String Self-energies on the Lightcone Worldsheet Lattice

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GP & C. Thorn

Outline

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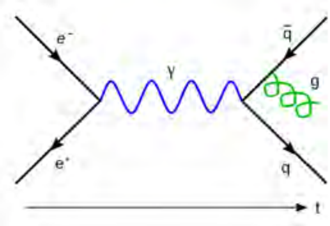

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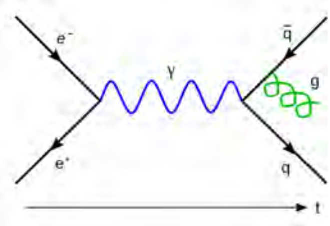

Our main tool for making sense of them is perturbation theory, which is however inapplicable at strong coupling, and cannot explain important physical phenomena such as confinement.[Millenium Prize]

Alternative approaches: Large $N \supset AdS/CFT$ OR lattice gauge theory, however quite challenging to combine both. N fixed at simulations.

Motivation

In this talk, we will view gauge theory as the infinite string tension T_0 ($\alpha' = (2\pi T_0)^{-1} \to 0$) limit of open string theory in flat space.[Scherk'71]

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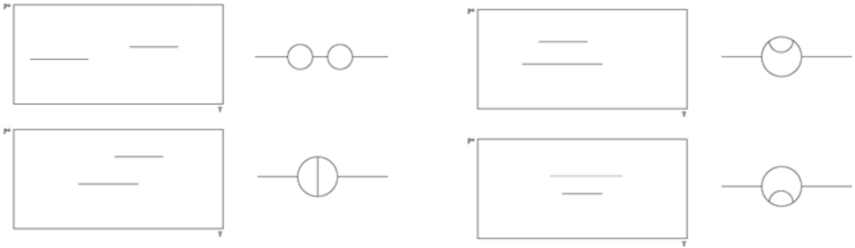

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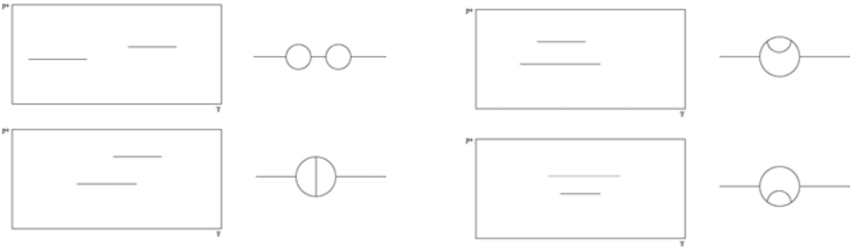

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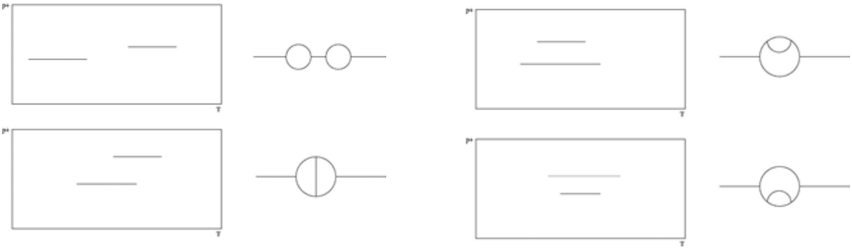

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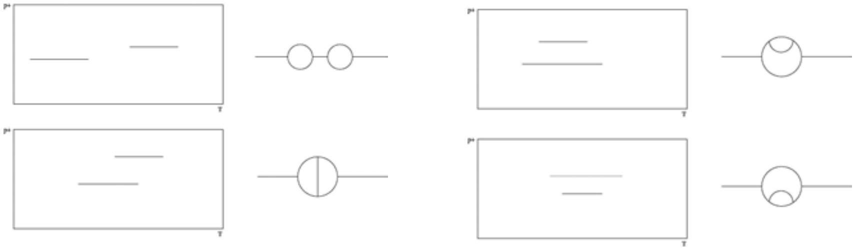

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Aim: Use lattice methods to sum planar multiloop string diagrams, and obtain information about large N QCD by taking $\alpha' \to 0$ at the end.

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- Left with unconstrained action for remaining transverse coordinates $\boldsymbol{x}$

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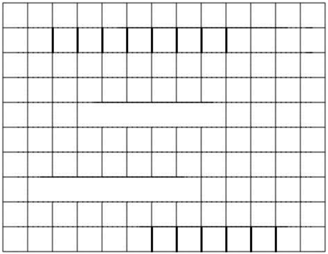

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- Extract string state energies by identifying exponential behaviors $e^{-a(N+1)E_\lambda(M)}$ of different P^- eigenvalues $E_\lambda(M)$. Typically

$$E_\lambda(M) \sim \alpha M + \beta + \frac{\gamma_\lambda}{M} + \dots .$$

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- First two terms lead to divergences in $m^2 = P^+P^-$, but at tree level can be canceled by two geometrical counterterms (bulk/boundary).

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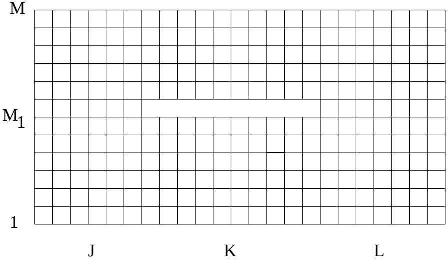

Self-energy diagram with

$$J + K + L = N + 1 \to \infty$$

and $J, L \sim N/2$

Hence characterized by K, M_1, M.

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$$-\frac{a\Delta P^-_{\text{tach}}}{M} \equiv \sum_{K=1}^{\infty} \delta P^-_K = \sum_{K=1}^{\infty}\left[\left(\frac{\coth(M\sinh^{-1}1)}{M\sqrt{2}}\right)^{1/2} \frac{e^{K\sum_{m=1}^{M-1}(\lambda^c_m-\lambda^o_m)-(K-1)(B_0+\epsilon)}}{\det A' \det B' \prod_{m=1,odd}^{M-1}(1-e^{-2K\lambda^o_m})}\right]^{12}$$

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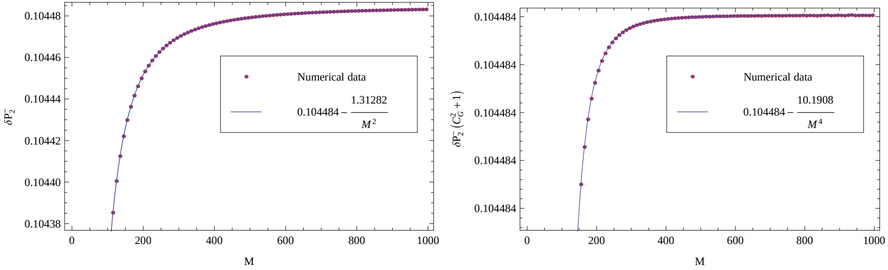
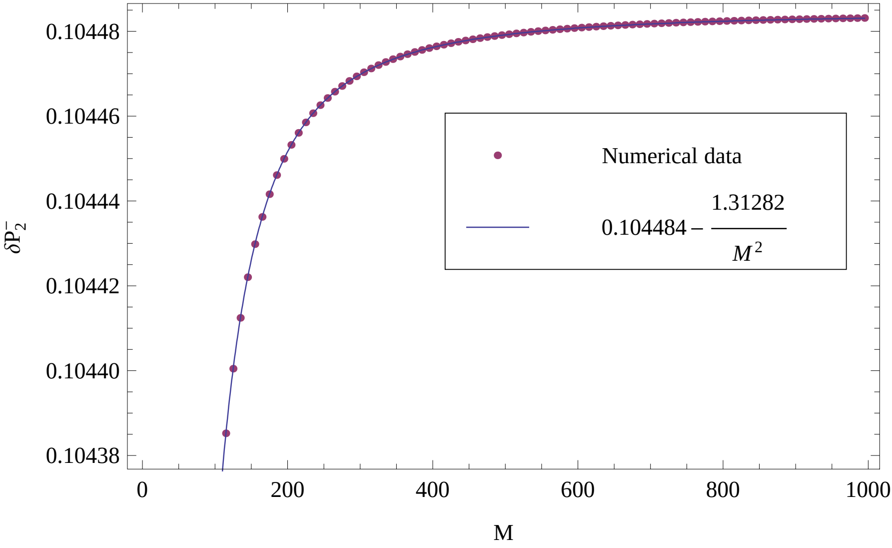
- We initially built up path integral from products of free string propagators.[GP, Thorn'12A]
- Obtained complicated formula involving M-dimensional determinants
- We numerically evaluated summand for wide range of M, K and found dependence by fitting with the help of `Mathematica`.

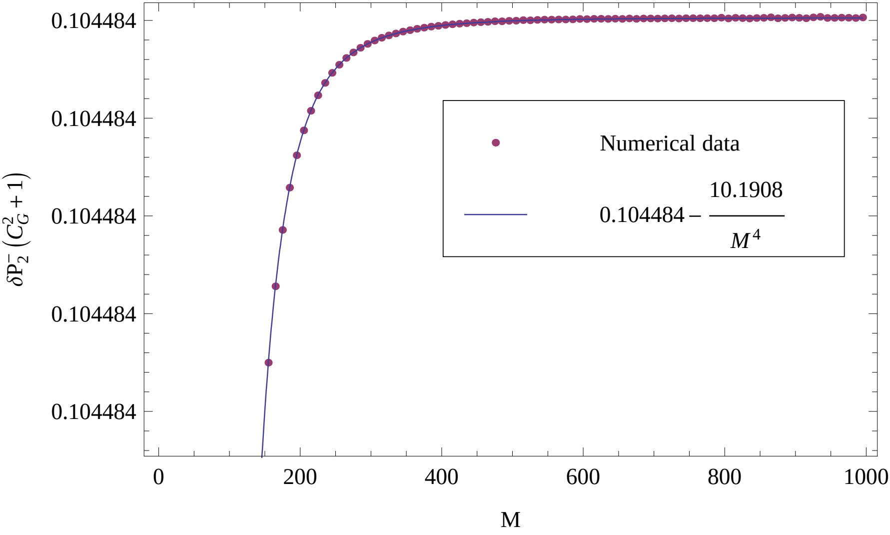


Analyzed ground (tachyon) and 1st excited (graviton) state, left- and right-hand side respectively

Numerics: Tachyon Self-energy Summand



Numerics: Graviton Self-energy Summand



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Then $\mathcal{D} = \mathcal{D}_0 \sum_{\{S\}} \det^{-12}(I + V\Delta) e^{-A(\{S\})}$, where

$$\Delta_{ij,kl} = T_0 \langle x_i^j x_k^l \rangle = T_0 \frac{\int \mathcal{D}x \;\; x_i^j x_k^l \;\; e^{-S}}{\int \mathcal{D}x \;\; e^{-S}}$$ WS propagator.

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Main result of paper: Closed WS propagator a simple sum

$$\Delta^c_{hj,kl} = \frac{N_T - |l-j|}{2M} + \frac{1}{2M}\sum_{m=1}^{M-1}\frac{e^{-|l-j|\lambda^c_m}}{\sinh\lambda^c_m}\exp\frac{2m(h-k)i\pi}{M}$$

where $2N_T = 2N + l - j$, and similarly for open string.

Testing Efficiency

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$$h_{lp} = \delta_{lp} + \Delta_{(k+1)l,kp} - \Delta_{kl,kp} + \Delta_{kl,(k+1)p} - \Delta_{(k+1)l,(k+1)p}\,.$$

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- Allows for asymptotic expansion in M with the Euler-Maclaurin formula,

$$\frac{1}{M}\sum_{m=0}^{M-1} f\left(\frac{m}{M}\right) = \int_0^1 dx f(x) - \frac{1}{2M}\, f(x)\big|_0^1 + \sum_{k=1}^{\infty} \frac{B_{2k}}{(2k)!} \frac{f^{(2k-1)}(x)\big|_0^1}{M^{2k}}$$

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- Allows for asymptotic expansion in M with the Euler-Maclaurin formula, and coefficients can be calculated exactly!

K	$\det(h_{lp}(x))$ up to $\mathcal{O}(x)$, $x = \frac{\pi}{6M^2}$
2	$\frac{1}{2} + x$
3	$-\frac{4}{\pi^2} + \frac{2}{\pi} + \frac{4x}{\pi}$
4	$-2 - \frac{64}{\pi^3} + \frac{16}{\pi^2} + \frac{8}{\pi} + \left(-4 + \frac{16}{\pi}\right) x$
5	$-16 - \frac{8192}{9\pi^4} - \frac{2048}{9\pi^3} + \frac{256}{\pi^2} + \frac{64}{3\pi} + \left(-64 - \frac{16384}{9\pi^3} + \frac{2048}{3\pi^2} + \frac{512}{3\pi}\right) x$

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With the help of the WS approach, similarly obtain [GP, Thorn'13]

$$h_{lp} = \int_0^{\lambda_0} d\lambda \frac{\sinh\frac{\lambda}{2}\cos\left[2(l-p)\sin^{-1}(\sinh\frac{\lambda}{2})\right]}{\pi\sqrt{1-\sinh^2\frac{\lambda}{2}}} \frac{\sinh\lambda(M-M_1)\sinh\lambda M_1}{\sinh(\lambda M/2)\cosh(\lambda M/2)}$$

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- Can be done by separating large M behavior of integral.
- Leading K-dependence of coefficients can also be calculated analytically! Due to $h_{lp} = h(l-p)$ being a **Toeplitz** matrix. [Fisher,Hartwig'68, Rambour,Seghier'09]

Open String Self-energy Summand

$$-\delta P_K^-(m) \simeq \frac{\sqrt{2}G^{36}}{e^3\pi^6}\left(\frac{M}{K^3} + b(K) + \frac{\pi^2}{8K}\frac{m-1}{M}\right) + \mathcal{O}(M^{-2})\,,$$

where $G \simeq 1.282$, $m = 0$ tachyon, $m = 1$ gluon,
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- $1/M$ term for closed string four times larger (and $b = 0$)
- Implies $\sum_K e^{-\epsilon(K-1)}/K \simeq -\log\epsilon$ can be consistently absorbed in renormalization of string tension T_0 as $\epsilon \to 0$.

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- Leading term same for $m = 0, 1$ states in both the closed and open string. Hence can be absorbed in bulk counterterm.
- $1/M$ term for closed string four times larger (and $b = 0$)
- Implies $\sum_K e^{-\epsilon(K-1)}/K \simeq -\log\epsilon$ can be consistently absorbed in renormalization of string tension T_0 as $\epsilon \to 0$.

Hence in the absense of D-branes, can sensibly study the sum of all planar diagrams of bosonic string theory with the help of Monte Carlo methods!

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- With Dp-brane, multiply by $(\ln q)^{(p-9)/2}$. In both cases, leading divergence $\mathcal{O}(M^0)$!

Conclusions & Future Directions

In this presentation we talked about

- How lattice-regularized string theory in the lightcone gauge can be used as a numerical tool for summing planar string diagrams, which via $\alpha' \to 0$ limit could teach us about large N QCD.
- How to test the reliability of the lattice as a regulator of divergences in bosonic string perturbation theory at 1-loop level.
- The fact that bosonic open string theory passes this test in the absence of D-branes, but not in their presence.

Next Stage

- Extend lattice model for the case of the superstring, which improves behavior of divergences and hence may restore power dependence on the regulator M in the presence of D-branes.
- Should this be possible, the next natural step would be the numerical evaluation of the full path integral with the help of Monte Carlo methods.

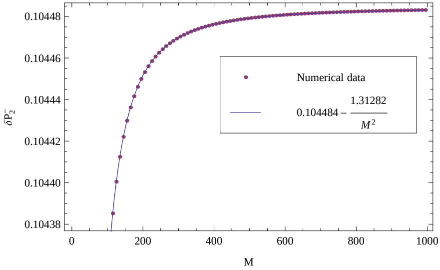
Closed Tachyon Self-Energy in String Field Theory Approach

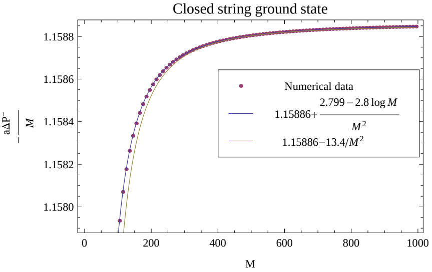
For value of boundary counterterm that makes tree-level Lorentz invariant,

$$\delta P^-_{\text{tach}}(K,M) \simeq c_1^K + \frac{2.8}{KM^2} + \mathcal{O}(1/M^3)$$

$$-\frac{a\Delta P^-_{\text{tach}}}{M} = c_1 + c_2\frac{1}{M^2} + c_3\frac{\log M}{M^2}$$

$$c_1 = 1.158863267 \pm 3\cdot 10^{-9}\,, \quad c_2 = 2.799 \pm 0.011\,, \quad c_3 = -2.800 \pm 0.002$$





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- Comes from $\sum e^{-\xi K/M}/K$.
- When we add $\epsilon > 0$ to b.c., $\log M \to \log(1/\epsilon)$, and then we obtain Lorentz invariant regularization.
- Divergence as $\epsilon \to 0$ simply renormalizes T_0.

Graviton Self-Energy in String Field Theory Approach

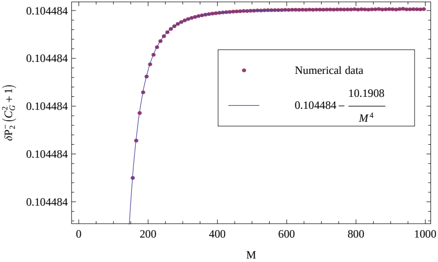
For value of boundary counterterm that makes tree-level Lorentz invariant,

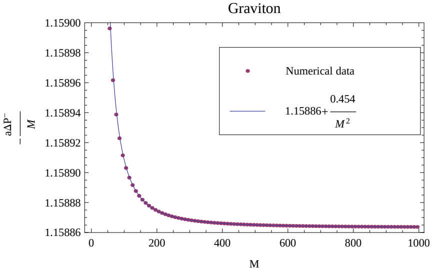
$$\delta P^-_{\mathsf{grav}}(K,M) \simeq c_1^K + \frac{\hat{c}_2 K}{M^4} + \mathcal{O}(1/M^5)$$

$$-\frac{a\Delta P^-_{\mathsf{grav}}}{M} = \tilde{c}_1 + \tilde{c}_2 \frac{1}{M^2}\,,$$

with

$$\tilde{c}_1 = 1.158863276 \pm 1.5 \cdot 10^{-8} \qquad \tilde{c}_2 = 0.454 \pm 0.004\,,$$





Worldsheet Propagator Interpretation

When transformed to Fourier space w.r.t. σ, inverse of lattice Laplacian,

$$(-\triangle + 4\sinh^2 \lambda/2)f_j \equiv 2f_j - f_{j+1} - f_{j-1} + 4f_j \sinh^2 \lambda/2\,.$$

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$$\begin{aligned}
2e^{-|l-j|\lambda} - e^{-|l+1-j|\lambda} - e^{-|l-1-j|\lambda} &= \begin{cases} e^{-(l-j)\lambda}\,(2-2\cosh\lambda) & l>j \\ e^{-(j-l)\lambda}\,(2-2\cosh\lambda) & l<j \\ 2-e^{-\lambda}-e^{-\lambda} & l=j \end{cases} \\
&= -4e^{-|l-j|}\sinh^2\frac{\lambda}{2} + 2\delta_{lj}\sinh\lambda\,, \\
\left(-\triangle + 4\sinh^2\frac{\lambda}{2}\right)\frac{e^{-|l-j|\lambda}}{2\sinh\lambda} &= \delta_{lj}\,.
\end{aligned}$$

Hence 2d propagator given as a simple sum or integral. Drastically improves calculational efficiency.

Closed Tachyon Self-Energy Summand

$$-\delta P^-_{\text{tach}} = \frac{e^{-24(K-1)B_0}}{\det^{12}(h_{lp})} = \frac{e^{-24(K-1)B_0}}{\det^{12}(c_{lp})}\left(1 - \frac{2\pi}{M^2}\sum_{l,s=1}^{K-1} c^{-1}_{ls}\right) + \mathcal{O}(1/M^4)\,,$$

$$c_{lp} = \int_0^1 dx \frac{\sin(\pi x/2)\cos\left[(l-p)\pi x\right]}{\sqrt{1+\sin^2(\pi x/2)}}\,, \quad l,p = 1,\ldots,K-1\,.$$

K	$-\delta P^-_{\text{tach}}$ fit	$-\delta P^-_{\text{tach}}$ actual
2	$0.1044844648 - 1.31291/M^2$	$0.104484465146 - 1.31299/M^2$
3	$0.027700432 - 0.9578/M^2$	$0.0277004334342 - 0.957933/M^2$
4	$0.010959556 - 0.7268/M^2$	$0.0109595576932 - 0.727031/M^2$
5	$0.005388196 - 0.5811/M^2$	$0.00538819758183 - 0.581471/M^2$
6	$0.003032942 - 0.4828/M^2$	$0.00303294412639 - 0.483277/M^2$

Results agree within margins of error, notice however difference increases with K. Due to systematic error from not taking into account $\mathcal{O}(1/M^4)$ term in the fits, whose relative size also increases with K.

Graviton Self-Energy Summand

This is equal to tachyon summand times

$$1 + 2\tilde{U} + 2\tilde{U}^2 \simeq 1 + \frac{2\pi}{M^2} \sum_{l,s=1}^{K-1} c_{ls}^{-1} + \mathcal{O}\left(\frac{1}{M^4}\right) ,$$

$$\tilde{U} = \frac{\sin \frac{\pi}{M}}{M\sqrt{1 + \sin^2 \frac{\pi}{M}}} \sum_{l,s=1}^{K-1} \left(\sin \frac{\pi}{M} + \sqrt{1 + \sin^2 \frac{\pi}{M}}\right)^{2(l-s)} h_{ls}^{-1} .$$

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Able to rigorously prove two important facts, for which we only had strong indications up to now:

1. Leading term in the M-expansion same for tachyon and graviton.
2. Nontrivial cancelation of $\mathcal{O}(1/M^2)$ term! Graviton massless in $K \ll M$ (UV) region.

K-Dependence of terms in asymptotic M-expansion

Governed by determinant and inverse of

$$c_{lp} = \int_0^1 dx \frac{\sin(\pi x/2)\cos\left[(l-p)\pi x\right]}{\sqrt{1+\sin^2(\pi x/2)}}\,, \quad l,p = 1,\ldots,K-1\,.$$

For any specific $n \equiv K-1$, can evaluate exactly as

$$\det(c_{lp}) = \sum_{r=0}^{n} \frac{n}{n+r} \left(\begin{array}{c} n+r \\ 2r \end{array} \right) 2^{2r} \frac{\Gamma(\frac{1}{2}+\frac{r}{2})}{2\sqrt{\pi}\Gamma(1+\frac{r}{2})} {}_2F_1(1-n,1+n;2;-1)\,.$$

But also for $n \gg 1$, can find leading analytic dependence! [Fisher,Hartwig'68]

$$\det(c_{lp}) = n^{\frac{1}{4}} \exp\left(-\log(1+\sqrt{2})n - \frac{1}{8}\log 2\right) \frac{G(\frac{3}{2})^2}{G(2)} \left(1+\mathcal{O}(n^{-1})\right)\,.$$

Similarly, [Rambour,Seghier'09] $$\sum_{l,p=1}^{n} (c^{-1})_{lp} \simeq \frac{\pi}{4} n^2$$

Continuum self-energies

In UV $q \sim 0$ region,

$$\Delta P^-_{\text{Tach}} = \frac{C_o}{2P^+} \int_0^1 \frac{dq}{q^3} \int_0^{2\pi} d\theta \left[\frac{1+24q^2}{4\sin^2(\theta/2)} - 2q^2 + \mathcal{O}(q^4) \right] ,$$

$$\Delta P^-_{\text{Gluon}} = \frac{C_o}{2P^+} \int_0^1 \frac{dq}{q^3} \int_0^{2\pi} d\theta \left[\frac{1+24q^2}{4\sin^2(\theta/2)} - 2q^2\cos\theta + \mathcal{O}(q^4) \right] .$$

For the gluon in RNS, after we compactify one of the transverse target space dimensions by imposing periodic (antiperiodic) boundary conditions on bosonic (fermionic) states in order to break supersymmetry,

$$\Delta P^- = \frac{C_s}{2P^+} \int \frac{dq}{q} \int d\theta \sum_{m=\text{odd}} q^{\frac{m^2R^2T_0}{4\pi^2}} \left(\frac{1-8q+36q^2}{4q\sin^2(\theta/2)} - 2q + 4q\sin^2\frac{\theta}{2} + \mathcal{O}(q^2) \right)$$

In all cases, introducing a Dp-brane implies the insertion of factors of $(-2\pi/\ln q)^{(D-p)/2}$ in the integrand.