Tr}\left[\left(\rho_H \mathbb{I}_{N\times N} + C_{BH}^{1/2}\Delta\rho_{OD}\not{\mathbb{I}}_{N\times N}\right)\ln\left(\rho_H \mathbb{I}_{N\times N} + C_{BH}^{1/2}\Delta\rho_{OD}\not{\mathbb{I}}_{N\times N}\right)\right]$$

$$= -\frac{1}{N}\mathrm{Tr}\left[\mathbb{I}_{N\times N}\ \rho_H \ln\rho_H\right] - \frac{1}{N}C_{BH}\mathrm{Tr}\left[\left(\Delta\rho_{OD}\not{\mathbb{I}}_{N\times N}\right)^2\left(\rho_H\ \mathbb{I}_{N\times N}\right)^{-1}\right]$$

$$+ \frac{C_{BH}}{2}\frac{1}{N}\mathrm{Tr}\left[\left(\rho_H\ \mathbb{I}_{N\times N}\right)\left(\Delta\rho_{OD}\not{\mathbb{I}}_{N\times N}\right)^2\left(\rho_H\ \mathbb{I}_{N\times N}\right)^{-2}\right] + \cdots$$

$$= -\mathrm{Tr}\left[\rho_H \ln\rho_H\right] - \frac{1}{2}NC_{BH}\mathrm{Tr}\left[\left(\Delta\rho_{OD}\right)^2\rho_H^{-1}\right] + \cdots$$

$$\frac{S}{N} = -\mathrm{Tr}\left[\rho^{(N)}\ln\rho^{(N)}\right]$$

$$\frac{S_H}{N} = -\frac{1}{N}\mathrm{Tr}\left[\mathbb{I}_{N\times N}\ \rho_H(\omega,\widetilde{\omega})\ln\rho_H(\omega,\widetilde{\omega})\right] = -\mathrm{Tr}\left[\rho_H(\omega,\widetilde{\omega})\ln\rho_H(\omega,\widetilde{\omega})\right]$$

Information released

$$I = S_H - S$$

$$I = \frac{1}{2} K \, N \, C_{BH} \, S_H$$

$$\frac{S}{N} = -\mathrm{Tr}\left[\rho_H \ln \rho_H\right] - \frac{1}{2} N C_{BH} \mathrm{Tr}\left[(\Delta\rho_{OD})^2 \rho_H^{-1}\right] + \cdots$$

$$\begin{aligned} S &= S_H \left(1 - \frac{1}{2} N C_{BH} \frac{\mathrm{Tr}\left[(\Delta\rho_{OD})^2 \rho_H^{-1}\right]}{-\mathrm{Tr}\left[\rho_H \ln \rho_H\right]}\right) \\ &= S_H \left(1 - \frac{1}{2} K \, N C_{BH}\right), \end{aligned}$$

$$K = \frac{\mathrm{Tr}\left[(\Delta\rho_{OD})^2 \rho_H^{-1}\right]}{-\mathrm{Tr}\left[\rho_H \ln \rho_H\right]}$$

Information released

Qualitative estimate of the rate of information release

$$I = S_H - S$$

$$I = \frac{1}{2} K\, N\, C_{BH}\; S_H$$

$$N(t) = S_{BH}(0) - S_{BH}(t) \sim S_H(t)$$

$$C_{BH}(t) = \frac{1}{S_{BH}(t)} \sim \frac{1}{S_{BH}(0) - S_H(t)}$$

$$I \sim \frac{K}{2} \frac{S_H^2}{S_{BH}(0) - S_H}$$

$$S_{BH}(t_{Page}) = \frac{1}{2} S_{BH}(0)$$

$$I(t_{Page}) \sim \frac{K}{2} \frac{S_H^2}{S_{BH}(0) - S_H} = \frac{K}{4} S_{BH}(0)$$

Information released

$$I = \frac{1}{2} K N C_{BH} S_H$$

$$I \sim \frac{K}{2} \frac{S_H^2}{S_{BH}(0) - S_H}$$

$$\frac{dI}{dS_H} \simeq \frac{K}{2} [2 + C_{BH} N] C_{BH} N$$

$$\left. \frac{dI}{dS_H} \right|_{t<t_{Page}} \sim \mathcal{O}[C_{BH}]$$

$$\left. \frac{dI}{dS_H} \right|_{t>t_{Page}} \sim 1$$

Information released: Purity of radiation

$$\mathrm{Tr}\left[\left(\rho^{(N)}\right)^2\right] \simeq \frac{1}{N^2}\mathrm{Tr}\left[N\rho_H^2 + N^2 C_{BH}(\Delta\rho_{OD})^2\right]$$

$$\frac{\mathrm{Tr}\left[\left(\rho^{(N)}\right)^2\right]}{\left(\mathrm{Tr}\ \rho^{(N)}\right)^2} \simeq \frac{1}{N}\mathrm{Tr}\ \rho_H^2\left(1 + NC_{BH}\frac{\mathrm{Tr}\left[\Delta\rho_{OD}\right]^2}{\mathrm{Tr}\ \rho_H^2}\right)$$

Consistent with

$$\left.\frac{dI}{dS_H}\right|_{t>t_{Page}} \sim 1$$

Additional “Paradoxes”

- More recently, “Firewall paradox”, etc. BH paradox recast in Quantum Information terms
- No-Hair theorem, absence of global symmetries in presence of BH’s
- Trans-Planckian problem
- Divergence of entanglement entropy

Conclusion

- Shell a quantum object → evolution consistent with standard rules of quantum mechanics – no need for another layer of unpredictability

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- Semiclassical radiation density matrix has small off-diagonal terms
- A finite fraction of the total information escapes before the BH completely evaporates
- Collecting the information is impractical

Backup: Firewall

- Discussion in the context of "complementarity" Susskind, Thorlacious, Uglum '93

  Stephens, 't Hooft, Whiting '93
- The basic suggestion is that information is duplicated, inside and outside the horizon, but it is still possible to tolerate this because no observer can observe both

Backup: Firewall

1) When a BH in a pure state evaporates, the Hawking radiation after the complete evaporation of the BH is in a pure state (unitarity)
2) The state seen by an observer at rest near the horizon is approximately a Rindler state
3) In regions of small curvature one can neglect most quantum gravitational effects

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Using information theory ideas → 1)+2)+3) are inconsistent

AMPS, 1207.3123

Backup: Firewall

- BH evaporation is a unitary process ➔ the state describing the vicinity of the horizon must store information about the collapsing matter system, information that is supposedly carried off by the outgoing Hawking modes.
- But, if a free-falling observer is to see nothing special on her way through the horizon ➔ Unruh vacuum is the quantum state. Anything else ➔ the observer would encounter a sea of high-energy quanta; that is, a firewall.
- However, the Unruh vacuum is independent of the matter that formed the BH and, as such, incapable of storing the requisite information.

Adapted from Avery, Chowdhury, Puhm 1210.6996

Backup: Firewall

- All observers must agree with an Unruh observer on transformation-invariant properties (such as the absence of a firewall)
- Only small deviations from the Unruh vacuum → The information about the collapsing matter might be safely stored in a non-trivial state.
- This stored information gets transferred into the outgoing radiation as told

Backup: Firewall

- Strong-subadditivity-of-entropy version of the firewall paradox, agree with Papadodimas+Raju: information stored in the various subsystems (early radiation, late radiation, in-falling partners of late radiation) cannot be cleanly separated.
- However, a true classical horizon - which acts as a rigid barrier between regions of spacetime - would allow for such a separation.
- For a quantum (fluctuating) horizon it is impossible, even in principle, to say what spacetime region the entanglement is “living”.

RB, J. Medved 1302.6086

Backup: No-Hair theorem revisited

The no-hair theorem

$$\frac{1}{\sqrt{-g}}\partial_\mu \sqrt{-g}\, g^{\mu\nu}\partial_\nu \phi = 0$$

$$\Longrightarrow$$

$$0 = \int_{R_c}^{\infty} dr\, \phi\, \partial_r \left[\sqrt{-g}\, g^{rr}\partial_r \phi\right]$$

$$= \int_{R_c}^{\infty} dr\, \phi\, \partial_r \left[r^2 F(r)\partial_r \phi\right]$$

$$[F(r) r^2 \phi \phi']_{R_c}^{\infty} = \int_{R_c}^{\infty} dr\, r^2 F(r)\, (\phi')^2$$

$$0 = \int_{R_c}^{\infty} dr\;\; r^2 F(r) (\phi')^2 \geq 0$$

$$\Longrightarrow$$

$$\phi(r) = \phi(\infty) = 0$$

Backup: No-Hair theorem revisited

Semiclassical hair

$$\left[F(r)r^2\phi\phi'\right]_{R_c}^{\infty} = \int_{R_c}^{\infty} dr\; r^2 F(r)\,(\phi')^2$$

$$\widehat{J}(r \to R_c; R) = \lim_{r \to R_c} \widehat{F}(r;R)\; r^2\; \phi(r)\phi'(r)$$

$$\langle\psi_{BH}|\widehat{F}(r;R)|\psi_{BH}\rangle = \frac{r-R}{r}$$

$$\langle\psi_{BH}|\widehat{J}|\psi_{BH}\rangle = 4\pi\mathcal{N}^{-1}\int_0^{\infty} dR\; R^2 \frac{(R_c - R)}{R_c} R_c^2\;\left[\phi\phi'\right]_{r=R_c}\; e^{-\frac{1}{C_{BH}}\frac{(R-R_c)^2}{R_c^2}}$$

Backup:No-Hair theorem revisited

Semiclassical hair

$$\langle\psi_{BH}|\widehat{J}|\psi_{BH}\rangle = 4\pi\mathcal{N}^{-1}\int_0^\infty dR\, R^2\frac{(R_c - R)}{R_c}R_c^2\ [\phi\phi']_{r=R_c}\ e^{-\frac{1}{C_{BH}}\frac{(R-R_c)^2}{R_c^2}}$$

$$\langle\psi_{BH}|\widehat{J}|\psi_{BH}\rangle = 4\pi\mathcal{N}^{-1}C_{BH}R_c^5\,[\phi\phi']_{r=R_c}\int_{-C_{BH}^{-1/2}}^{\infty} dl\ \left(1 + 2C_{BH}^{1/2}l + C_{BH}l^2\right) l\ e^{-l^2}$$

$$\begin{aligned}\langle\psi_{BH}|\widehat{J}|\psi_{BH}\rangle &= C_{BH}R_c^2\,[\phi\phi']_{r=R_c}^2 \\ \langle\psi_{BH}|\widehat{J}^2|\psi_{BH}\rangle &= 4\pi\mathcal{N}^{-1}C_{BH}^{3/2}R_c^7\,[\phi\phi']_{r=R_c}^2\int_{-C_{BH}^{-1/2}}^{\infty} dl\ l^2\ e^{-l^2} \\ &= \frac{1}{2}C_{BH}R_c^4\,[\phi\phi']_{r=R_c}^2 + \mathcal{O}[C_{BH}^2]\ ,\end{aligned}$$

Backup: No-Hair theorem revisited

Significance

$$\langle\psi_{BH}|\widehat{J}|\psi_{BH}\rangle = C_{BH} R_c^2 \left[\phi\phi'\right]^2_{r=R_c}$$

$$\left[F(r) r^2 \phi\phi'\right]^{\infty}_{R_c} = \int_{R_c}^{\infty} dr\, r^2 F(r) \left(\phi'\right)^2$$

$$\langle\psi_{BH}|\widehat{J}^2|\psi_{BH}\rangle = \frac{1}{2} C_{BH} R_c^4 \left[\phi\phi'\right]^2_{r=R_c}$$

$0 \rightarrow 1/S_{BH}$

- Charge measured quantum mechanically
- BH's *do not forbid* global symmetries
  - Baryon number can be a symmetry