Three-point functions in $\mathcal{N} = 2$ Higher-Spin Holography

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Motivation: A more tractable AdS/CFT?

- Usual AdS/CFT involves CFT in 4d and String Theory in 10d. Both hard to understand
- Would be nice to find a holographic setting where both sides are more tractable
- Proposal: Higher Spin gravity on $\mathrm{Ad}S_3$ is equivalent to a 2d Minimal Model CFT with $\mathcal{W}$-algebra symmetry

  [Gopakumar-Gaberdiel '11, see also Rajesh's talk]
- The boundary CFT can be described by a WZW coset:

$$\frac{\widehat{\mathfrak{su}}(N)_k \times \widehat{\mathfrak{su}}(N)_1}{\widehat{\mathfrak{su}}(N)_{k+1}}$$

- The HS theory is described by Vasiliev theory (more later) in the 't Hooft limit $0 \leq \lim_{N,k\to\infty} \frac{N}{N+k} \leq 1 \quad$ fixed
- Supersymmetry not necessary

The $\mathcal{N}=2$ Higher-Spin Duality

- The duality can be extended to include supersymmetry. We will focus on the $\mathcal{N}=2$ case [Creutzig, Hikida,Rønne '11]
- Boundary theory: The $\mathcal{N}=(2,2)$ CP^N Kazama-Suzuki Model

$$\frac{\widehat{\mathfrak{su}}(N+1)_k \times \widehat{\mathfrak{so}}(2N)_1}{\widehat{\mathfrak{su}}(N)_{k+1} \times \widehat{\mathfrak{u}}(1)_{N(N+1)(k+N+1)}}$$

- Bulk: The $\mathcal{N}=2$ Prokushkin-Vasiliev theory [PV '98]
- 't Hooft coupling $\lambda = \frac{N}{2(N+k)}$
- 't Hooft limit: $0 \leq \lim_{N,k\to\infty} \lambda \leq \frac{1}{2}$ fixed
- More symmetry$\to$ some calculations easier. SUSY allows for finer checks of the duality. Connections to string theory?

Symmetries

- The massless sector of the HS theory can be formulated as a Chern-Simons theory with $\text{shs}[\lambda]_{k_{CS}} \times \text{shs}[\lambda]_{-k_{CS}}$ gauge group
- The HS fields are coupled to two massive $3d$ hypermultiplets, $(\phi_\pm, \psi_\pm)$ and $(\tilde{\phi}_\pm, \tilde{\psi}_\pm)$, with two complex scalars and two fermions in each. Their masses are:

$$(M_+^B)^2 = 4(\lambda^2 - \lambda), \qquad (M_-^B)^2 = 4\lambda^2 - 1, \qquad (M_\pm^F)^2 = (2\lambda - \tfrac{1}{2})^2,$$

- Asymptotically, the Lie algebra $\text{shs}[\lambda]_{k_{CS}}$ induces a non-linear $\mathcal{SW}_\infty[\lambda]$ algebra. [Hanaki, Peng '12]
- The chiral algebra of the KS coset is $\mathcal{SW}_N$ [Ito '91], which gives $\mathcal{SW}_\infty[\lambda]$ in the 't Hooft limit [Candu, Gaberdiel '12]
- $\text{shs}[\lambda]$ is a subalgebra of $\mathcal{SW}_\infty[\lambda]$ when $c \to \infty$, when restricting to the "wedge" $|m| \le s - 1$

The (modified) Prokushkin-Vasiliev theory

- Look at linearised level in the matter fields
- A, C: One-form and scalar generating functions

$$dA + A \star \wedge A = 0, \qquad d\bar{A} + \bar{A} \star \wedge \bar{A} = 0.$$

$$dC + A \star C - C \star \bar{A} = 0,$$
$$d\tilde{C} + \bar{A} \star \tilde{C} - \tilde{C} \star A = 0,$$

- A, C can be expanded in a basis of shs$[\lambda]$

$$A = \sum_{s=1}^{\infty} \sum_{|m| \leq s-1} A^s_m \, L^{(s)}_m + \sum_{s=\frac{3}{2}}^{\infty} \sum_{|r| \leq s-\frac{3}{2}} A^s_r \, G^{(s)}_r,$$

$$C = \sum_{s=1}^{\infty} \sum_{|m| \leq s-1} C^s_m \, L^{(s)}_m + \sum_{s=\frac{3}{2}}^{\infty} \sum_{|r| \leq s-\frac{3}{2}} C^s_r \, G^{(s)}_r,$$

The $\mathcal{SB}[\mu]$ Algebra

- Convenient to define shs$[\lambda]$ through the associative $\star$-product of an algebra called $\mathcal{SB}[\mu]$
- This is defined through the relations:

$$L_m^{(s)} \star L_n^{(t)} = \sum_{u=1}^{s+t-1} g_u^{st}(m,n;\lambda)\, L_{m+n}^{(s+t-u)}, \qquad L_m^{(s)} \star G_q^{(t)} = \sum_{u=1}^{s+t-1} h_u^{st}(m,q;\lambda)\, G_{m+q}^{(s+t-u)},$$

$$G_p^{(s)} \star G_q^{(t)} = \sum_{u=1}^{s+t-1} \tilde{g}_u^{st}(p,q;\lambda)\, L_{p+q}^{(s+t-u)}, \qquad G_p^{(s)} \star L_n^{(t)} = \sum_{u=1}^{s+t-1} \tilde{h}_u^{st}(p,n;\lambda)\, G_{p+n}^{(s+t-u)}$$

- Explicitly,

$$g_u^{st}(m,n;\lambda) = \sum_i F_{st}^u \left[h\Big(u + \tfrac{1}{2}\tilde{h}\Big(s+t+\tfrac{1}{2}\Big)\Big) i + \tilde{h}(s)\tilde{h}\Big(u + \tfrac{1}{2}\tilde{h}\Big(s+t+\tfrac{1}{2}\Big)\Big); \lambda \right]$$

$$\times (m - \lfloor s \rfloor + 1)_{\lceil i,u,s,t \rceil_1} \, (n - \lfloor t \rfloor + 1)_{\lfloor u \rfloor - 1 + \tilde{h}\left(s+\frac{1}{2}\right)\tilde{h}\left(t+\frac{1}{2}\right) - \tilde{h}\left(u+\frac{1}{2}\right)\tilde{h}\left(s+t+\frac{1}{2}\right) - \lceil i,u,s,t \rceil_1}$$

where the range of the sum is

$$0 \le i \le h\Big(u + \tfrac{1}{2}\tilde{h}(s+t)\Big)(\lfloor u \rfloor - 1) + \tilde{h}(u)\tilde{h}\Big(s+t+\tfrac{1}{2}\Big) - \tilde{h}(s)\tilde{h}\Big(u + \tfrac{1}{2}\tilde{h}\Big(s+t+\tfrac{1}{2}\Big)\Big)\tilde{h}\Big(u+\tfrac{1}{2}\Big).$$

Some definitions

$$F^u_{st}(i,\lambda) = F^u_{st}(\lambda)(-1)^{\lfloor i/2\rfloor + 2i(s+u)} \begin{bmatrix} u-1 \\ i/2 \end{bmatrix} (\lfloor 2s-u \rfloor)_{\lfloor u-1-i/2\rfloor + |2u|_2 |2u-2-i|_2} \times (\lfloor 2t-u \rfloor)_{\lfloor i/2 \rfloor + |2u|_2 |i|_2}$$

- $$h(u) = \lceil u - \lfloor u \rfloor + 1 \rceil \quad , \quad \tilde{h}(u) = \lceil u - \lfloor u \rfloor \rceil \quad , \quad |n|_2 = n - 2\lfloor n/2 \rfloor$$

- $$\lceil i,u,s \rfloor = \left\lceil h(u) \frac{\left[i + \tilde{h}\left(u+\frac{1}{2}\right)\tilde{h}(s)\right]}{2} \right\rceil \quad , \quad \lceil i,u,s,t \rceil_1 = \left\lceil i, u + \tfrac{1}{2}\tilde{h}\left(s+t+\tfrac{1}{2}\right), s \right\rceil$$

($\lceil u \rceil$ and $\lfloor u \rfloor$ are the ceiling and floor functions)

- $$F^u_{st}(\lambda) = (-1)^{\lfloor s+t-u-1 \rfloor} \frac{(2s+2t-2u-2)!}{(2s+2t-\lfloor u \rfloor -3)!} \sum_{i=0}^{2s-2} \sum_{j=0}^{2t-2} \delta(i+j-2s-2t+2u+2) \times A^i(s, \tfrac{1}{2}-\lambda) A^j(t,\lambda)(-1)^{2s+2i(s+t-u)},$$

- $$A^i(s,\lambda) = (-1)^{\lfloor s \rfloor + 1 + 2s(i+1)} \begin{bmatrix} s-1 \\ i/2 \end{bmatrix} \frac{([(i+1)/2] + 2\lambda)_{\lfloor s-1/2 \rfloor - \lfloor (i+1)/2 \rfloor}}{(\lfloor s+i/2 \rfloor)_{2s-1-\lfloor s+i/2 \rfloor}}$$

Computations will clearly get a bit involved!

The shs[λ] **Algebra**

- shs[λ] is an infinite-dimensional Lie superalgebra, with commutators

$$\left[L_m^{(s)}, L_n^{(t)}\right] = \sum_{u=1}^{s+t-1} \hat{g}_u^{st}(m,n;\lambda)\, L_{m+n}^{(s+t-u)}, \qquad \left[L_m^{(s)}, G_q^{(t)}\right] = \sum_{u=1}^{s+t-1} \hat{h}_u^{st}(m,q;\lambda)\, G_{m+q}^{(s+t-u)},$$

$$\left\{G_p^{(s)}, G_q^{(t)}\right\} = \sum_{u=1}^{s+t-1} \hat{\hat{g}}_u^{st}(p,q;\lambda)\, L_{p+q}^{(s+t-u)}, \qquad \left[G_p^{(s)}, L_n^{(t)}\right] = \sum_{u=1}^{s+t-1} \hat{\hat{h}}_u^{st}(p,n;\lambda)\, G_{p+n}^{(s+t-u)}$$

- All definitions as above, with the replacement

$$F_{st}^u(\lambda) \to f_{st}^u(\lambda)\ ,$$

where

$$f_{st}^u(\lambda) = F_{st}^u(\lambda) + (-1)^{\lfloor -u \rfloor + 4(s+u)(t+u)}\, F_{st}^u(\frac{1}{2} - \lambda).$$

- Similar expressions for $G_p^{(s)} \star G_q^{(t)}$, $L_m^{(s)} \star G_q^{(t)}$, $G_p^{(s)} \star L_n^{(t)}$

Scalars on AdS_3

- As an example, consider how the PV theory leads to scalars with specific masses living on AdS_3
- The connection corresponding to AdS_3 is

$$\begin{aligned} A &= e^{\rho}\, L^{(2)}_{1}\, dz + L^{(2)}_{0}\, d\rho \\ \bar{A} &= e^{\rho}\, L^{(2)}_{-1}\, d\bar{z} - L^{(2)}_{0}\, d\rho \end{aligned} \qquad \Rightarrow \qquad ds^2 = d\rho^2 + e^{2\rho}\, dz d\bar{z},$$

where $g_{\mu\nu} = \frac{1}{2}\mathrm{tr}(e_\mu e_\nu)$, $e = \frac{1}{2}(A - \bar{A})$

- Plugging C into $dC + A \star C - C \star \bar{A} = 0$ gives

$$\sum_{s=1}^{\infty} \sum_{|m|\le s-1} \Big(\mathrm{d}C^s_m\, L^{(s)}_m + e^{\rho}\, C^s_m\, L^{(2)}_1 \star L^{(s)}_m\, \mathrm{d}z - e^{\rho}\, C^s_m\, L^{(s)}_m \star L^{(2)}_{-1}\, \mathrm{d}\bar{z} + C^s_m \left\{ L^{(2)}_0 \star L^{(s)}_m + L^{(s)}_m \star L^{(2)}_0 \right\} \mathrm{d}\rho \Big) = 0.$$

Scalars on AdS_3 (cont.)

- Focus on fields in the wedge, i.e. $C^s_m = 0$ if $|m| > s-1$. We get:

$$\partial_\rho C^s_m + 2\left[C^{s-1}_m + C^{s+1}_m\, g^{s+1,2}_3(m,0) + C^{s-\frac{1}{2}}_m\, g^{s-\frac{1}{2},2}_{\frac{3}{2}}(m,0) + C^{s+\frac{1}{2}}_m\, g^{s+\frac{1}{2},2}_{\frac{5}{2}}(m,0)\right] = 0,$$

$$\partial C^s_m + e^\rho \left[C^{s-1}_{m-1} + g^{2,s}_2(1,m-1) C^s_{m-1} + g^{2,s+1}_3(1,m-1) C^{s+1}_{m-1} \right.$$
$$\left. + g^{2,s-\frac{1}{2}}_{\frac{3}{2}}(1,m-1) C^{s-\frac{1}{2}}_{m-1} + g^{2,s+\frac{1}{2}}_{\frac{5}{2}}(1,m-1) C^{s+\frac{1}{2}}_{m-1}\right] = 0,$$

$$\bar\partial C^s_m - e^\rho \left[C^{s-1}_{m+1} + g^{s,2}_2(m+1,-1) C^s_{m+1} + g^{s+1,2}_3(m+1,-1) C^{s+1}_{m+1} \right.$$
$$\left. + g^{s-\frac{1}{2},2}_{\frac{3}{2}}(m+1,-1) C^{s-\frac{1}{2}}_{m+1} + g^{s+\frac{1}{2},2}_{\frac{5}{3}}(m+1,-1) C^{s+\frac{1}{2}}_{m+1}\right] = 0.$$

- All fields auxiliary apart from C^1_0 and $C^{\frac{3}{2}}_0$ $\Rightarrow$ Need to isolate the equations involving these two fields.
- Reduce to just two coupled equations:

$$\Box C^1_0 + 6\lambda\,(1-2\lambda)\, C^1_0 + 2\lambda\,(1-6\lambda+8\lambda^2)\, C^{3/2}_0 = 0,$$

$$\Box C^{3/2}_0 - \frac{1-4\lambda}{6\lambda(1-2\lambda)}\,\Box C^1_0 + \frac{2}{3}(1+\lambda-2\lambda^2)\, C^{3/2}_0 = 0,$$

where $\Box = \partial^2_\rho + 2\,\partial_\rho + 4\,e^{-2\rho}\,\partial\bar\partial$.

Scalars on AdS_3 (cont.)

- Can rewrite the above as

$$\Box \boldsymbol{C} + \begin{bmatrix} 6\lambda(1-2\lambda) & 2\lambda(1-6\lambda+8\lambda^2) \\ 1-4\lambda & 1-2\lambda+4\lambda^2 \end{bmatrix} \boldsymbol{C} = 0, \qquad \boldsymbol{C} = \begin{pmatrix} C_0^1 \\ C_0^{\frac{3}{2}} \end{pmatrix}.$$

  and convert to mass eigenstates $\phi_\pm$

$$C_0^1 = (2\lambda - 1)\,\phi_+ + 2\,\lambda\,\phi_-, \qquad C_0^{\frac{3}{2}} = \phi_+ + \phi_-.$$

  satisfying

$$\left[\Box - 4\,\left(\lambda^2 - \lambda\right)\right]\phi_+ = 0, \qquad \left[\Box - \left(4\,\lambda^2 - 1\right)\right]\phi_- = 0.$$

- We conclude that the $\mathcal{N} = 2$ theory contains two physical scalars with masses:

$$(M_+^B)^2 = 4(\lambda^2 - \lambda) \qquad \text{and} \qquad (M_-^B)^2 = 4\lambda^2 - 1,$$

  (Usual convention would take $\lambda \to \lambda/2$)

Holographic OPE's

- We expect to see an $\mathcal{N}=2\ \mathcal{SW}_\infty[\lambda]$ symmetry arising as an asymptotic symmetry of the bulk higher-spin theory
- Has previously been shown by a Brown-Henneaux-type analysis [Hanaki,Peng '12]
- We provided an alternative derivation using holographic Ward identities (following [Gutperle,Kraus '11])
- Write the connections as

$$\begin{aligned} A &= b^{-1}ab + b^{-1}\mathrm{d}b, \\ \bar{A} &= b\bar{a}b^{-1} + b\mathrm{d}b^{-1}, \end{aligned} \qquad \text{where} \quad b = e^{\rho\, L_0^{(2)}}.$$

where the sources appear in the antiholomorphic part

$$a = \left(L_1^{(2)} + \frac{2\pi}{k} \sum_{s\geq\frac{3}{2}} \left[\frac{1}{N_s^B} \mathcal{L}_s\, L^{(s)}_{-\lfloor s\rfloor+1} + \frac{1}{N_s^F}\, \psi_s\, G^{(s)}_{-\lceil s\rceil+\frac{3}{2}} \right] \right) \mathrm{d}z$$

$$+ \left(\sum_{s\leq\frac{3}{2}} \sum_{|m|\leq\lfloor s\rfloor-1} \mu_m^s\, L_m^{(s)} + \sum_{s\leq\frac{3}{2}} \sum_{|r|\leq\lceil s\rceil-\frac{3}{2}} \nu_r^s\, G_r^{(s)} \right) \mathrm{d}\bar{z},$$

Holographic OPE's

- Flatness conditions $\Rightarrow$ Holographic Ward identities

$$\partial a_{\bar{z}} - \bar{\partial} a_z + [a_z, a_{\bar{z}}] = \sum_{s \geq \frac{3}{2}} \left[\sum_{|m| \leq \lfloor s \rfloor - 1} c^B_{s,m} \, L^s_m + \sum_{|r| \leq \lceil s \rceil - \frac{3}{2}} c^F_{s,r} \, G^s_r \right] = 0$$

- The boundary conserved currents fall into $\mathcal{N} = 2$ multiplets

$$\left(W^{s-}, G^{(s+\frac{1}{2})-}, G^{(s+\frac{1}{2})+}, W^{(s+1)+} \right), \qquad s \in \mathbb{Z}_{\geq 1}$$

- The super-Virasoro multiplet is $\left(j, G^{\frac{3}{2}\pm}, T \right)$
- Now turn on sources, e.g. for the Virasoro algebra $\left(\mu^1_0, \nu^{\frac{3}{2}}_{\pm\frac{1}{2}}, \nu^2_{\pm\frac{1}{2}}, \mu^2_{\pm 1} \right)$, and solve the flatness conditions
- Need to do a Sugawara redefinition $T(z) = \tilde{T}(z) + \frac{1}{4k} [jj](z)$

Holographic OPE's

- Finally find the $\mathcal{N}=2$ superconformal algebra:

$$j(z)j(w) \sim \frac{c/3}{(z-w)^2}, \qquad j(z)G^{\frac{3}{2}\pm}(w) \sim \frac{1}{z-w}\, G^{\frac{3}{2}\mp}(w),$$

$$T(z)T(w) \sim \frac{c/2}{(z-w)^4} + \frac{2}{(z-w)^2}\, T(w) + \frac{1}{z-w}\, \partial T(w),$$

$$T(z)G^{\frac{3}{2}\pm}(w) \sim \frac{3/2}{(z-w)^2}\, G^{\frac{3}{2}\pm}(w) + \frac{1}{z-w}\, \partial G^{\frac{3}{2}\pm}(w),$$

$$G^{\frac{3}{2}\pm}(z)G^{\frac{3}{2}\pm}(w) \sim \frac{\mp 2c/3}{(z-w)^3} + \frac{\mp 2}{z-w}\, T(w),$$

$$G^{\frac{3}{2}\pm}(z)G^{\frac{3}{2}\mp}(w) \sim \frac{\pm 2}{(z-w)^2}\, j(w) + \frac{\pm 1}{z-w}\, \partial j(w),$$

$$T(z)j(w) \sim \frac{1}{(z-w)^2}\, j(w) + \frac{1}{z-w}\, \partial j(w).$$

- Reproduce the Brown-Henneaux central charge

$$c = \frac{3l}{2G}.$$

- By turning on HS sources, can similarly recover the structure of $\mathcal{SW}_\infty[\lambda]$ (Though no full analysis yet)

Bulk Correlation Functions

- Consider a background with one spin-s charge turned on:

$$A = \left(e^{\rho}\, L_1^{(2)} + \frac{2\pi}{N_s^B}\, e^{-(\lfloor s \rfloor - 1)\rho} \mathcal{L}_s\, L^{(s)}_{-\lfloor s \rfloor + 1} \right) \mathrm{d}z + \sum_{|m| \leq \lfloor s \rfloor - 1} e^{m\rho}\, \mu_m^s\, L_m^{(s)}\, \mathrm{d}\bar{z} + L_0\, \mathrm{d}\rho$$

- Would like to compute three-point functions of the type

$$\left\langle \mathcal{O}_\Delta(z_1, \bar{z}_1) \bar{\mathcal{O}}_\Delta(z_2, \bar{z}_2) J^s(z_3) \right\rangle$$

  where O_Δ are bosonic operators in the CFT and J^s are higher-spin currents
- Standard AdS/CFT methods too cumbersome and don't make full use of the higher-spin gauge symmetry
- Will use a trick due to [Ammon, Kraus, Perlmutter '11]: The higher-spin background is related to AdS_3 by a gauge transformation

Bulk Correlation Functions

- Start with the AdS_3 connection

$$A = e^{\rho}\, L_1^{(2)}\, dz + L_0^{(2)}\, d\rho, \qquad \bar{A} = e^{\rho}\, L_{-1}^{(2)}\, d\bar{z} - L_0^{(2)}\, d\rho$$

- Perform a gauge transformation leading to the spin-s background

$$\Lambda(\rho, z, \bar{z}) = \sum_{m=0}^{\lfloor s\rfloor -1} \frac{1}{(\lfloor s\rfloor - m - 1)!} (\text{-}\partial)^{\lfloor s\rfloor - m - 1} \Lambda^s\, e^{m\rho}\, L_m^{(s)} + \sum_{m=0}^{\lfloor s\rfloor -1} \tilde{F}^s_{-m}\, e^{-m\rho}\, L_{-m}^{(s)}$$

- Need to find how the scalars transform under this transformation

$$\widehat{C} = C + \delta_s C, \qquad \delta_s C = C \star \bar{\Lambda} - \Lambda \star C = -\Lambda \star C$$

Bulk Correlation Functions

- We find:

$$\delta_s C = -\sum_{t=1}^{\infty} \sum_{|n|\leq \lfloor t\rfloor -1} \sum_{m=0}^{\lfloor s\rfloor -1} \frac{(\text{-}\partial)^{\lfloor s\rfloor - m-1}\Lambda^s}{(\lfloor s\rfloor - m - 1)!} C_n^t e^{m\rho} L_m^{(s)} \star L_n^{(t)} + \underbrace{\ldots}_{m<0}$$

$$= \delta_s C_0^1 L_0^{(1)} + \delta_s C_0^{\frac{3}{2}} L_0^{(\frac{3}{2})} + \ldots$$

where

$$L_m^{(s)} \star L_n^{(t)} = \sum_{u=1}^{\text{Min}(2s-1,2t-1)} g_u^{st}(m,n;\lambda) L_{m+n}^{(s+t-u)}$$

- After (quite) some work we obtain for $\delta_s\phi_i = \widehat{\phi}_i - \phi_i$

$$\delta_s\phi_i = \tilde{a}_i \delta_s C_0^1 + \tilde{b}_i \delta_s C_0^{\frac{3}{2}},$$

$$= -\sum_{m=0}^{\lfloor s\rfloor -1} \frac{(\text{-}\partial)^{\lfloor s\rfloor - m-1}\Lambda^s}{(\lfloor s\rfloor - m-1)!} e^{m\rho} \Big(\tilde{a}_i C^s_{-m} g^{ss}_{2s-1}(m,\text{-}m;\lambda) + \tilde{b}_i \Big[C^s_{-m} g^{ss}_{2s-\frac{3}{2}}(m,\text{-}m;\lambda)$$

$$+ C^{s-1/2}_{-m} g^{ss-1/2}_{2s-2}(m,\text{-}m;\lambda)\chi_{[0,\lfloor s-1/2\rfloor -1]}(m) + C^{s+1/2}_{-m} g^{ss+1/2}_{2s-1}(m,\text{-}m;\lambda)\Big]\Big)$$

Bulk Correlation Functions

- Now apply the standard AdS/CFT procedure

$$\phi_i(\rho, z) = \int d^2 z' \, G_{b\partial}(\rho, z; z') \, \phi_i^{\partial}(z'),$$

with the AdS_3 bulk-to-boundary propagator

$$G_{b\partial}(\rho, z; z') = c_{\pm} \left(\frac{e^{-\rho}}{e^{-2\rho} + |z - z'|^2} \right)^{\Delta_{\pm}},$$

- Near boundary expansion:

$$\phi_i(\rho, z) \longrightarrow r^{d-\Delta_{\pm}} \left(\phi_i^{\partial}(z) + o(r) \right) + r^{\Delta_{\pm}} \left(\frac{1}{2\Delta_{\pm} - d} \left\langle \mathcal{O}_{\Delta_{\pm}}(z) \right\rangle + o(r) \right),$$

- Now gauge transform:

$$\phi_i(\rho, z) \longrightarrow \widehat{\phi}_i(\rho, z) = \phi_i(\rho, z) + \delta_s \phi_i(\rho, z)$$

to get

$$\widehat{\phi}_i(\rho, z) \longrightarrow r^{d-\Delta_{\pm}} \left(\widehat{\phi}_i^{\partial}(z) + o(r) \right) + r^{\Delta_{\pm}} \left(\frac{1}{2\Delta_{\pm} - d} \left\langle \mathcal{O}_{\Delta_{\pm}}(z) \right\rangle_{\mu} + o(r) \right).$$

Bulk Correlation Functions

- $\langle \mathcal{O}_{\Delta_\pm}(z) \rangle_\mu$ is the vev in the presence of a HS source
- It leads directly to the required three-point functions:

$$\langle \mathcal{O}_{\Delta_\pm}(z_1,\bar{z}_1) \rangle_\mu = \mu_\phi \langle \mathcal{O}_{\Delta_\pm}(z_1,\bar{z}_1)\, \bar{\mathcal{O}}_{\Delta_\pm}(z_2,\bar{z}_2) \rangle + \mu_\phi\, \mu_s \langle \mathcal{O}_{\Delta_\pm}(z_1,\bar{z}_1)\, \bar{\mathcal{O}}_{\Delta_\pm}(z_2,\bar{z}_2)\, J^s(z_3) \rangle + \ldots .$$

- After some computation we find

$$\langle \mathcal{O}_{\Delta_\pm}(z_1) \rangle_\mu = \frac{\mu_\phi\, \mu_s\, B^\pm_m\, c_\pm\, (-1)^{\lfloor s \rfloor -1}}{2\pi\, |z_{12}|^{2\Delta_\pm}} \sum_{m=0}^{\lfloor s \rfloor -1} \frac{1}{z_{12}^m} \Big\{ f_s^{s,i}(\lambda, -\Delta_\pm)\, \frac{\Gamma(\Delta_\pm + m)}{\Gamma(\Delta_\pm)}\, \frac{(\lfloor s \rfloor - m - 1)!}{z_{13}^{\lfloor s \rfloor - m}} - f_m^{s,i}(\lambda, -\Delta_\mp)\, \frac{1}{z_{23}^{\lfloor s \rfloor - m}} \sum_{j=0}^{m} (-1)^j \binom{m}{j} \frac{\Gamma(\Delta_\pm + m - j)}{\Gamma(\Delta_\pm)} (\lfloor s \rfloor - m - 1 + j)! \left(\frac{z_{12}}{z_{23}}\right)^j \Big\}$$

- Can also factor out the z-dependence

$$\langle \mathcal{O}_\Delta(z_1,\bar{z}_1) \bar{\mathcal{O}}_\Delta(z_2,\bar{z}_2) J^{(s)}(z_3) \rangle = \langle \mathcal{O}_\Delta \bar{\mathcal{O}}_\Delta J^{(s)} \rangle \left(\frac{z_{12}}{z_{13} z_{23}} \right)^s \langle \mathcal{O}_\Delta(z_1,\bar{z}_1) \bar{\mathcal{O}}_\Delta(z_2,\bar{z}_2) \rangle$$

Bulk Correlation Functions

- Final results for the three-point coefficients

$$\langle \mathcal{O}^{\mathcal{B}}_{\Delta_+} \bar{\mathcal{O}}^{\mathcal{B}}_{\Delta_+} W^{s+} \rangle = (-1)^s \frac{\Gamma^2(s)}{\Gamma(2s-1)} \frac{\Gamma(s-2\lambda+1)}{\Gamma(2-2\lambda)},$$

$$\langle \mathcal{O}^{\mathcal{B}}_{\Delta_-} \bar{\mathcal{O}}^{\mathcal{B}}_{\Delta_-} W^{s+} \rangle = (-1)^s \frac{\Gamma^2(s)}{\Gamma(2s-1)} \frac{\Gamma(s-2\lambda)}{\Gamma(1-2\lambda)},$$

$$\langle \tilde{\mathcal{O}}^{\mathcal{B}}_{\Delta_+} \bar{\tilde{\mathcal{O}}}^{\mathcal{B}}_{\Delta_+} W^{s+} \rangle = (-1)^{s-1} \frac{\Gamma^2(s)}{\Gamma(2s-1)} \frac{\Gamma(-2\lambda+1)}{\Gamma(-2\lambda-s+2)},$$

$$\langle \tilde{\mathcal{O}}^{\mathcal{B}}_{\Delta_-} \bar{\tilde{\mathcal{O}}}^{\mathcal{B}}_{\Delta_-} W^{s+} \rangle = (-1)^{s-1} \frac{\Gamma^2(s)}{\Gamma(2s-1)} \frac{\Gamma(-2\lambda)}{\Gamma(-2\lambda-s+1)},$$

$$\langle \mathcal{O}^{\mathcal{B}}_{\Delta_+} \bar{\mathcal{O}}^{\mathcal{B}}_{\Delta_+} W^{s-} \rangle = (-1)^{s-1} \frac{\Gamma^2(s)}{\Gamma(2s-1)} \frac{\Gamma(s-2\lambda+1)}{\Gamma(2-2\lambda)} \frac{s-1+2\lambda}{2s-1},$$

$$\langle \mathcal{O}^{\mathcal{B}}_{\Delta_-} \bar{\mathcal{O}}^{\mathcal{B}}_{\Delta_-} W^{s-} \rangle = (-1)^s \frac{\Gamma^2(s)}{\Gamma(2s-1)} \frac{\Gamma(s-2\lambda)}{\Gamma(1-2\lambda)} \frac{s-2\lambda}{2s-1},$$

$$\langle \tilde{\mathcal{O}}^{\mathcal{B}}_{\Delta_+} \bar{\tilde{\mathcal{O}}}^{\mathcal{B}}_{\Delta_+} W^{s-} \rangle = (-1)^s \frac{\Gamma^2(s)}{\Gamma(2s-1)} \frac{\Gamma(-2\lambda+1)}{\Gamma(-2\lambda-s+2)} \frac{s-1+2\lambda}{2s-1},$$

$$\langle \tilde{\mathcal{O}}^{\mathcal{B}}_{\Delta_-} \bar{\tilde{\mathcal{O}}}^{\mathcal{B}}_{\Delta_-} W^{s-} \rangle = (-1)^{s-1} \frac{\Gamma^2(s)}{\Gamma(2s-1)} \frac{\Gamma(-2\lambda)}{\Gamma(-2\lambda-s+1)} \frac{s-2\lambda}{2s-1}.$$

Boundary Correlation Functions

- Would like to match the above bulk results by calculating the same three-point functions from the CFT side
- Recall:

$$\left\langle \mathcal{O}_{\Delta_\pm}(z_1,\bar{z}_1)\bar{\mathcal{O}}_{\Delta_\pm}(z_2,\bar{z}_2)J^{(s)}(z_3)\right\rangle$$
$$= A_\pm(s)\left(\frac{z_{12}}{z_{13}z_{23}}\right)^{\lfloor s\rfloor}\left\langle \mathcal{O}_{\Delta_\pm}(z_1,\bar{z}_1)\bar{\mathcal{O}}_{\Delta_\pm}(z_2,\bar{z}_2)\right\rangle.$$

- So the coefficient is given by the OPE:

$$J^{(s)}(z)\mathcal{O}_\Delta(w,\bar{w}) \sim \frac{A(s)}{(z-w)^s}\,\mathcal{O}_\Delta(w,\bar{w}) + \dots.$$

- Need to compute OPE's of HS currents with CFT operators
- In principle, should calculate in the Kazama-Suzuki CFT and take $N, k \to \infty$
- Can we compute directly at $N, k = \infty$?

Boundary Correlation Functions

- [Candu-Gaberdiel]: In the 't Hooft limit, the Kazama-Suzuki algebra extends to $\mathcal{SW}_\infty[\lambda]$
- This algebra becomes linear for $c \to \infty$, with shs[λ] arising as a wedge subalgebra

$$L_m^{(s)} \quad \text{with} |m| \leq s - 1$$

- So representation theory of shs[λ] should be enough
- Even simpler: Construct a CFT that realises shs[λ] as a subalgebra.
- There exists a very simple free-field construction of such a CFT [Bergshoeff, de Wit, Vasiliev '91]

Boundary Correlation Functions

- Ghost Action

$$S = \frac{1}{\pi} \int d^2 z \left\{ b\bar{\partial} c + \beta \bar{\partial} \gamma + \tilde{b} \partial \tilde{c} + \tilde{\beta} \partial \tilde{\gamma} \right\}$$

with

$$\gamma(z)\beta(w) \sim \frac{1}{z-w} \ , \quad \text{and} \quad c(z)b(w) \sim \frac{1}{z-w}$$

Conformal weights:

	b	c	β	γ	$\tilde{b}$	$\tilde{c}$	$\tilde{\beta}$	$\tilde{\gamma}$
h	$\lambda + \frac{1}{2}$	$\frac{1}{2} - \lambda$	λ	$1 - \lambda$	0	0	0	0
$\bar{h}$	0	0	0	0	$\lambda + \frac{1}{2}$	$\frac{1}{2} - \lambda$	λ	$1 - \lambda$

Boundary correlation functions

- Combine to create duals of bulk fields

$$\mathcal{O}^{\mathcal{B}}_{\Delta_+}(z,\bar z) = \gamma(z)\otimes\tilde\gamma(\bar z), \qquad \mathcal{O}^{\mathcal{F}}_{\Delta_+}(z,\bar z) = c(z)\otimes\tilde\gamma(\bar z),$$
$$\mathcal{O}^{\mathcal{B}}_{\Delta_-}(z,\bar z) = c(z)\otimes\tilde c(\bar z), \qquad \mathcal{O}^{\mathcal{F}}_{\Delta_-}(z,\bar z) = \gamma(z)\otimes\tilde c(\bar z),$$

and

$$\tilde{\mathcal{O}}^{\mathcal{B}}_{\Delta_+}(z,\bar z) = \beta(z)\otimes\tilde\beta(\bar z), \qquad \tilde{\mathcal{O}}^{\mathcal{F}}_{\Delta_+}(z,\bar z) = \beta(z)\otimes\tilde b(\bar z),$$
$$\tilde{\mathcal{O}}^{\mathcal{B}}_{\Delta_-}(z,\bar z) = b(z)\otimes\tilde b(\bar z), \qquad \tilde{\mathcal{O}}^{\mathcal{F}}_{\Delta_-}(z,\bar z) = b(z)\otimes\tilde\beta(\bar z).$$

- Higher Spin Current

$$V^{(s)+}_{\lambda}(z) = \sum_{i=0}^{s-1} a^i(s,\lambda)\partial^{s-1-i}\left\{(\partial^i\beta)\gamma\right\} + \sum_{i=0}^{s-1} a^i(s,\lambda+\tfrac{1}{2})\partial^{s-1-i}\left\{(\partial^i b)c\right\}$$

- Similarly for $V^{(s)-}_{\lambda}(z)$ and $Q^{(s)\pm}_{\lambda}(z)$

Correlators from the CFT

- Now use free-field OPE's, for example

$$V_\lambda^{(s)+}(z)\beta(w) \sim a^0(s,\lambda)\frac{(-1)^{s-1}(s-1)!}{(z-w)^s}\beta(w) + \cdots$$

- First Multiplet with $V_\lambda^{(s)+}$

$$\left\langle \mathcal{O}^{\mathcal{B}}_{\Delta_+}\bar{\mathcal{O}}^{\mathcal{B}}_{\Delta_+}V_\lambda^{(s)+}\right\rangle = (-1)^s\frac{\Gamma^2(s)}{\Gamma(2s-1)}\,\frac{\Gamma(s-2\lambda+1)}{\Gamma(2-2\lambda)},$$

$$\left\langle \mathcal{O}^{\mathcal{B}}_{\Delta_-}\bar{\mathcal{O}}^{\mathcal{B}}_{\Delta_-}V_\lambda^{(s)+}\right\rangle = (-1)^s\frac{\Gamma^2(s)}{\Gamma(2s-1)}\,\frac{\Gamma(s-2\lambda)}{\Gamma(1-2\lambda)}.$$

- Precise match with the bulk calculation!
- Found agreement for all other boson-boson-hs 3-point functions
- Simultaneous work by [Creutzig, Hikida, Rønne '12]

Summary

- We provided a detailed check of the $\mathcal{N}=2$ version of the MM/HS duality
- Holographic construction of the asymptotic symmetries, showed how the $\mathcal{N}=2$ $\mathcal{SW}_\infty[\lambda]$ symmetry arises.
- Computed scalar-scalar-hs three-point functions in the bulk and matched to boson-boson-hs current correlators in the CFT
- A modification of the Prokushkin-Vasiliev theory greatly simplified the bulk computations
- For the boundary calculation, we used a free-field ghost CFT with shs[λ] symmetry
- Our computation can easily be extended to include fermions, some of these correlators were found in [Creutzig, Hikida, Rønne '12]

Outlook

- Extend to other correlators, such as three matter fields $\Rightarrow$ Would need to go beyond the linearised Vasiliev equations
- Not all quantities can be captured by the free-field CFT. E.g. four-point functions would be sensitive to the fact that the spectrum is different from that of the CP^N model
- Would we need to calculate at finite N, k and take the 't Hooft limit? Or can we constrain the CFT so as to obtain $\mathcal{SW}_\infty[\lambda]$ directly in the 't Hooft limit?
- Models with different amounts of supersymmetry
- BH backgrounds?
- Some of these techniques were also recently used in [Creutzig, Hikida, Rønne '13] in the context of matrix higher-spin theory