A holographic model for the fractional quantum Hall effect

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WIP with Elias Kiritsis, Matthew Lippert and Tassos Taliotis, 1307.xxxx

Outline

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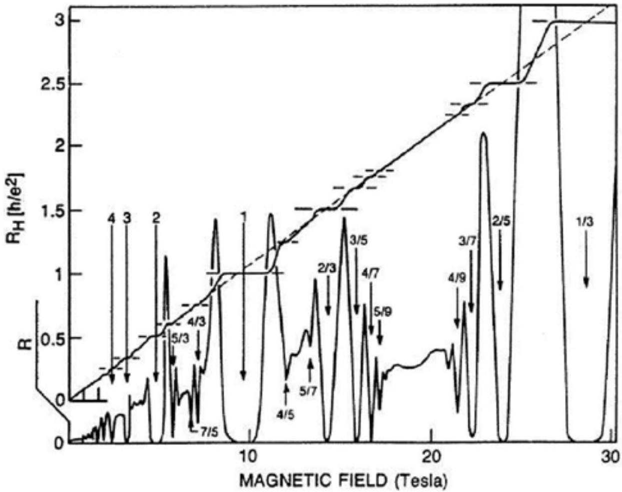
The Fractional Quantum Hall Effect

- In systems with 2D electron gases, at very low temperatures, high magnetic fields, clean samples :



Stormer (1992)

The Fractional Quantum Hall Effect

- FQHE states are gapped states with quantized Hall conductvitiy

$$\sigma_{xy} = \frac{p}{q}\left(\frac{e^2}{h}\right)\,, \quad p, q \in \mathbb{Z}\,, \quad q \text{ odd}$$

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- Physics of pseudoparticle excitations invariant under Modular Group Action : $\sigma = \sigma_{xy} + i\sigma_{xx}$

$$\sigma \mapsto \frac{a\sigma + b}{c\sigma + d}, \quad \begin{pmatrix} a & b \\ c & d \end{pmatrix} \in \Gamma_0(2) \subset SL(2, \mathbb{Z}), \quad c \text{ even}$$

- $\sigma \mapsto \sigma + 1$ Landau Level addition (T)
- $-\frac{1}{\sigma} \mapsto 2 - \frac{1}{\sigma}$ 2π Statistical Flux attachment (ST^2S)

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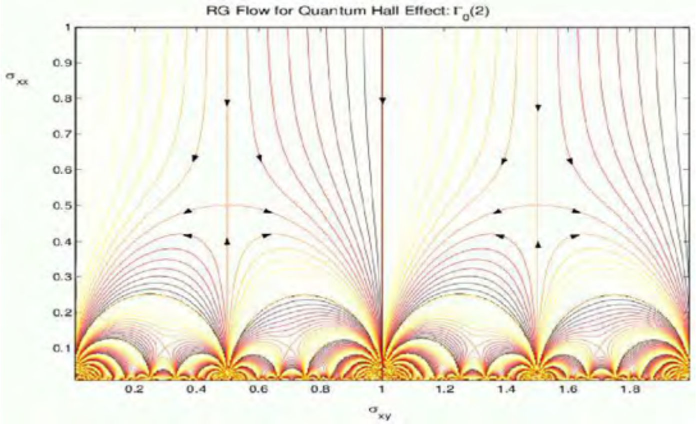
  - $\sigma \mapsto \sigma + 1$ Landau Level addition (T)
  - $-\frac{1}{\sigma} \mapsto 2 - \frac{1}{\sigma}$ 2π Statistical Flux attachment (ST^2S)

- Group action commuting with the RG flow implies that RG fixed points are $\Gamma_0(2)$ fixed points, structure imprinted on σ flows in $\sigma_{xx} - \sigma_{xy}$ plane

The Fractional Quantum Hall Effect

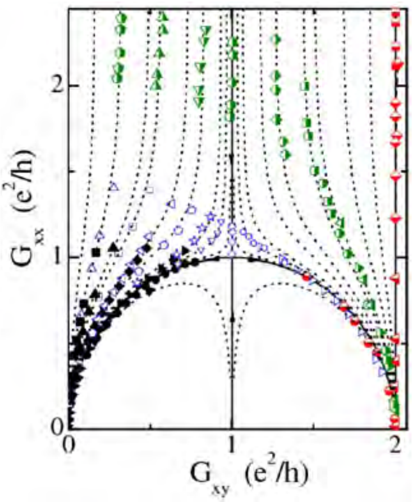
- Assume:
  - Integers are attractive
  - $\sigma_{xx} \downarrow$ as $T \downarrow$ (semi-conductor behaviour)

Modular symmetry $\Rightarrow$ Even denominators repulsive



[Burgess+Lutken 1997, Dolan 1999, Lutken+Ross 2009]

The Fractional Quantum Hall Effect



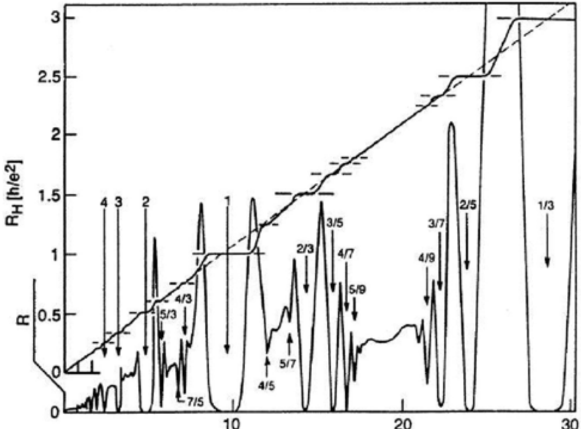
[S.S. Murzin et al 2002]

The Fractional Quantum Hall Effect

- Any transition can be reached from $\sigma : 0 \to 1$ by a $\Gamma_0(2)$:

$$\sigma' = \frac{(p'-p)\sigma + p}{(q'-q)\sigma + q} \quad \Rightarrow \quad \gamma = \begin{pmatrix} p'-p & p \\ q'-q & q \end{pmatrix} \in \Gamma_0(2)$$

$\Rightarrow$ Selection Rule: $p'q - pq' = 1$ (e.g. $1/3 \to 2/5$) [Dolan 1998]



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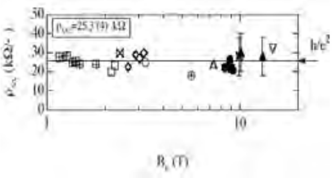

[cond-mat/9805143]

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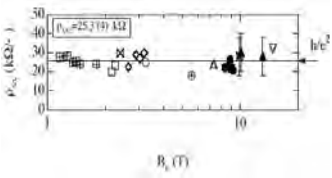

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- Transition is a 2nd order QPT : [Fisher '90]
  - Simple scaling $\Rightarrow \sigma(T, \Delta B, n, ...) = \sigma(\Delta B/T^{\kappa}, n/T^{\kappa'}, ...)$
  - Superuniversality: κ and κ' are same for all transitions
  - Experimentally: $\kappa = \kappa' = 0.42 \pm 0.01$ [Wanli et al 2009]
  - Nonlinear inversion symmetry around critial point

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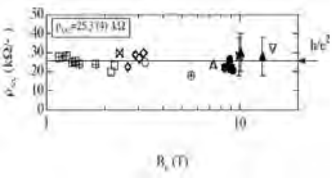

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- CAN WE REPRODUCE THIS IN A SINGLE HOLOGRAPHIC MODEL BY USING MODULAR INVARIANCE?

Outline

$SL(2, R)$ and Black Hole Charges

- The main idea of [1007.2490,1008.1917,WIP] is to use an SL(2,R) or SL(2,Z) invariant gravity action. These groups act on the electric and magnetic charges of the black hole solutions, which label the QH plateaux with charge density n and external magnetic field B. The filling fraction n/B inherits the group action, as do the conductivity and other observables.

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- The starting point of [1007.2490] is the SL(2,R) invariant action

$$S = \int d^4x\sqrt{-g}\left[R - 2\Lambda - \frac{1}{2}(\partial\phi)^2 - \frac{1}{2}e^{2\phi}(\partial a)^2 - \frac{1}{4}(e^{-\phi}F^2 + aF\tilde{F})\right]$$

- SL(2,R) acts on the fields as

$$\tau = a + ie^{-\phi} = \tau_1 + i\tau_2\,, \quad \tau \to \frac{a\tau + b}{c\tau + d} \quad \text{and}$$

$$F \to F' = (c\tau_1 + d)F - c\tau_2\tilde{F} \qquad ds^2 \to ds^2$$

$SL(2,R)$ and Black Hole Charges

- Dyonic black branes generated from purely electric ones by $SL(2,R)$. [1007.2490]

$$Q'_e = aQ_e\,, \quad Q'_m = cQ_e$$

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$$Q'_e = aQ_e\,, \quad Q'_m = cQ_e$$

- Any filling fraction can be generated in this way. The electric solution flows in the IR to $\tau_{1*} = 0\,, \tau_{2*} = +\infty$, which after SL(2,R) becomes $\tau'_{1*} = \frac{a}{c}$ and $\tau'_{2*} = {\tau_{2*}}^{-1} = 0$. The filling fraction in the IR is hence equal to the value of the transformed axion

$$\nu = \frac{Q'_e}{Q'_m} = \frac{a}{c} = \tau'_{1*}\,,$$

which can be roughly though of setting the Chern-Simons level in the dual field theory. [1007.2490] .

$SL(2,R)$ and Black Hole Charges

- Are these black branes the QH plateaux?

  Evidence 1: They are unique IR attractors by the attractor mechanism (in the absence of a scalar).

  Evidence 2: Hall Conductivity [1007.2490] used the known AC conductivity of the purely electric solution,

$$\sigma_{xx} = C' \frac{T^2}{\mu^2} + iC'' \frac{\mu}{\omega} + \ldots , \quad \sigma_{yx} = 0$$

  and the action of $SL(2,R)$ on the AC conductivity to show that at low frequencies the Hall conductivity agrees with the filling fraction and also the axionic attractor value. .

$$\boxed{\sigma'_{yx} = \frac{a}{c}\left(1 + O(\omega^2)\right) , \quad \sigma'_{xx} = \frac{16}{i(Q'_m)^2 C''} \frac{\omega}{\mu} \left(1 + O(\omega)\right) .}$$

  The DC conductivity vanishes exactly.

  N.B.: No ω^{-1} pole since momentum is nonconserved in external magnetic field.

Summary: SL(2,R) invariant model of [1007.2490]

- We have seen that the model has attractor solutions with the right filling fractions (if we restrict SL(2,R) to $\Gamma_0(2)$) and Hall conductivities. [1007.2490].

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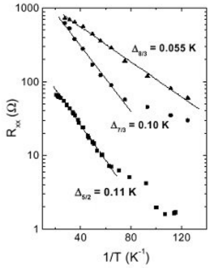

[Pan etal PRL 83 1999]

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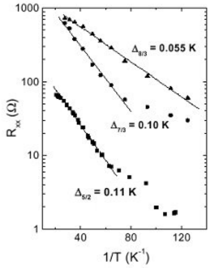
    1. There is no hard gap in the charged excitations, i.e. σ_{DC} does not vanish as $e^{-\frac{\Delta}{T}}$ at low temperatures ($T \ll \mu$), but as a power law.
    2. Performing a SL(2,R) trafo from one filling fraction to another, $\sigma_{DC}(T=0)=0$ along the way, while the Hall conductivity changes. This is not the experimentally observed behavior.



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Outline

Gapped Spectra in Charged Systems and the FQHE

[WIP with Elias Kiritsis, Matthew Lippert, Anastasios Taliotis]

- In string theory, SL(2,R) is usually broken to SL(2,Z) by nonperturbative effects. This typically will generate a SL(2,Z) invariant potential for the axio-dilaton $(\tau = \tau_1 + i\tau_2 = a + ie^{\gamma\phi})$

$$\boxed{S = M_{Pl}^2 \int d^4x \sqrt{-g} \left[R - \frac{1}{2\gamma^2} \frac{\partial\tau\partial\bar{\tau}}{\tau_2^2} + V(\tau,\bar{\tau}) - \frac{1}{4}\left(\tau_2 F^2 + \tau_1 F\tilde{F}\right)\right]}$$

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- A simple choice is the real-analytic Eisenstein series

$$V(\tau,\bar{\tau}) = E_s(\tau,\bar{\tau}) = \sum_{m,n\in\mathbb{Z}^2/0,0} \left(\frac{|m+n\tau|}{\tau_2}\right)^{-s}$$

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$$E_s = 2\zeta(2s)\tau_2^s + 2\sqrt{\pi}\frac{\Gamma(s-1/2)}{\Gamma(s)}\zeta(2s-1)\tau_2^{1-s} + \text{instanton contributions}$$

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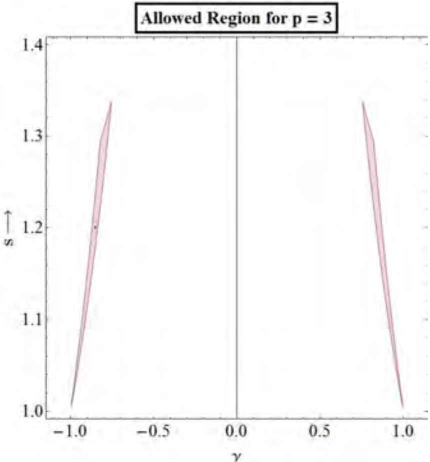
- We tune the two parameters (γ, s) such that the dyonic ground state has a gapped charged excitation spectrum. The $SL(2,Z)$ dual CDBH we start from has a gapped and discrete spectrum.

Gapped Spectra in Charged Systems and the FQHE

- For large τ_2 the potential is dominated by $V \sim e^{\gamma s \phi}$. CDBHs with this potential were extensively analyzed in [1005.4690] . Gubser's constraint, thermodynamic instability of small black holes, consistency of the spin 1 fluctuation problem and existence of a discrete and gapped spectrum restrict (γ, s):



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- QH Plateaux? Since E_s is SL(2,Z) invariant it has runaway minima at $\tau_1 = \frac{p}{q}$, $\tau_2 = 0$, the images of the CDBH at $\tau_2 = \infty$. Their charges fulfill

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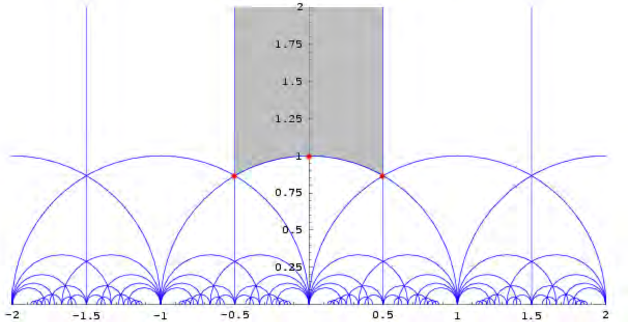
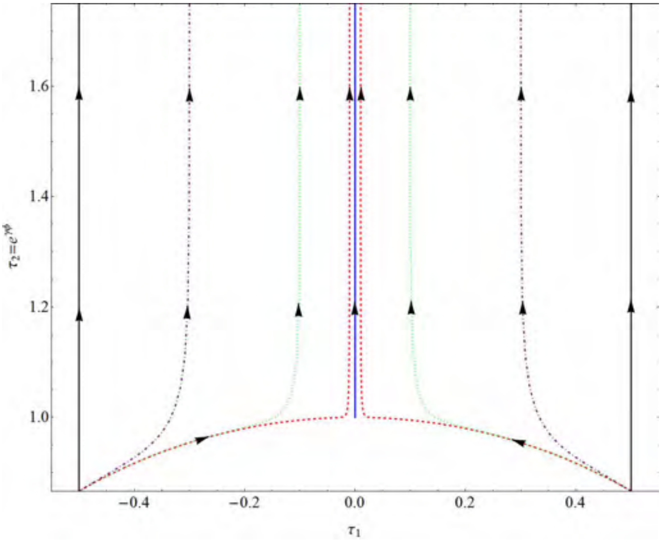
- RG Flows: E_s is stationary in the fundamental domain at the SL(2,Z) fixed points:



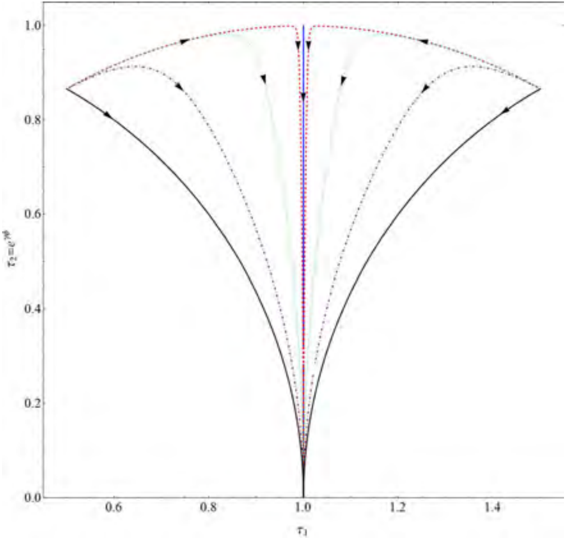
Gapped Spectra in Charged Systems and the FQHE

- Since SL(2,Z) commutes with the RG flow it suffices to construct the RG flows inside the fundamental domain:



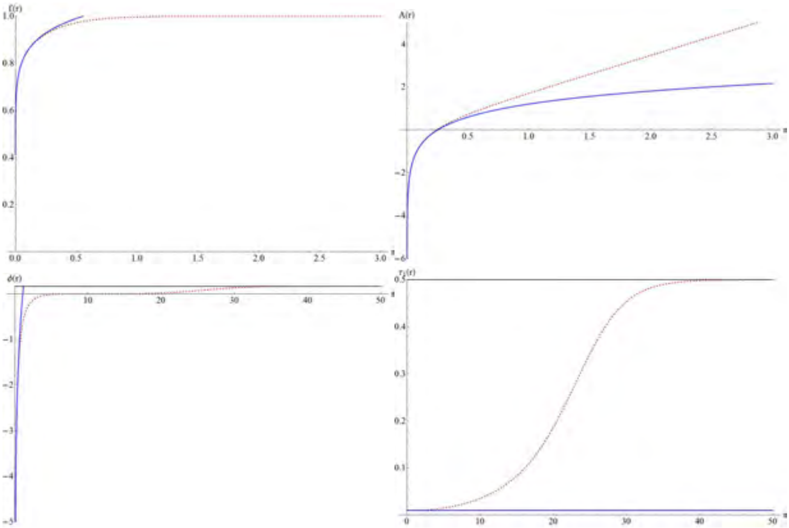
Gapped Spectra in Charged Systems and the FQHE

- By SL(2,Z) we can generate flows to any QH plateaux $\tau_1 = p/q$. E.g. $\nu = 1$:



Gapped Spectra in Charged Systems and the FQHE

- Our flows are the IR scaling geometries of [1005.4690]



Gapped Spectra in Charged Systems and the FQHE

- Conductivities: At low enough temperatures the purely electric state is discrete and gapped. Hence the conductivity at small ω is dominated by the contribution from translation invariance:

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- Thus after $SL(2, Z)$ the QH plateaux have the correct Hall conductivity

$$\sigma_{xy} = \frac{a}{c}$$

But are they gapped as well?

Gapped Spectra in Charged Systems and the FQHE

- We calculate the charged spectrum in the dyonic geometry directly. We fluctuate

$$\delta A_x , \quad \delta A_y , \quad \delta g_{tx} , \quad \delta g_{ty} \, .$$

In general dyonic solutions with running scalars the equations can be decoupled into a single second order equation after taking linear combinations [0910.0645]

$$E_z = \omega(\delta A_x + i\delta A_y) + h g_{rr}(\delta g^x{}_t - i\delta g^y{}_t) \, .$$

The fluctuations obey a single 2nd order ODE

$$\boxed{E_z'' + F(r)E_z' + G(r)E_z = 0}$$

This is equivalent to the Schrödinger problem

$$-\Psi'' + V(r,w)\Psi = 0$$

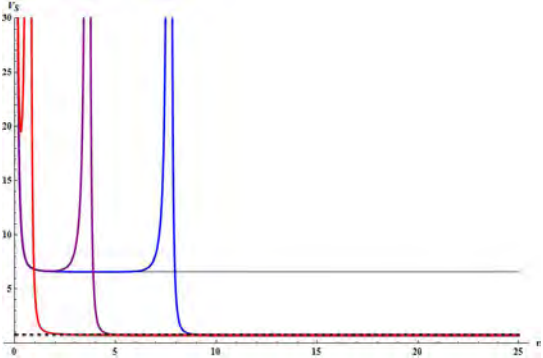
if we set $\Psi(r) = E_z(r)e^{\frac{1}{2}\int drF(z)}$

Gapped Spectra in Charged Systems and the FQHE

- For our choice of γ, s the potential

$$\boxed{V(r,w) = \frac{1}{4}\left(F^2 - 4G + 2F'\right)}$$

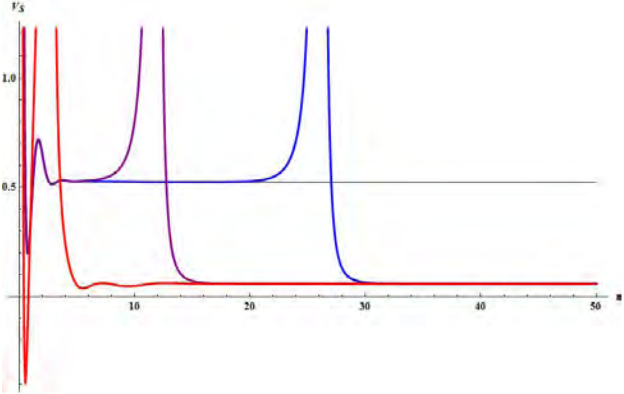
diverges in the IR and approaches $\frac{1}{4L^2}$ in the UV. The spectrum is hence gapped in the QH state. .
E.g. Flow to filling fraction one from image of $\tau = i$:



(blue $w = 10^{-5}$, purple $w = 10^{-2}$, red $w = 1$)

Gapped Spectra in Charged Systems and the FQHE

- Flows with varying axion: Bound states possible at larger frequencies



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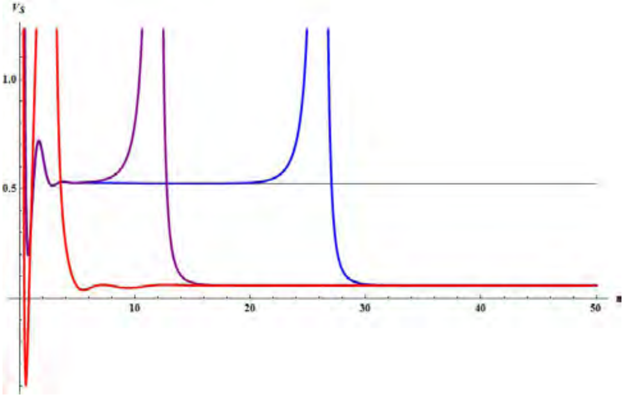

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- N.B.: The singularity in the potential is an accessory singularity, i.e. there is no monodromy, and the singularity is traversable by the wavefunctions. This was not appreciated in e.g. [0910.0645]]
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Conclusions

- Several holographic bottom-up models of the FQHE so far have employed SL(2,R) transformations to infer the properties of the QH state from an ungapped state at zero magnetic field. The resulting QH state was ungapped. [1007.2490(,1008.1917)]

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- We use a SL(2,Z) invariant Eisenstein potential which allows us to tune the electric state to have a gapped and discrete charge spectrum at low temperatures. We constructed the RG flows to CDBHs in the fundamental domain, and hence all RG flows to QH plateaux states, and showed that the QH states have the correct Hall conductivity, and a real gap (no $\delta(\omega)$ pole).

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  2. IR fixed points (AdS_2 and running) and thermodynamics

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- We use a SL(2,Z) invariant Eisenstein potential which allows us to tune the electric state to have a gapped and discrete charge spectrum at low temperatures. We constructed the RG flows to CDBHs in the fundamental domain, and hence all RG flows to QH plateaux states, and showed that the QH states have the correct Hall conductivity, and a real gap (no $\delta(\omega)$ pole).
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  2. IR fixed points (AdS_2 and running) and thermodynamics
  3. Transitions between QH Plateaux as a QPT?

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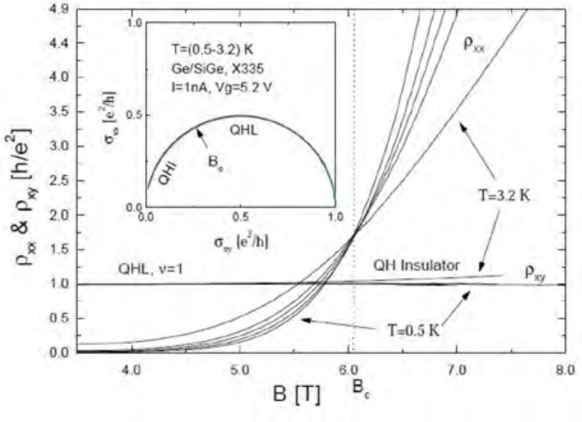
  5. Phenomenology? Subgroups such as $\Gamma_0(2)/\Gamma_\theta(2)$?
- STAY TUNED!

Backup Slides

The Fractional Quantum Hall Effect

- Semicircle law: Conductivity sweeps out a semicircle in σ plane during QH transitions [e.g. Burgess etal 1008.1917]



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$$\tau_{2*} \partial_\tau V|_* = \frac{i(q - h\bar{\tau}_*)^2}{2\tau_{2*}} \,, \quad L_{AdS_2}^{-2} = V_*$$

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  These are gapless and come in SL(2,Z) orbits $\rightarrow$ QCPs for Plateaux Transitions?
- For the leading exponential potential in the Eisenstein series $E_s = e^{\gamma s \phi} + \ldots$ there are also more general running scalar solutions where both the axion and the dilaton runs:

$$g_{rr} = b_0 r^b , \quad g_{tt} = r^d , \quad g_{xx} = r^c , \quad \phi = \kappa \log r , \quad a = a_0 r^\lambda$$

  We are classifying all of them along the lines of [Gouteraux & Kiritsis '12] in order to find all possible translation and rotationally invariant ground states.

SL(2,R) invariant probe branes [1008.1917]

- [Burgess etal 1008.1917] realized that the DC conductivity in the QH state of [1007.2490] vanishes due to the momentum-conservation pole in $\Im\sigma_{xx}$ of the purely electric solution.

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- [Burgess etal 1008.1917] realized that the DC conductivity in the QH state of [1007.2490] vanishes due to the momentum-conservation pole in $\Im \sigma_{xx}$ of the purely electric solution.
- They introduce dissipation by separating the sector that generates the gravity background of [1007.2490] from the sector of charge carriers, which they model using a SL(2,R) invariant probe brane

$$\begin{aligned} S &= M_{Pl}^2 \int d^4x \sqrt{-g} \left[R - 2\Lambda - \frac{1}{2} \left((\partial\phi)^2 + e^{2\phi} (\partial a)^2 \right) \right] + \\ &\quad + M_{Pl}^2 S_{Lifshitz} + S_{gauge} \end{aligned}$$

The first two terms are assumed to be separately SL(2,R) invariant, and $S_{Lifshitz}$ to be chosen such as to generate the metric of the $z = 5$ Lifshitz black hole of [1007.2490], together with an appropriate axio-dilaton profile.

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- S_{gauge} is taken to be a SL(2,R) invariant version of the DBI action, treated in the probe limit:

$$\begin{aligned} S_{gauge} &= -T\int d^4x\left[\sqrt{-det\left(g_{\mu\nu}+\ell^2 e^{-\phi/2}F_{\mu\nu}\right)}-\sqrt{-g}\right] \\ &\quad -\frac{1}{4}\int d^4x\sqrt{-g}aF_{\mu\nu}\tilde{F}^{\mu\nu} \end{aligned}$$

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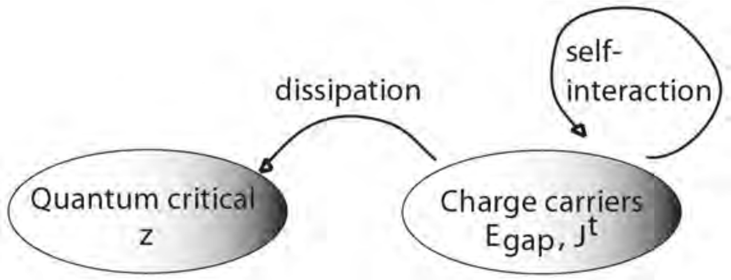
- This describes self-interacting charge carriers coupled to a large reservoir of quantum critical excitations into which they can loose energy via dissipation:



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$$\sigma_{xx} = \frac{\sigma_0}{d^2 + c^2\sigma_0^2}\,, \quad \sigma_{xy} = \frac{ac\sigma_0^2 + bd}{d^2 + c^2\sigma_0^2}\,,$$

with $\sigma_0(T/\mu)$ the DC conductivity of the probe brane in the purely electric state (with $\sigma_{yx} = 0$). For probe branes in Lifshitz backgrounds like

$$ds_z^2 = L^2\left[-h(r)\frac{dt^2}{r^{2z}} + \frac{dr^2}{r^2 h(r)} + \frac{dx^2 + dy^2}{r^2}\right]$$

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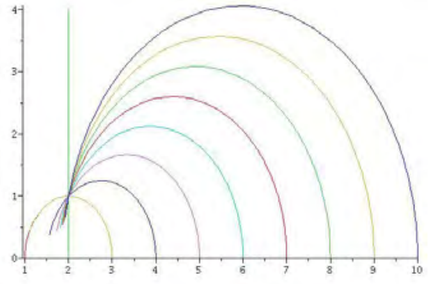
- This temperature flow commutes with SL(2,R) or any subgroup.

SL(2,R) invariant probe branes [1008.1917]

- The four parameters of the necessary SL(2,R) transformation are fixed by the data of the endpoint $(Q'_e, Q'_m, a, e^{-\phi})$. The temperature flow of the conductivities then trace out semi-circles in the σ plane, and for small T asymptote to (in linear response)

$$\sigma^{xx} \sim \frac{\rho T^{2/z}}{B^2} \to 0$$
$$\sigma^{xy} = \nu = \frac{a}{c}$$



This also predicts the superuniversality exponents $\kappa \approx \frac{2}{z} = \kappa'$ close to the measured value if $z = 5$ as in [1007.2490] .

However there is still no hard gap. .