Thermalization on the Probe Brane

Hajar Ebrahim

Based on 1203.3425 & 1211.1637 with M. Ali-Akari

IPM

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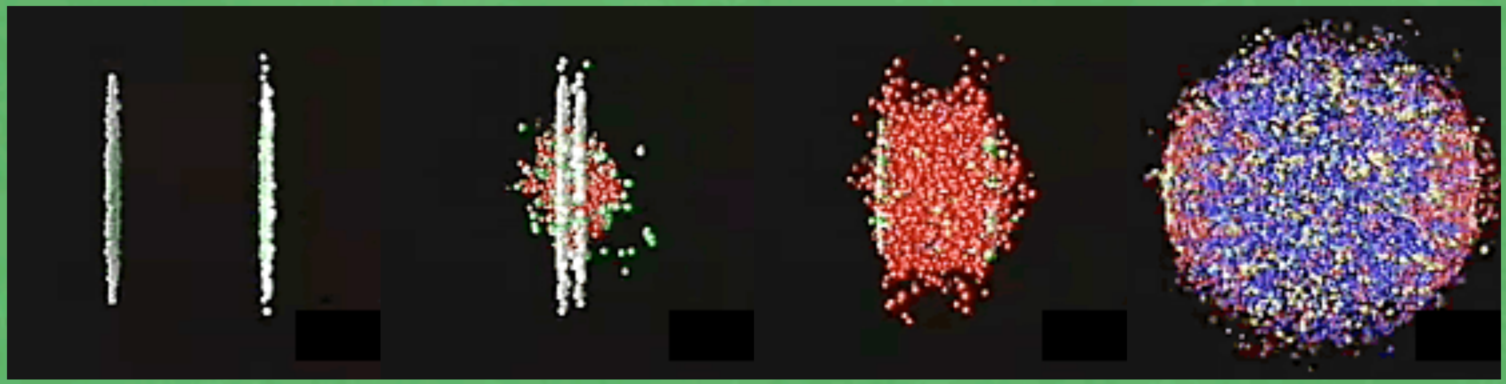
Outline:

- Introduction
  - Relativistic Heavy Ion Collision
    - What do we mean by Thermalization?
  - Gauge/Gravity Correspondence
- Thermalization in Gauge/Gravity
  - On the Probe Brane
    - The Effect of Magnetic Field
- Future Directions

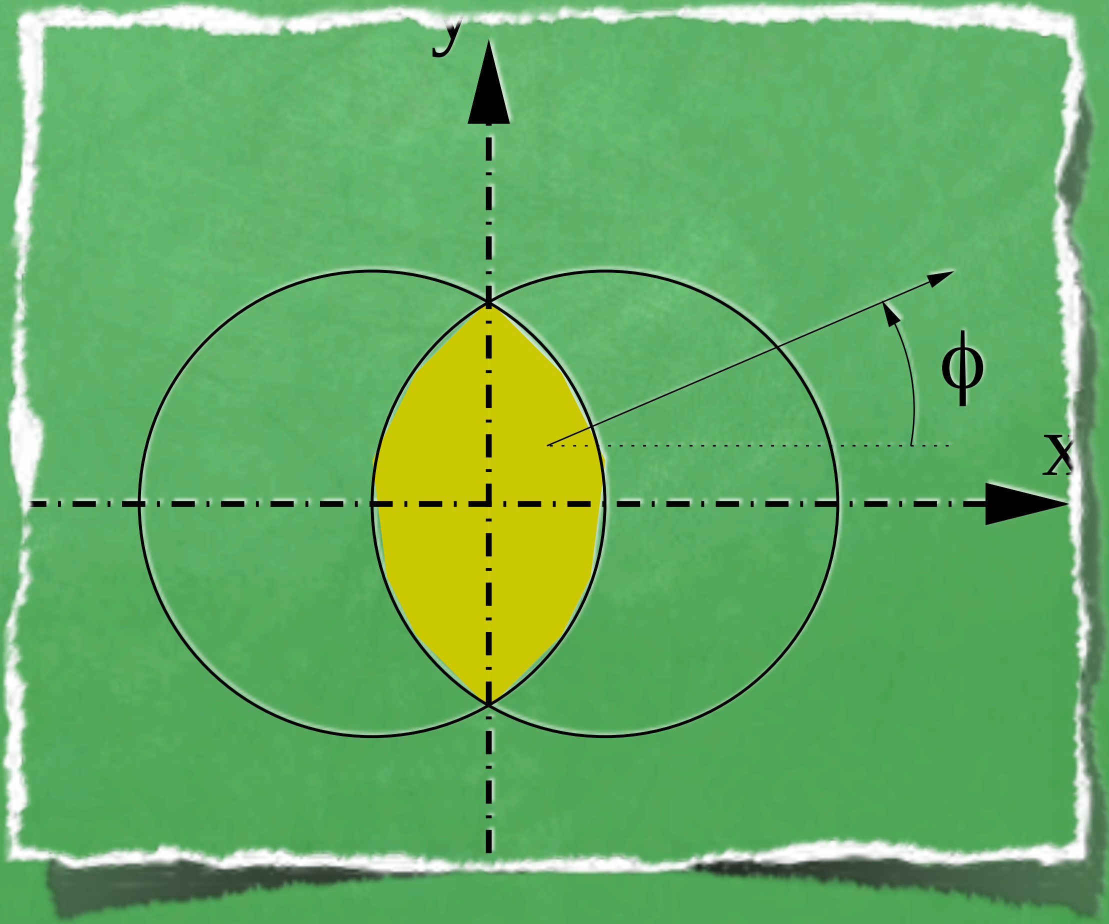
Relativistic Heavy Ion Collision

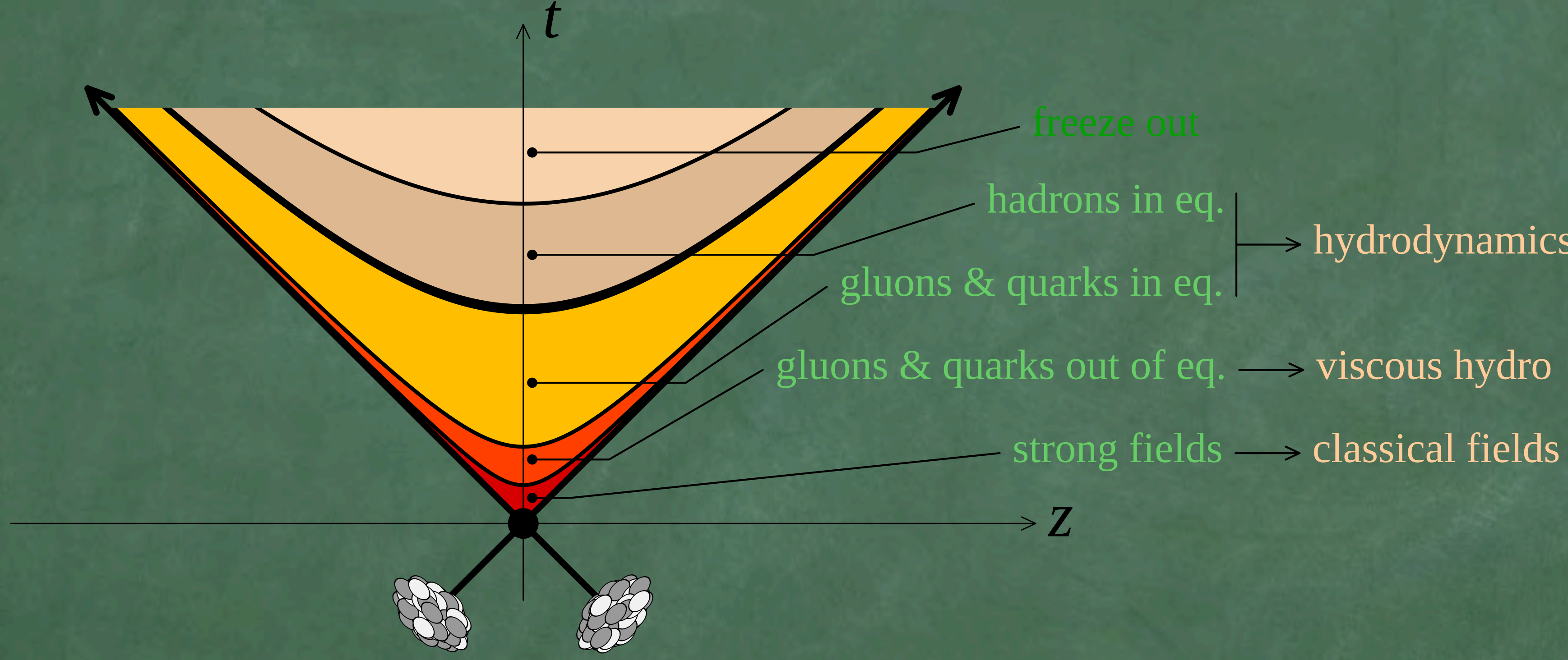
- Collision of two heavy nuclei (Gold or Lead) in relativistic speeds: Pancakes



- Center of Mass Energy:
  RHIC = 200 Gev
  LHC = 2.7 TeV

- Non-zero impact parameter ⇨ Anisotropic Quark-Gluon Plasma



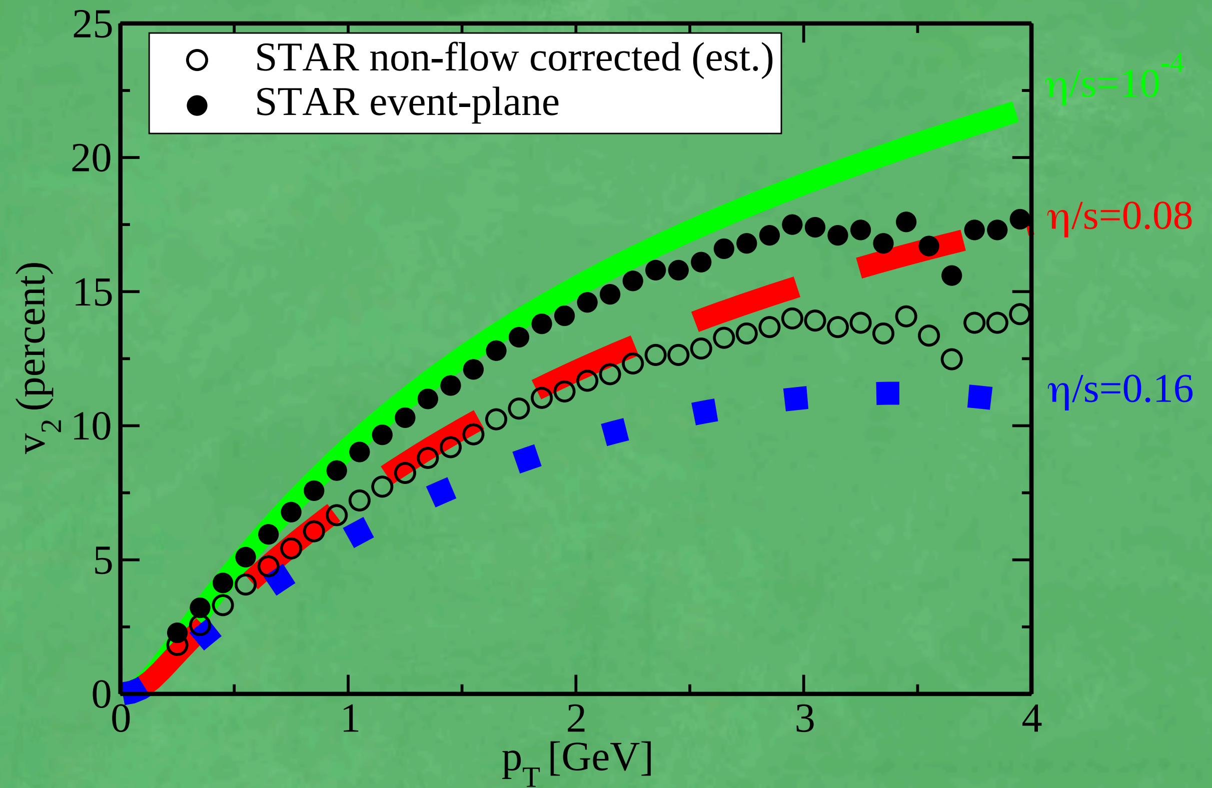


$$\tau = \sqrt{t^2 - z^2}$$

- Strongly Coupled Quark-Gluon-Plasma with very low viscosity:

$$0.08 \lesssim \frac{\eta}{s} \lesssim 0.4$$

- By 1 fm/c after the collision the matter is flowing like a fluid, obeying Hydrodynamic Equations: $\partial_\mu T^{\mu\nu} = 0$



v_2 elliptic flow

$$\frac{\mathrm{d}N}{\mathrm{d}^2\mathbf{p}_t\,\mathrm{d}y} = \frac{1}{2\pi p_T}\frac{\mathrm{d}N}{\mathrm{d}p_T\,\mathrm{d}y}\left[1 + 2v_1\cos(\phi - \Phi_R) + 2v_2\cos 2(\phi - \Phi_R) + \cdots\right]$$

- Hydrodynamization before Isotropization, viscous hydrodynamics

$$\mathbf{p} = \left(p_T\cos\phi, p_T\sin\phi, \sqrt{p_T^2 + m^2}\sinh y\right)$$

Rapid Thermalization

- Far From Equilibrium System

Quark-Gluon Plasma
produced at
Heavy Ion Colliders

> Linear Response Theory ✗
> Hydrodynamics ✗
> Strongly Coupled Plasma

perturbation away from equilibrium

- Gauge/Gravity :

type IIB string theory on $AdS_5 \times S^5$ ⟷ N=4 su(N) Conformal SYM

$$\frac{L^4}{l_s^4} \sim g_s N \sim g^2 N \gg 1$$ ⟶ Strong/Weak Duality

Strongly Coupled CFT
⟶ Classical Gravity on Asymptotically AdS Space-time

- Different asymptotically AdS spacetimes manifest themselves by different states in the boundary field theory.

Vacuum State in FT ⟷ Pure AdS Spacetime

Thermal State in FT ⟷ AdS Black Hole

- The dual gravitational description of a strongly coupled gauge theory provides an efficient way to study the thermodynamic properties of gauge theories.

- **Thermalization**

In QFT: Pure State at T=0 ⟶ Thermal State

In Gravity: Pure AdS Spacetime ⟶ AdS-Schwarzschild

Black Hole (Horizon) Formation

Question: Can we observe rapid thermalization in AdS/CFT Models?

Thermalization in Gauge/Gravity

- Horizon Formation in the Bulk

Gluon Sector

P. Chesler, L. Yaffe, 2008-10; S. Bhattacharyya, S. Minwalla, 2009;

- Horizon Formation on the Probe Brane

Das, Nishioka, Takayanagi, 2010; Hashimoto, Iizuka, Oka, 2011

Meson Sector

- Injecting Energy by time-dependent source
- Producing out of equilibrium modes
- Thermal state at equilibrium
- start from the ground state
- Turn on time-dependent source term, non-normalizable modes, collapse of matter
- Analyze the subsequent evolution of the system

- Out of Equilibrium Initial Conditions

Heller, Janik, Witaszczyk, 2012;

Horizon Formation on the Probe Brane

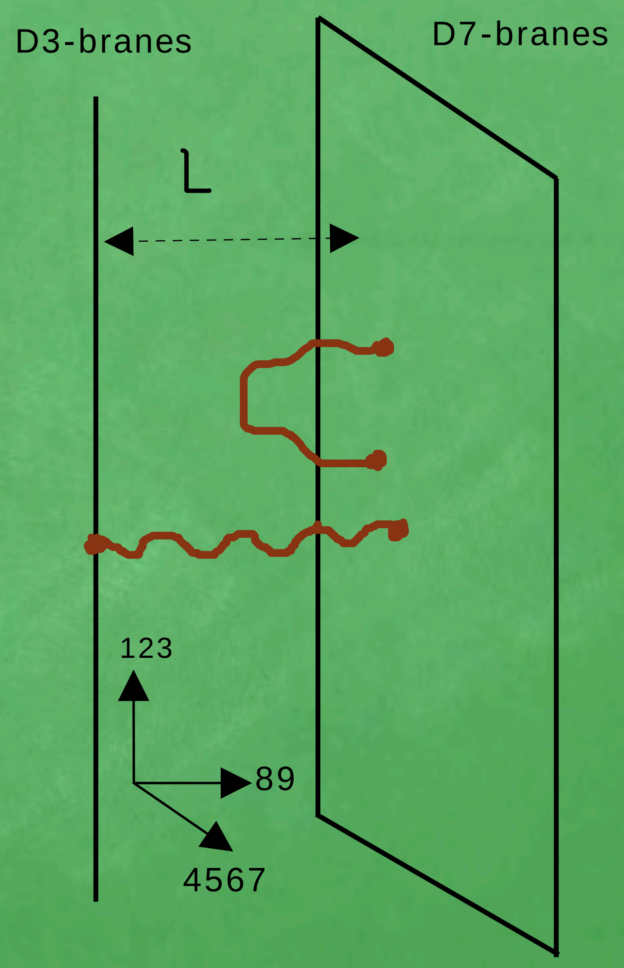
Das, Nishioka, Takayanagi, 2010; Hashimoto, Iizuka, Oka, 2011

- Introducing fundamental matter into the gauge theory
- $\mathcal{N}=4$ SU(N) SYM coupled to $\mathcal{N}=2$ Fundamental matter
- Injection of energy by introducing a time-dependent coupling ⇨ Non-trivial time-dependent classical solutions
- Horizon formation on the probe brane ⇨ Dissipation of energy into the field theory

D3 - D7 system

	0	1	2	3	4	5	6	7	8	9
$D3$	$\times$	$\times$	$\times$	$\times$						
$D7$	$\times$	$\times$	$\times$	$\times$	$\times$	$\times$	$\times$	$\times$		

- N=2 Fundamental Hypermultiplet: Flavour Sector A. Karch and E. Katz, 2002
- quarks : end points of the strings stretched between the D7 and D3-branes
- mesons : strings with both ends on the D7-brane



D7-Brane Embedding:

M. Kruczenski, D. Mateos, R. C. Myers and D. J. Winters, 2003

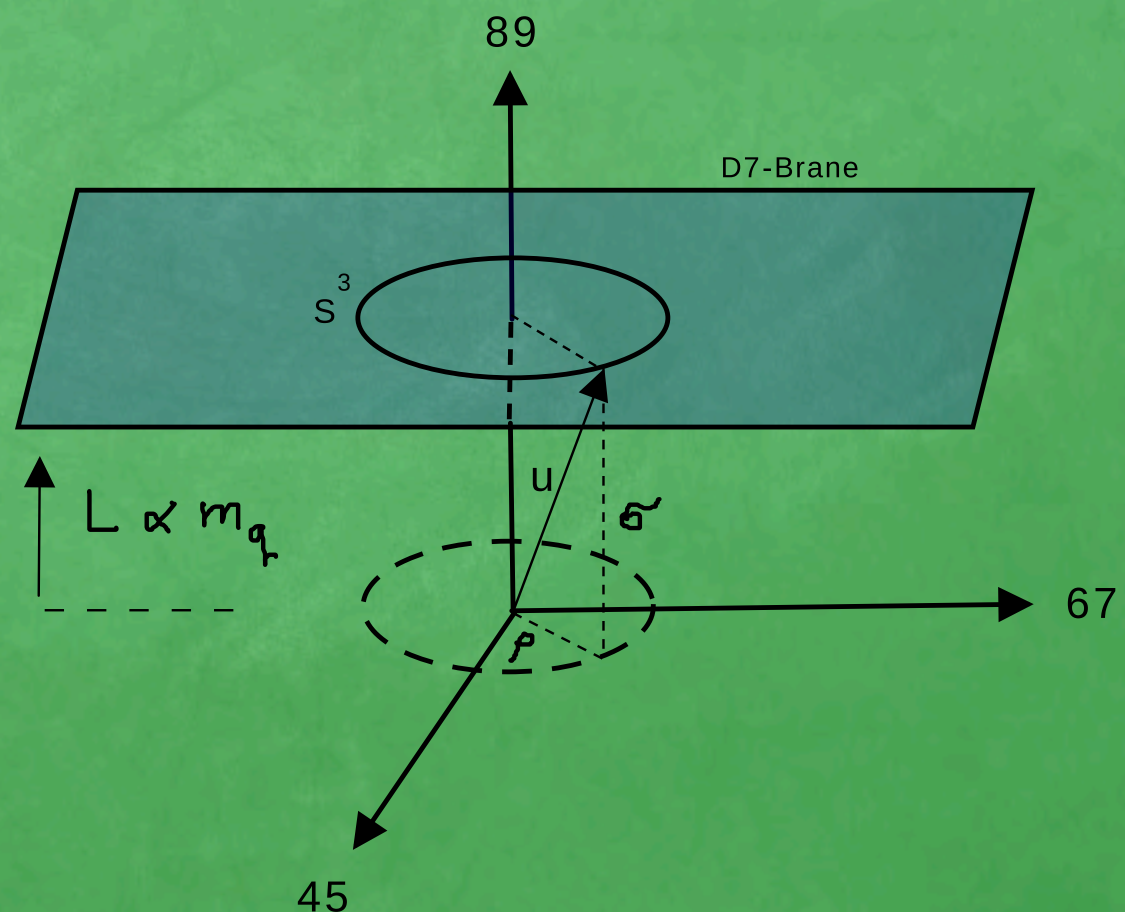
$$g_s C_{(4)} = \frac{u^4}{R^4} dx^0 \wedge dx^1 \wedge dx^2 \wedge dx^3$$

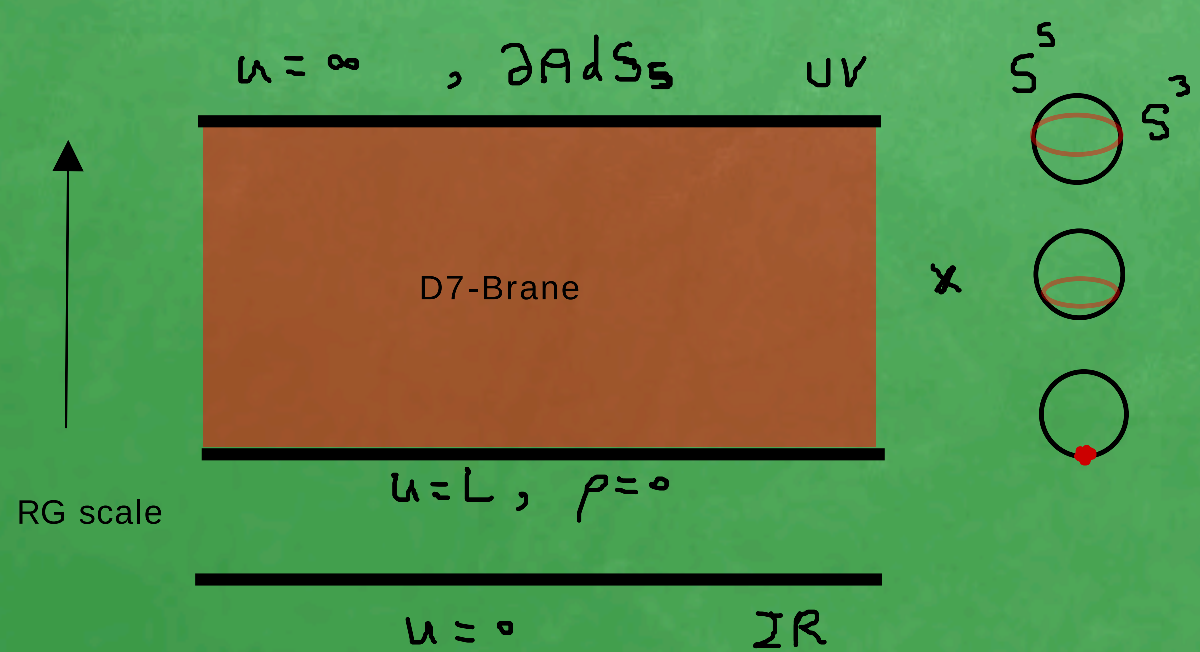
$$ds^2 = (\frac{u}{R})^2(-dt^2 + d\vec{x}^2) + (\frac{R}{u})^2(d\rho^2 + \rho^2 d\Omega_3^2 + d\sigma^2 + \sigma^2 d\varphi^2)$$

$$u^2 = \rho^2 + \sigma^2$$

$$\sigma(\rho) = m + \frac{c}{\rho^2} + \dots$$

$$c = (2\pi\alpha')^3\langle\bar{q}q\rangle$$





Shape of the Probe Brane:

- DBI action:

$$S_{\rm DBI} = -\mu_7 \int d^8\xi \; e^{-\phi}\sqrt{-\det(g_{ab} + B_{ab} + 2\pi\alpha' F_{ab})}$$

$$g_{ab} = G_{MN}\partial_a X^M \partial_b X^N$$

$$B_{ab} = B_{MN}\partial_a X^M \partial_b X^N$$

$B_{MN} = 0$

$AdS_5 \times S^5$

- $S_{cs} = 0$

- Massless Quarks: $\sigma = 0 + \delta\sigma(x)$

> – Adding a time-dependent source term to the action
> – Solving the e.o.m to obtain the time-dependent gauge field solution
> – Calculate the metric observed by the probe brane fluctuations

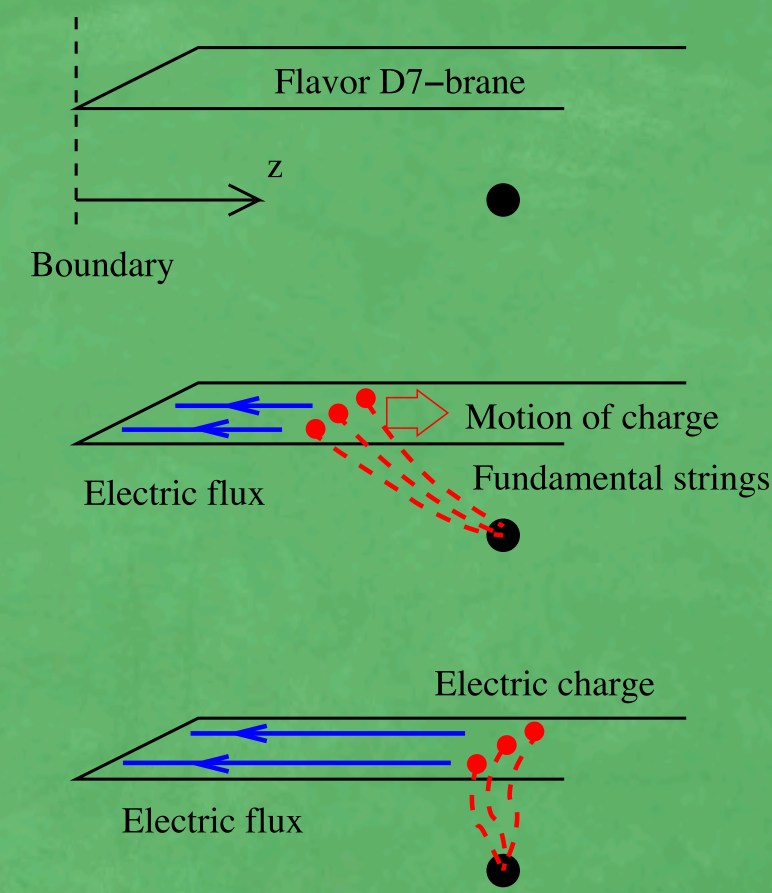
Baryon Injection:

- Adding the source term

$$\delta S = \mu_7 V_3 \mathrm{Vol}(\Omega_3) \int dt d\rho (A_t j^t + A_\rho j^\rho)$$

$$x^{\pm} \;=\; t \pm z = t \mp \int H^{\frac{1}{2}} d\rho \qquad H = (R/u)^4$$

$$j^\rho = -H^{\frac{-1}{2}} j^t = g'(x^-)$$: Massless Quarks



$$(2\pi\alpha')F_{t\rho} = \frac{g}{\sqrt{g^2 + (2\pi\alpha')^2 \rho^6}}$$

← Solution to the e.o.m.

$$g(x^-) = (2/\pi)(2\pi\alpha')^4 \lambda n_B(x^-)$$: Time-Dependent Chemical Potential

Baryon Number Density

Identifies how the baryons are injected

$$g(x^-) = \begin{cases} 0 & x^- < 0 \\ g_{max}\, \omega x^- & 0 < x^- < \frac{1}{\omega} \\ g_{max} & \frac{1}{\omega} < x^- \end{cases}$$

Meson Thermalization - Dissociation

- scalar mesons (scalar fluctuations)

$$S = -\int dt d\rho d^3x^i d^3\theta^\alpha \sqrt{-\tilde{g}}\; \tilde{g}^{ab}\partial_a\delta\sigma\partial_b\delta\sigma$$

open string metric $\tilde{g} = g - (2\pi\alpha')^2 F g^{-1} F$

- Apparent Horizon : A surface whose area variation vanishes along the null ray.

$$V_{\text{surface}} = \int d^3x^i d^3\theta^\alpha \Big(\prod_{i=1}^{3}\tilde{g}_{ii}\prod_{\alpha=1}^{3}\tilde{g}_{\alpha\alpha}\Big)^{\frac{1}{2}}$$

$$= V_3 \mathrm{Vol}(S^3)\frac{\mu_7}{2\pi\alpha'}H^{\frac{1}{2}}\sqrt{g^2 + (2\pi\alpha')^2\rho^6}$$

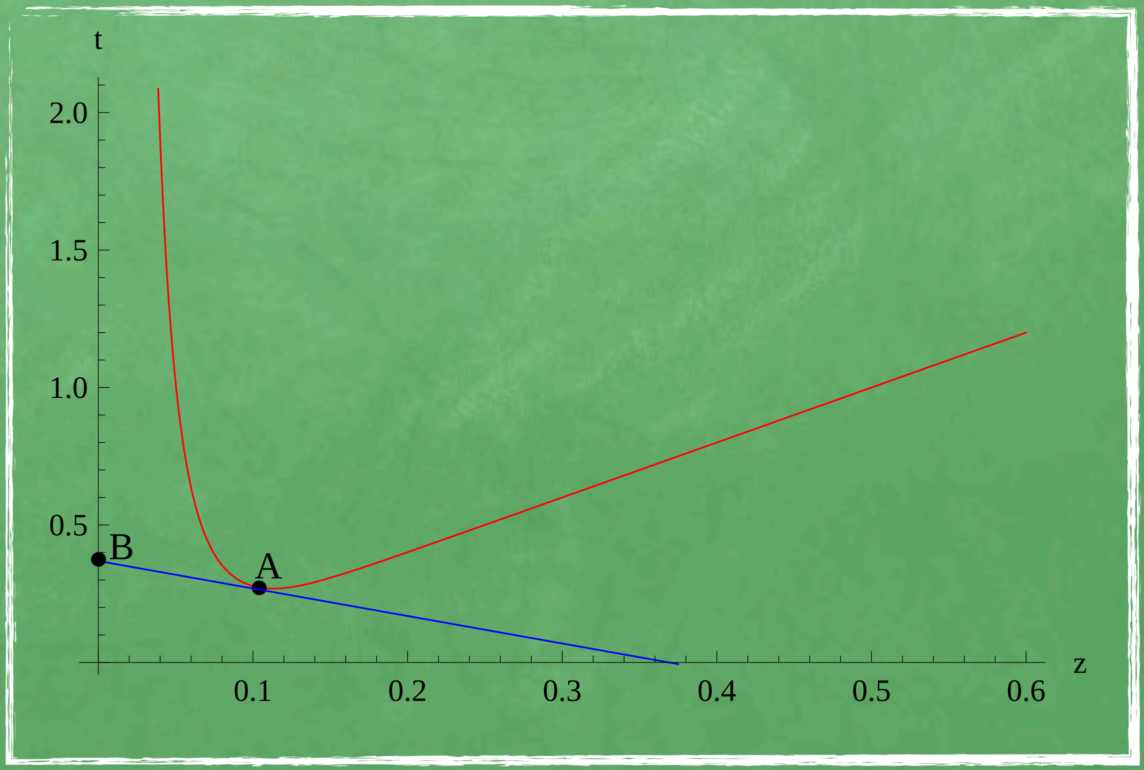
$$dV\big|_{dt=-dz} = 0$$

Apparent horizon equation leads to :

$$4gg'H^{\frac{1}{2}}\rho - 4g^2 + 2(2\pi\alpha')^2\rho^6 = 0$$

Thermalization time : The time the earliest null ray tangent to t-z graph reaches the boundary.

$$t_{th} \sim \left(\frac{\lambda}{n_B^2\omega^2}\right)^{\frac{1}{8}}$$



general Dp-Dq

Ali-Abrari, Ebrahim, 2012

R. C. Myers, R. M. Thomson, 2006
D. Arean, A. V. Ramallo, 2006

q=p+4

	0	1	$\cdots$	p	$p+1$	$p+2$	$p+3$	$p+4$	$p+5$	$\cdots$	9
Dp	$\times$	$\times$	$\cdots$	$\times$							
$D(p+4)$	$\times$	$\times$	$\cdots$	$\times$	$\times$	$\times$	$\times$	$\times$			

Only Transverse Fluctuations:

$$t_{th} \sim \left(\frac{\lambda^{\frac{2(p-2)}{5-p}}}{n_B^2 \omega^2} \right)^{\frac{5-p}{2(11-p)}}$$

q=p+2

	0	1	$\cdots$	$p-1$	p	$p+1$	$p+2$	$p+3$	$p+4$	$\cdots$	9
Dp	$\times$	$\times$	$\cdots$	$\times$	$\times$						
$D(p+2)$	$\times$	$\times$	$\cdots$	$\times$		$\times$	$\times$	$\times$			

Both Transverse and Parallel Fluctuations:

$$t_{th} \sim \left(\frac{\lambda^{\frac{2(p-3)}{5-p}}}{n_B^2 \omega^2} \right)^{\frac{5-p}{2(9-p)}}$$

q=p

	0	1	$\cdots$	$p-2$	$p-1$	p	$p+1$	$p+2$	$p+3$	$\cdots$	9
background	$\times$	$\times$	$\cdots$	$\times$	$\times$	$\times$					
probe	$\times$	$\times$	$\cdots$	$\times$			$\times$	$\times$			

Both Transverse and Parallel Fluctuations:

$$t_{th} \sim \left(\frac{\lambda^{\frac{2(p-4)}{5-p}}}{n_B^2 \omega^2} \right)^{\frac{5-p}{2(7-p)}}$$

- 4-Dim N=4 SYM

$$t_{th} \sim \begin{cases} (\frac{\lambda}{n_B^2\omega^2})^{\frac{1}{8}}, & \mathrm{D3-D7}, \quad 3+1 \\ (\frac{1}{n_B^2\omega^2})^{\frac{1}{6}}, & \mathrm{D3-D5}, \quad 2+1 \\ (\frac{1}{n_B^2\omega^2\lambda})^{\frac{1}{4}}, & \mathrm{D3-D3}. \quad 1+1 \end{cases}$$

No Dependence on SUSY or Conformal Symmetry

- (2+1)-Dim Defect

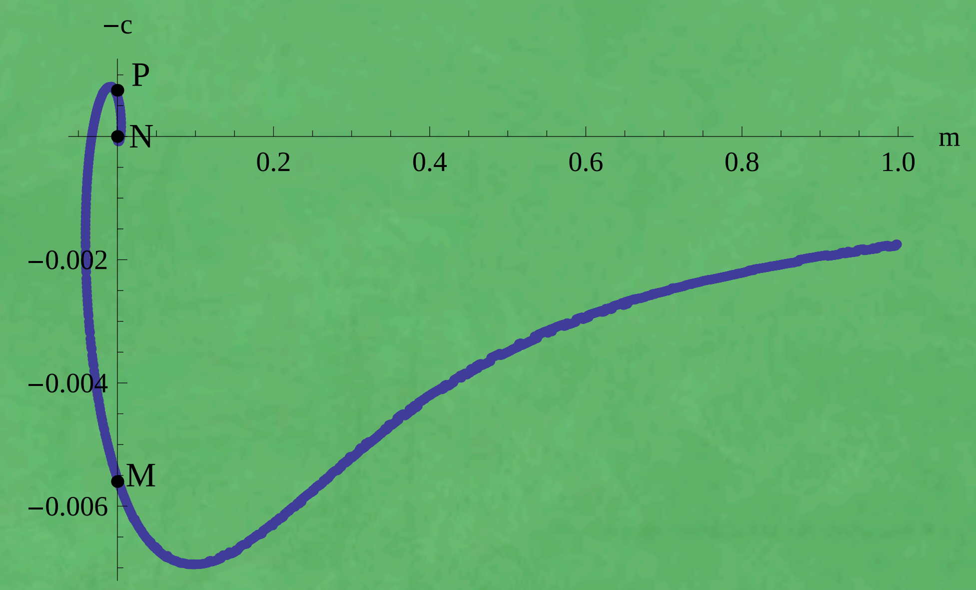
D2-D6

D3-D5 $\quad (\frac{1}{n_B^2\omega^2})^{\frac{1}{6}}$

D4-D4

The Effect of the Magnetic Field on Thermalization Time:

Ali-Abrari, Ebrahim, 2012

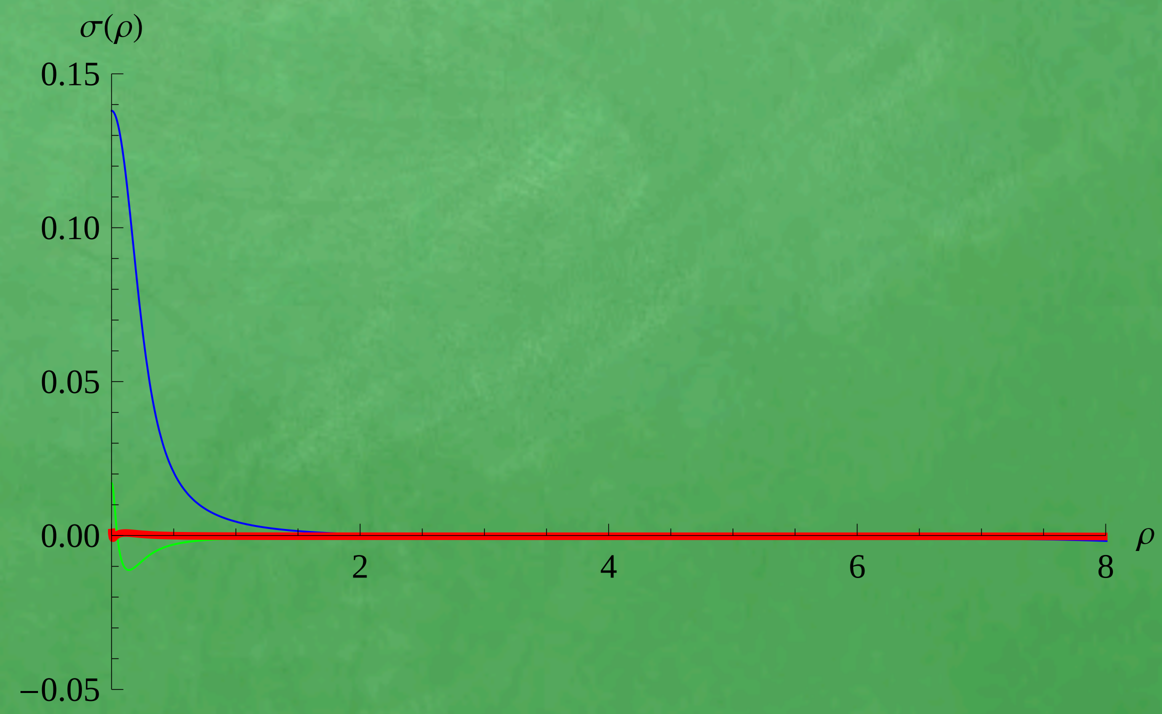


- Presence of a magnetic field at the early stages of QGP production

  Kharzeev, McLerran, Warringa, 2007

- Ansatz for the Magnetic field:

  $$B = F_{xy}$$



- Existence of the Massless solution

  $$\sigma(\rho) = m + \frac{c}{\rho^2} + ...$$

- Change in the shape of the brane

- The classical solution is non-zero

transverse direction : $x^I = x_0^I + y^I$

classical solution (x_0^I) — fluctuation (y^I)

- DBI Action:

$$S_{DBI} = S_0 + S_1 + \ldots$$
$$= S_0 - \frac{\mu_7}{2}\int d^8\xi\sqrt{\gamma_0}\,\Big(\gamma_0^{ab}M_{ba} + \gamma_0^{ab}N_{ba} - \frac{1}{2}\gamma_0^{ab}M_{bc}\gamma_0^{cd}M_{da} + \frac{1}{4}(\gamma_0^{ab}M_{ba})^2 + \ldots\Big)$$

$$S_0 = -\mu_7\int d^8\xi\sqrt{\gamma_0}$$

$$\gamma_{0\ ab} = G_{ab} + G_{IJ}\partial_a x_0^I \partial_b x_0^J + (2\pi\alpha')F_{ab}$$

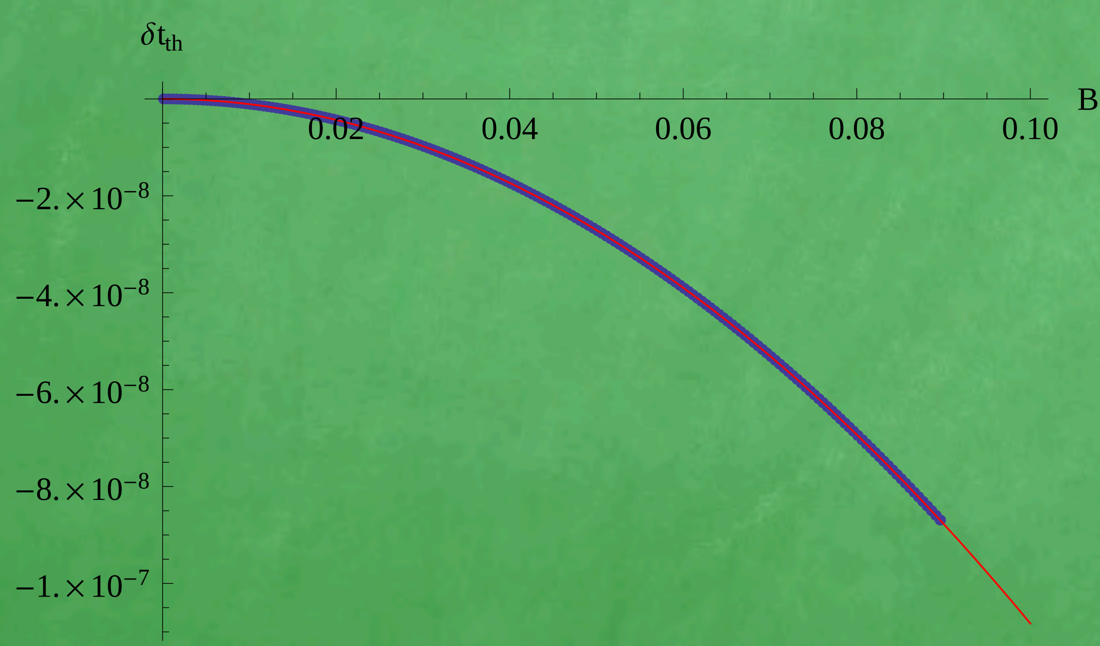
$$M_{ab} = \partial_I G_{ab} y^I + G_{IJ}\partial_a x_0^I \partial_b y^J + G_{IJ}\partial_a y^I \partial_b x_0^J + \partial_K G_{IJ}\partial_a x_0^I \partial_b x_0^J y^K$$

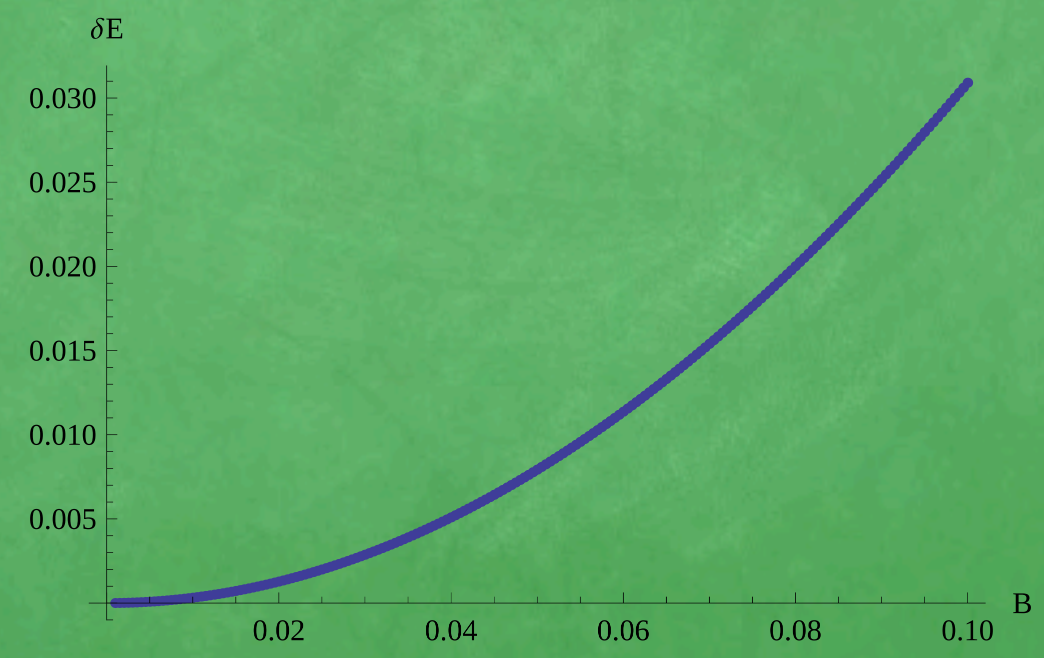
$$N_{ab} = \frac{1}{2}\partial_I\partial_J G_{ab} y^I y^J + G_{IJ}\partial_a y^I \partial_b y^J + \partial_K G_{IJ}\partial_a x_0^I \partial_b y^J y^K + \partial_K G_{IJ}\partial_a y^I \partial_b x_0^J y^K + \frac{1}{2}\partial_K\partial_L G_{IJ}\partial_a x_0^I \partial_b x_0^J y^K y^L$$

- Weak magnetic field limit: $2\pi\alpha' B \ll 1$

$$t_{th}^{B} = \left(1 - 29 \times 10^{-6} (2\pi\alpha')^2 B^2\right) t_{th}^{B=0}$$

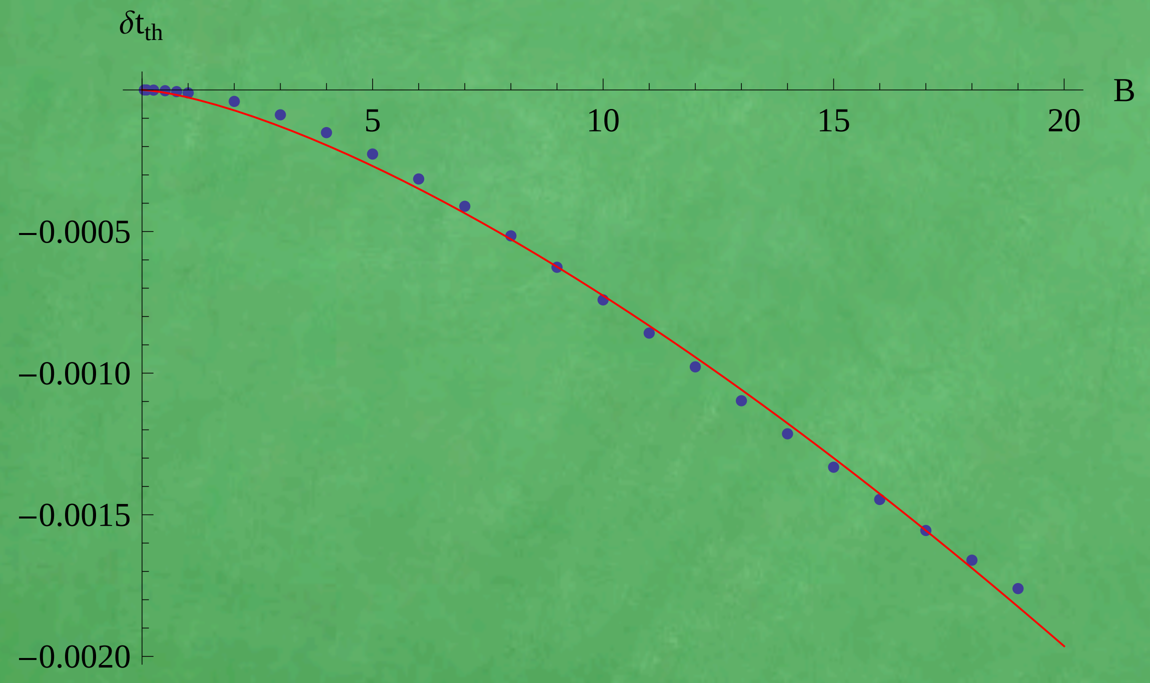
$$\delta t_{th} = t_{th}^{B} - t_{th}^{B=0}$$





- General values of the magnetic field

$$t_{th}^{B} = \left(1 - 4\times10^{-5}(2\pi\alpha')^2 B^{1.435}\right) t_{th}^{B=0}$$



Thermalization happens faster in the presence of magnetic field.

Conclusion and Future Directions:

- Meson thermalization time depends only on ω, λ and n_B; Faster Thermalization in External Magnetic Field
- Generalizing the calculation to more general bulk metrics such as black hole or Lifshitz
- Trying to understand the field theory picture better
- away from large N or large t'Hooft coupling
- Developing numerical techniques to include more parameters, ...

Thank You