Bold Diagrammatic Monte Carlo Study of φ^4 Theory

Ali Davody

IPM

Evaluation of Partition Function in strong coupling regime

- Analytical solutions are limited to special cases (exactly solvable models – Holography)

- Using numerical methods to calculate physical quantities

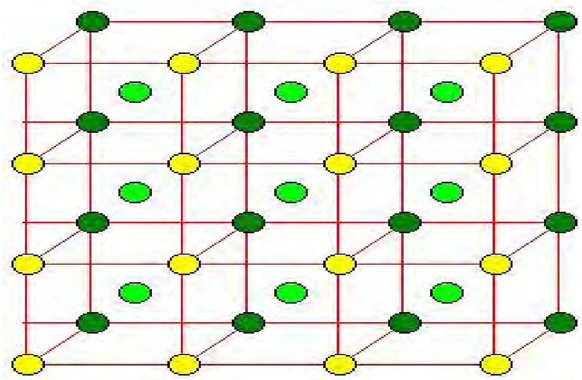
Lattice Field Theory

- Based on path integral formulation of field theory

$$Z = \int D\phi \; e^{-S} = \sum_{lattice} e^{-S}$$

- Summation all possible configuration is not possible

- Using Monte Carlo techniques to find the sum



Shortcomings of lattice approach

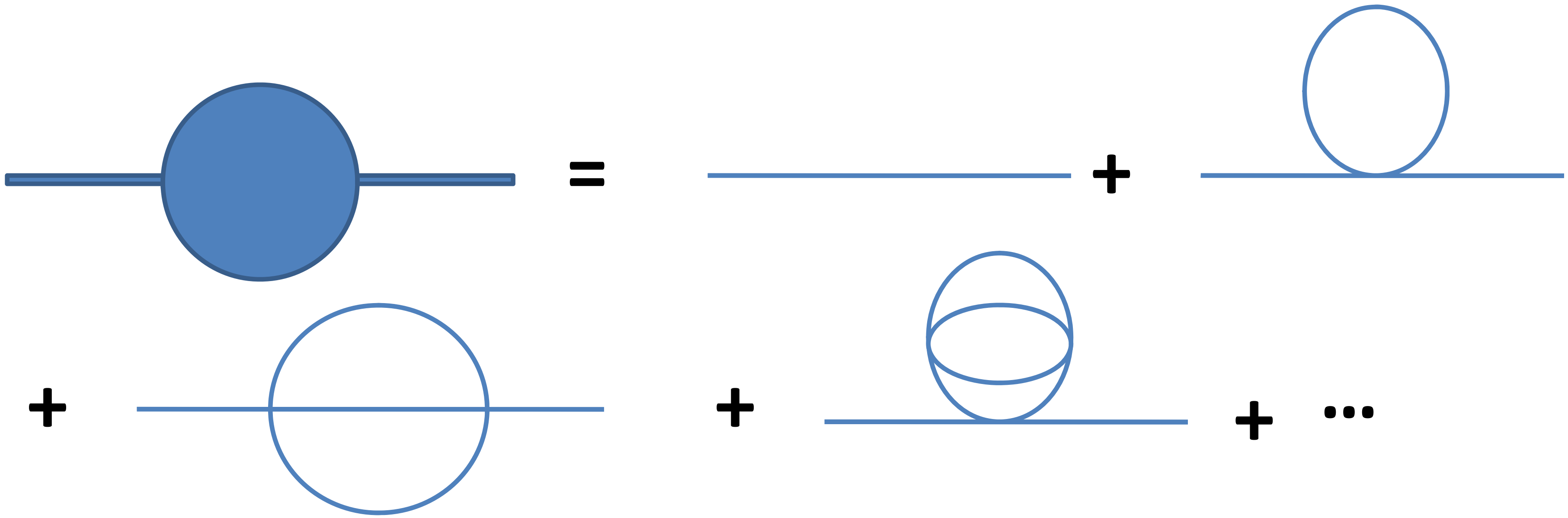
- Finite size effects

- Sign problem
  - Finite chemical potential
  - Real time field theory

Diagrammatic Monte Carlo

- Using diagrammatic formulation of field theory

- stochastically sampling Feynman diagrams



$$Q(y) = \sum_{m=0}^{\infty} \sum_{\xi_m} \int D(\xi_m, y, x_1, x_2, \ldots, x_m) \quad dx_1 dx_2 \cdots dx_m$$

Using diagrammatic rules to evaluate these functions

- Generate a representative ensemble of random diagrams

$$(\xi_m, y, x_1, x_2, \ldots, x_m)$$

and sample $Q(y)$ from this ensemble

- advantage of DMC:

  calculating physical quantities in thermodynamic limit

  (no finite size effect)

- shortcoming :

  due to divergence of perturbation series a resummation technique is required to make the Scheme convergent

Bold Diagrammatic Monte Carlo

- A way to improve the convergence of diagrammatic Monte Carlo scheme is to expand physical quantities in term of full- screened (bold) correlation function instead of free correlators

A simple example

$$f = a - au + au^2 - au^3 + \cdots$$

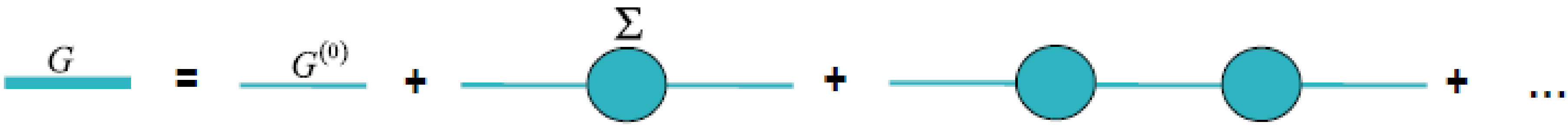
↓

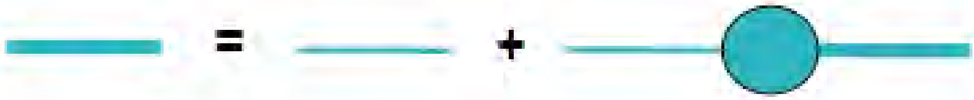
|u|>1, divergent series:
resummation techniques

$$f = a - u f$$

The simplest case of self-consistent ("Bold") Diagrammatic MC

Dyson summation





$$G = G^{(0)} + G^{(0)} \Sigma G$$

Green's function

bare (non-perturbed) Green's function

self-energy

Dyson equation

Bold Diag MC scheme

Sample $\Sigma[G]$ by Diag MC using current knowledge of G.

Improve knowledge of G by solving Dyson equation with current Σ.

$$G = G^{(0)} + G^{(0)}\Sigma G$$

updated G

troublesome sign problem becomes an advantage for convergence of the MC scheme

Sign Problem is Welcome!

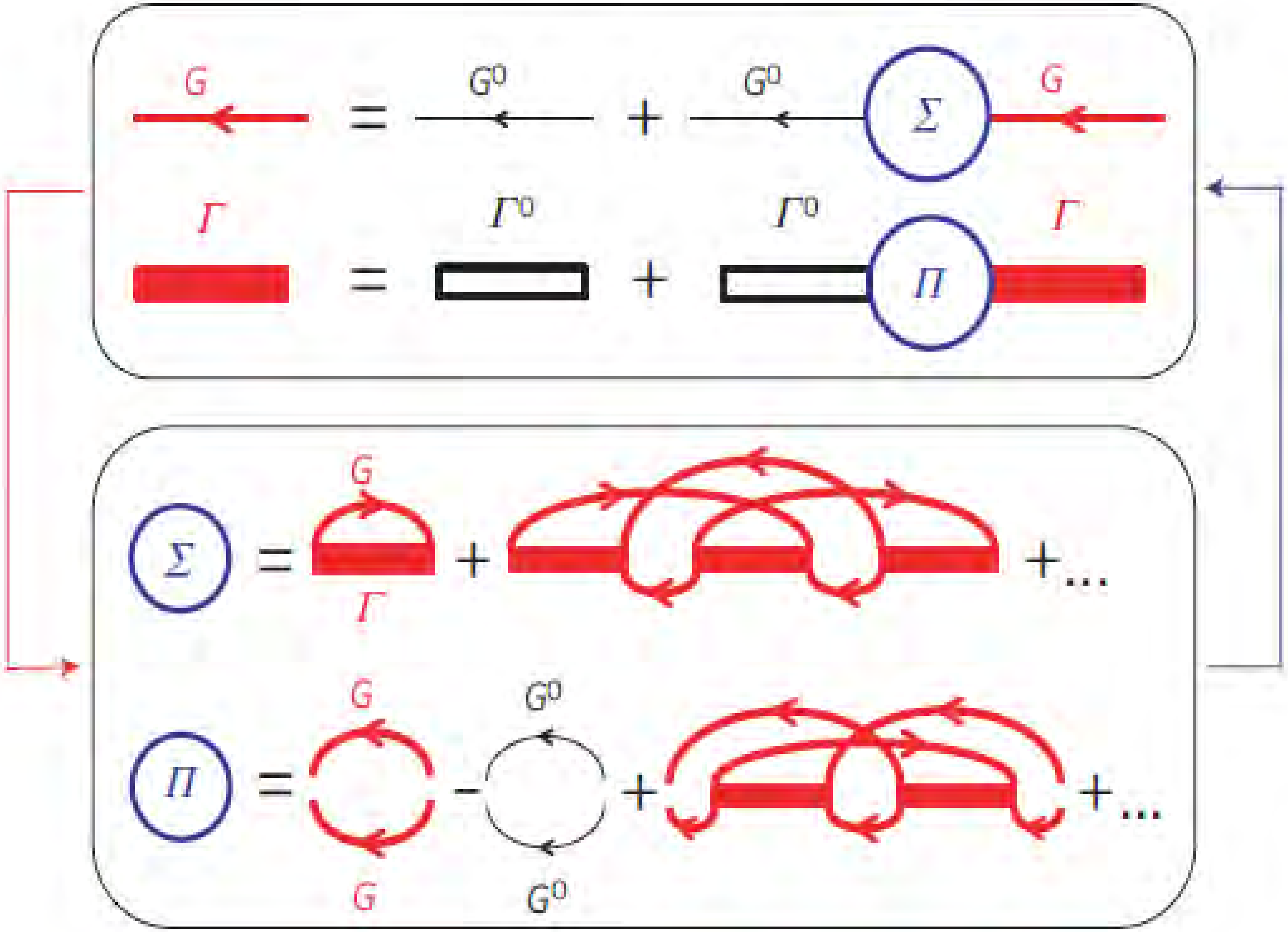
Unitary Fermi gas

- A strongly interacting fermionic system

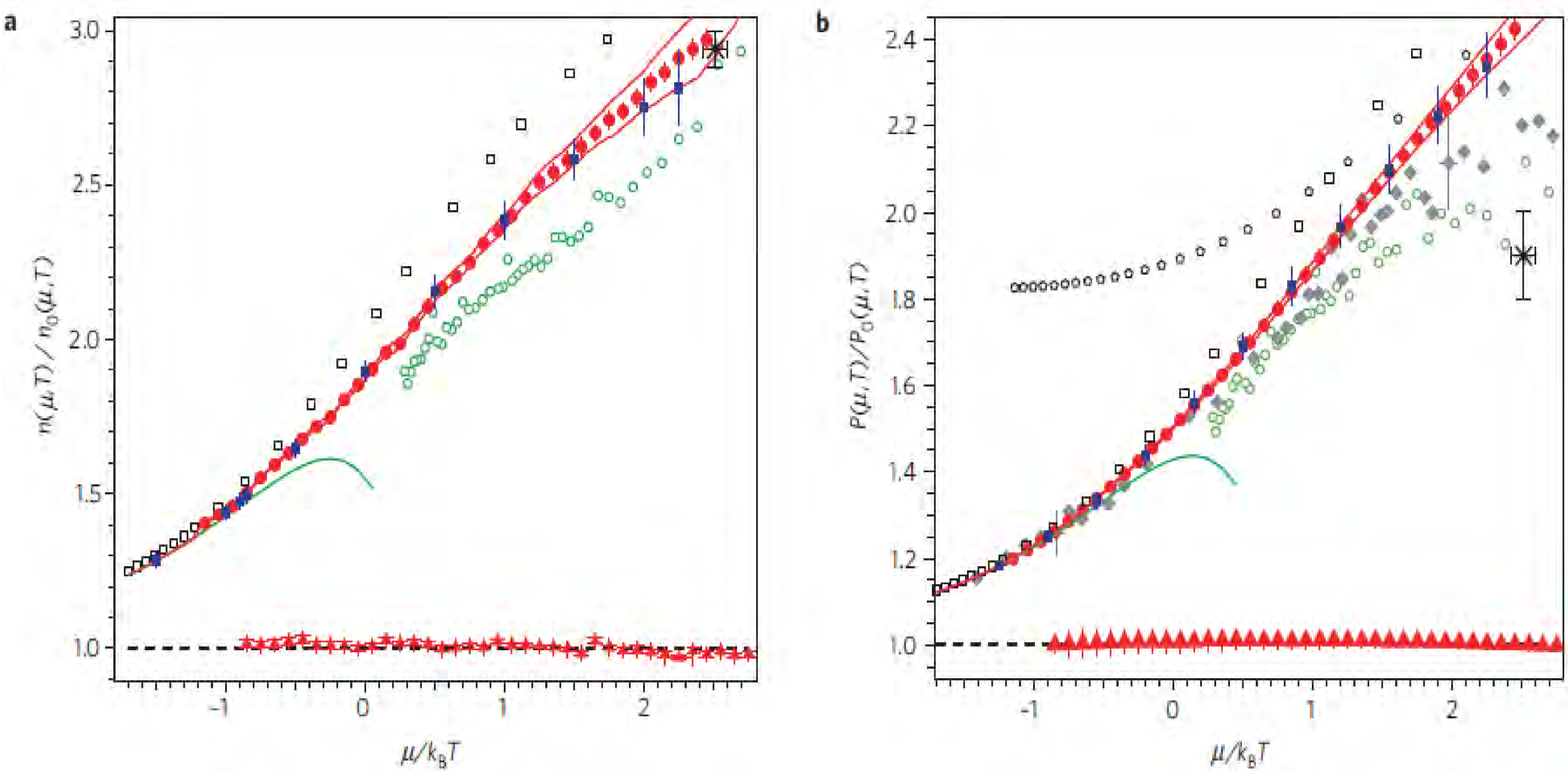
- The equation of state at finite chemical potential

$$S_F = \int d^d x \, d\tau \left\{ \sum_\sigma \psi_\sigma^* \left(\partial_\tau - \frac{\nabla^2}{2m} - \mu_\sigma \right) \psi_\sigma + g_0 \psi_\uparrow^* \psi_\downarrow^* \psi_\downarrow \psi_\uparrow \right\}$$

K. Van Houcke et al. , Nature Phys. 8, 366 (2012)


G
G^0
G^0
Σ
G
Γ
Γ^0
Γ^0
Π
Γ
Σ
G
Γ
+...
Π
G
G
G^0
G^0
+...

Equation of state of the unitary Fermi gas in the normal phase.



K. Van Houcke et al. , Nature Phys. 8, 366 (2012)

BDMC & High Energy physics

- Applying BDMC method for field theories which are more relevant to high energy physics

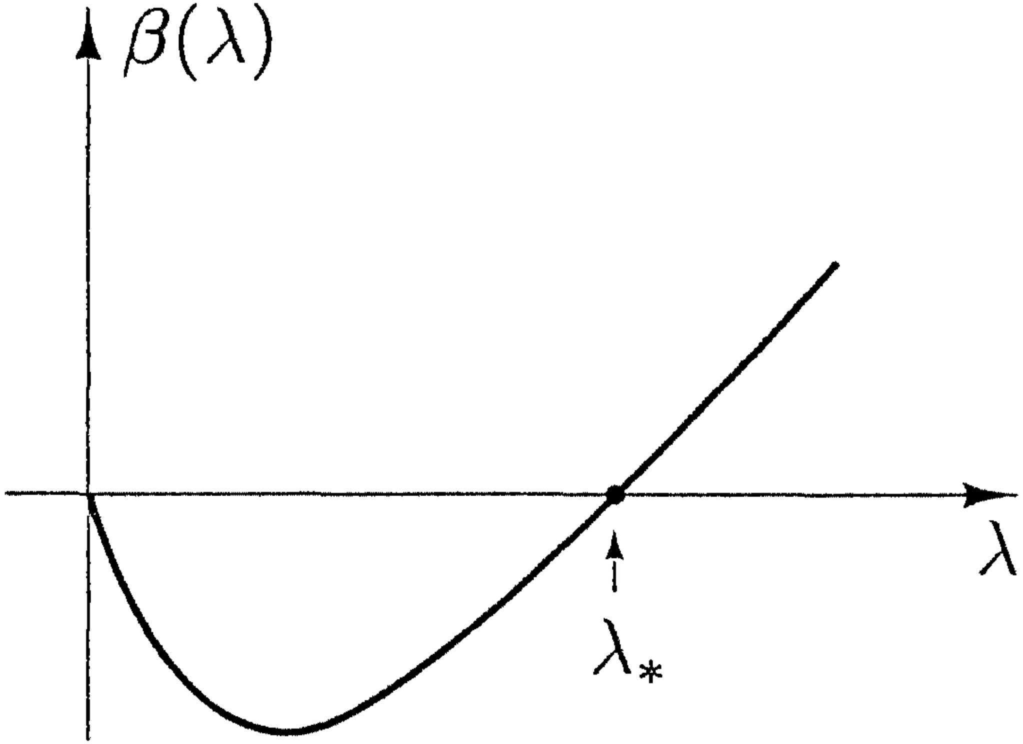
- Incorporating Renormalization procedure into BDMC

Phi^4 theory in three dimensions

$$S_b[\phi_b] = \int d^3x \left\{ \frac{1}{2}\phi_b(x)\left(-\partial^2 + m_0^2\right)\phi_b(x) + \frac{g_0}{4!}\phi_b^4(x) \right\}$$

- The theory is super-renormalizable.
- All vertex diagrams are finite.
- IR fixed point (Wilson-Fisher)

There is a nontrivial IR fixed point

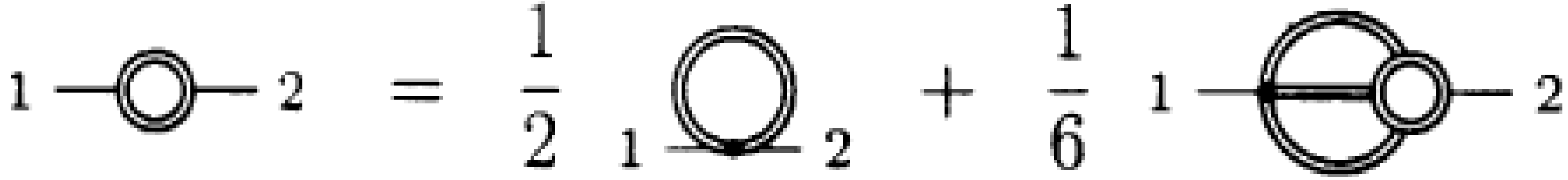


$$g_0 \to \infty \quad \Rightarrow \quad g = g^* + \frac{1}{g^{\omega}} + \frac{1}{g^{2\omega}} + \cdots$$

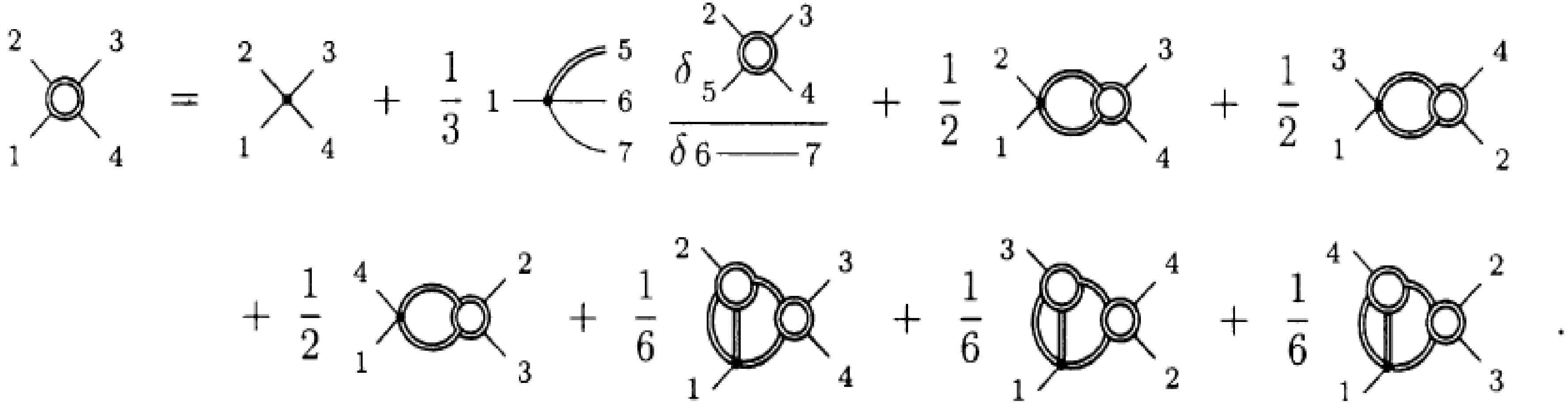
A set of Schwinger-Dyson equation s

$$E[\phi] = \frac{1}{2}\int_{12} G_{12}^{-1}\phi_1\phi_2 + \frac{1}{24}\int_{1234} V_{1234}\phi_1\phi_2\phi_3\phi_4$$

$$\begin{aligned}\boldsymbol{\Sigma}_b(p) &\equiv \Gamma_b(p) - \boldsymbol{\Gamma}_b(p)\\ &= -\frac{g_0}{2}\int\frac{d^dk}{(2\pi)^d}\boldsymbol{G}_b(k) + \frac{g_0}{6}\int\frac{d^dk}{(2\pi)^d}\frac{d^dq}{(2\pi)^d}\boldsymbol{G}_b(k)\,\boldsymbol{G}_b(q)\boldsymbol{G}_b(Q)\boldsymbol{\Gamma}_b(k,q,Q,p)\end{aligned}$$



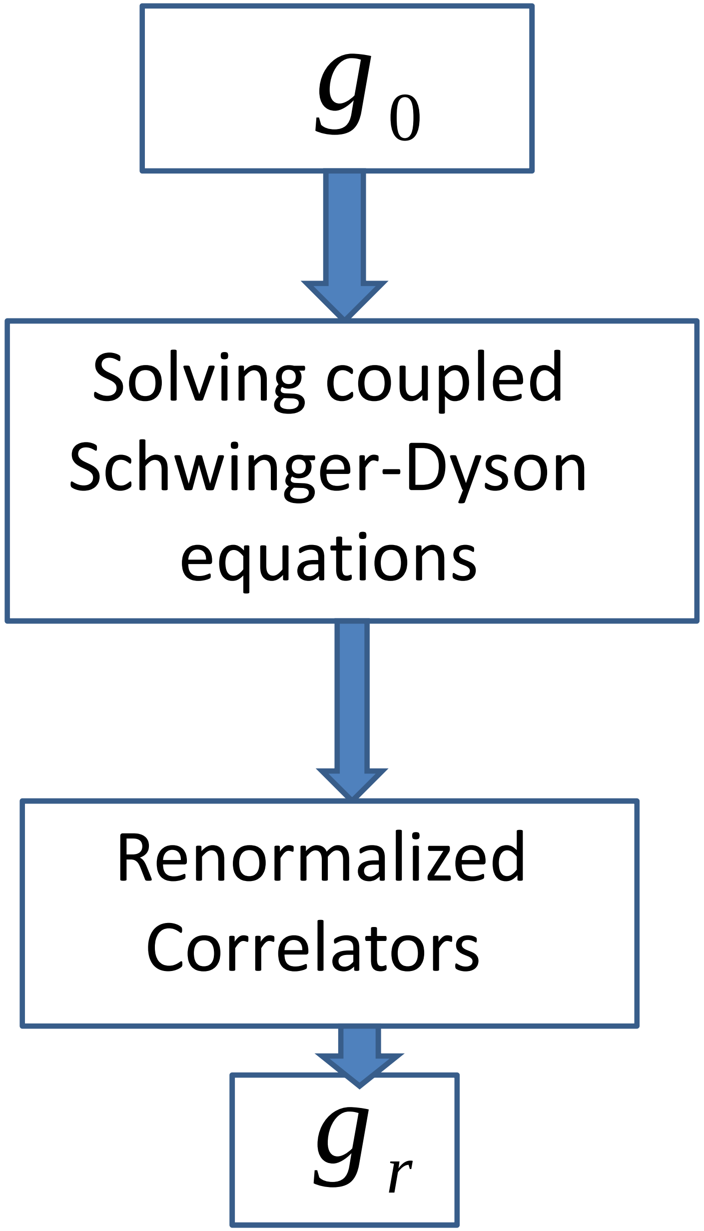
$$
\begin{aligned}
\boldsymbol{\Gamma}_{b,1234} \;=\; & V_{b,1234} - \frac{1}{3}\int_{567890} V_{b,1567}\boldsymbol{G}_{b,58}G_{69}G_{70}\frac{\delta\boldsymbol{\Gamma}_{b,8234}}{\delta G_{90}} - \frac{1}{2}\int_{5678} V_{b,1256}\boldsymbol{G}_{b,57}\boldsymbol{G}_{b,68}\boldsymbol{\Gamma}_{b,7834} \\
& - \frac{1}{2}\int_{5678} V_{b,1356}\boldsymbol{G}_{b,57}\boldsymbol{G}_{b,68}\boldsymbol{\Gamma}_{b,7824} - \frac{1}{2}\int_{5678} V_{b,1456}\boldsymbol{G}_{b,57}\boldsymbol{G}_{b,68}\boldsymbol{\Gamma}_{b,7823} \\
& + \frac{1}{6}\int_{56789 0\bar{1}\bar{2}} V_{b,5167}\boldsymbol{G}_{b,69}\boldsymbol{G}_{b,70}\boldsymbol{\Gamma}_{b,902\bar{1}}\boldsymbol{G}_{b,\bar{1}\bar{2}}\boldsymbol{\Gamma}_{b,\bar{2}348}\boldsymbol{G}_{b,85} \\
& + \frac{1}{6}\int_{56789 0\bar{1}\bar{2}} V_{b,5167}\boldsymbol{G}_{b,69}\boldsymbol{G}_{b,70}\boldsymbol{\Gamma}_{b,903\bar{1}}\boldsymbol{G}_{b,\bar{1}\bar{2}}\boldsymbol{\Gamma}_{b,\bar{2}248}\boldsymbol{G}_{b,85} \\
& + \frac{1}{6}\int_{56789 0\bar{1}\bar{2}} V_{b,5167}\boldsymbol{G}_{b,69}\boldsymbol{G}_{b,70}\boldsymbol{\Gamma}_{b,904\bar{1}}\boldsymbol{G}_{b,\bar{1}\bar{2}}\boldsymbol{\Gamma}_{b,\bar{2}238}\boldsymbol{G}_{85}\,.
\end{aligned}
$$



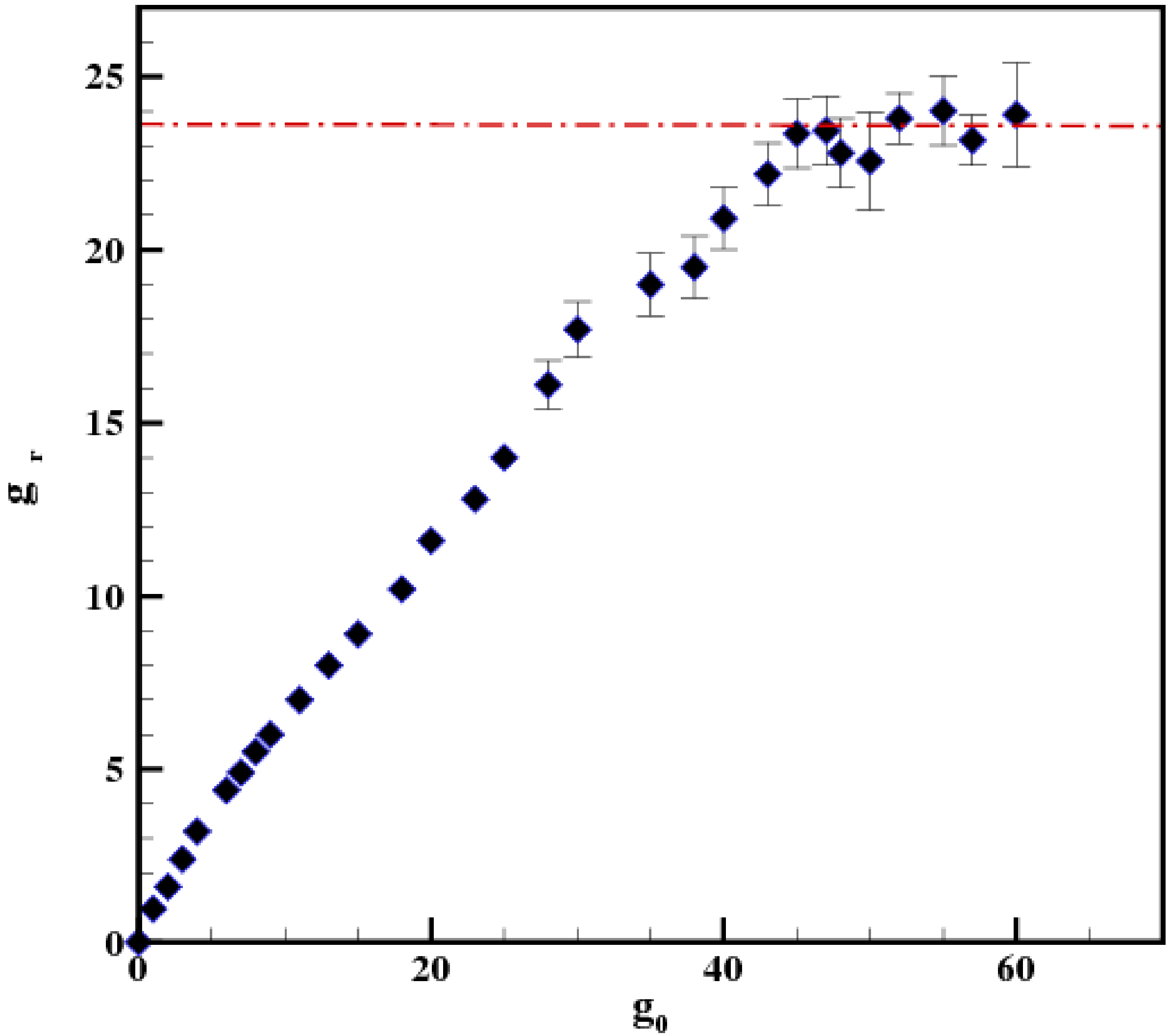
Renormalization

$$
\begin{aligned}
\phi_b &= Z^{\frac{1}{2}}(g)\,\phi \\
m_0^2 &= m^2\,\frac{Z_m(g)}{Z(g)} \\
g_0 &= g\; m^{4-d}\,\frac{Z_g(g)}{Z^2(g)}
\end{aligned}
$$

$$
\begin{aligned}
\mathbf{\Gamma}_r^{(2)}\,(p=0) &= m^2 \\
\frac{\partial}{\partial p^2}\mathbf{\Gamma}_r^{(2)}\,(p)\big|_{p=0} &= 1 \\
\mathbf{\Gamma}_r^{(4)}(0,0,0,0) &= m^{4-d}\,g
\end{aligned}
$$



g_0
Solving coupled Schwinger-Dyson equations
Renormalized Correlators
g_r



25
20
15
10
5
0
0
20
40
60
g_r
g_0

Lattice Field Theory	**BDMC**
IR Cutoff	No Finite Size effect
UV Cutoff	A Continues Scheme
Sign problem	Sign Problem is welcome
Convergent	Re-summation is required