Brane Tilings Specular Duality

Amihay Hanany
with Rak-Kyeong Seong

Conformal Field Theories

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NSVZ Beta Function

$$\beta_{\frac{1}{g^2}} = \frac{1}{8\pi^2}\frac{3N - \sum_M \mu[\mathcal{R}_M](1-\gamma_M(g))}{1 - g^2 N/8\pi^2}$$

$$D = 1 + \frac{\gamma}{2} = \frac{3}{2}r$$

Vanishing Beta Function

$$\sum_{B \in bifund[a,b]} N_b(1 - r_{B,ab}) = 2N_a$$

All ranks equal

$$\sum_{B \in bifund} (1 - r_B) = 2$$

A superpotential term

$$\sum_{B \in monomial} r_B = 2$$

Conditions for conformal invariance

$$\sum_{B \in bifund} (1 - r_B) = 2$$

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- 3 objects in N=1 supersymmetry in 3+1d:

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- gauge fields - vector multiplets
- matter fields - chiral multiplets

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- 3 objects in N=1 supersymmetry in 3+1d:
- gauge fields - vector multiplets
- matter fields - chiral multiplets
- interactions - superpotential

Brane Tilings

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- bi-partite tilings of the torus

Brane Tilings

- bi-partite tilings of the torus
- or periodic bi-partite tilings of the plane

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- bi-partite tilings of the torus
- or periodic bi-partite tilings of the plane
- Write a Lagrangian according to the rules:

Brane Tilings Dictionary

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- Face (tile) - U(N) Gauge group; U(N) V-plet

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- Face (tile) - U(N) Gauge group; U(N) V-plet
- Edge - A bi-fundamental chiral multiplet
- Node - Interaction term in W
- +(-) sign for a white (black) node

Conditions for conformal invariance

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- Locally flat tiles (NSVZ)

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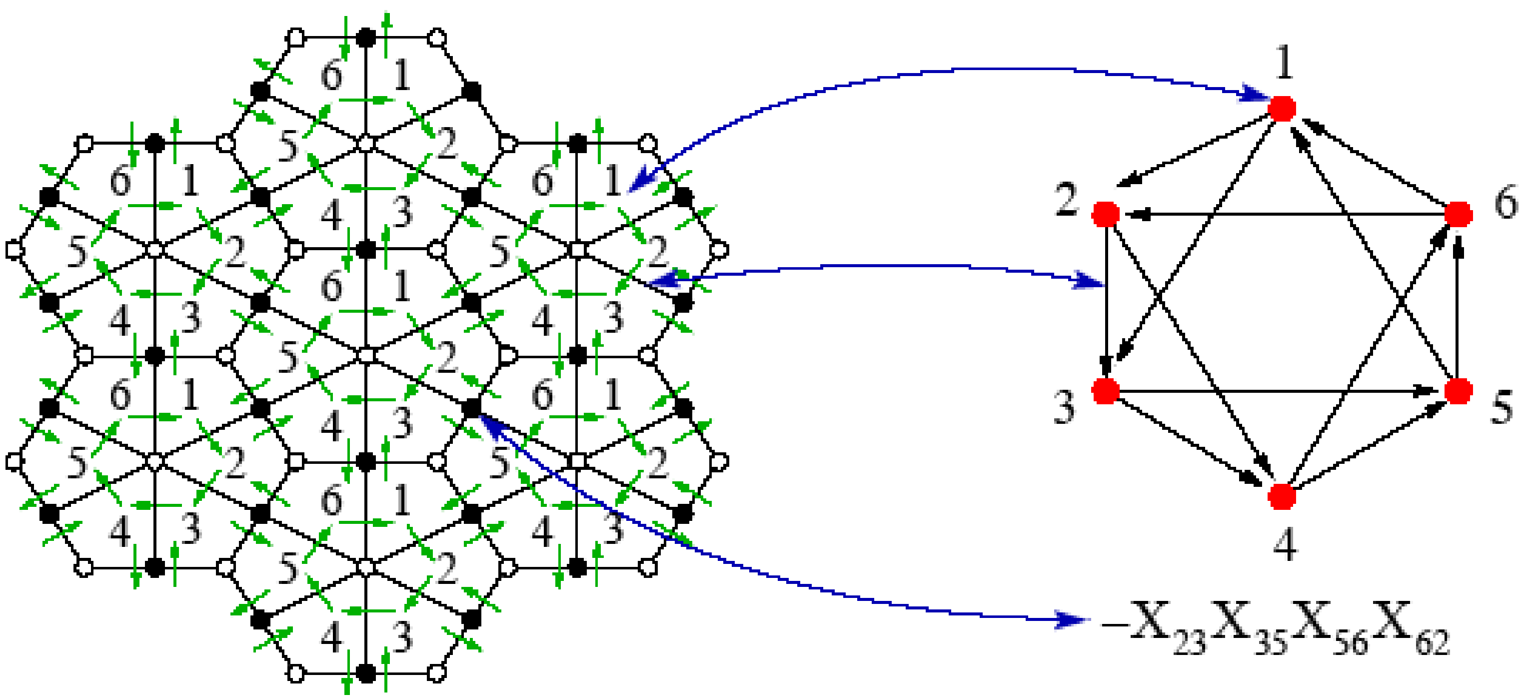
- Locally flat tiles (NSVZ)
- Locally flat nodes (W has R charge 2)
- Periodic, bi-partite, 2d tilings
- R charges are angles in the tiling

Example: dP3 tiling and quiver



Example Quiver, Tiling, W

3 Model 1: $\mathbb{C}^3/\mathbb{Z}_3 \times \mathbb{Z}_3$ $(1,0,2)(0,1,2)$

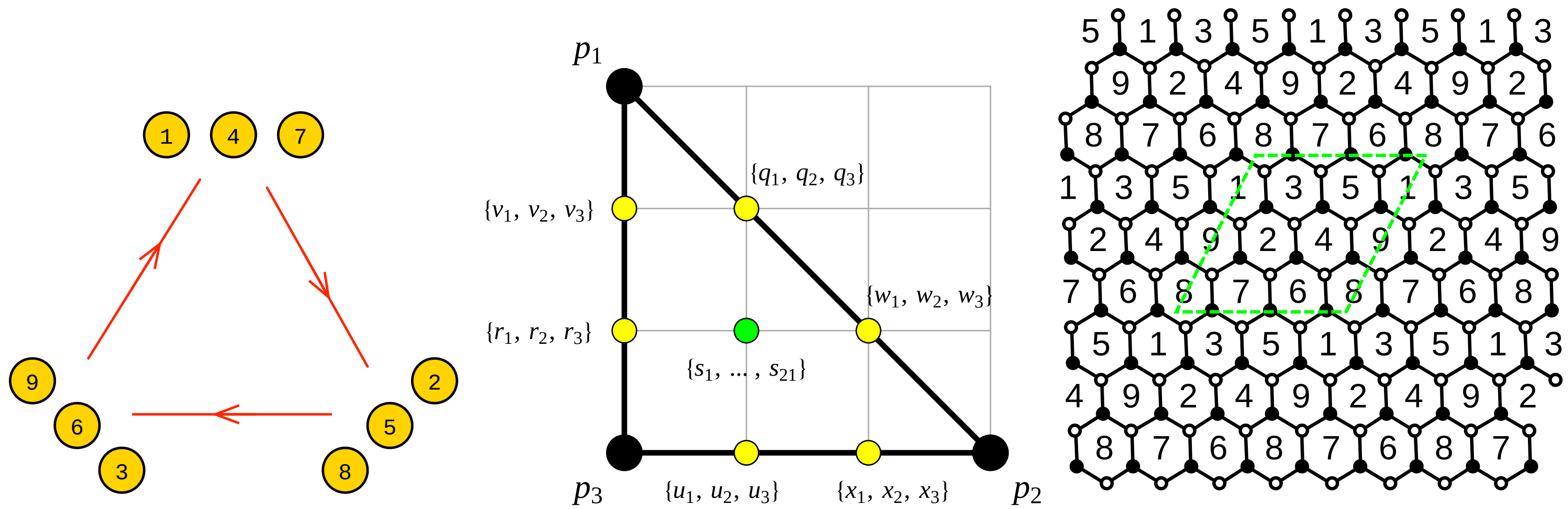


Figure 4. The quiver, toric diagram, and brane tiling of Model 1. The red arrows in the quiver indicate all possible connections between blocks of nodes.

The superpotential is

$$
\begin{aligned}
W = &+X_{15}X_{56}X_{61} + X_{29}X_{91}X_{12} + X_{31}X_{18}X_{83} + X_{42}X_{23}X_{34} + X_{53}X_{37}X_{75} + X_{67}X_{72}X_{26} \\
&+X_{78}X_{89}X_{97} + X_{86}X_{64}X_{48} + X_{94}X_{45}X_{59} - X_{15}X_{59}X_{91} - X_{29}X_{97}X_{72} - X_{31}X_{12}X_{23} \\
&-X_{42}X_{26}X_{64} - X_{53}X_{34}X_{45} - X_{67}X_{75}X_{56} - X_{78}X_{83}X_{37} - X_{86}X_{61}X_{18} - X_{94}X_{48}X_{89}
\end{aligned}
$$

Quiver, Toric diagram, Tiling, W

18 Model 16: $\mathbb{C}^3/\mathbb{Z}_3$ $(1,1,1)$, $\mathbf{dP}_0$

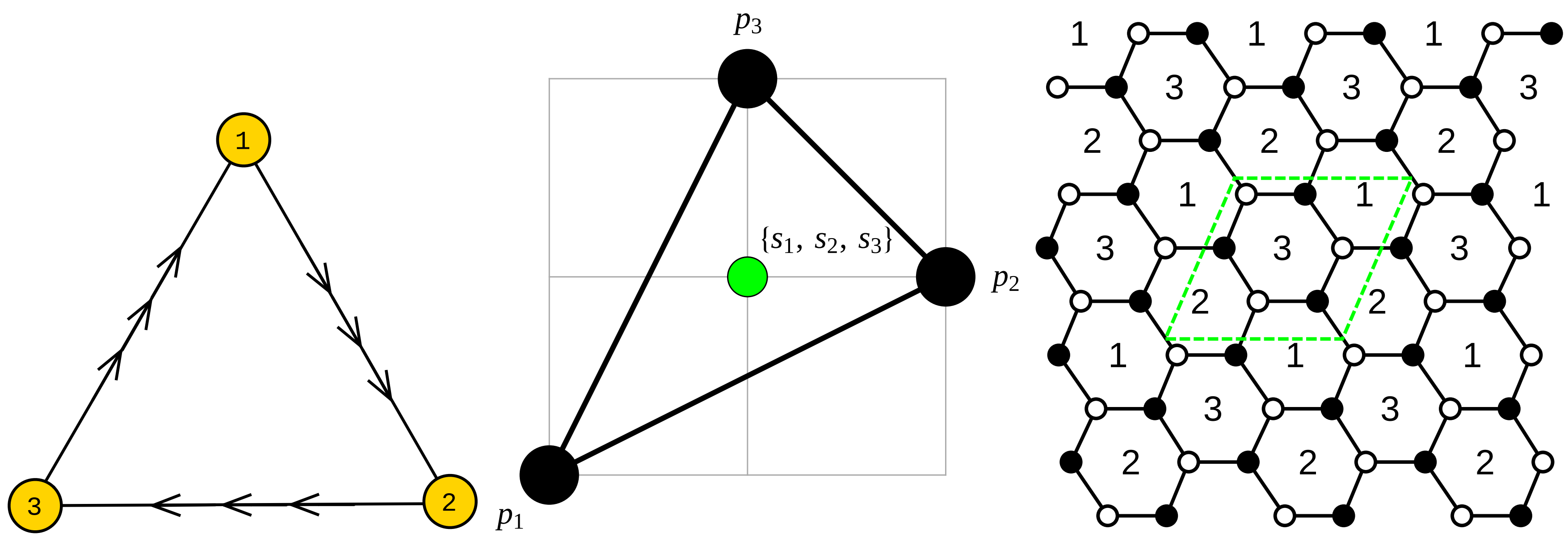
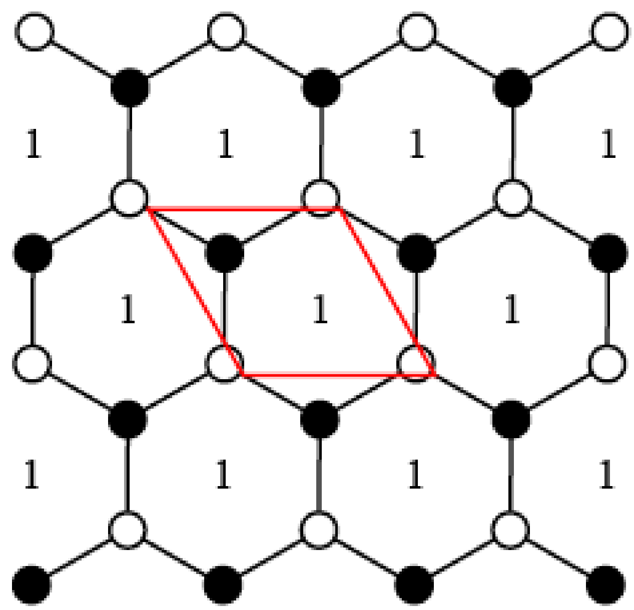


Figure 36. The quiver, toric diagram, and brane tiling of Model 16.

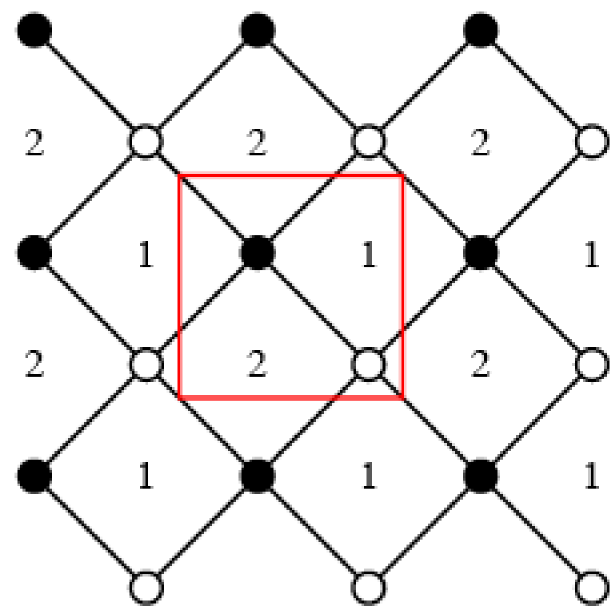
The superpotential is

$$W = +X_{12}^1 X_{23}^3 X_{31}^2 + X_{12}^2 X_{23}^1 X_{31}^3 + X_{12}^3 X_{23}^2 X_{31}^1 \\ -X_{12}^1 X_{23}^1 X_{31}^1 - X_{12}^3 X_{23}^3 X_{31}^3 - X_{12}^2 X_{23}^2 X_{31}^2 \quad (18.1)$$

Brane Tilings
N_T=2, G=1,2

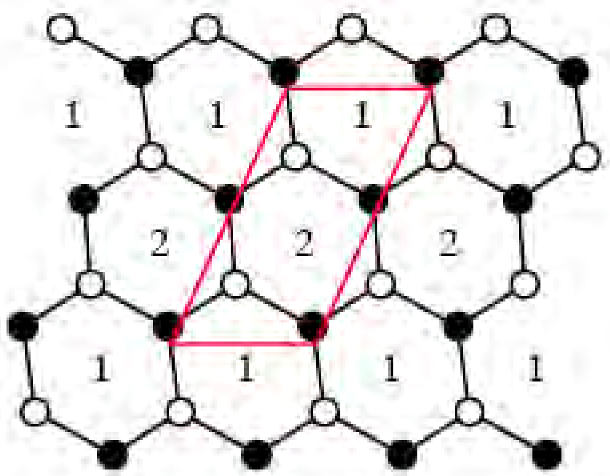


(1.1) $\mathbb{C}^3$

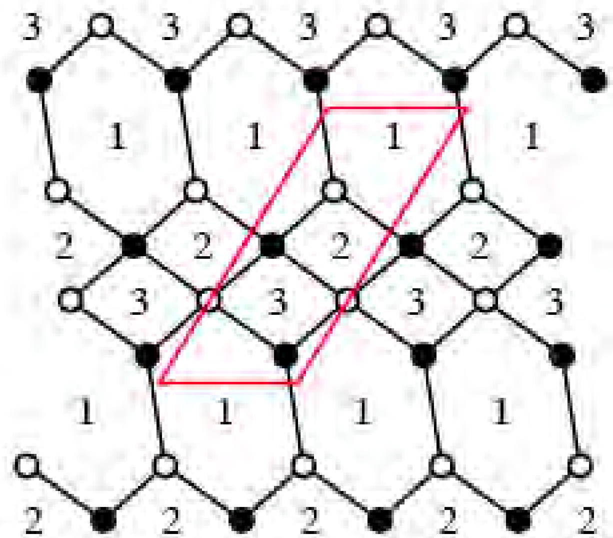


(1.2) $\mathcal{C}$

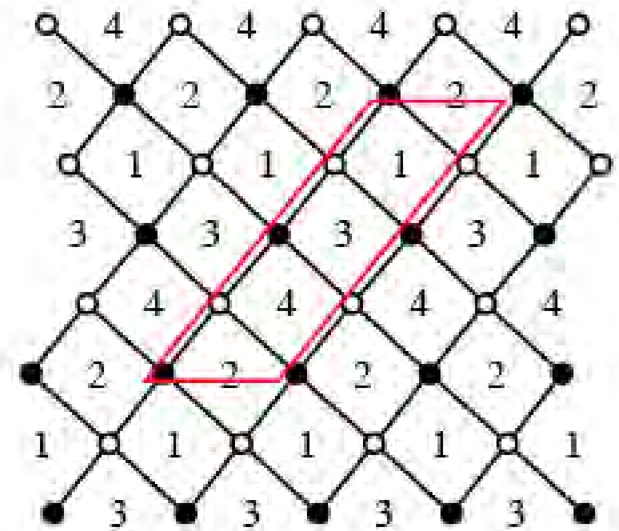
Brane Tilings
N_T=4, G=2,3,4



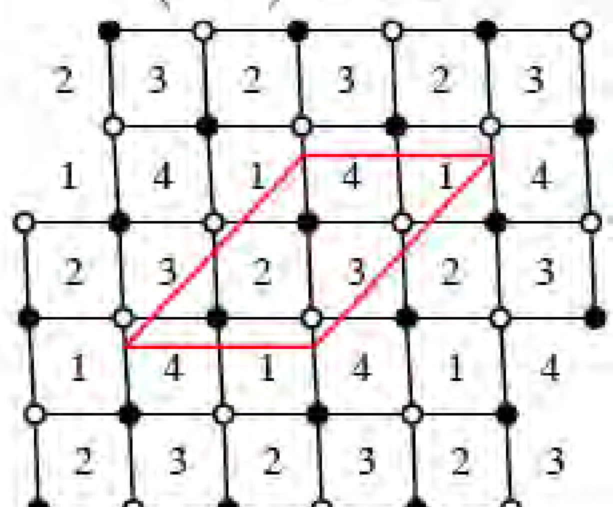
(2.1) $\mathbb{C}^2/\mathbb{Z}_2 \times \mathbb{C}$



(2.2) SPP

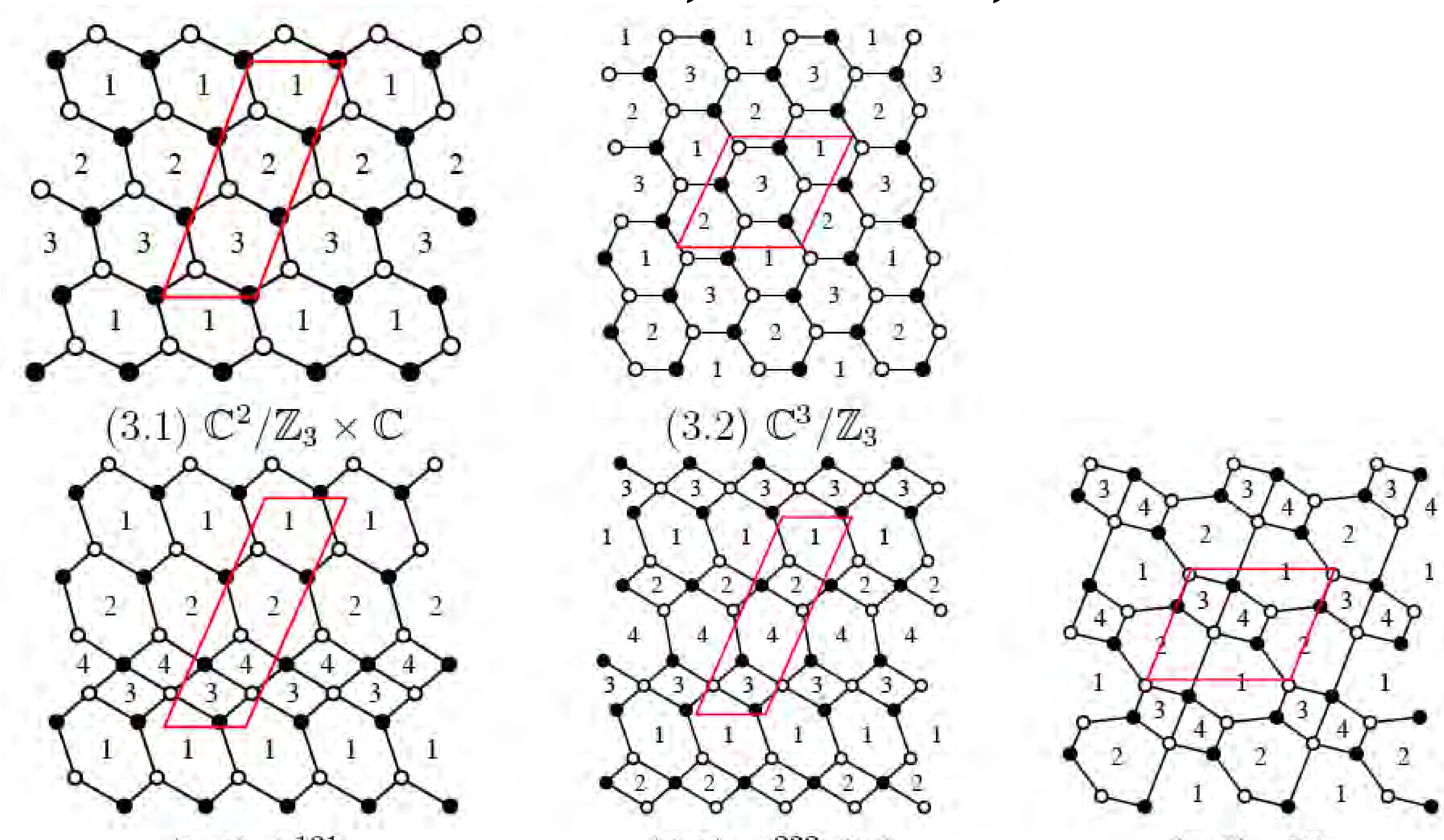


(2.4) L^{222} (I)



(2.5) $\mathbb{F}_0$ (I)

Brane Tilings
N_T=6, G=3,4



(3.1) $\mathbb{C}^2/\mathbb{Z}_3 \times \mathbb{C}$

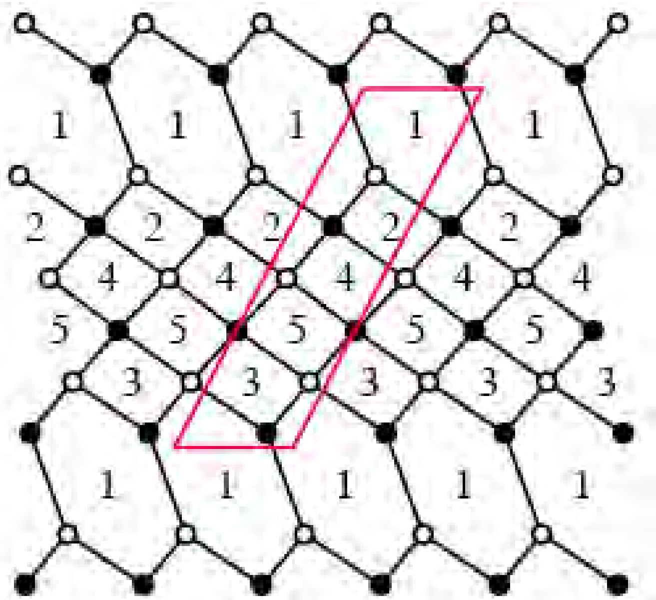
(3.2) $\mathbb{C}^3/\mathbb{Z}_3$

(3.4) L^{131}

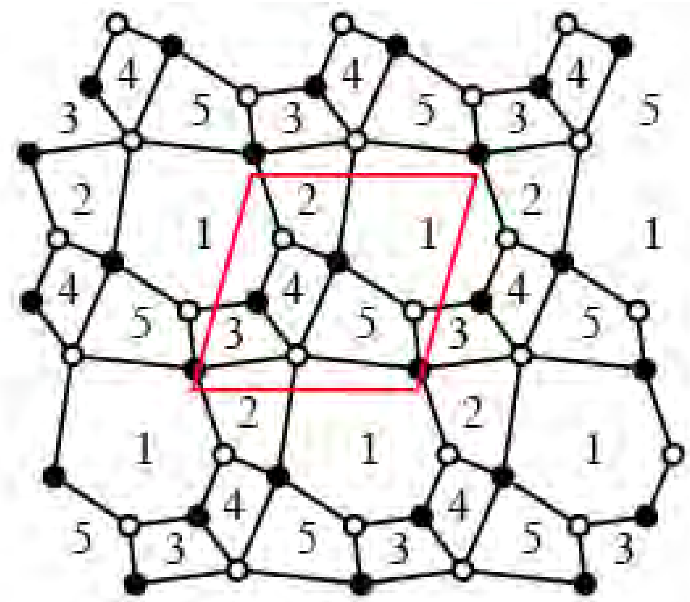
(3.5) L^{222} (II)

(3.6) dP_1

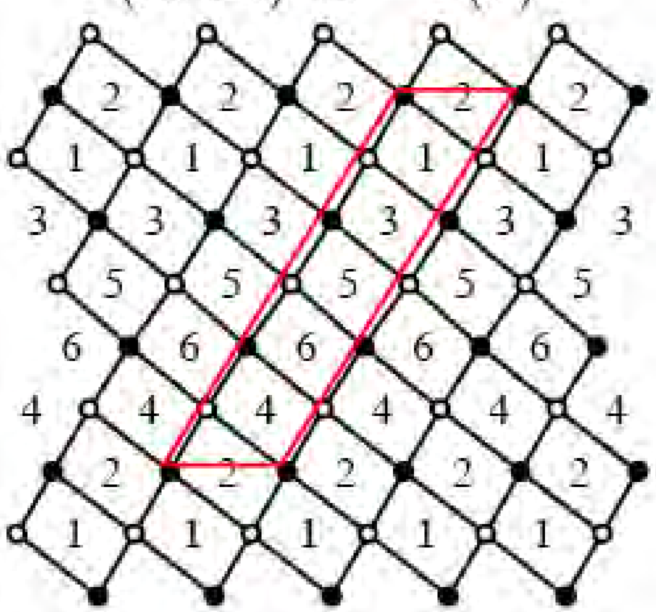
Brane Tilings
N_T=6, G=5,6



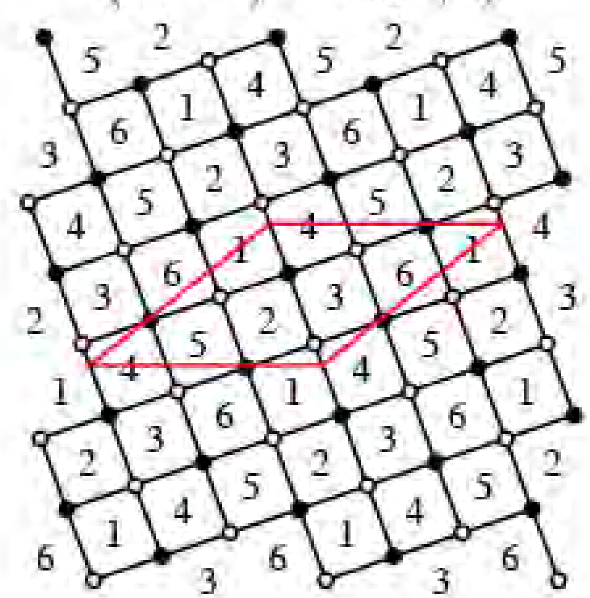
(3.13) L^{232} (I)



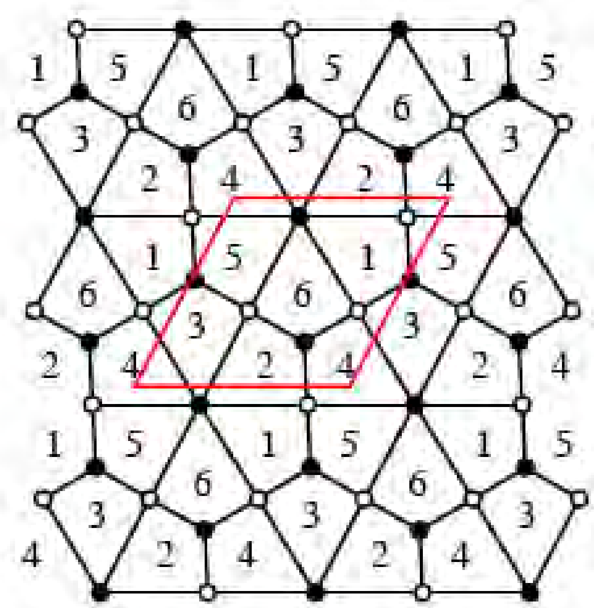
(3.14) dP_2 (I)



(3.26) L^{333} (I)



(3.27) $Y^{3,0}$ (I)



(3.28) dP_3 (I)

N=1 supersymmetric gauge theories

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- Consider an SU(n) gauge theory

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- Two types of gauge invariant operators

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- Baryons - epsilon contraction

N=1 supersymmetric gauge theories

- Consider an SU(n) gauge theory
- Two types of gauge invariant operators
- Mesons - delta contraction
- Baryons - epsilon contraction
- Goes through for a product gauge group

Moduli Space of Vacua

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- Given a supersymmetric gauge theory

Moduli Space of Vacua

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- Study its moduli space

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Moduli Space of Vacua

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- Study its moduli space
- VEV to mesons - Mesonic moduli space
- VEV to baryons - Baryonic moduli space
- Combined mesonic baryonic moduli space

Brane Tilings Mesonic Moduli space

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Brane Tilings Mesonic Moduli space

- Mesonic moduli space is singular toric CY3
- Type IIB D3 brane at the tip of this cone
- AdS_5 x SE^5
- N branes mesonic moduli space is $S^N(CY3)$

Moduli Space

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- An N=1 supersymmetric theory

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- Set of all fields subject to F terms - F flat

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- Set of all fields subject to F terms - F flat
- D terms divided into Abelian & non Abelian
- mesonic space: gauge invariants under both
- combined space: under non Abelian only

Dimensions

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- Mesonic moduli space: 3N

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- Set G the number of gauge groups

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- Combined space has dimension 3N+G-1

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- Set G the number of gauge groups
- Number of Abelian D terms is G-1
- Combined space has dimension 3N+G-1
- singular CY cone

Master Space

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- For one brane, N=1,

Master Space

- For one brane, N=1,
- master space of dimension G+2

Master Space

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Master Space

- For one brane, N=1,
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- singular toric CY cone
- set of fields subject to F terms (F flat)

Master Space Examples

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- For C^3 no baryonic moduli

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- G=1, master space is C^3
- For conifold, G=2, master space is C^4

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- For C^3/Z_2, G=2: conifold x C

Master Space Examples

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- For C^3/Z_3, G=3, master space is

Master Space Examples

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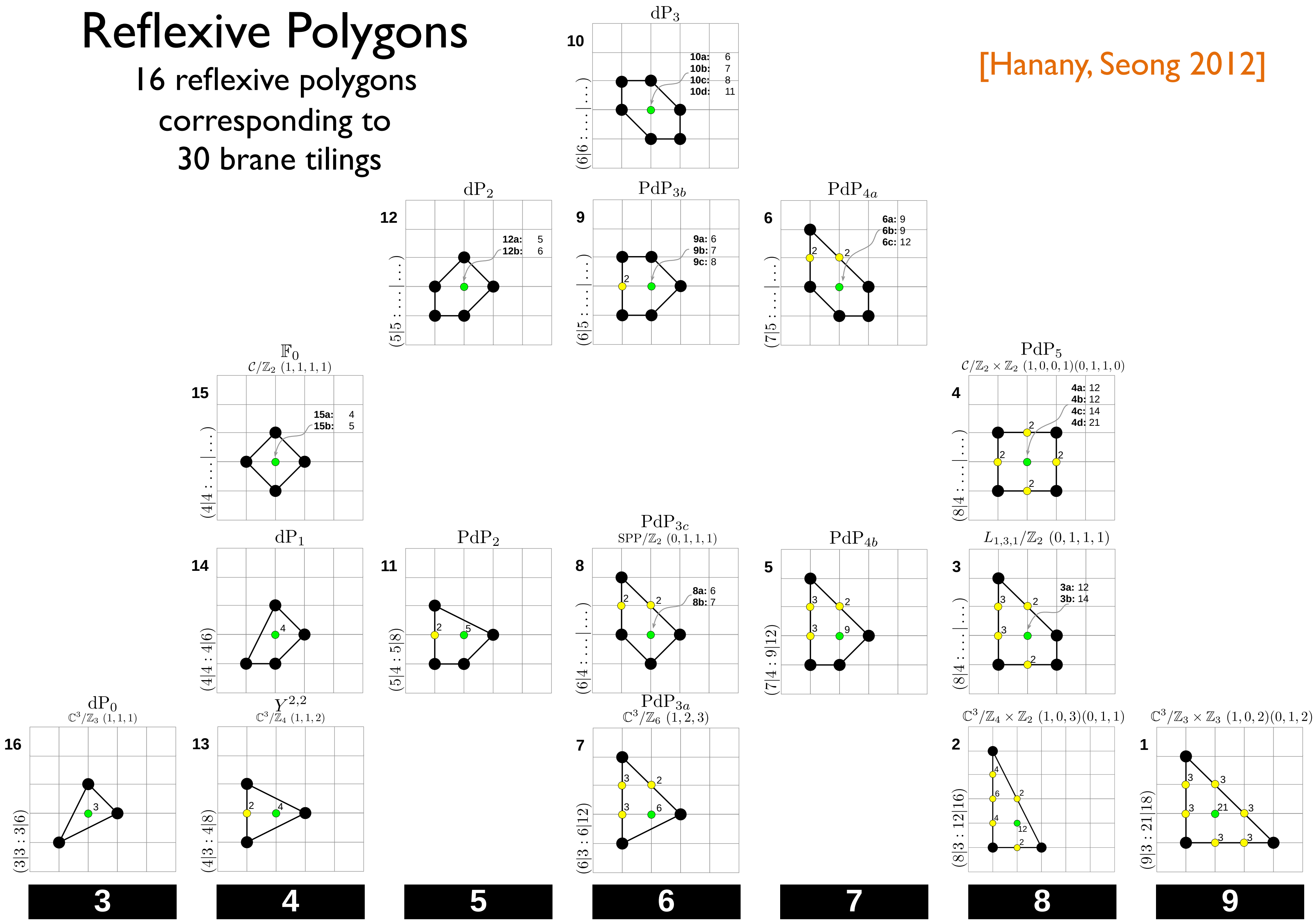
- G=1, master space is C^3
- For conifold, G=2, master space is C^4
- For C^3/Z_2, G=2: conifold x C
- For C^3/Z_3, G=3, master space is
- complex cone over $P^2 \times P^2$

Reflexive Polygons

16 reflexive polygons
corresponding to
30 brane tilings

[Hanany, Seong 2012]



Monday, June 17, 13

Reflexive Duality

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- Chiral ring of the mesonic moduli space

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- generated by chiral gauge inv operators

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- Form a lattice

Reflexive Duality

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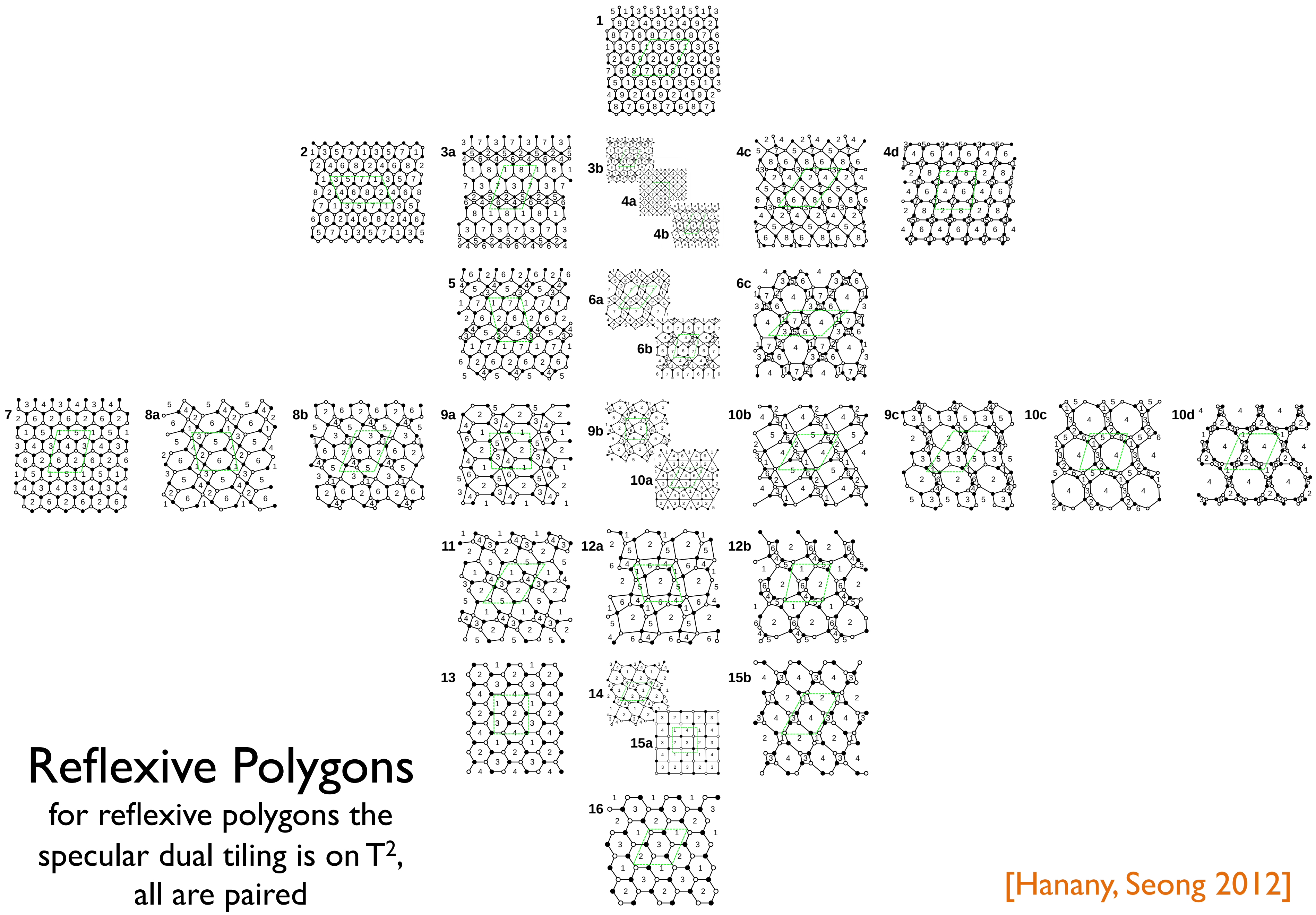
- generated by chiral gauge inv operators
- charged under 3 U(1) symmetries
- Form a lattice
- Reflexive dual

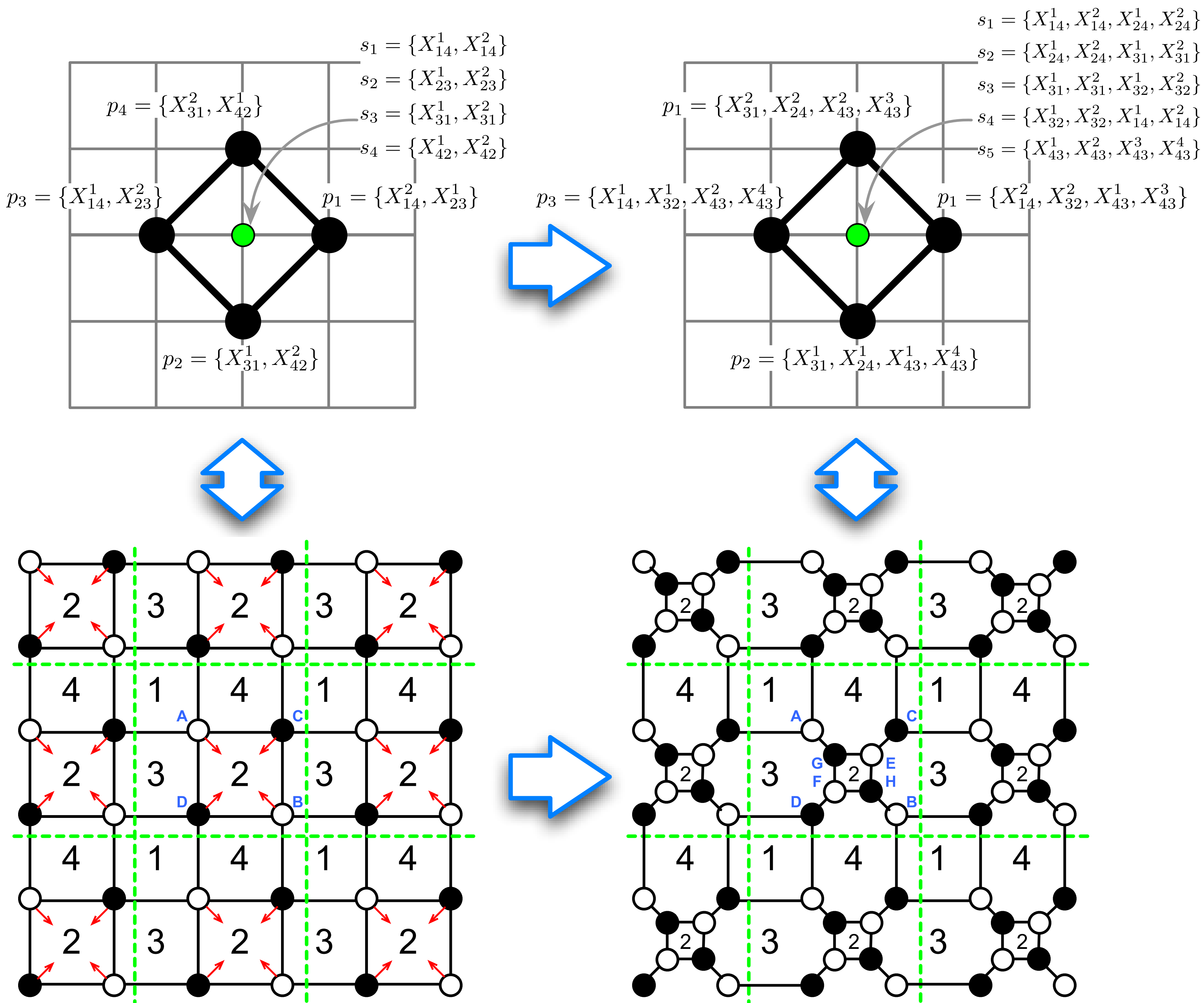


Reflexive Polygons

for reflexive polygons the
specular dual tiling is on T^2,
all are paired

[Hanany, Seong 2012]

Seiberg Duality F0



F0 I

17 Model 15: $\mathcal{C}/\mathbb{Z}_2\ (1,1,1,1),\ \mathbb{F}_0$

17.1 Model 15 Phase a

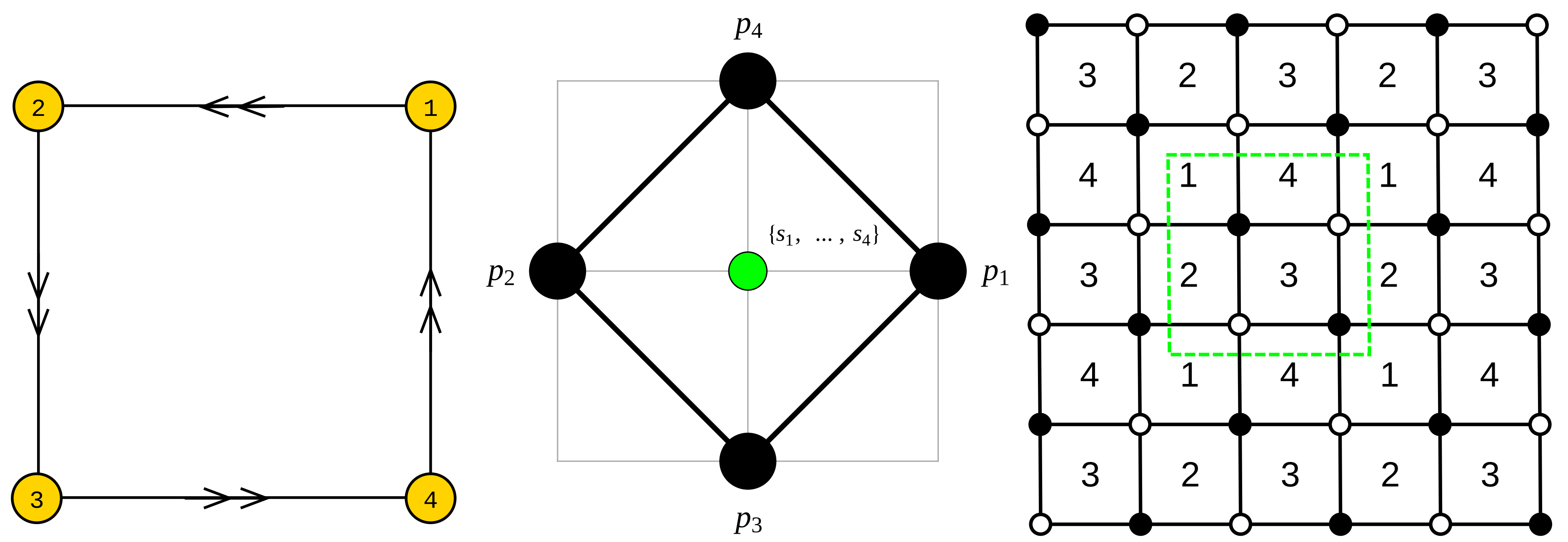


Figure 34. The quiver, toric diagram, and brane tiling of Model 15a.

The superpotential is

$$W = +X_{12}^1 X_{23}^1 X_{34}^2 X_{41}^2 + X_{12}^2 X_{23}^2 X_{34}^1 X_{41}^1 - X_{12}^1 X_{23}^2 X_{34}^2 X_{41}^1 - X_{12}^2 X_{23}^1 X_{34}^1 X_{41}^2 \quad . \tag{17.1}$$

F0 II

17.2 Model 15 Phase b

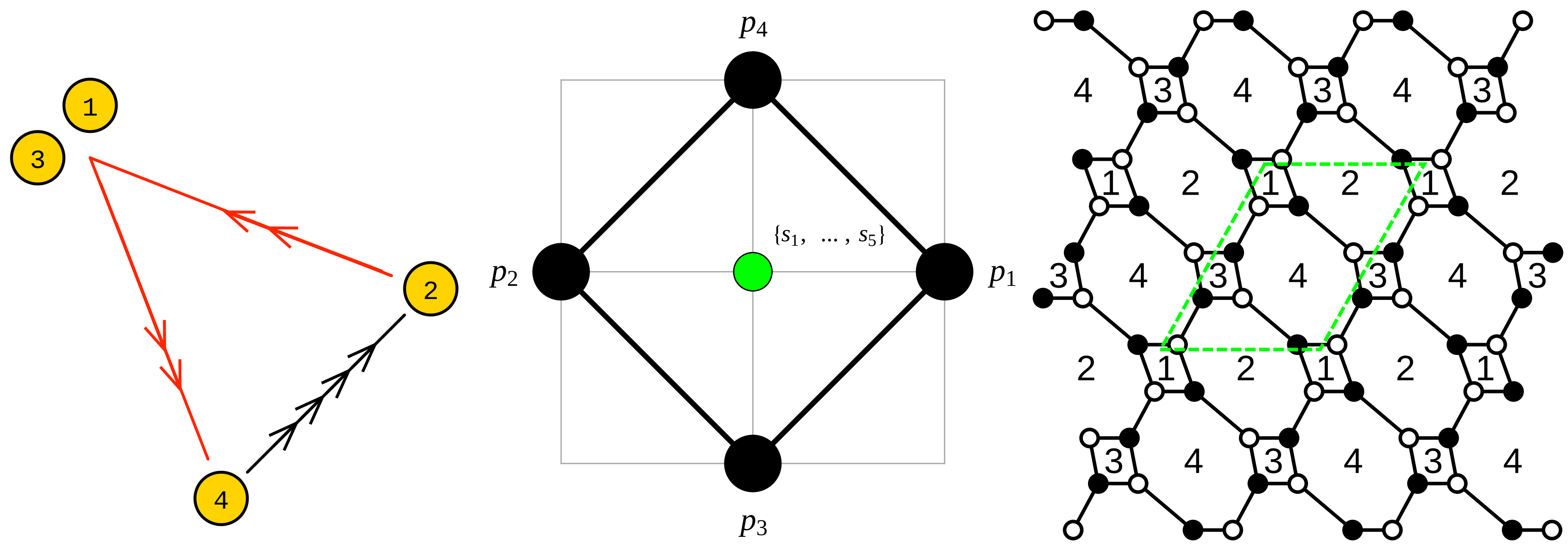
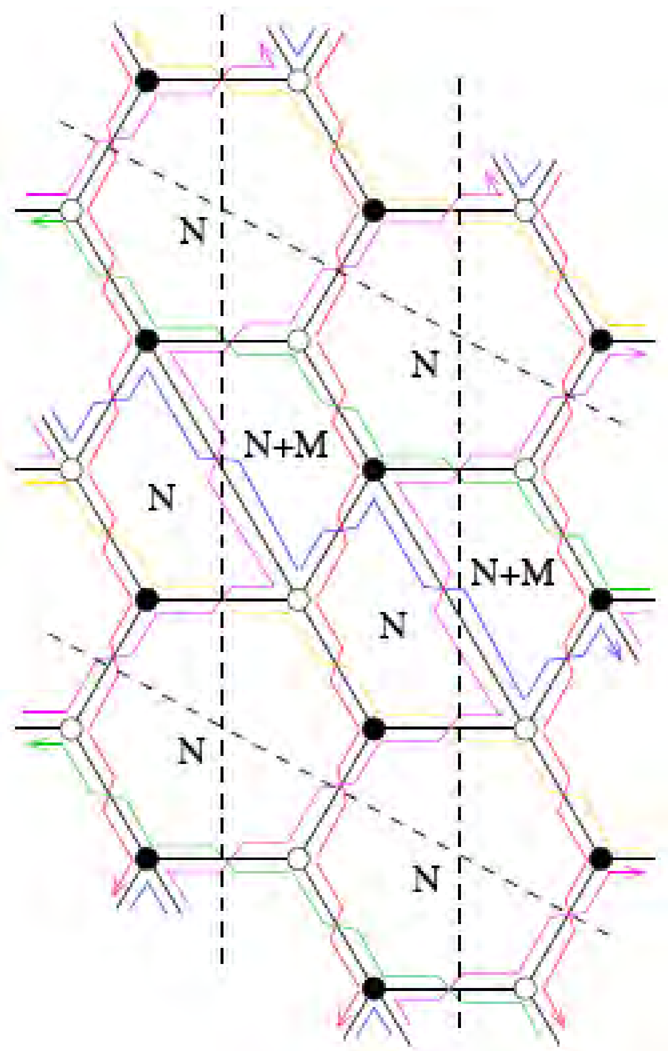


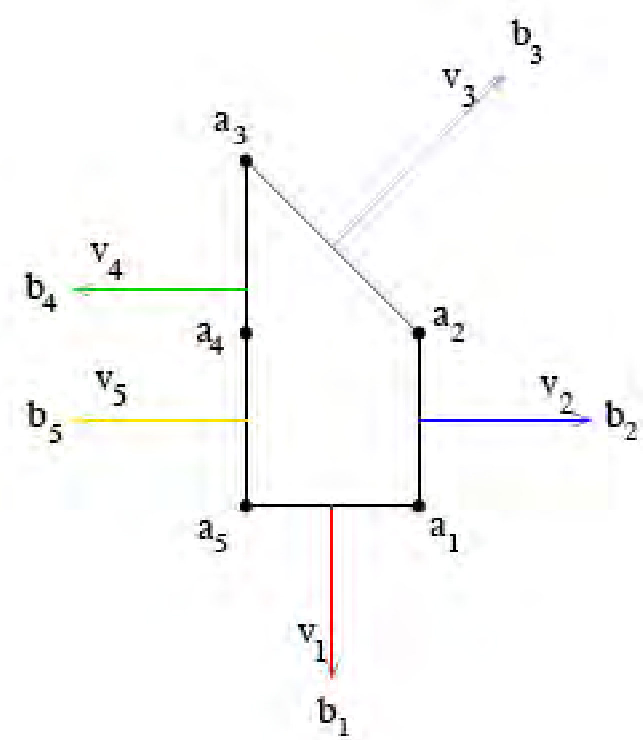
Figure 35. The quiver, toric diagram, and brane tiling of Model 15b. The red arrows in the quiver indicate all possible connections between blocks of nodes.

The superpotential is

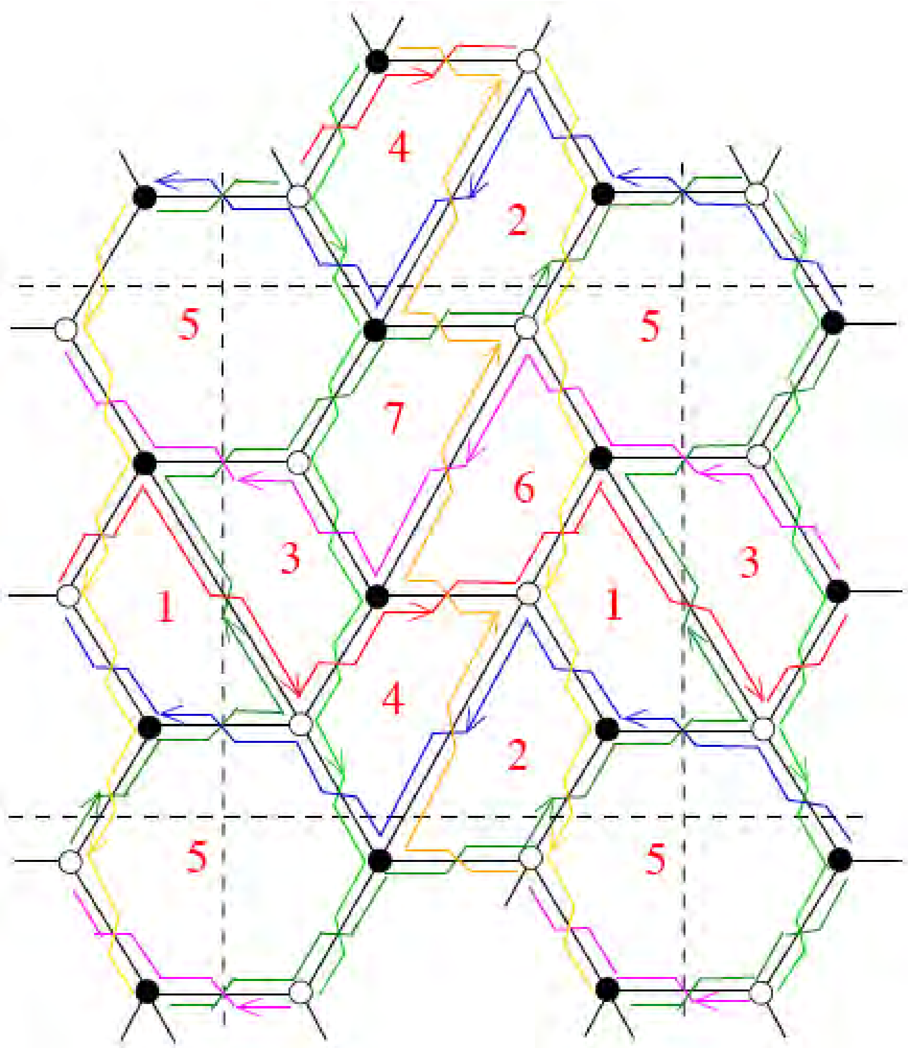
$$\begin{aligned} W = & +X_{21}^1 X_{14}^1 X_{42}^1 + X_{21}^2 X_{14}^2 X_{42}^2 + X_{23}^1 X_{34}^2 X_{42}^3 + X_{23}^2 X_{34}^1 X_{42}^4 \\ & -X_{21}^1 X_{14}^2 X_{42}^3 - X_{21}^2 X_{14}^1 X_{42}^4 - X_{23}^1 X_{34}^1 X_{42}^2 - X_{23}^2 X_{34}^2 X_{42}^1 \end{aligned} \quad (17.13)$$

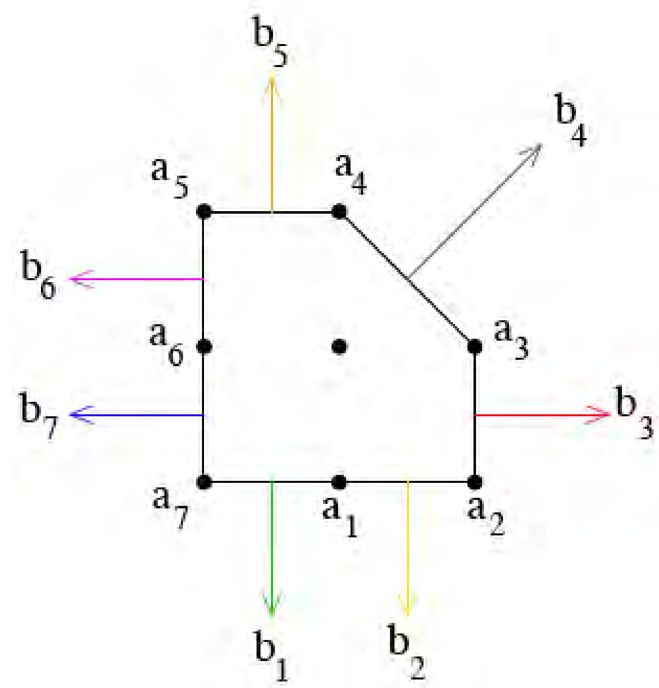
Zig Zag Paths SPP





Zig Zag paths PdP4





Zig Zag path

Zig Zag path

- Each edge at the intersection of precisely 2 zig zag paths

Zig Zag path

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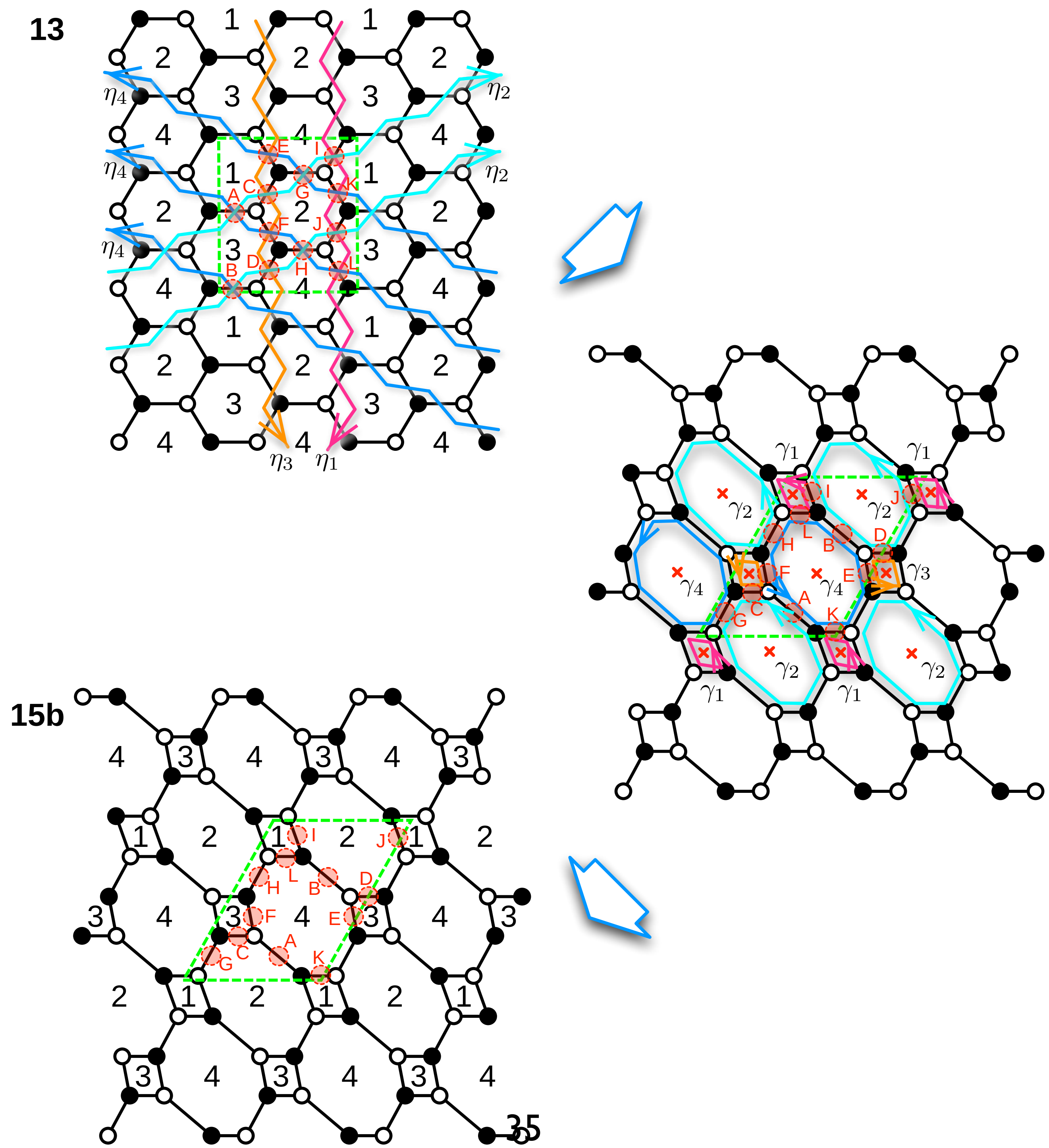
- 1-1 with external legs in the dual to the toric diagram

Zig Zag path

- Each edge at the intersection of precisely 2 zig zag paths
- 1-1 with external legs in the dual to the toric diagram
- closed paths

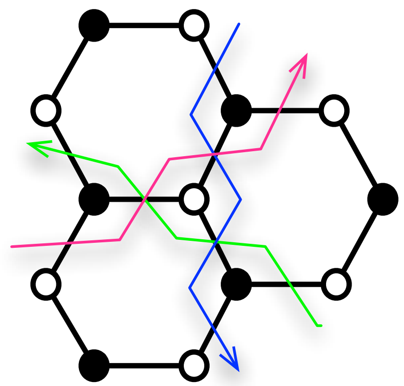
Specular Duality

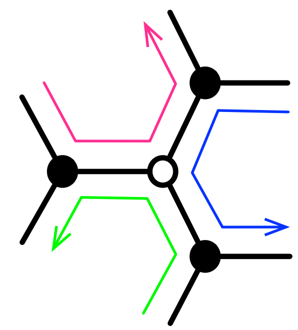


Specular Duality

Brane Tilings can have also the *same* Master Space (N=1)

zig-zag paths





faces

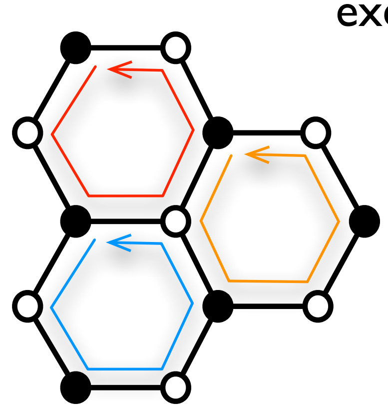
Mutation

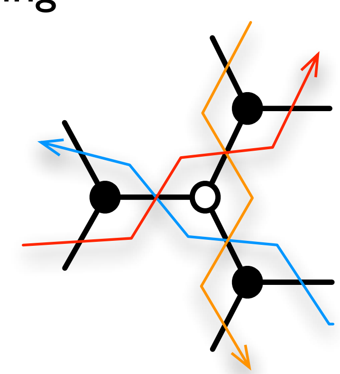
zig-zag paths and faces are exchanged while preserving adjacency

=

=

faces





zig-zag paths

Brane Tilings (Quiver Theories) are not necessary the same

Specular Duality

[Feng, He, Kennaway, Vafa 2005]
[Hanany, Seong 2012]

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- Master space is isomorphic

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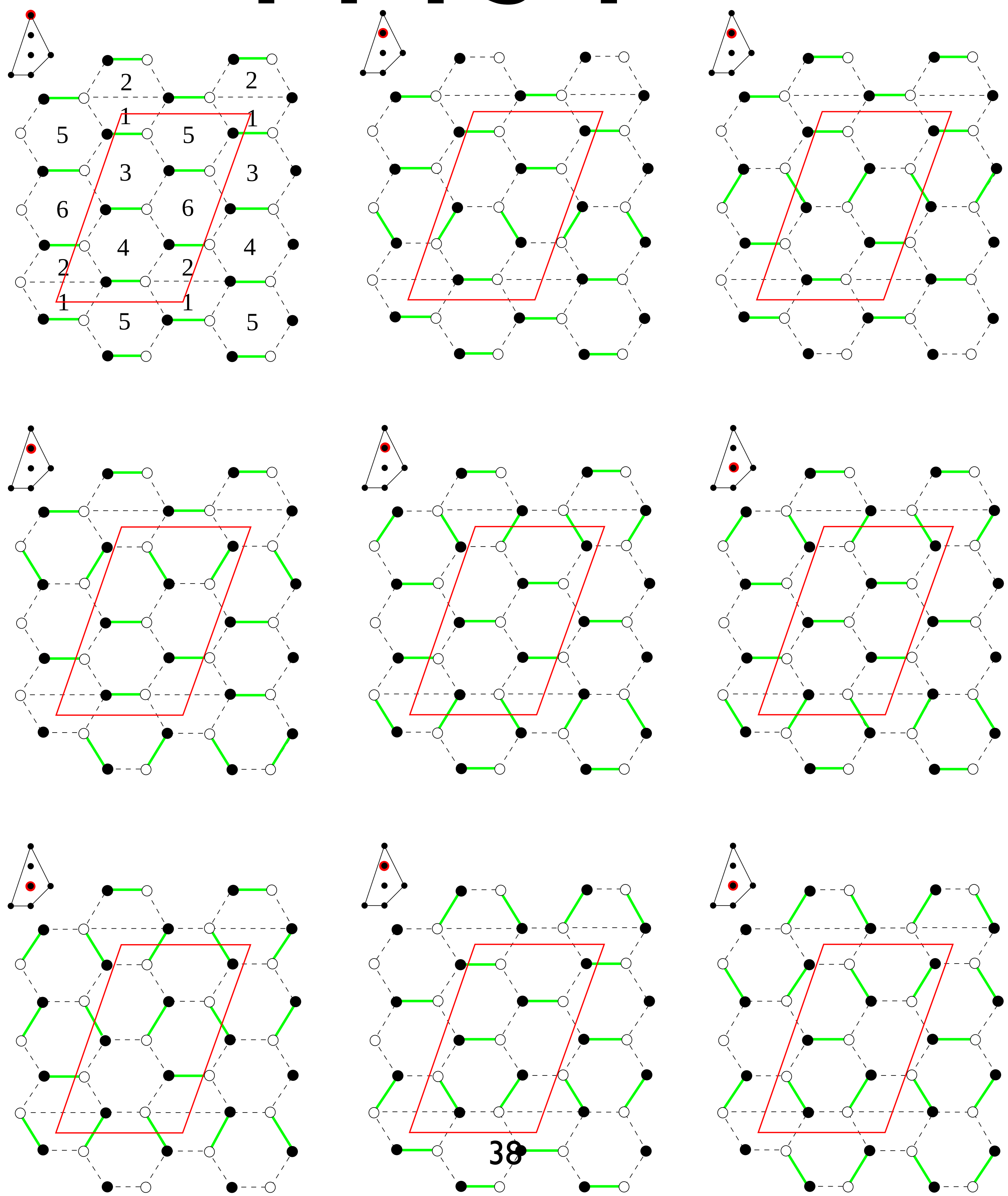
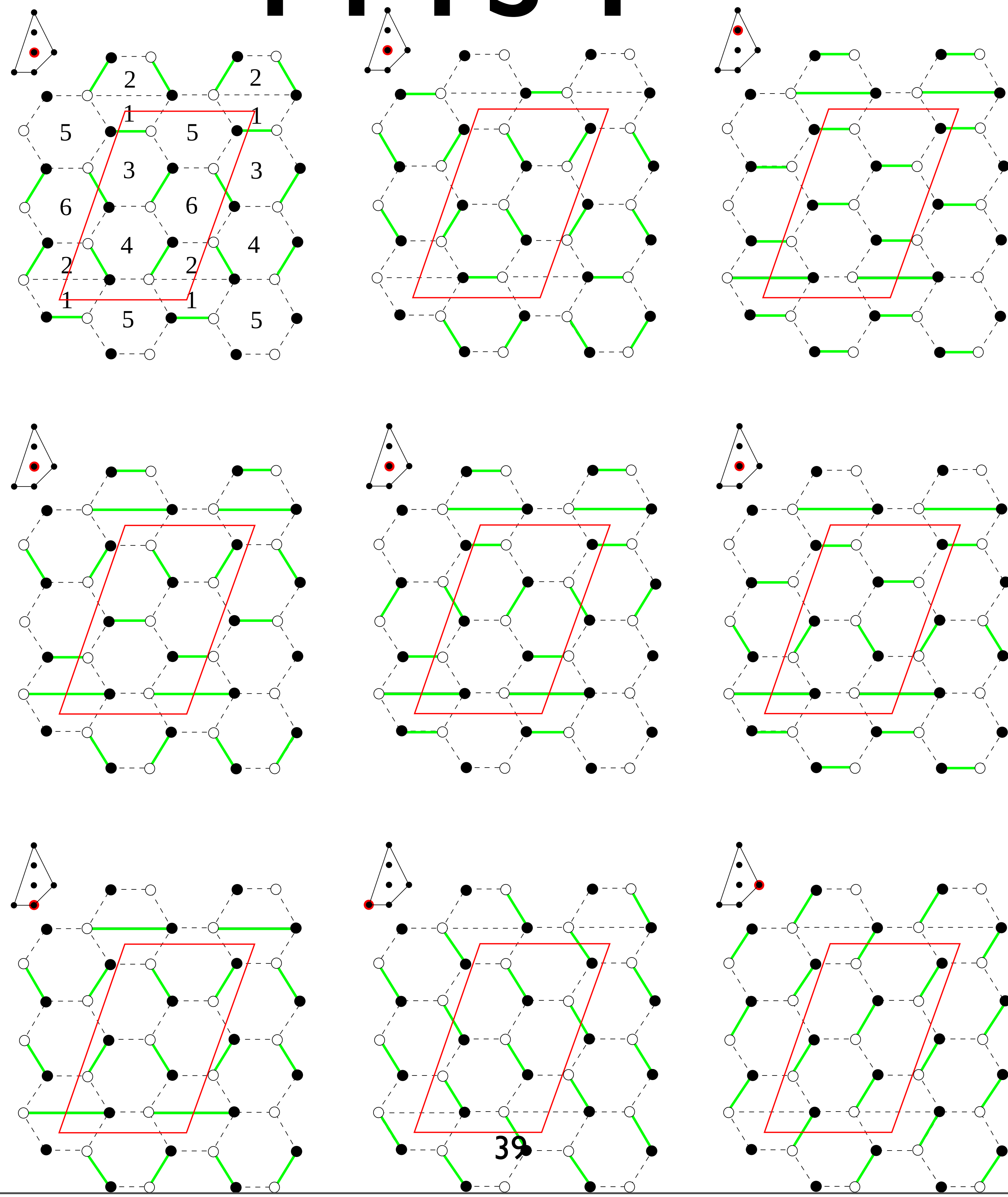
- W of Abelian theory remains the same
- internal pm's exchange with external pm's
- baryonic symmetry -- mesonic symmetry

PM's Y^{32}



PM's Y^{32}



Perfect Matchings

Perfect Matchings

- generators on the master space

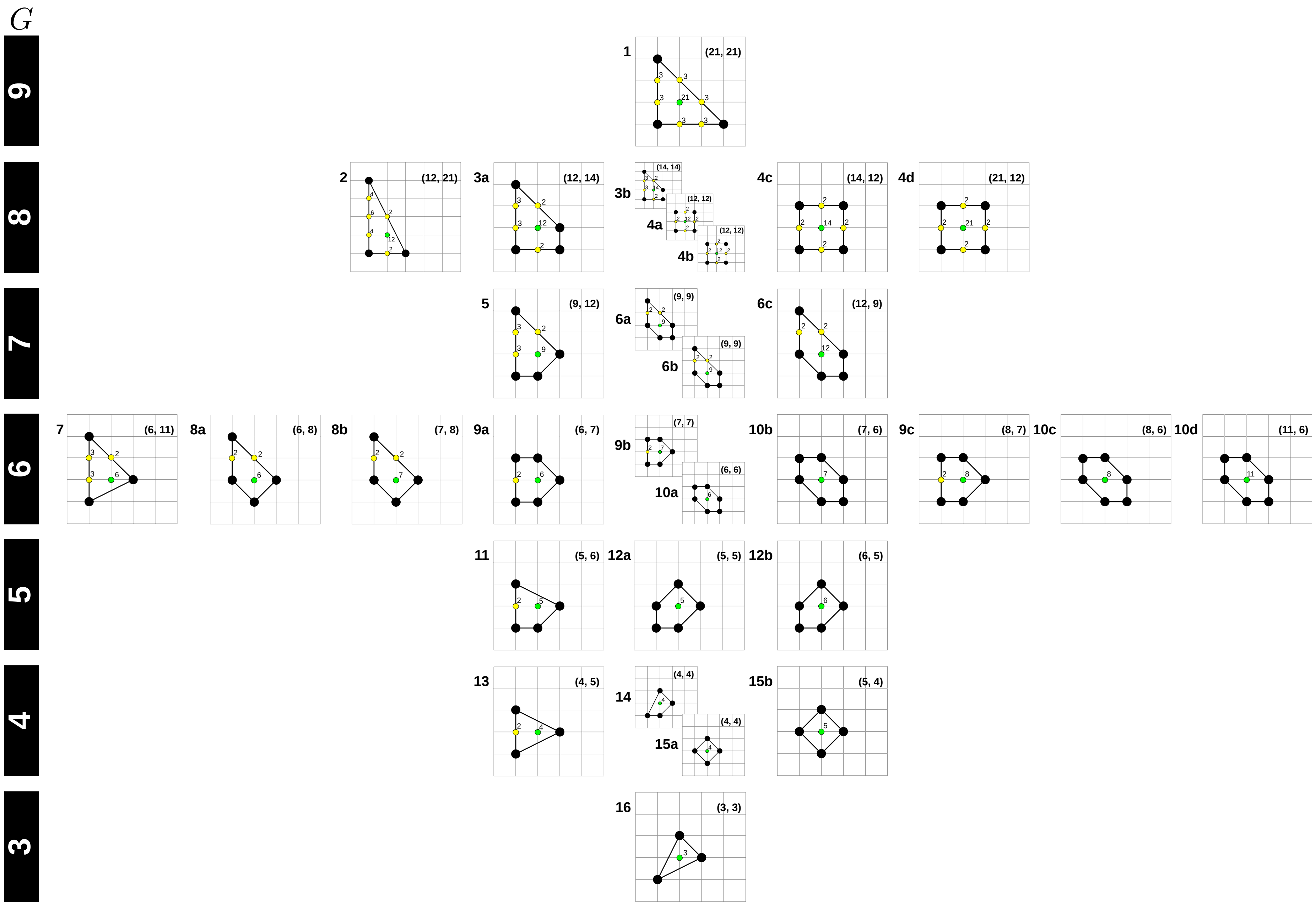
Perfect Matchings

- generators on the master space
- A solution to F term equations

Perfect Matchings

- generators on the master space
- A solution to F term equations
- points in toric diagram

Specular Duality



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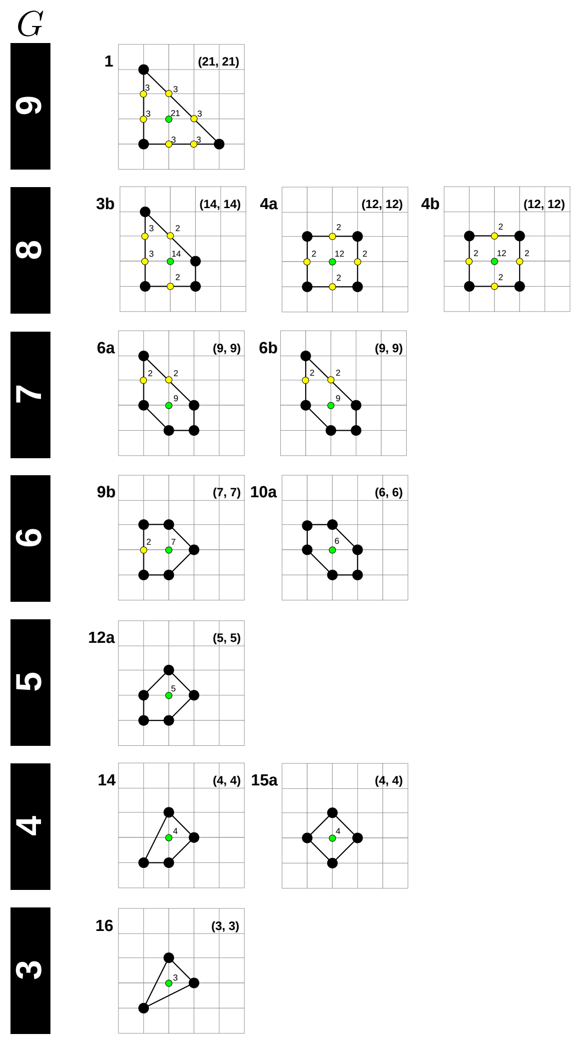
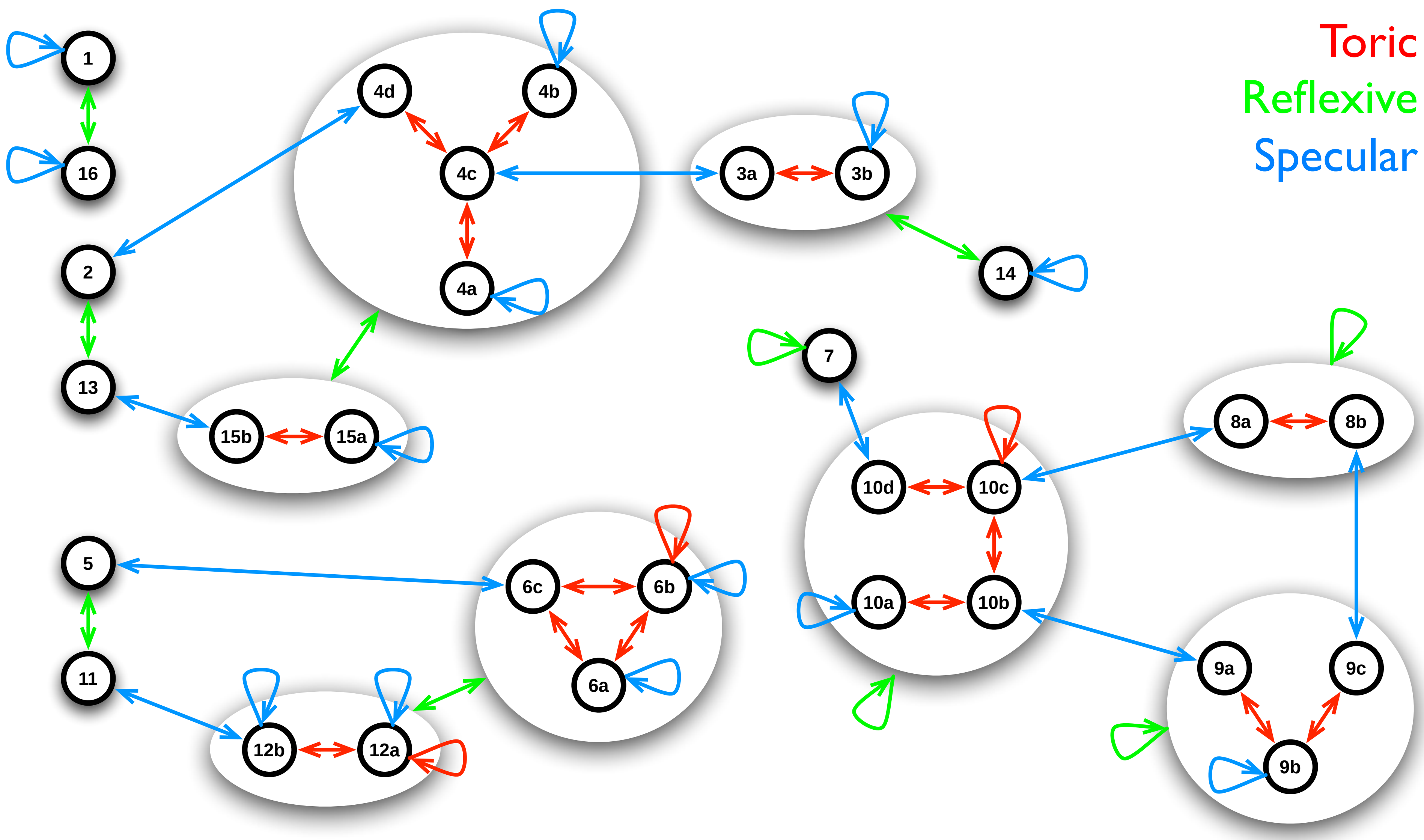
Specular Self duals



Specular Duality



Summary

Summary

- Brane Tilings

Summary

- Brane Tilings
- Mesonic, Baryonic, combined

Summary

- Brane Tilings
- Mesonic, Baryonic, combined
- Master Space, G+2, singular toric CY cone

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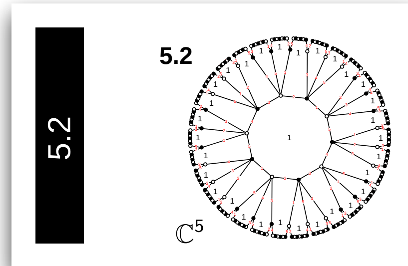
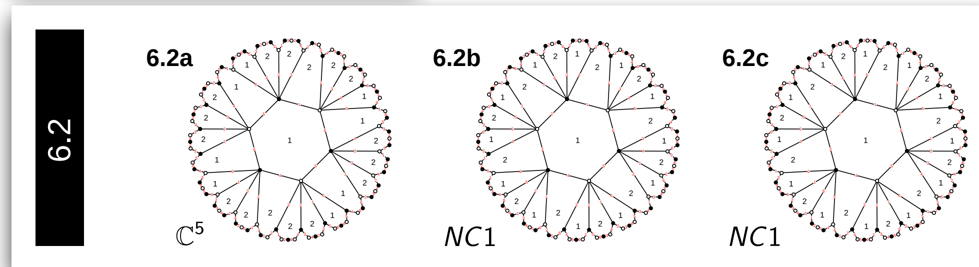
- Specular Duality (baryons vs mesons)

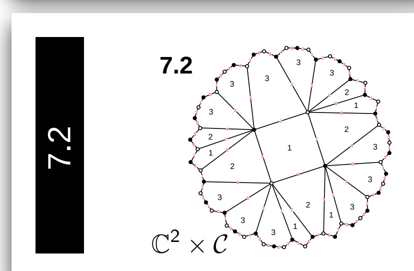
$\mathcal{M}_{3,2} \equiv \mathbb{C}^5/\langle abc - de \rangle$

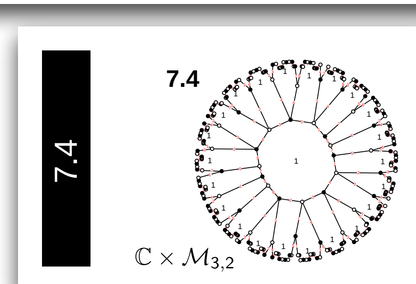
$\mathcal{M}_{3,3} \equiv \mathbb{C}^6/\langle abc - def \rangle$

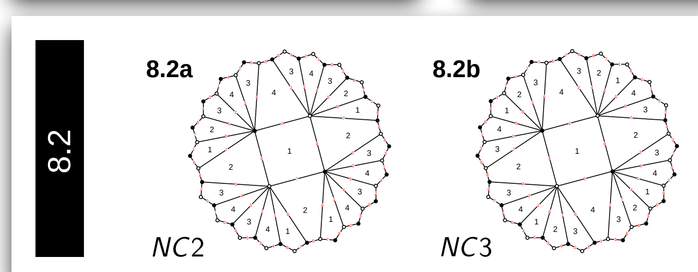
$\mathcal{M}_{4,2} \equiv \mathbb{C}^6/\langle abcd - ef \rangle$

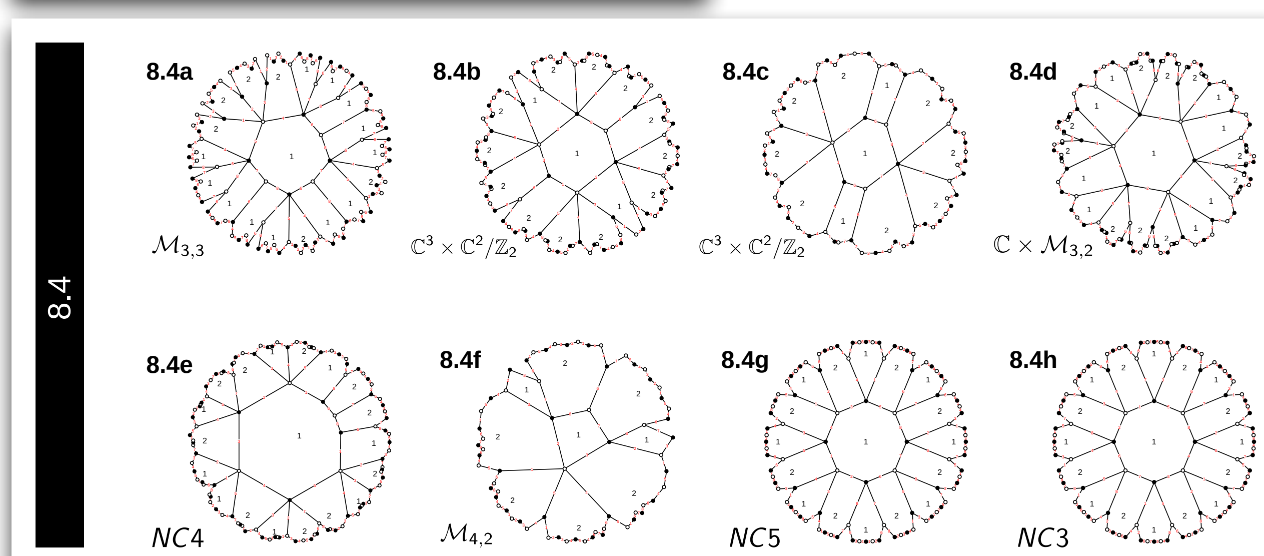












Thank You!