M2-branes ending on M5-branes

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based on recent work with **K. Siampos**

1302.0854, ``*The black M2-M5 ring intersection spins*'' Proceedings Corfu Summer School, 2012

1206.2935, ``*Entropy of the self-dual string soliton*'', JHEP 1207 (2012) 134

1205.1535, ``*M2-M5 blackfold funnels*'', JHEP 1206 (2012) 175

and older work with **R. Emparan, T. Harmark and N. A. Obers** ➢ **blackfold theory**

1106.4428, ``*Blackfolds in Supergravity and String Theory*'', JHEP 1108 (2011) 154

0912.2352, ``*New Horizons for Black Holes and Branes*'', JHEP 1004 (2010) 046

0910.1601, ``*Essentials of Blackfold Dynamics*'', JHEP 1003 (2010) 063

0902.0427, ``*World-Volume Effective Theory for Higher-Dimensional Black Holes*'', PRL 102 (2009)191301

0708.2181, ``*The Phase Structure of Higher-Dimensional Black Rings and Black Holes*'' + **M.J. Rodriguez** JHEP 0710 (2007) 110

Important lessons about the fundamentals of string/M-theory (and QFT) are obtained by studying the low-energy theories on D-branes and M-branes.

Most notably in M-theory, recent progress has clarified the low-energy QFT on N M2-brane and the $N^{3/2}$ dof that it exhibits.

Bagger-Lambert '06,
Gustavsson '07, ABJM '08
Drukker-Marino-Putrov '10

Our understanding of the M5-brane theory is more rudimentary, but efforts to identify analogous properties, *e.g.* the N^3 scaling of the massless dof, is underway.

Douglas '10
Lambert,Papageorgakis,Schmidt-Sommerfeld '10
Hosomichi-Seong-Terashima '12
Kim-Kim '12
Kallen-Minahan-Nedelin-Zabzine '12
...

It is believed that the M5 theory is a theory of strings.

M2-branes can end on M5-branes

Townsend '95, Strominger '95, Becker-Becker '96 …

	0	1	2	3	4	5	6	7	8	9	10
M2 :	•	•					•				
M5 :	•	•	•	•	•	•					

just like F-strings can end on D-branes in string theory.

The above intersection is 1/4-BPS (preserves 8 supercharges).

The IR dynamics is controlled by a (1+1)-dim *(4,4)* SCFT that lives in the intersection.

We would like to identify this theory, or key features of this theory
e.g. *how does the central charge of this CFT scale with* N_2 , N_5 ?

Technically, we can approach this question in two different ways:

- from a microscopic analysis of the M5/M2 brane physics
- from a supergravity analysis of the corresponding black brane intersection (e.g. near-extremal black brane thermodynamics gives
  for M2-branes $S \sim N^{\frac{3}{2}} T^2$
  for M5-branes $S \sim N^3 T^5$)

e.g. *Klebanov-Tseytlin '96*

Both approaches are technically complicated.

I will describe work in the SUGRA approach.

M5 point of view: the Howe-Lambert-West solution

Descriptions of the intersection are possible either from the M2 or M5 brane point of view.

I will not say much about the M2 point of view, as it will be less relevant for the approach of this talk.
(ABJM model with boundary conditions...)

Basu-Harvey '04,
Berman et al. '06, '09 ...

From the M5 point of view the string intersection appears as a solitonic solution of the M5 brane worldvolume theory.

Howe, Sezgin, West,
Bandos, Lechner, Nurmagambetov,
Pasti, Sorokin, Tonin,
Aganagic, Park, Poperscu, Schwarz
'96, '97

The Howe-Lambert-West solution: $N_5=1$, $N_2>0$.

The abelian worldvolume theory on a single M5 brane is known.

It is a theory of a self-dual 3-form field strength and 5 transverse scalars (plus their fermion partners).

Key point: this theory is the leading term in a long-wavelength derivative expansion (analogous to the DBI theory for D-branes).

The 1/4-BPS self-dual string soliton solution

Howe-Lambert-West '97

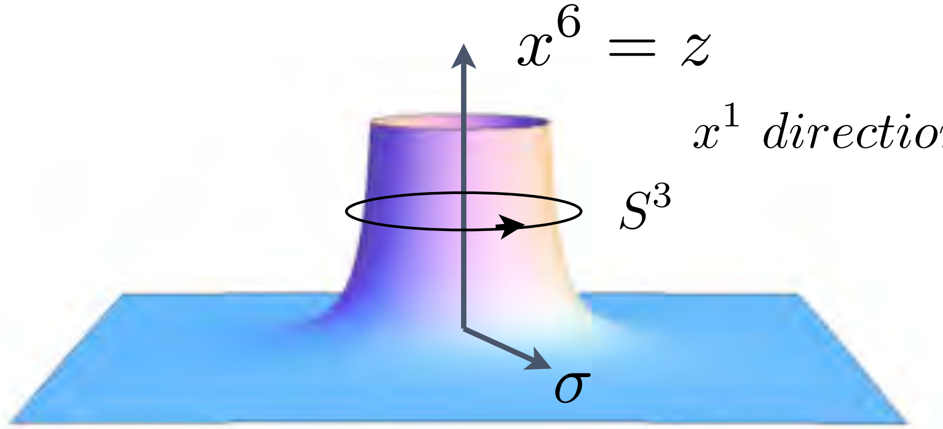
(M-theory analog of BIon solution)

Callan-Maldacena '97

Generalizations
Gauntlett-Lambert-West '98

$$z(\sigma) = \frac{2Q_{sd}}{\sigma^2} \ , \quad z := x^6$$

$$H_{(3)} = *_6 H_{(3)} = *_4 dz$$



An S^3 spike describes N_2 M2-branes ending on N_5=1 M5-branes.

The solution (and the associated derivative expansion) breaks down at some radius, but a **`miracle'** happens:

at the tip of the spike one recovers the tension of the orthogonal M2 branes.

The leading order solution works much better than naively expected.

We will re-encounter and extend this feature below.

The non-abelian M5-brane worldvolume theory remains a mostly open subject.

Lambert,Papageorgakis '10
Saemann-Wolf '11, '12
Ho-Huang-Matsuo '11
Bonetti-Grimm-Hohenegger '12
Samtleben-Sezgin-Wimmer-Wulff '11, '12
Chu-Ko '12
...

This obstructs a similar analysis at generic N_2, N_5 , but recent progress has been possible.

Chu-Ko-Vanichchapongjaroen '12

Chu-Vanichchapongjaroen '13

Managed to reproduce one of our supergravity predictions...

Known supergravity solutions

Supergravity allows us to examine the system in the limit $N_2, N_5 >> 1$.

Brane intersections in supergravity is a subject with a long history and impressive achievements.

Nevertheless, in many cases it is technically challenging to find explicit solutions that describe fully localized intersections.

☞ The Einstein equations imply complicated systems of PDEs.

The challenge is greater as we reduce the amount of supersymmetry, or if we have no supersymmetry at all, e.g. for non-extremal solutions.

In the case of the 1/4-BPS orthogonal M2-M5 intersection

	0	1	2	3	4	5	6	7	8	9	10
M2 :	•	•					•				
M5 :	•	•	•	•	•	•					

we are looking for a solution with SO(1,1) x SO(4) x SO(4) symmetry.

A partially localized solution with metric element

$$ds^2 = H_2^{1/3} H_5^{2/3} \Big[(H_2 H_5)^{-1}(-dt^2 + (dx^1)^2) + H_5^{-1}((dx^2)^2 + \ldots + (dx^5)^2) + H_2^{-1}(dx^6)^2 + (dx^7)^2 + \cdots + (dx^{10})^2 \Big] \, ,$$

$$\nabla^2_{(789(10))} H_5 = 0 \, , \quad \Big(H_5 \nabla^2_{(2345)} + \nabla^2_{(789(10))} \Big) H_2 = 0$$

is known (delocalized along the 6-direction).

Duff-Ferrara-Khuri-Rahmfeld '95
Tseytlin '96, Gauntlett '97
Youm '99, Smith '02

Recent progress towards a fully localized solution

☞ **Lunin:** 0704.3442 (near-horizon), 0706.3396 (asymptotically flat)

derives the appropriate PDEs, solutions are not explicit

☞ **D'Hoker-Estes-Gutperle-Krym**

(near-horizon) 0810.1484, 0806.0605, 0810.4647, 0906.0596
& follow-up: ***Estes-Feldman-Krym*** 1209.1845,
Berdichevsky-Dahan 1304.4389

explicit families of solutions, but so far unclear which solution describes the orthogonal M2-M5,...

I will describe a different treatment of the fully localized solution in SUGRA that

- works within a long-wavelength expansion scheme
(and is thus technically & conceptually closer to the non-gravitational M5 worldvolume description)

- gives immediate intuitive information, and

- extends easily to more complicated (less symmetric) configurations that are well beyond the reach of current exact solution generating techniques.

analogous description of BIon (F1-D3) solution: Grignani, Harmark, Marini, Obers, Orselli '10,'11

Part of a more general still-developing programme in (super)gravity to:

☞ **systematically** capture the dynamics of (black) brane solutions with long-wavelength expansion schemes around exact solutions

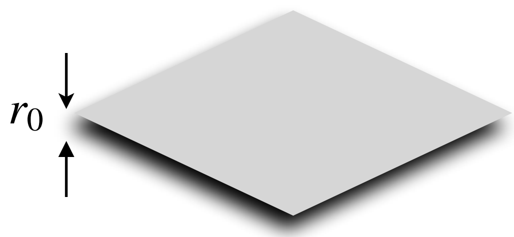
☞ develop the gravitational analog of DBI-theory for supergravity solutions

Blackfold theory basics

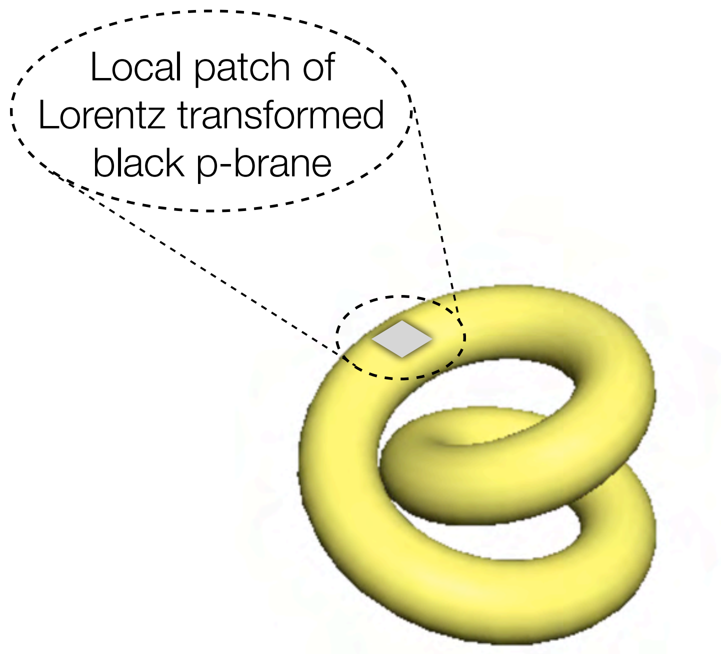
Emparan, Harmark, VN, Obers

Blackfolds provide a general effective (long-wavelength) worldvolume description of black brane dynamics

They describe how a black p-brane fluctuates, spins and bends



flat black p-brane



a thin curved blackfold

The **fluid/gravity correspondence** illustrates nicely the idea.

- a spin-off of the AdS/CFT correspondence

Bhattacharyya-Hubeny-Minwalla-Rangamani '07,...

- it describes temperature and velocity fluctuations of AdS black branes in the long-wavelength approximation $\lambda \gg \frac{1}{T} \sim \frac{L^2_{AdS}}{r_0}$ in terms of a relativistic conformal fluid. ← $\nabla_\mu T^{\mu\nu} = 0$

- the fluid lives on a time-like surface in the asymptotic region of the black hole spacetime (AdS boundary)

- there is a constructive perturbative procedure that maps uniquely the solutions of the fluid equations to regular bulk spacetimes

Blackfolds add co-dimension to the fluid-gravity correspondence
➠ a mix of fluid dynamics + `DBI'.

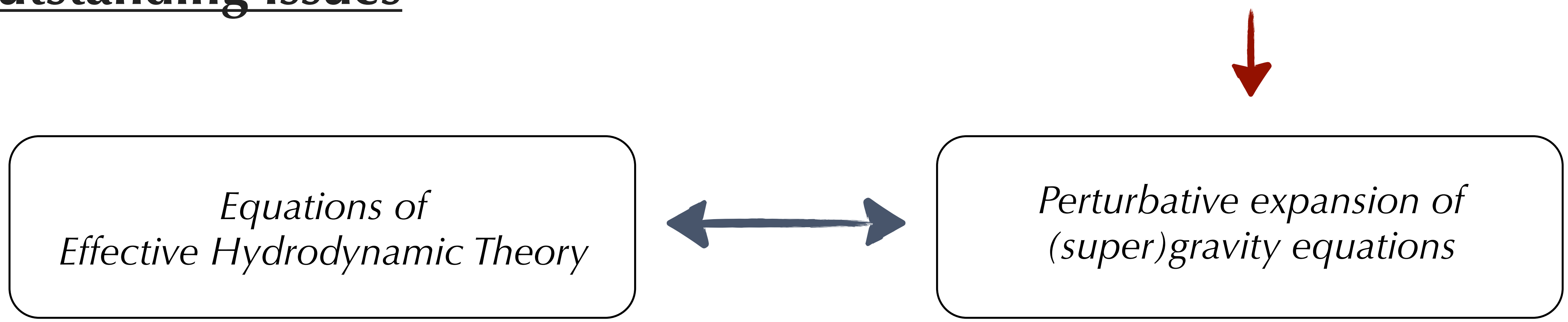
Long-wavelength fluctuations of a black brane are captured by an effective hydrodynamic theory that describes a fluid on an *dynamical* worldvolume.

The effective degrees of freedom are the slowly-varying temperature, fluid velocity, worldvolume bending scalars, etc.

The equations of motion of the effective degrees of freedom are conservation equations for the stress-energy tensor, charge currents...

Outstanding issues

matched asymptotic expansions



☞ Effective Hydro theory has been constructed only at few leading orders.

Emparan-Harmark-VN-Rodriguez-Obers '07
Camps, Emparan '12

☞ In progress: effective hydro theory at all orders,

general map between hydro solutions and regular bulk spacetimes.

We will focus on the leading order of this expansion scheme.
0th order blackfold solution ➠ 1st order thermodynamics

M2-M5 blackfold funnels

We want to describe a spiky deformation of the planar M5 black brane (with dissolved M2 brane charge)

➠ how the planar black M2-M5 bound state deforms

Izquierdo-Lambert-Papadopoulos-Townsend '95
Russo-Tseytlin '96

$$
\begin{aligned}
ds_{11}^2 =& (HD)^{-1/3}\Big[-f dt^2 + (dx^1)^2 + (dx^2)^2 + D\left((dx^3)^2 + (dx^4)^2 + (dx^5)^2\right) \\
&+ H\left(f^{-1}dr^2 + r^2 d\Omega_4^2\right)\Big] ,
\end{aligned}
$$

$$
C_3 = -\sin\theta(H^{-1}-1)\coth\alpha\, dt\wedge dx^1\wedge dx^2 + \tan\theta D H^{-1} dx^3\wedge dx^4\wedge dx^5 ,
$$

$$
C_6 = \cos\theta D(H^{-1}-1)\coth\alpha\, dt\wedge dx^1\wedge\cdots\wedge dx^5 ,
$$

$$
H = 1 + \frac{r_0^3 \sinh^2\alpha}{r^3} , \quad f = 1 - \frac{r_0^3}{r^3} , \quad D^{-1} = \cos^2\theta + \sin^2\theta H^{-1} .
$$

$$
F_4 = dC_3 + D^{-1} \star dC_6
$$

The parameters of the planar M2-M5 bound state are promoted to slowly-varying functions of the effective 5-brane worldvolume coordinates

Effective degrees of freedom

$$r_0(\hat{\sigma}^a)\ ,\ \alpha(\hat{\sigma}^a)\ ,\ \theta(\hat{\sigma}^a)\ ,\ u^a(\hat{\sigma}^a)\ ,\ X^{\perp}(\hat{\sigma}^a)\ ,\ \hat{V}_{(3)}$$

local temperature

M2, M5 charges

local boosts

transverse scalars

unit 3-volume form controls embedding of M2 charge in 5-brane wv

Leading order blackfold equations

$K_{ab}{}^i$: extrinsic curvature tensor

`DBI' part extrinsic equations

`hydrodynamic' part intrinsic equations

$$K_{ab}{}^{i}T^{ab} = 0 \qquad D_a T^{ab} = 0$$

$$d * J_3 = 0 \ , \quad J_3 = \mathcal{Q}_2 \hat{V}_{(3)}$$

$$d * J_6 = 0 \ , \quad J_6 = \mathcal{Q}_5 \hat{V}_{(6)}$$

local stress-energy tensor, constitutive relations,...

$$T_{ab} = \mathcal{T} s \left(u_a u_b - \frac{1}{3}\gamma_{ab} \right) - \sum_{q=2,5} \Phi_q \mathcal{Q}_q h_{ab}^{(q)} \qquad .....$$

anisotropic perfect fluid with p-form charge currents

new generalized hydrodynamics

...

$SO(1,1) \times SO(4) \times SO(4)$ *symmetry*

We are looking for a static solution with a single `excited′ transverse scalar

$$x^6 = z(\sigma) \ , \quad \sigma^2 = (x^2)^2 + (x^3)^2 + (x^4)^2 + (x^5)^2$$

For stationary configurations the intrinsic eqs can be solved generically, and we end up with DBI-like eqs for the transverse scalars

For extremal *T=0* configurations we solve the eom of the Dirac action

$$I \simeq \int d\sigma \, \sigma^3 \sqrt{1 + \frac{\kappa^2}{\sigma^6}} \sqrt{1 + z'^2} \ , \quad \kappa = 4\pi \frac{N_2}{N_5} \ell_P^3$$

We recover the extremal (1/4-BPS) 3-sphere spike solution

$$z(\sigma) = 2\pi \frac{N_2}{N_5} \frac{\ell_P^3}{\sigma^2}$$

reproduced from 1304.4322 Chu-Vanichchapongjaroen

The blackfold derivative expansion breaks down when derivatives become large. The characteristic breakdown scale is:

$$\sigma_c = \left(\frac{\pi N_5}{\sqrt{2}}\right)^{\frac{1}{3}} \left(1 + \sqrt{1 + \frac{4}{\lambda^2}}\right)^{\frac{1}{6}} \ell_P \ , \qquad \frac{1}{\lambda} := \frac{4N_2}{N_5^2}$$

Derivative corrections are controlled by the ratio:

$$\frac{1}{\lambda} = \frac{4N_2}{N_5^2} \ll 1$$

controls how strongly the M2s deform the M5 geometry

➠ we work in the large-*N* limit

$$N_2,\ N_5 \gg 1 \ , \quad N_2 \ll N_5^2$$

Despite the breakdown of the effective theory the usual **miracle** happens.

The *leading-order* solution reproduces correctly the tension of M2-branes at the tip of the spike (at any λ)

$$\frac{1}{L_t L_{x^1}} \frac{dM}{dz}\bigg|_{\sigma=0} = Q_2 = N_2 T_{M2}$$

(We have also observed this matching for **non-SUSY extremal** configurations)

Thermalizing the spike

Spikes at finite temperature can be obtained by solving the eom of the action

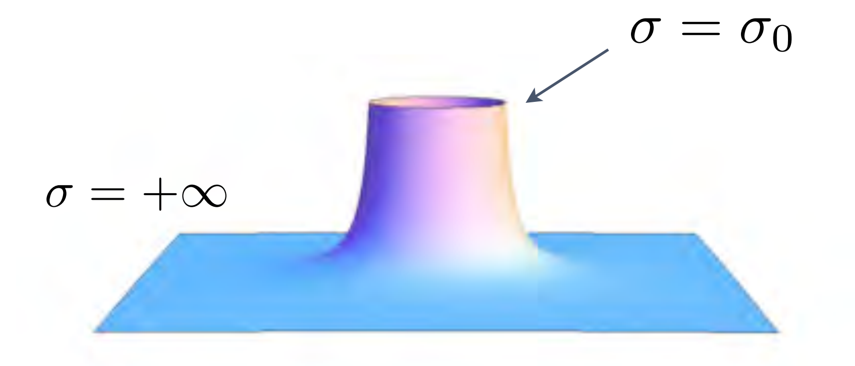
$$I \simeq \int d\sigma \sqrt{1+z'^2}\, F(\sigma;\beta)\ , \quad \beta = \frac{3}{4\pi T}$$

$$q_5 = \frac{16\pi G}{3\Omega_{(4)}} Q_5$$

$$F(\sigma) = \sigma^3 \left(\frac{1+\frac{\kappa^2}{\sigma^6}}{1+\sqrt{1-\frac{4q_5^2}{\beta^6}\left(1+\frac{\kappa^2}{\sigma^6}\right)}} \right)^{\frac{3}{2}} \left(-2 + \frac{3\beta^6}{2q_5^2} \frac{1+\sqrt{1-\frac{4q_5^2}{\beta^6}\left(1+\frac{\kappa^2}{\sigma^6}\right)}}{1+\frac{\kappa^2}{\sigma^6}} \right)$$

- an analogous non-gravitational description is not easy to find
- the exact non-extremal SUGRA solution is not known

General boundary conditions for spiky solutions



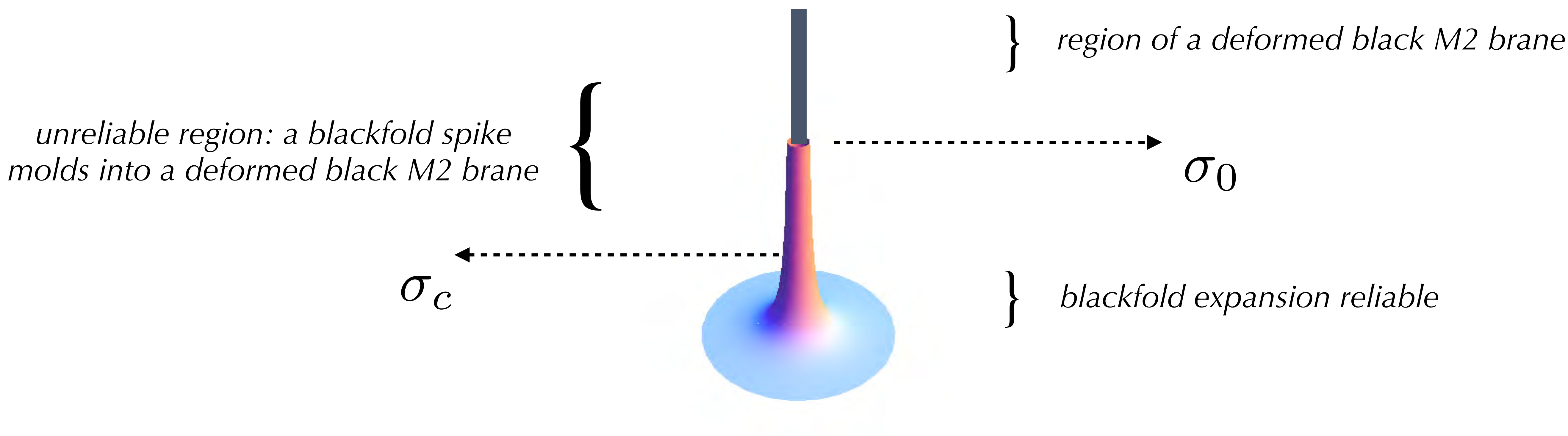
$$\lim_{\sigma\to+\infty} z(\sigma) = 0 \ , \quad \lim_{\sigma\to\sigma_0^+} z'(\sigma) = -\infty$$

With these boundary conditions the general solution is

$$z(\sigma) = \int_\sigma^{+\infty} ds \left(\frac{F(s)^2}{F(\sigma_0)^2} - 1 \right)^{-\frac{1}{2}}$$

Data beyond the perturbative regime are needed to fix $\sigma_0(T)$.

This involves a more impressive (and potentially dangerous) set of extrapolations of the leading order theory beyond its regime of validity away from extremality!



An approximate matching of the leading order blackfold solution to a planar black M2 at σ_0 continues to work impressively well and determines $\sigma_0(T)$.

Indeed, matching the thermodynamic data of the near-extremal spike with those of the emerging M2 we obtain

$$\left(\frac{1}{L_{x_1}}\frac{dM}{dz}\Big|_{\sigma=\sigma_0^+}\right)_{M2-M5} = \left(\frac{M}{L_{x_1}L_z}\right)_{M2} \ , \qquad \left(\frac{1}{L_{x_1}}\frac{dS}{dz}\Big|_{\sigma=\sigma_0^+}\right)_{M2-M5} = \left(\frac{S}{L_{x_1}L_z}\right)_{M2}$$

$$\sigma_0^{(M)} = \frac{q_2^{\frac{1}{4}}}{\beta^{\frac{1}{2}}}\left(c_1^{(M)} + c_2^{(M)}\frac{q_2^{\frac{1}{2}}}{\beta^3} + \mathcal{O}(\beta^{-6})\right) \ , \quad c_1^{(S)} \simeq 1.234 \ , \quad c_2^{(M)} \simeq -0.068$$

$$\sigma_0^{(S)} = \frac{q_2^{\frac{1}{4}}}{\beta^{\frac{1}{2}}}\left(c_1^{(S)} + c_2^{(S)}\frac{q_2^{\frac{1}{2}}}{\beta^3} + \mathcal{O}(\beta^{-6})\right) \ , \quad c_1^{(S)} \simeq 1.189 \ , \quad c_2^{(M)} \simeq 0.052$$

$$\Rightarrow \quad \sigma_0(T) \simeq c_1\frac{q_2^{\frac{1}{4}}}{\beta^{\frac{1}{2}}} \ , \quad c_1 \simeq 1.2 \ , \quad q_2 = \frac{16\pi G}{3\Omega_{(3)}\Omega_{(4)}}Q_2$$

The leading order coefficients $(c_1^{(M)}, c_1^{(S)})$ agree within 4% !

Entropy of thermal spikes

Given a solution of the blackfold equations the formalism provides specific formulae for thermodynamic data. For the entropy in this particular application

$$\cosh\alpha = \frac{\beta^3}{\sqrt{2}q_5}\sqrt{\frac{1+\sqrt{1-\frac{4q_5^2}{\beta^6}(1+\frac{\kappa^2}{\sigma^6})}}{1+\frac{\kappa^2}{\sigma^6}}}$$

$$\frac{S}{L_{x_1}} = \frac{\Omega_{(3)}\Omega_{(4)}\beta^4}{4G}\int_{\sigma_0}^{+\infty} d\sigma\,\sigma^3\,\frac{F(\sigma)}{\sqrt{F^2(\sigma)-F^2(\sigma_0)}}\,\frac{1}{\cosh^3\alpha(\sigma)}$$

Expanding in positive powers of *T* we wish to identify the leading small temperature *O*(*T*) contribution from the (1+1)-dimensional intersection.

The contributions far from the core (*M*5) and close to the core (*M*2) are subleading in the temperature.

The leading order contribution to *S* is indeed *O(T)* (as expected!)

$$\frac{S}{L_{x^1}} = \frac{8\sqrt{\pi}\Gamma(\frac{1}{3})\Gamma(\frac{1}{6})}{135 c_1^8} \frac{N_2^2}{N_5} T + \mathcal{O}(T^4) \ , \quad c_1 \simeq 1.2$$

Comparing to the Cardy formula for two-dimensional CFT

$$\frac{S}{L_{x^1}} = \frac{\pi c}{6} T$$

we find an expression for the central charge *c*

$$c \simeq 0.6 \, \frac{N_2^2}{N_5} + \ldots$$

These results indicate that the *d=2 large N=(4,4)* SCFT at the intersection has a strong t′ Hooft like expansion

$$N_2 \ , \ N_5 \gg 1 \ , \quad \lambda \sim \frac{N_5^2}{N_2} \gg 1$$

- *How does the $\frac{1}{\lambda}$ expansion arise in field theory?*
  Is this a tractable new corner of this SCFT?

In this limit the leading order contribution to the central charge takes the highly suggestive form

from dimensional analysis !!!

$$c \simeq 0.6 \, \frac{N_2^2}{N_5} + \ldots = 0.04 \, \frac{N_5^{3}}{\lambda^2} + \ldots = 0.3 \, \frac{N_2^{\frac{3}{2}}}{\sqrt{\lambda}} + \ldots$$

- *What is the field theory interpretation of this result?*
  What does it teach us about M2 and M5 brane physics?

Further work

Much remains to be done:

1) Part of the result relied on a set of interesting `miracles' and extrapolations. The extrapolations are the price we have to pay in order to get around the hard problem. Extra checks are needed and under consideration.

- Approximate reconstruction of the full bulk geometry from the leading EFT analysis ➠ approximate extremal near-horizon geometry...

- Holography for *large d=2 N=(4,4)* SCFTs...

in type II string theory
... '98, '99
Gukov-Martinec-Moore-Strominger '04

2) The same technology allows us to probe more complicated configurations of the intersection.

For example, we searched for a closed M2-M5 string intersection in SUGRA.
➠ A rotating black M2 cylinder ending on a black M5.

- The configuration preserves less symmetry: SO(1,1) x SO(3) x SO(4)
- The configuration is stationary: the blackfold fluid rotates.
- The black M5 has to be also cylindrical
- The extremal configuration carries a null momentum wave along the intersection ➠ M2-M5-KKW ring intersection.
- Surprisingly, although non-SUSY it exhibits many of the miracles of supersymmetric configurations (e.g. thermo data at the tip of the spike)
- The identification of near-horizon data is now harder...

3) How one formulates the 2D CFT at the intersection and how this meshes with the M2 and M5 brane physics is the most outstanding question...

Extending the gravity analysis and combining it with non-perturbative QFT tools gives hope for further progress...