QCD in strong magnetic fields

Umut Gürsoy

Kolymbari, 22.6.2013

Part I: **arXiv:1212.3894 with I. Iatrakis, E. Kiritsis, F. Nitti, A. O' Bannon**
Part II: **ongoing with D. Kharzeev and K. Rajagopal**

External Magnetic Fields

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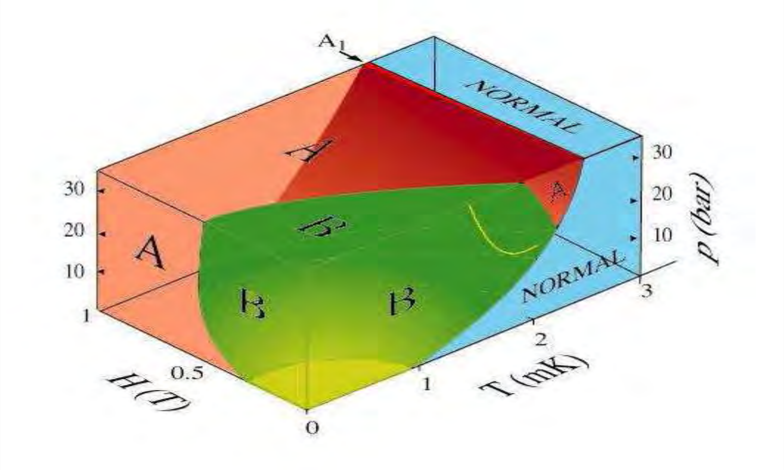
- An excellent tool to study phases of matter, e.g. He3



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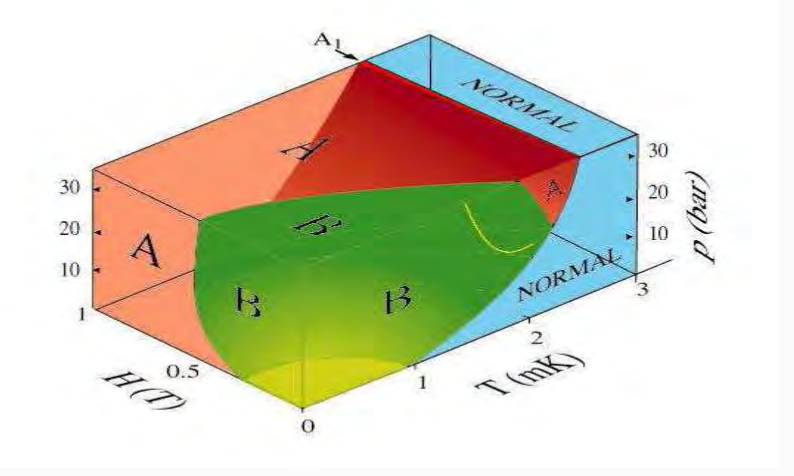
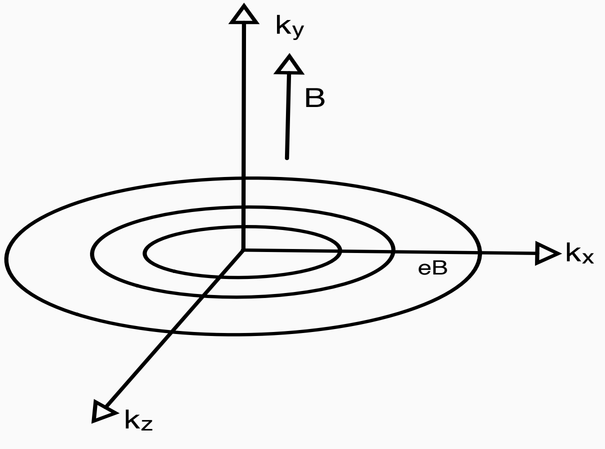


- Strong magnetic fields ⇒ drastic effects:



- Energy $E = \sqrt{m^2 + 2e|B|n}$, $n = 0, 1, \cdots$
- Transverse momentum phase space $S_n \propto nB$
- For $|B| \gg k_F^2$, effectively 1+1 D system!

QCD under magnetic fields

- Schwinger pair production if $F > m_e^2/e$ for $eB \approx 10^{13}$ G.
- Magnetic catalysis: B (de)catalyzes $\langle \bar{q}q \rangle$, $T_c(B)$ is complicated
  Bali et al '12
- rho-meson condensation $\Rightarrow$ superconducting QCD vacuum!
  Chernodub '10
- Phase diagram of QCD in $\mu - T - B$?

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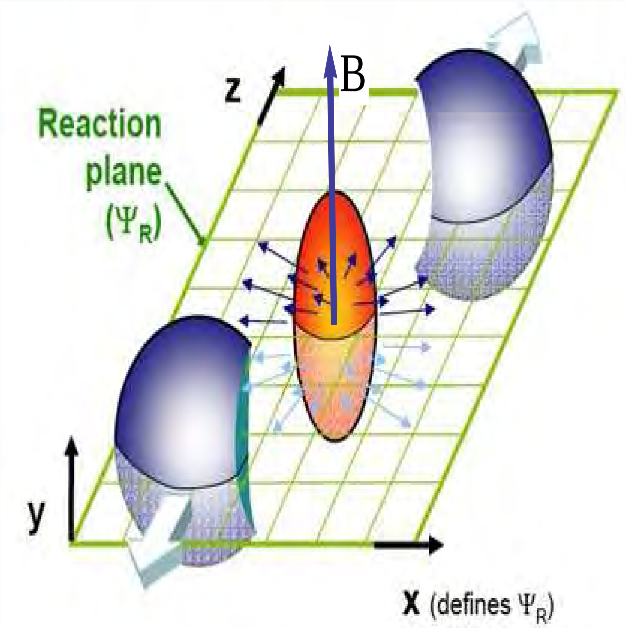
This talk:

- Chiral Magnetic effect Kharzeev, McLerran, Warringa '07
- Induced currents in the quark-gluon plasma U.G, Kharzeev, Rajagopal, ongoing

Heavy ion collisions and magnetic fields



- Initial magnitude of B
- Bio-Savart: $B_0 \sim \gamma Z e \frac{b}{R^3} \Rightarrow \sim 10^{18} (10^{19})$ G at RHIC (LHC).
- $B_0 \sim 10^{10} - 10^{13}$G (neutron stars), 10^{15} (magnetars)
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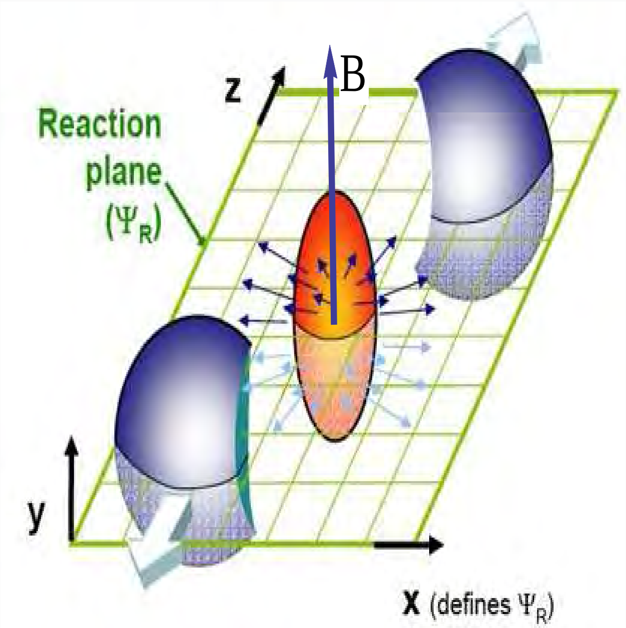

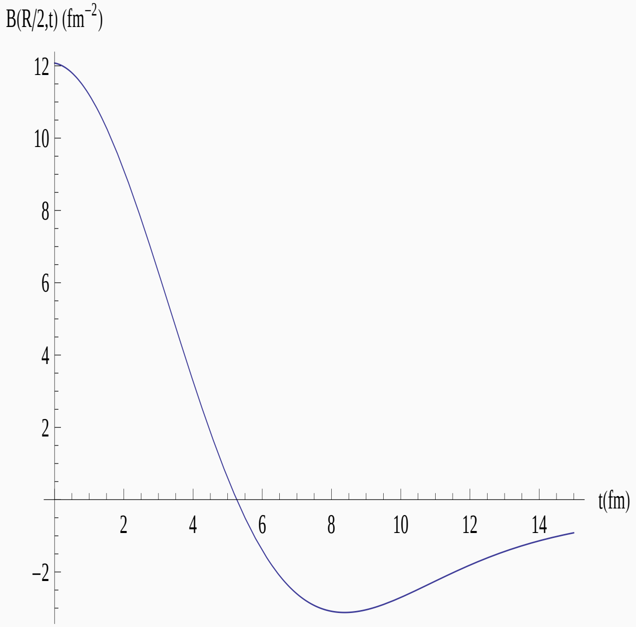

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  - More relevantly $eB \approx 5 - 15 \times m_\pi^2$ RHIC (LHC).
- Subsequent evolution of B in charged medium
- Solve $\nabla^2 \vec{B} = \mu\epsilon \frac{\partial^2 \vec{B}}{\partial t^2} + \mu\sigma \frac{\partial \vec{B}}{\partial t}$
- with $\vec{B}(\rho, t_0) = \hat{y} B_0 e^{-\frac{\rho^2}{R^2}}$.

Quark-gluon plasma under strong B in the lab!

PART I: Chiral Magnetic Effect

- *Classically* QGP chiral symmetric: $N_L = N_R$
  as $T \approx 500$ MeV $\gg m_u,\ m_d$
- Axial current $\partial_\mu J^{\mu 5} = \partial_\mu \left(\langle \bar{\psi}\gamma^\mu \psi \rangle_L - \langle \bar{\psi}\gamma^\mu \psi \rangle_R \right) = 0$
- Chiral imbalance only due to QM anomaly.
- Under B spin degeneracy of quarks lifted due $H \sim -q\vec{s}\cdot\vec{B}$:

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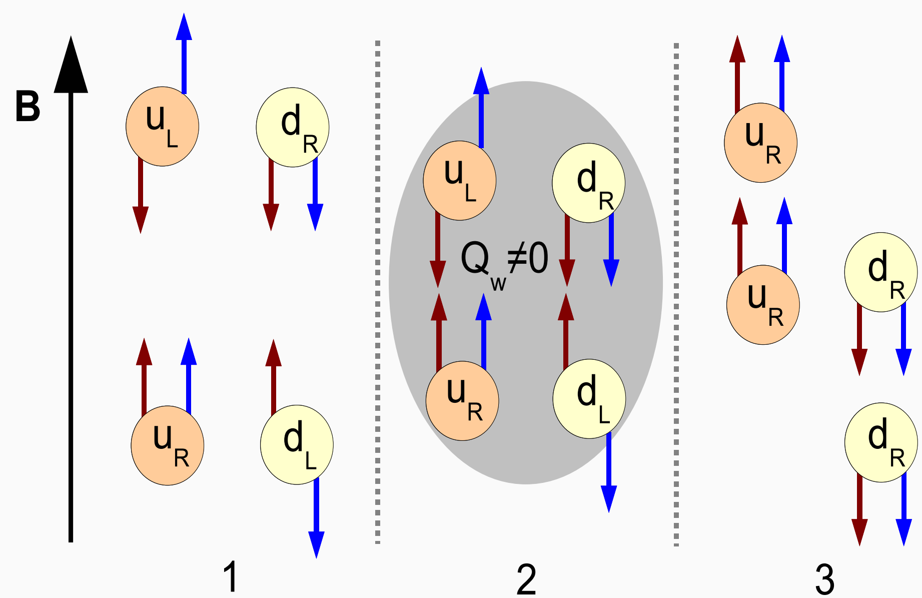

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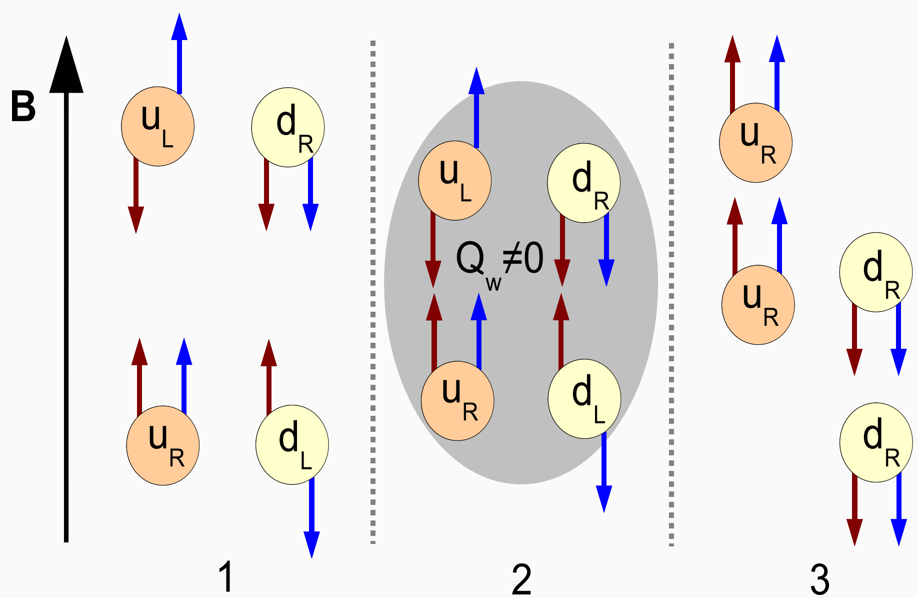

- Macroscopic manifestation of the axial anomaly
- Anomalous magnetohydrodynamics: $\vec{J} = \frac{e^2}{2\pi^2}\, \mu_5\, \vec{B}$

  Kharzeev et al ’07, Son, Surowka ’09
- μ_5 encodes the imbalance $N_L \neq N_R$

How to generate chiral imbalance in the first place?

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- Answer: topologically non-trivial gluon configurations
- Gluon winding number: $Q_w = \frac{g^2}{32\pi^2} \int \mathrm{d}^4x \, F^a_{\mu\nu} \tilde{F}^{\mu\nu}_a \in \mathbb{Z}$.
- Anomaly $\partial_\mu (j^\mu_L - j^\mu_R) = -\frac{N_f g^2}{16\pi^2} F^a_{\mu\nu} \tilde{F}^{\mu\nu}_a \Rightarrow \Delta(N_L - N_R) = 2N_f Q_w$.

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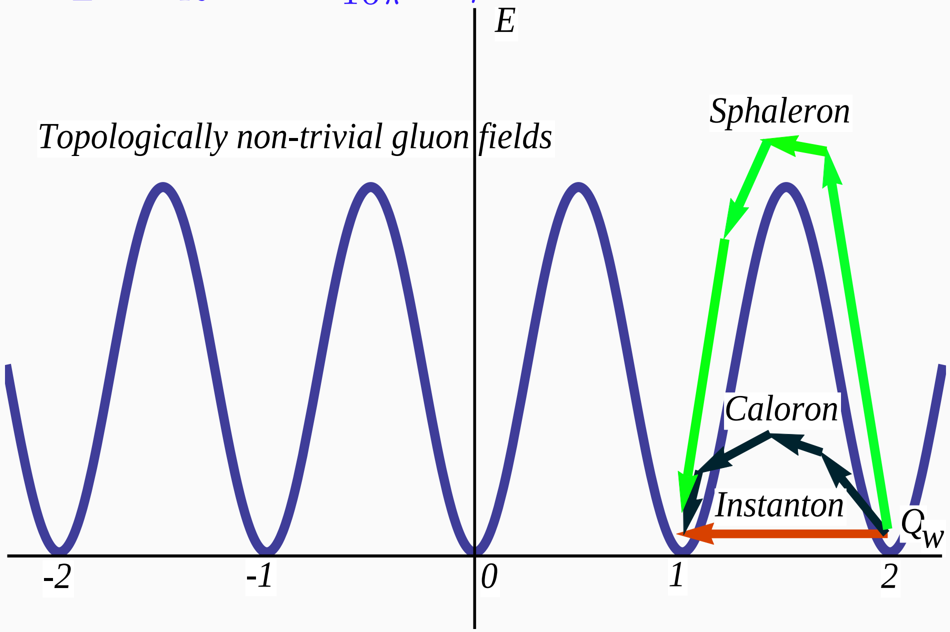

- Sphalerons ("ready to roll"): thermally induced changes in Q_w
- pQCD: sphalerons the most dominant Q_w decay: Moore et al '97
- Sphaleron decay rate: $\frac{d(N_L - N_R)}{dt d^3x} \approx 192.8\, \alpha_s^5\, T^4$

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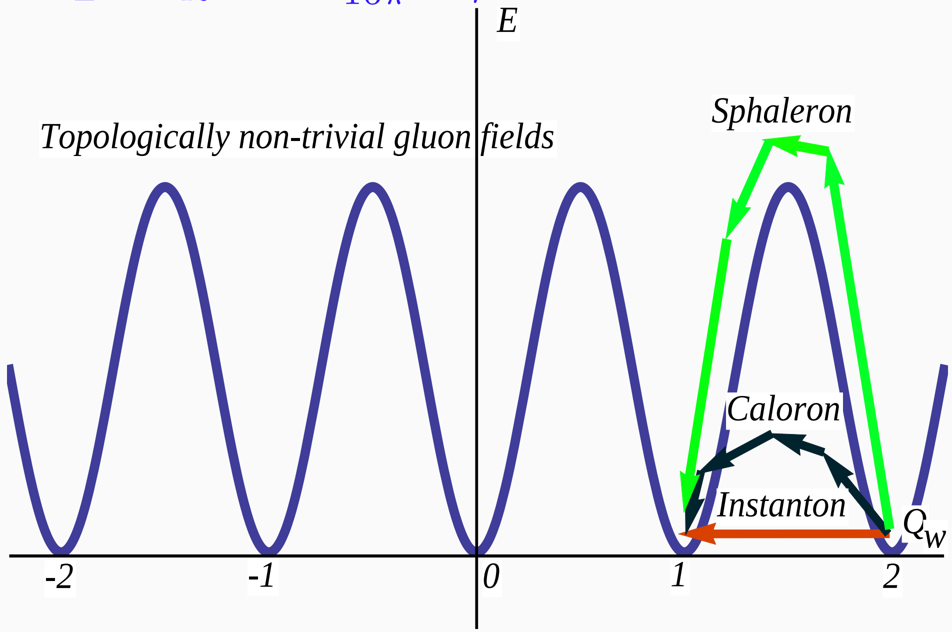

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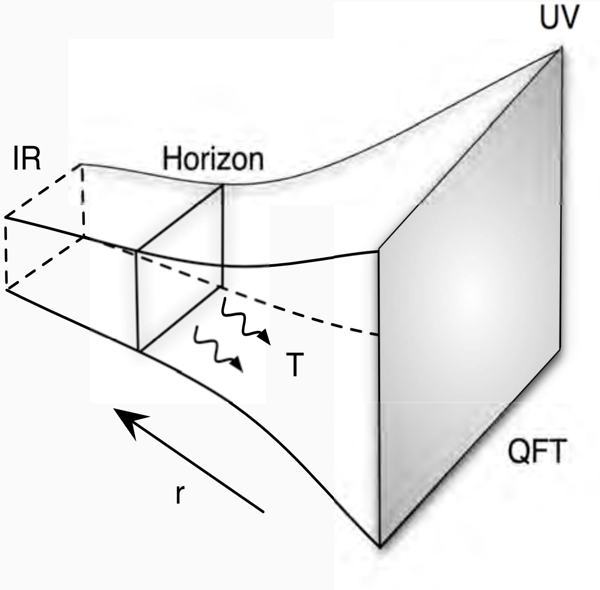
But should we trust pQCD in the Quark-gluon plasma?

Holographic calculation



Finite T, $N_c \gg 1$, $\alpha_s \gg 1$ QFT $\Leftrightarrow$ GR
on black holes in 5D

Maldacena '97; Witten; Gubser, Klebanov, Polyakov '98

1. A bulk fluctuation $\phi(x, r) \Leftrightarrow \mathcal{O}(x)$ on the boundary.

$$\exp(-S_{GR}[\phi(x, r) \to \phi_0(x)]) = \langle \exp(-\int dx \mathcal{O} \phi_0) \rangle$$

2. e.g. $\langle \mathcal{O}(x_1)\mathcal{O}(x_2) \rangle$ computed from $\hat{\nabla}^2 \phi = m^2 \phi$ on the BH.

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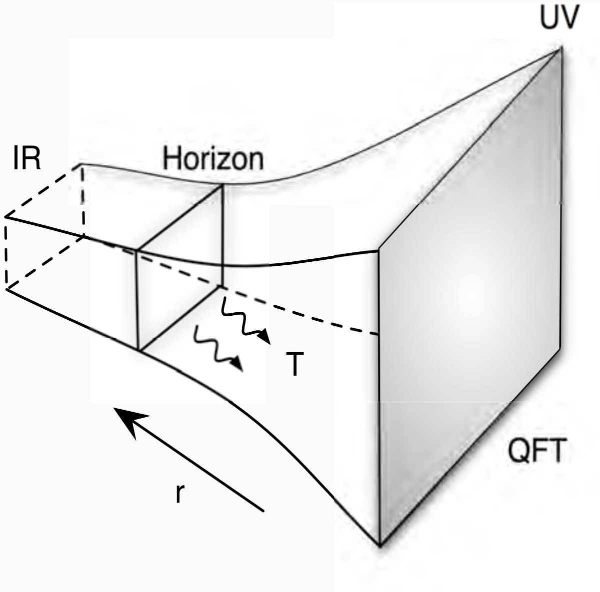

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2. e.g. $\langle \mathcal{O}(x_1)\mathcal{O}(x_2) \rangle$ computed from $\hat{\nabla}^2\phi = m^2\phi$ on the BH.
3. Recall $\omega J^{\mu 5}(\omega) \propto \mathrm{Tr} F\tilde{F}(\omega)$. Introduce CP odd axion $a(r,x)$
4. The source term $\int d^4x a_0(x) \mathrm{Tr} F\tilde{F}(x)$ with $a(r,x) \to a_0(x)$ at the boundary.

Holographic calculation of $\Delta(N_L - N_R)$

- Initial excess $N^5 \equiv N_L - N_R$ near thermal equilibrium.
- Described by perturbation $\mathcal{L} \to \mathcal{L} + \epsilon \mathrm{Tr} F\tilde{F}$
- Linear response theory: $\frac{d}{dt}N_5 \to \omega J^{05} \propto \langle \mathrm{Tr}F\tilde{F}\,\mathrm{Tr}F\tilde{F}(\omega)\rangle N_5$
- Should calculate the decay rate $\Gamma_{CS} \sim \langle \mathrm{Tr}F\tilde{F}\,\mathrm{Tr}F\tilde{F}(\omega)\rangle$
- Holography: Study $\hat{\nabla}^2 a(r,x) = 0$ on the 5D BH.

Holographic calculation of $\Delta(N_L - N_R)$

AdS/CFT: $\Gamma_{CS} = \frac{(g^2 N_c)^2}{256\pi^3} T^4$, Son, Starinets '02

Phenomenologically interesting region $T \approx T_c$ where conformality breaks down.

- Work it out in non-CFT: $\Gamma_{CS}(T_c) \geq s(T_c) T_c C$

  U.G, Iatrakis, O'Bannon, Kiritsis, Nitti '12

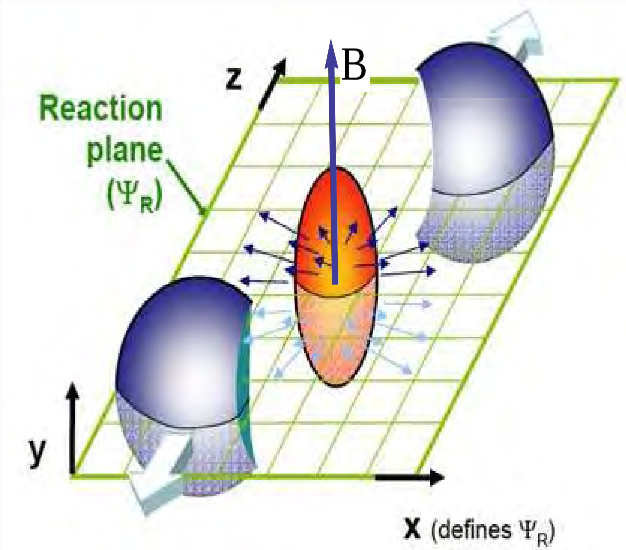
See talk by I. Iatrakis

- Comparison of AdS/CFT with non-AdS/non-CFT at T_c: $\Gamma_{CS}^{CFT} \approx 0.045 T_c^4$ vs. $\Gamma_{CS} \approx 1.64 T_c^4$
- Conclusion: both pQCD and "realistic" holography in favor of the chiral magnetic effect in HICs

PART II: Induced currents in the QGP

ongoing work with D. Kharzeev and K. Rajagopal

Biggest problem in search for magnetic effects in QGP: how to distinguish effects of B from effect of *anisotropy*



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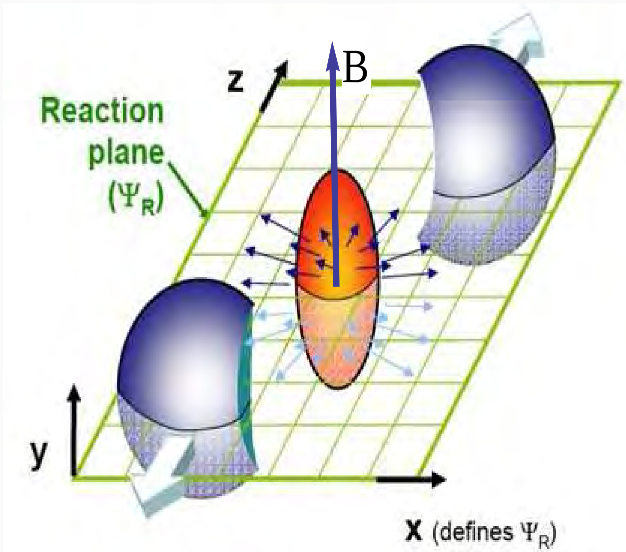
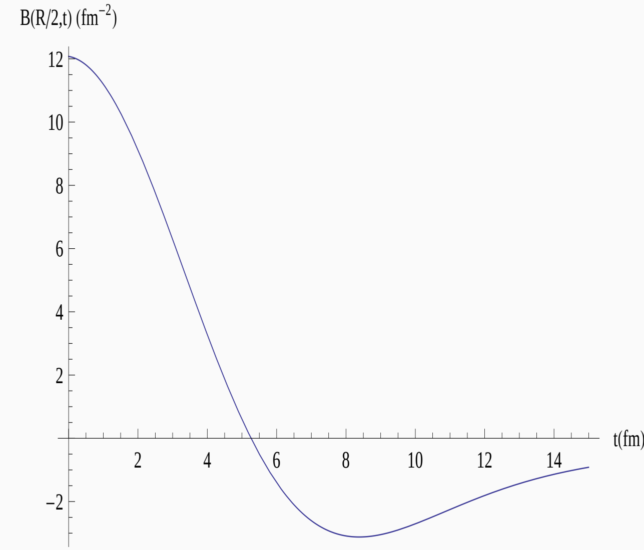
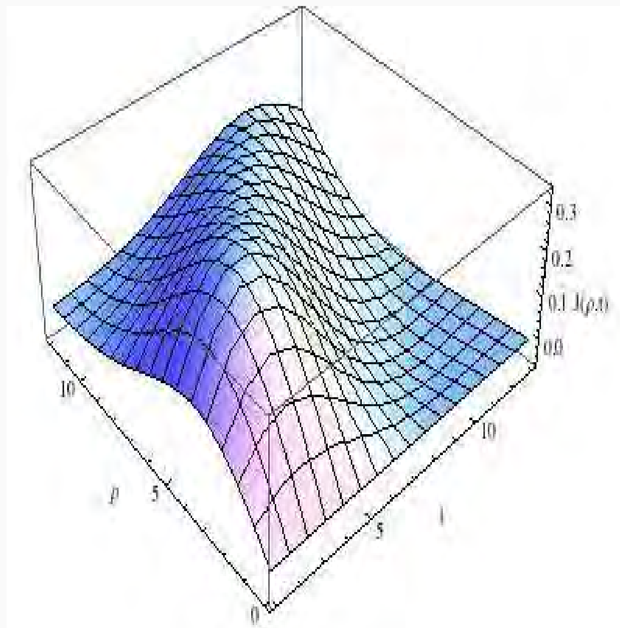




- Charged medium $\Rightarrow$ t-dependence of B induces circular currents around y-axis.

- For initial $\vec{B}(\rho, t_0) = \hat{y} B_0 e^{-\frac{\rho^2}{R^2}}$ straightforward to calculate:
- $J^\theta(\rho, t) = \frac{B_0}{2} R^2 \sigma\, e^{-\frac{\sigma t}{2}} \int_0^\infty dp_\perp\, p_\perp\, \Psi_2(-p_\perp^2 \rho^2/4) e^{-\frac{1}{4} p_\perp^2 R^2}$
$$\times \left(\frac{\sigma}{2} \cos\left(t\sqrt{p_\perp^2 - \sigma^2/4} \right) + \sqrt{p_\perp^2 - \sigma^2/4}\, \sin\left(t\sqrt{p_\perp^2 - \sigma^2/4} \right) \right)$$



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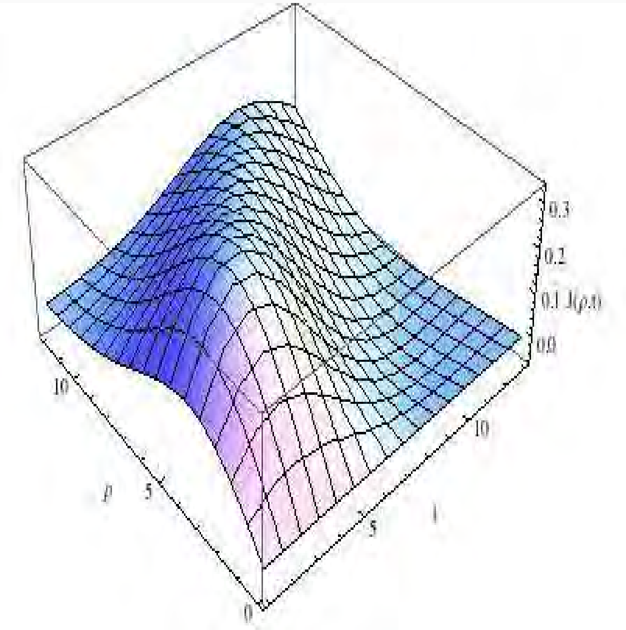
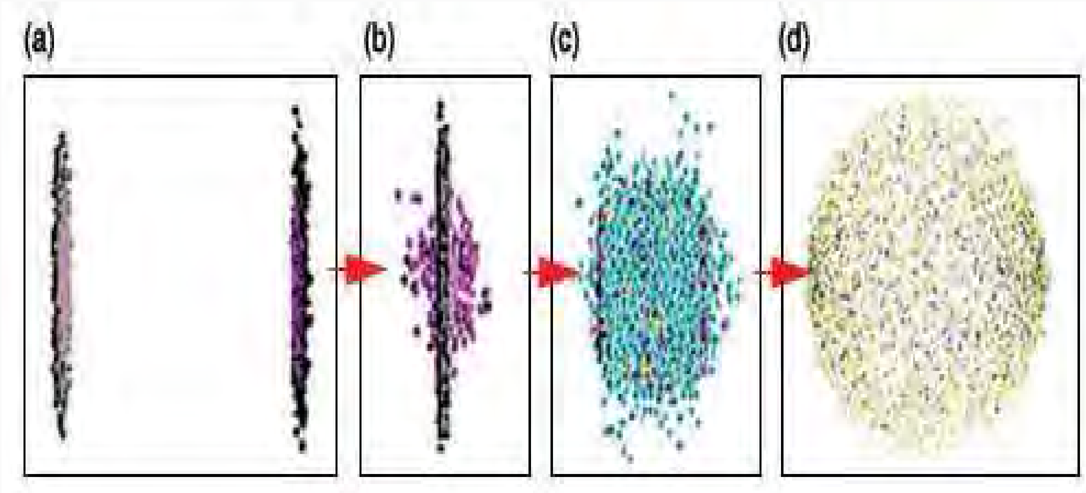


- Circular velocity component in QGP due to B: $\vec{v}_B$.



- However QGP is an expanding fluid with 4-velocity $u^\mu(x)$
- One has to add the circular velocity to $u^\mu(x)$
- Suppose we know u^μ, assume $|\vec{v}_B| \ll |\vec{u}|$
- Treat v_B as perturbation, ignore backreaction on u:

$$\vec{V}^i = \frac{\vec{u}^i\left(1+\frac{\vec{u}\cdot\vec{v}_B}{\vec{u}\cdot\vec{u}}\right)+\sqrt{1-\vec{u}\cdot\vec{u}}\left(\delta_{ij}-\frac{u^i u^j}{\vec{u}\cdot\vec{u}}\right)v_B^j}{1+\vec{u}\cdot\vec{v}_B}\,.$$

- Construct the total 4-velocity V^μ: contains all observable information on time varying B.

Do we know u^μ of this expanding fluid?

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- As a first step, assume: Bjorken '83
  1. Boost invariance along z: $\xi = z\partial_t + t\partial_x$
  2. Rotation around z: $\xi = x\partial_y - y\partial_x$
  3. Translations in transverse plane: $\xi = \partial_x$ and $\xi = \partial_y$
- Solution to $[\xi, u] = 0$ is $u = \partial_\tau$ $(ds^2 = -d\tau^2 + \tau^2 d\eta^2 + dx_\perp^2 + x_\perp^2 d\phi^2)$
- Hydrodynamics: $\nabla_\mu T^{\mu\nu} = 0$ with
  $T_{\mu\nu} = \epsilon u_\mu u_\nu + p(g_{\mu\nu} + u_\mu u_\nu)$ + visc.
- Bjorken's flow: fine except transverse translations

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- Gubser's flow Gubser '10
- Replace $\xi = \partial_x,\ \partial_y$ with $\xi_i = \partial_i + q^2\left[2x^i x^\mu \partial_\mu - x^\mu x_\mu \partial_i\right]$
- Solution to $[\xi, u] = 0$ is $u = \cosh\kappa\partial_\tau + \sinh\kappa\partial_\perp$ with
  $\kappa = \frac{2q^2\tau x_\perp}{1+q^2\tau^2+q^2x_\perp^2}$
- Solution to Hydrodynamics: $\nabla_\mu T^{\mu\nu} = 0$ with
  $\epsilon = \frac{\hat{\epsilon}_0}{\tau^{4/3}} \frac{(2q)^{8/3}}{\left[1+2q^2(\tau^2+x_\perp^2)+q^4(\tau^2-x_\perp^2)^2\right]^{4/3}}$

How to test Gubser's flow?

- Hadron spectrum from hydrodynamic flow: Cooper-Frye:

$$S_i = p^0 \frac{dN_i}{dp^3} = -\frac{g_i}{(2\pi)^3} \int d\Sigma_\mu p^\mu F\left(\frac{p^\mu V_\mu}{T_f}\right)$$

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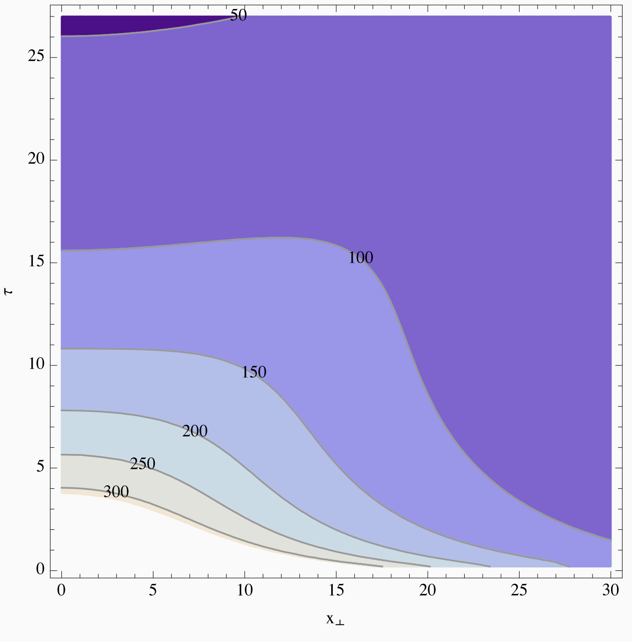

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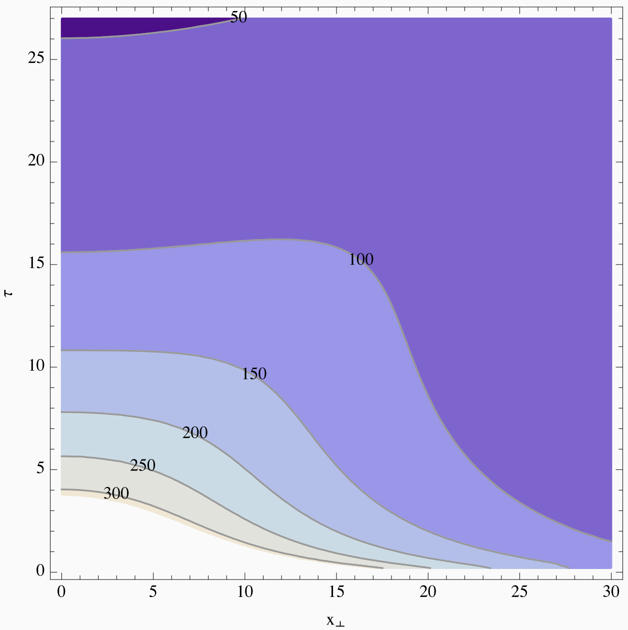

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- $S_i(p_T) =$
  $\frac{g_i}{2\pi^2} \int dx_\perp x_\perp \tau_f \left\{ K_1(\frac{m_T u^\tau}{T_f}) I_0(\frac{p_T u^\perp}{T_f}) - \tau_f' p_T K_0(\frac{m_T u^\tau}{T_f}) I_1(\frac{p_T u^\perp}{T_f}) \right\}$
- Gubser's flow is independent of Φ_p and Y

Comparison to data

Need to fix parameters q and $\hat{\epsilon}_0$. Some tension between realistic spectrum and hydronization temperature $T_h \approx 400 - 550$ MeV $\Rightarrow$ Optimal solution $q = 1/11$ fm^{-1} and $\hat{\epsilon}_0 = 15^4$

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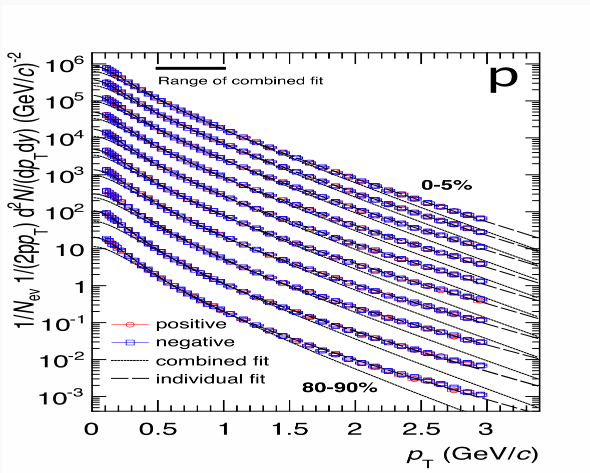

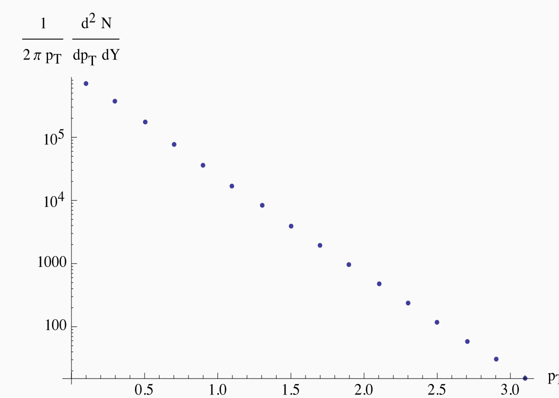

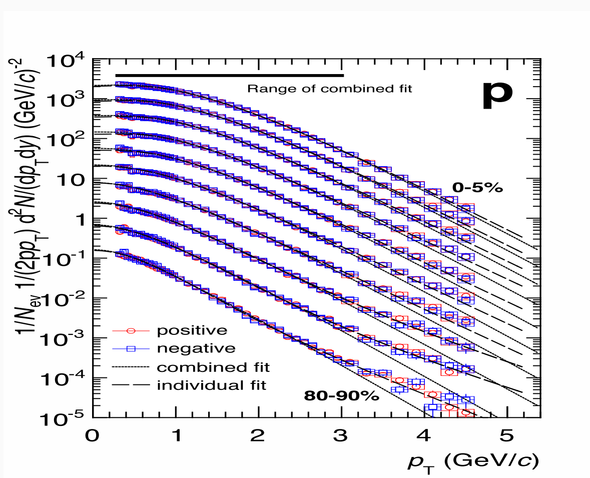

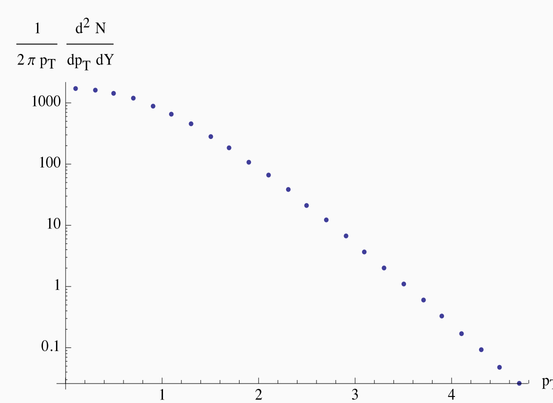

Effect of circular current on spectra

- Decompose spectrum in flow parameters:
  $S_i = v_0 \left(1 + v_1(p_T, Y)\cos(\phi_p) + \cdots\right)$
- $v_1|_{\text{Gubser}} = 0$ but $v_1|_{\text{Gubser}+J} \neq 0$, solely due to B!

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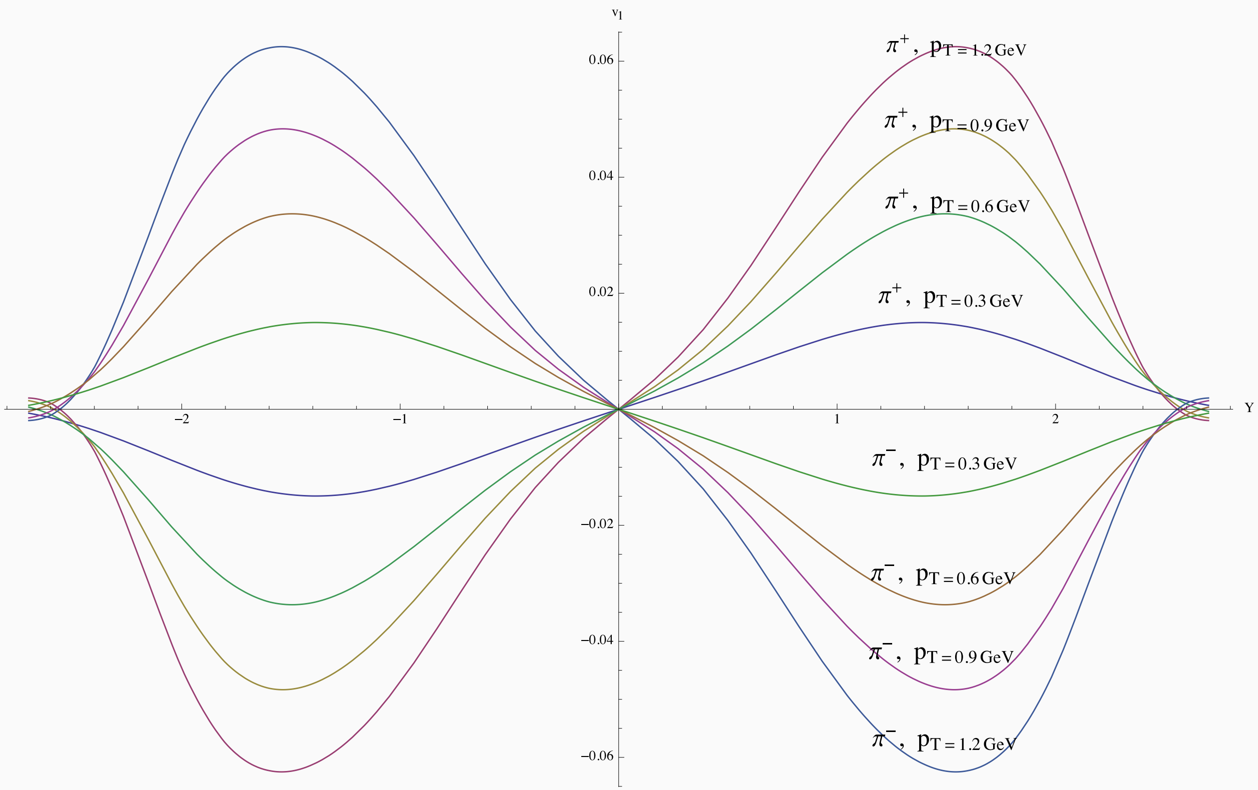

A (very preliminary) prediction for LHC.

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  - Holographic calculation prefers large decay rate $\Rightarrow$ significant charge separation at HICs
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  - To be compared with data
- Outlook:
  - Relax rotational symmetry along z and find analytic hydro solution?
  - Going beyond the perturbative treatment $\Rightarrow$ full magnetohydrodynamics.

- Summary:
  - QFT anomalies + sphalerons $\Rightarrow$ chiral magnetic effect
  - Holographic calculation prefers large decay rate $\Rightarrow$ significant charge separation at HICs
  - To distinguish anisotropies in QGP from effects of B, calculate v_1 from Gubser's flow
  - To be compared with data
- Outlook:
  - Relax rotational symmetry along z and find analytic hydro solution?
  - Going beyond the perturbative treatment $\Rightarrow$ full magnetohydrodynamics.
  - Transport coefficients e.g. $\eta(B)$
  - Anomalous transport
  - Analogies with cosmology?

THANK YOU !