EMERGENT TIME & THE M5-BRANE

EMERGENT TIME & SPACE

- Time and space may not be fundamental
- Cosmology: Time or Space might "emerge" from non-geometric phase e.g. matrix model
- Planck scale: physics may be discrete, with geometry emerging at larger "scales". Discrete theory may resolve singularities
- Holography: 4D SYM gains 6 dimensions: 10D string, $AdS_5 \times S^5$

A LA RECHERCHE DU TEMPS PERDU

- Can a timeless Euclidean theory gain a "hidden" time dimension in the quantum theory?
- e.g. lose time by compactification on time dimension to Euclidean theory
- New insights into time and dynamics?
- Work with Neil Lambert, to appear at some time

DIMENSIONS EMERGENT AT STRONG COUPLING

- Some strong coupling limits (e.g. IIA string) give extra space dimension.
- Can some give extra time dimension?
- Can theory in Euclidean space R^d become a theory in d+1 dimensional Minkowski space at strong coupling?
- Controlled situation to study "emergent" time, giving insight into time and emergent theory?

SPACE DIMENSION FROM STRONG COUPLING

- Tower of 0-branes. BPS, so can extrapolate to strong coupling
- Infinite tower of 0-branes become massless at infinite coupling
- Interpret as decompactification of extra dimension, with 0-branes as Kaluza-Klein modes
- Further checks: e.g. compare BPS spectra, seek evidence of higher dimensional Lorentz symmetry

M-THEORY FROM IIA STRING

- IIA string: D0 brane states $M \propto \frac{n}{g_s}$
- Interpret as KK states for circle $M \propto \frac{n}{R}$
- IIA string at strong coupling is M-theory on spatial circle $R \propto g_s$
- Problem: do not have intrinsic formulation of M-theory, or of IIA string at strong coupling

(2,0) THEORY FROM SYM

Rozali, Witten

- Super Yang-Mills in 4+1 with 16 SUSY, SO(5) R-symmetry
- Soliton: (YM instanton in R^4) × (time) $\quad M = \frac{4\pi^2|n|}{g^2}$
- Interpret as KK states for circle $M \propto \frac{n}{R}$
- SYM at strong coupling is (2,0) theory on spatial S^1 $\quad R = \frac{g^2}{4\pi^2}$
- Problem: do not have intrinsic formulation of (2,0) theory, or of SYM at strong coupling

5D SYM

- SYM in 4+1 non-renormalisable, 6-loop divergence Bern et al
- Embed in UV complete theory, e.g. string theory.
- D4 brane world-volume theory $g^2 = (2\pi)^2 {\alpha'}^{1/2} g_s$
- At strong coupling: M5 brane wrapped on M-theory S^1
- (2,0) theory (decoupling limit of) M5-brane world-volume theory

(2,0) THEORY

- Superconformal theory in 5+1 dimensions, 16 SUSYs and SO(5) R-symmetry
- Abelian: self-dual tensor multiplet: 5 scalars X^I, H=*H, H=dB
- Non-abelian: no conventional field theory formulation(?)
- Construct from IIB on K3 at ADE singularity, or M5-brane world volume, or matrix model, or CFT dual to $AdS_7 x S^4$

SYM IN 5 EUCLIDEAN DIMENSIONS

- Super Yang-Mills in 5 Euclidean dimensions with 16 SUSY, SO(5) R-symmetry
- Seek evidence that at strong coupling an extra TIME dimension opens up $$R = \frac{g^2}{4\pi^2}$$
- Euclidean SYM at strong coupling is (2,0) theory on timelike S^1
- Simple argument: This theory and usual 4+1 SYM defined by SAME Euclidean path integral, but with different continuations back to real section. If one gets extra dimension, both do. SUSY fixes signature.

CONSTRUCTING EUCLIDEAN SYM

- Reduce SYM from 9+1 on d+1 dimensions gives ESYM in D=9-d with SO(d,1) R-symmetry. D=5: SO(4,1) R-symmetry
- Compactify (2,0) on time: 5+0 SYM with SO(5) R-symmetry. Want this Euclidean SYM, not one with SO(4,1) R symmetry
- Can get this theory from SYM in 5+5 dimensions by reducing on 5 time dimensions

EUCLIDEAN SYM

$$S = \frac{1}{4g^2} tr \int d^5x \Big[\frac{1}{4} F_{ij}F^{ij} - \frac{1}{2} D_i X^I D^i X^I + \frac{1}{4}[X^I, X^J]^2$$
$$- \frac{i}{2}\psi^T \Gamma_0 \Gamma^i D_i \psi - \frac{1}{2}\psi^T \Gamma^I [X^I, \psi] \Big]$$

Space indices i,j=1,..,5

Internal indices I,J=6,7,...,10

Conserved currents T_{ij} $J = \frac{1}{8g^2} * tr(F \wedge F)$

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Analytic continuation to positive bosonic action

$$X^I \to iX^I$$

Same Euclidean action as Wick rotation of 4+1 SYM

ABELIAN (2,0) THEORY

$$S^{(abelian)} = -\int d^6x \Big[\frac{1}{12}H_{\mu\nu\rho}H^{\mu\nu\rho} + \frac{1}{2}\partial_\mu X^I \partial^\mu X^I\Big]$$

Reduce on time

$$S = -\int d^5x \Big[\frac{1}{12}H_{ijk}H^{ijk} + \frac{1}{2}\partial_i X^I \partial^i X^I\Big]$$

Dualise $\quad F = *H$

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5+1 EM tensor $\Theta_{\mu\nu}$ $\Theta_{ij} = T_{ij},$ $\Theta_{0i} = J_i$

SUSY, gauge inv. ⇒ true for non-abelian theory too

$$\Theta_{00} = tr\left(\frac{1}{4g^4} F_{ij} F^{ij} + \frac{1}{2} D_i X^I D^i X^I - g^4 [X^I, X^J]^2\right)$$

4+1 SYM & NEW SPACE DIM.

Topological current $J = \frac{1}{2g^2} * tr(F \wedge F)$

Charge $K \equiv \int d^4 x J_0 = \frac{4\pi^2 n}{g^2}$

(YM instanton in R^4) x (time): BPS soliton $M = |K|$

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(YM instanton in R^4) x (time): BPS soliton $M = |K|$

Interpret as KK states for circle in x^5 direction

$$P_5 = \frac{n}{R} \qquad M = |P_5| \qquad R = \frac{g^2}{4\pi^2}$$

$P_5 = \int d^4 x dx^5\, \Theta_{05}$ so $P_5 \sim K$ if $\Theta_{05} \propto J_0$

5+0 SYM & NEW TIME DIM?

Topological current $J = \frac{1}{2g^2} * tr(F \wedge F)$

Charge $K = \int d^4 x J_5 = \frac{4\pi^2 n}{g^2}$

(YM instanton in R^4) x (R^1): BPS solution extended in x^5

Interpret as KK states for circle in x^0 direction

$$E = \frac{n}{R} \qquad\qquad R = \frac{g^2}{4\pi^2}$$

If $\Theta_{05} \propto J_5$ $\qquad K \sim \int d^4 x \Theta_{05}$

Is this an energy E or a momentum P_5 ? What if n -ve?

QUANTISATION IN EUCLIDEAN SIGNATURE

- One way: choose a Euclidean “time” e.g. $\tau = x^5$
- Canonical formalism, Poisson brackets etc based on τ
- Path integral defined by slicing wrt Euclidean “time” τ
- Natural if τ is Wick rotated time, but not for intrinsically Euclidean theories, as it breaks rotation invariance
- Useful for subsector extended along τ direction.

A TIMELY RESOLUTION

- Treat x^5 as a Euclidean time, use canonical formalism for x^5
- Interchange roles of x^5, x^0 in usual picture, get same story
- E is P^0 in x^5 formalism (has different meaning in x^0 picture)
- Null state in 5+1 with $x^0=x^5$ gives world line along x^0 in 4+1, or E1-brane along x^5 in 5+0
- Ep-brane: extended along p spatial dimensions

EUCLIDEAN CHARGES

$$x^i = (\tau, x^a) \qquad a = 1, 2, 3, 4$$

Restrict to configurations that fall off sufficiently rapidly in transverse R^4.

$$\hat{q} = \int d^4x \, j_\tau$$

Integral over 4-surface of fixed τ. Conserved: $\partial_\tau \hat{q} = 0$

$$\hat{P}_i = \int d^4x \; T_{\tau i}$$

Generates translations through τ Poisson brackets

$$[\hat{P}_i, \psi] = \partial_i \psi$$

Charge for branes from instantons in transverse R^4

$$K = \frac{1}{8g^2}\int d^4x \, J_\tau$$

Supercharges

$$\hat{Q} = \int d^4x \; \mathcal{S}_\tau$$

$$\{\hat{Q}_\alpha, \hat{Q}_\beta\} = 2(\Gamma^i C^{-1})_{\alpha\beta}\hat{P}_i - 2\delta_{\alpha\beta}K + (\Gamma^i\Gamma^I C^{-1})_{\alpha\beta}\hat{Z}^I_i + \dots$$

Use K as 0-component:

$$\hat{P}^\mu = (\hat{P}^0 = K, \hat{P}^i)$$

Whole superalgebra can be written in a way suggestive of 5+1 dimensions

$$\{\hat{Q}_\alpha, \hat{Q}_\beta\} = 2(\Gamma^\mu C^{-1})_{\alpha\beta}\hat{P}_\mu + (\Gamma^\mu\Gamma^I C^{-1})_{\alpha\beta}\hat{Z}^I_\mu + (\Gamma^{\mu\nu\lambda}\Gamma^{IJ} C^{-1})_{\alpha\beta}\hat{Z}^{IJ}_{\mu\nu\lambda}$$

$$\hat{P}_a = tr\int d^4x D_a X^I D_\tau X^I - \frac{1}{g^4} F_{ab} F_\tau{}^b$$

$$\hat{P}_\tau = tr\int d^4x \frac{1}{4g^4} F_{ab}F^{ab} - \frac{1}{2g^4} F_{\tau a} F_\tau{}^a - \frac{g^2}{4}[X^I, X^J][X^I, X^J] + \frac{1}{2} D_\tau X^I D_\tau X^I - \frac{1}{2} D_a X^I D^a X^I$$

$$\hat{P}_0 = \frac{1}{8g^4} tr\int \varepsilon_{\tau bcde} F^{bc} F^{de}$$

$$\hat{Z}^I_0 = \frac{2}{g^2} tr\int d^4x F_{\tau a} D^a X^I - 4ig^4 D_\tau X^J [X^J, X^I]$$

$$\hat{Z}^I_a = \frac{1}{g^2} tr\int d^4x \ \varepsilon_{\tau abcd} F^{cd} D^b X^I$$

$$\hat{Z}^I_\tau = 0$$

$$\hat{Z}^{IJ}_{0a\tau} = -ig^2 \varepsilon^{IJKLM} tr\int d^4x \ [X^K, X^L] D_a X^M$$

$$\hat{Z}^{IJ}_{0ab} = -\frac{i}{2} \varepsilon_{\tau abcd} tr\int d^4x \ F^{cd}[X^I, X^J]$$

$$\hat{Z}^{IJ}_{ab\tau} = itr\int d^5x \ F_{ab}[X^I, X^J]$$

LIFT TO 5+1

$$x^i = (\tau, x^a) \qquad a = 1, 2, 3, 4$$

Restrict to configurations falling off rapidly in transverse R^4
Expect to lift to configs in 5+1 falling off in transverse R^4

$$x^\mu = (x^0 = t, x^a, x^5 = \tau)$$

Expect quantization based on time τ to lift to quantization in 5+1 based on τ instead of t.

τ–independent charges $\qquad \hat{q} = \int d^4x dt \, j_\tau$

instead of t-independent charges $\qquad q = \int d^4x d\tau \, j_t$

$$\hat{P}_\mu = \int d^4x dt\ \Theta_{\mu\tau}$$

generates translations through τ–Poisson brackets

$$\hat{P}_0 = \int d^4x dt\ \Theta_{0\tau}$$

reduces to K-charge in 5+0 as $\Theta_{i0} \to J_i$

$$K = \frac{1}{8g^2}\int d^4x\, J_\tau \qquad\qquad J = \frac{1}{2g^2} * tr(F \wedge F)$$

As t periodic, charge is quantised

$$\hat{P}_0 = \frac{n}{R}$$

Suggests topological charge K identified with $\hat{P}_0$ for KK modes of 5+1 (2,0) theory compactified on time $R = \frac{g^2}{4\pi^2}$

MOMENTA

$$P_\mu = \int d^4x d\tau \; \Theta_{\mu t} \qquad \hat{P}_\mu = \int d^4x dt \; \Theta_{\mu\tau}$$

for configurations that only depend on

$$x^a, t+\tau \quad (a = 1,2,3,4)$$

$$P_\tau = \int d^4x d\tau \; \Theta_{\tau 0} = \int d^4x dt \; \Theta_{\tau 0} = \hat{P}_0$$

so quantized charge arising from instantonic E1-branes is conventional momentum in τ direction but is momentum in time direction for τ formalism

TIMES AND QUANTIZATIONS

- **t canonical quantization** breaks SO(d,1) to SO(d), need to prove Lorentz covariance
- Well suited for states extended along time (world-lines), but perhaps issues for states (if any) localised in time?
- τ **canonical quantization** breaks SO(d,1) to SO(d-1,1), need to establish Lorentz covariance
- Good for states extended along τ, but issues for others?
- 5+0 SYM: all BPS states extended along at least one direction

MATCHING BPS STATES

Charged string of (2,0) in 5+1

In abelian phase, charged self-dual strings, Charge Z_i^I

i=1,...,5 labels direction in space

I=6,...,10 labels R-charge

String with charge Z_5^6 matches with:

Charged E1-brane in ESYM

$$F_{a5} = D_a X^6$$

$$A_5 = X^6 = \langle X^6 \rangle - \frac{Q_E}{4\pi^2 r^2}\ , \qquad A_a = 0$$

Electrically charged, 1/2 BPS $\quad Z_5^6 \to \hat{Z}_0^6 \qquad \hat{P}_5 = \frac{1}{2}|\hat{Z}_0^6|$

Boosted charged string of (2,0) in 5+1

String with charge Z_5^6 along x^5, boosted in x^4 direction
1/2 BPS, charges P_0, P_4, Z_5^6

Charged Solution in ESYM

Expect version of charged E1-brane with momentum P_4 from K-charge of instantonic E1-brane along x^4 direction.
Solution extended in x^4, x^5, depends only on x^1, x^2, x^3

$$F_{mn} = \varepsilon_{mnp} D^p \Phi \qquad A_5 = \frac{1}{v}\Phi \qquad X^6 = \frac{1}{v\gamma}\Phi$$

Parameters v,γ $\qquad F_{m5} = \frac{1}{\gamma g^2} D_m X^6$

If v=1, instanton in 1235-plane $\qquad F_{mn} = \varepsilon_{mnp} F_{p5}$

$$F_{mn} = \varepsilon_{mnp} D^p \Phi \qquad A_5 = \frac{1}{v}\Phi \qquad X^6 = \frac{1}{v\gamma}\Phi$$

1/2 BPS if $\gamma^2 = 1/(1-v^2)$

$$\Phi = v\gamma g^2 \langle X^6 \rangle - \frac{Q_M}{2r} + \mathcal{O}(1/r^2) \ , \quad r \to \infty$$

Identify with string in 5+1 boosted to velocity v
Spectrum covariant under SO(1,1) acting as boosts in x^4 direction

$$P_0^2 = P_4^2 + \frac{1}{4}(Z_5^6)^2$$

FROM (2,0) A THEORY

Lambert+Papageorgakis

- A 5+1 field theory with (2,0) SUSY, non-abelian gauge symmetry, adjoint self-dual tensors Has vector field C that takes constant expectation value
- If C spacelike, get 4+1 SYM in orthogonal $R^{4,1}$
- If C timelike, get 5+0 SYM in orthogonal R^5
- Gives suggestive setting of SYM in 5+1 formalism

TIME FOR M-THEORY

Hull

- Compactify D=11 sugra on time → IIA_E sugra in 10+0 dimensions

- Does M-theory on timelike circle give sensible quantum theory? If so, gives IIA_E string in 10+0, fundamental E2-brane

- At strong coupling, IIA_E string gives M-theory on time circle

$$R = {\alpha'}^{1/2} g_s$$

- RR E1-branes lift to M-theory KK modes on time circle

M-THEORY, BRANES AND TIME

- M5 branes wrap time circle to give E5 branes of IIA_E string
- At strong coupling, E5 branes of IIA_E string become M5-branes
- E5-brane world-volume theory is ESYM with SO(5) R-symm.
- So ESYM at strong coupling should give (2,0) theory on time circle

$$g^2 = (2\pi)^2 {\alpha'}^{1/2} g_s \qquad R = {\alpha'}^{1/2} g_s, \implies R = \frac{g^2}{4\pi^2}$$

CONCLUSIONS

- Evidence for conjecture that SYM in 5+0 gets an extra time dimension at strong coupling, giving (2,0) theory on time circle
- Similar to evidence for 4+1 SYM getting an extra space dim
- 5+0 and 4+1 SYM governed by same path integral, so both should grow a dimension if one does
- Detailed matching of BPS spectra in 5D and 6D
- BPS spectra SO(5,1) covariant

- Follows from strong coupling limits of IIA,IIA_E giving M-theory
- Don't know what (2,0) theory or M-theory are, but these strong coupling limits give useful information
- Emergent time is a circle. There are issues in meaning of quantum theory with periodic time. (Probability? Measurement?)
- Intimate relation between time and quantum theory. How to quantise a theory without time (without resorting to a Euclidean time)? For SYM, use quantisation of (2,0) on time circle?