Non-equilibrium phase transition from AdS/CFT

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Seventh Crete regional meeting on string theory
Kolymbari 2013

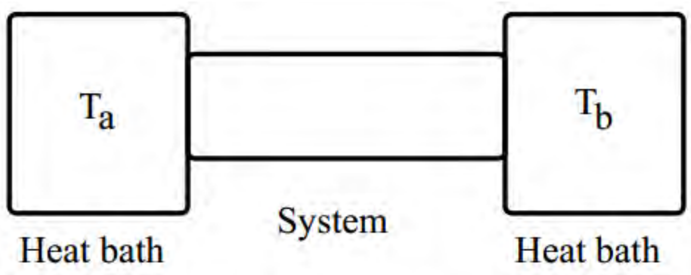
Outline

1. Non-equilibrium system: non-equilibrium steady sates.
2. Non-equilibrium phase transition of differential conductivity.
3. AdS/CFT correspondence + matter fields
4. Non-linear conductivity in the presence of magnetic field:
   3.1: D3-D7 system.
   3.2: D3-D5(supersymmetric and non-supersymmetrc) systems.

Non-equilibrium system

Challenging problem: Physical system at non-equilibrium phase.

1. Local equilibrium ⟶ Hydrodynamics
2. Non-equilibrium steady states:
   They have no macroscopically observable time dependence.



A system in contact with two heat baths at temperatures T_a and T_b.

Y. Oono, and M. Paniconi, “Steady state thermodynamics,” Prog. Theor. Phys. Suppl. **130**, 29--44 (199
S. Sasa and H. Tasaki, ``Steady state thermodynamics,'' [cond-mat/0411052].
Z. Racz, ``Nonequilibrium phase transition,'' [cond-mat/0210435].
B. Derrida, ``Non equilibrium steady states: fluctuations and large deviations of the density and of th

Conductivity: $\sigma(T) = \frac{J}{E}$

Non-linear conductivity: $\sigma(T, E) = \frac{J}{E}$

Duo to the non-linearity of conductivity, differential conductivity may be either positive or negative.

$\frac{\partial J}{\partial E} > 0$ Positive differential conductivity(PDC)

~~Phase transition~~

$\frac{\partial J}{\partial E} < 0$ Negative differential conductivity(NDC)

→ **Strong interaction**

↓

AdS/CFT correspondence as a theoretical framework

Y. Taguchi, T. Matsumoto, Y. Tokura, "Dielectric breakdown of one-dimensional Mott insulators Sr_2CuO_3 and $SrCuO_2$," Phys. Rev. B 62, 7015-7018 (2000)

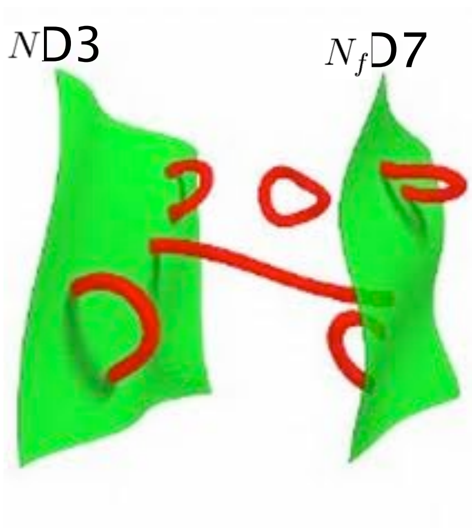
Takashi Oka, Hideo Aoki, " Nonequilibrium Quantum Breakdown in a Strongly Correlated Electron System", arXiv:0803.0422

AdS/CFT correspondence

IIB string theory on AdS(5)xS(5) $\longleftrightarrow$ N=4 D=4 superconformal Su(N) gauge theory

D3-D7:

	X_0	X_1	X_2	X_3	X_4	X_5	X_6	X_7	X_8	X_9
D3	$\times$	$\times$	$\times$	$\times$						
D7	$\times$	$\times$	$\times$	$\times$	$\times$	$\times$	$\times$	$\times$		



$$\xrightarrow[\lambda \to \infty,\ N \to \infty]{\text{Probe limi} \frac{N_f}{N} \ll 1}$$

AdS background + D7-brane

$\updownarrow$

Strongly coupled Su(N) gauge theory + fundametal matters

A. Karch and E. Katz, JHEP 0206, 043 (2002) [hep-th/0205236].

AdS background + D7-brane
(A reminder)

$$ds^2 = \frac{dz^2}{z^2} - \frac{1}{z^2}\frac{(1-z^4/z_h^4)^2}{1+z^4/z_h^4}dt^2 + \frac{1}{z^2}\left(1+\frac{z^4}{z_h^4}\right)d\vec{x}^2 + d\Omega_5^2$$
$$d\Omega_5^2 = d\theta^2 + \sin^2\theta d\psi^2 + \cos^2\theta d\Omega_3^2$$

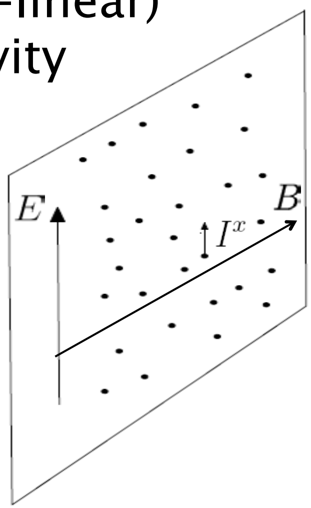
D7-brane is extendec $AdS_5 \times S^3$.

$$S = S_{\rm DBI} + S_{\rm CS}\ ,$$
$$S_{\rm DBI} = -N_f T_{D7}\int d^8\xi\ e^{-\phi}\sqrt{-\det(g_{ab}+B_{ab}+2\pi\alpha' F_{ab})}\ ,$$
$$S_{\rm CS} = N_f T_{D7}\int P[\Sigma C^{(n)}e^B]e^{2\pi\alpha' F}\ ,$$

$$g_{\mu\nu} = G_{MN}\partial_\mu X^M \partial_\nu X^N$$
$$B_{\mu\nu} = B_{MN}\partial_\mu X^M \partial_\nu X^N$$

Gauge theory side

1. Strongly coupled thermal field theory.
2. Explicit charge carriers.
3. Electric and magnetic fileds.
4. (non-linear) Conductivity $\sigma(T) = \frac{J}{E}$.



Gravity side

1. BH -AdS Sch.
2. Time component of the gauge field on the D7-branes (A_0).
3. The spatial components of the g... $A_x(t,z) = -Et + a_x(z), \;\; A_y(x) = Bx, \;\; \theta(z).$

   $\theta(z)$ → Mass of the fundamental matter
4. $J = ?$

A. Karch and A. O'Bannon, ``Metallic AdS/CFT,'' JHEP **0709**, 024 (2007), [arXiv:0705.3870 [hep-th]].

A. O'Bannon, ``Hall Conductivity of Flavor Fields from AdS/CFT,'' Phys. Rev. D **76**, 086007 (2007), [arXiv:0708.1994 [hep-th]].

D7-branes action

$$S_{D7} = \int dtdz\ \mathcal{L}$$
$$= -\frac{1}{2\pi\alpha'}\int dtdz\ g_{zz}^{1/2}|g_{tt}|^{-1/2}g_{xx}^{-1}\sqrt{\xi\chi},$$
$$\xi = |g_{tt}|g_{xx}^2 + (2\pi\alpha')^2(|g_{tt}|B^2 - g_{xx}E^2)$$
$$\chi = \mathcal{N}^2(2\pi\alpha')^2|g_{tt}|g_{xx}^2\cos^6\theta - C^2,\ \ \mathcal{N} = \frac{\lambda N_c N_f}{(2\pi)^4}$$

Conductivity equations

$$|g_{tt}|g_{xx}^2 + (2\pi\alpha')^2(|g_{tt}|B^2 - g_{xx}E^2) = 0$$
$$\mathcal{N}^2(2\pi\alpha')^2|g_{tt}|g_{xx}^2\cos^6\theta(z_*) - C^2 = 0$$

Current

$$J \equiv \langle J\rangle = \frac{\delta S_{D7}}{\delta A_x} = C$$

Asymptotic behaviour

$$A_x(t,z) = -Et + \frac{1}{2}\frac{\langle J\rangle}{\mathcal{N}(2\pi\alpha')^2}z^2 + O(z^4),$$
$$\theta(z) = \theta_0 z + \theta_2 z^3 + O(z^5).$$

Mass

$$(2\pi\alpha')m = \theta_0$$

A. Karch and A. O'Bannon, ``Metallic AdS/CFT,'' JHEP **0709**, 024 (2007), [arXiv:0705.3870 [hep-th]].
A. O'Bannon, ``Hall Conductivity of Flavor Fields from AdS/CFT,'' Phys. Rev. D **76**, 086007 (2007),

z_*

$$z_*^4 = \left(2F(e,b) - \sqrt{(2F(e,b)-1)^2 - 1} - 1\right) z_h^4$$

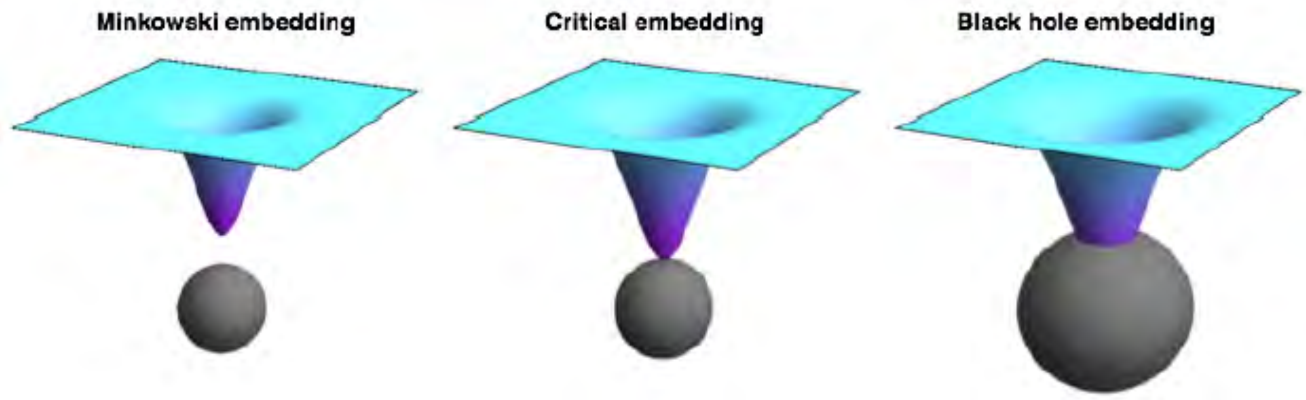
$$F(e,b) = \frac{1}{2}\left(1 + e^2 - b^2 + \sqrt{(e^2-b^2)^2 + 2(e^2+b^2) + 1}\right),$$

$$e = \frac{E}{\frac{\pi}{2}\sqrt{\lambda}T}, \quad b = \frac{B}{\frac{\pi}{2}\sqrt{\lambda}T}.$$

Current $$\langle J\rangle^2 = \mathcal{N}^2(2\pi\alpha')^2|g_{tt}|g_{xx}^2\cos^6\theta(z_*)$$

We need to solve the equation of $\theta(z)$tion for to find its asymptotic value
for given E, B and temperature.

S. Nakamura, ``Nonequilibrium Phase Transitions and Nonequilibrium Critical Point from AdS/CFT,''
Phys. Rev. Lett. **109**, 120602 (2012) [arXiv:1204.1971 [hep-th]].
S. Nakamura, ``Negative Differential Resistivity from Holography,'' Prog. Theor. Phys. **124**, 1105 (2010) [arXiv:1006.4105 [hep-th]].
M. Ali-Akbari and A. Vahedi, ``Non-equilibrium Phase Transition from AdS/CFT,'' arXiv 1305.3713 [hep-th].



$\frac{m}{T} \gg 1$: Insulator phase

$\frac{m}{T} \ll 1$: Conductor phase

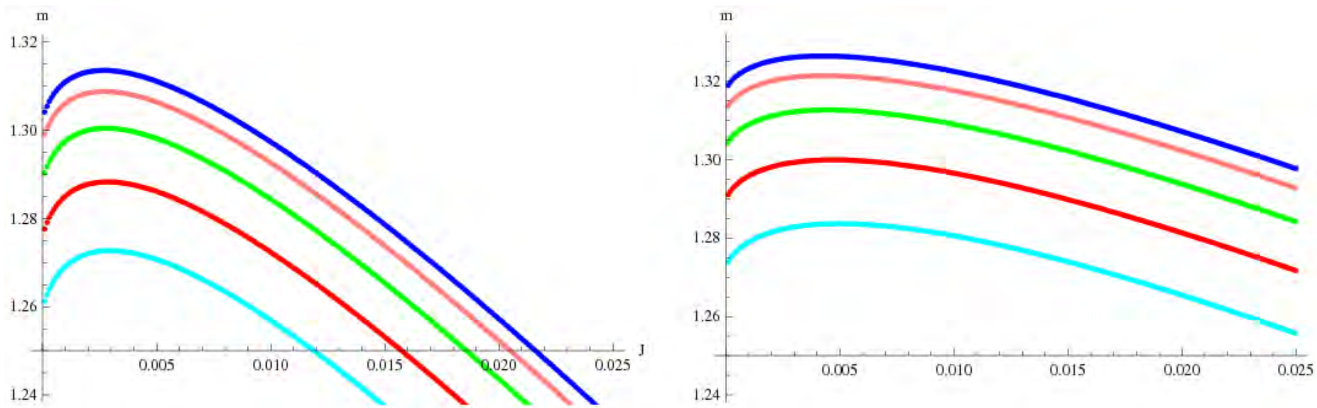


Figure 1: The value of mass as a function of current has been plotted for B=0, 0.3, 0.5, 0.7, 0.9 (top to bottom) and $T = 0.450158$. We set $E = 0.1$ ($E = 0.2$) in the left (right) figure.

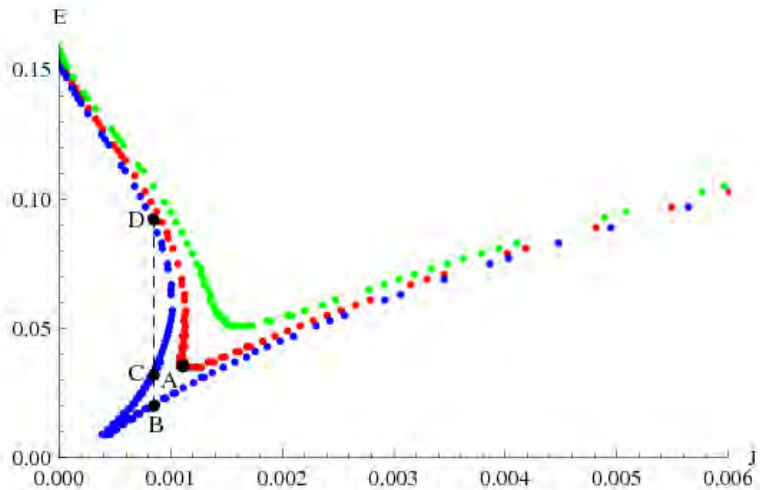


Figure 2: For various temperatures the valve of electric field E versus J has been showed. The curves with $T = 0.449350 > T_c$, $T_c = 0.449250$ and $T = 0.449100 < T_c$ are presented by the blue, red and green, respectively. We have chosen $m = 1.302$ and $B = 0.3$.

Small-J region: NDC
Large-J region: PDC

Blue curve : First order phase transition
Red curve: Second order phase transition
Green curve: Crossover

Hamiltonian

$$\mathcal{H} = \dot{A}_x \frac{\partial \mathcal{L}}{\partial \dot{A}_x} - \mathcal{L}$$
$$= \sqrt{\frac{g_{zz} g_{tt}}{g_{xx}} \frac{(\mathcal{N}^2 (2\pi\alpha')^2 |g_{tt}| g_{xx}^2 \cos^6\theta(z) - J^2)(g_{xx}^2 + (2\pi\alpha')^2 B^2)^2}{|g_{tt}| g_{xx}^2 - (2\pi\alpha')^2 g_{xx} E^2 + (2\pi\alpha')^2 |g_{tt}| B^2}}$$

Energy

$$E = \lim_{\epsilon \to 0} \left(\int_\epsilon^{z_h} dz\, \mathcal{H} - L_c(\epsilon) \right)$$

It numerically turns out that the solutions with the largest value of the electric field have the lowest energy.

A. Karch and A. O'Bannon, ``Metallic AdS/CFT,'' JHEP **0709**, 024 (2007), [arXiv:0705.3870 [hep-th]].
A. O'Bannon, ``Hall Conductivity of Flavor Fields from AdS/CFT,'' Phys. Rev. D **76**, 086007 (2007), [arXiv:0708.1994 [hep-th]]

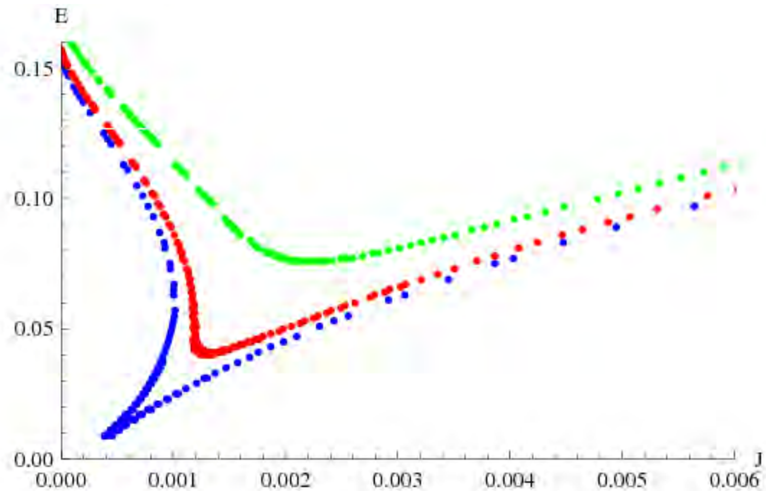


Figure 4: At a fixed temperature, $T = 0.449350$, the valve of electric field E versus J has been showed. The different curves correspond to different magnetic fields, with blue, red and green corresponding to $B = 0.30, 0.315$ and 0.36, respectively. We set $m = 1.302$.

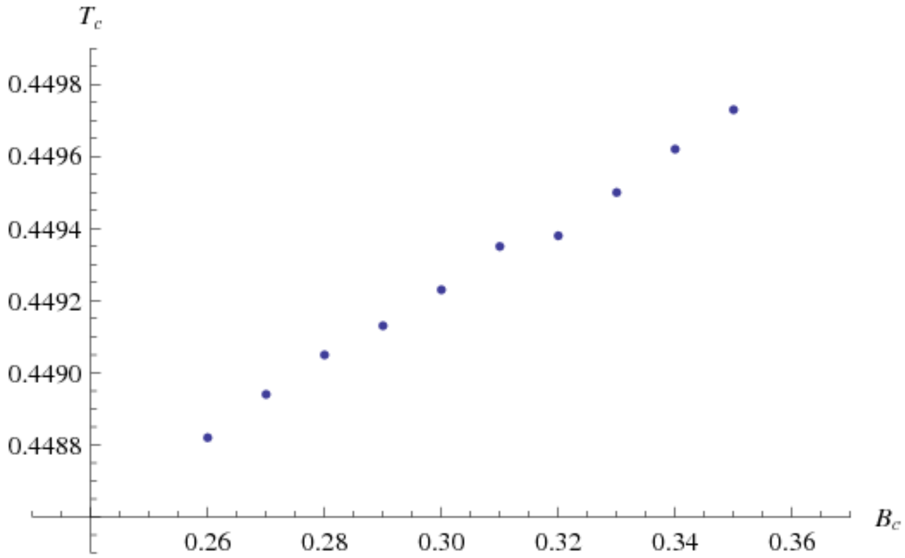


Figure 5: T_c as a function of B_c for $m = 1.302$.

D3-D5 systems

Supersymmetric D3-D5

	t	x	y	w	z	S^2	θ	S^2
$D3$	$\times$	$\times$	$\times$	$\times$				
$D5$	$\times$	$\times$	$\times$		$\times$	$\times$		

$\downarrow$ 2+1 dimensional subspace on which the matter fields live.

Current $\quad J^2 = \mathcal{N}^2(2\pi\alpha')^2|g_{tt}|g_{xx}\cos^4\theta(z_*)$

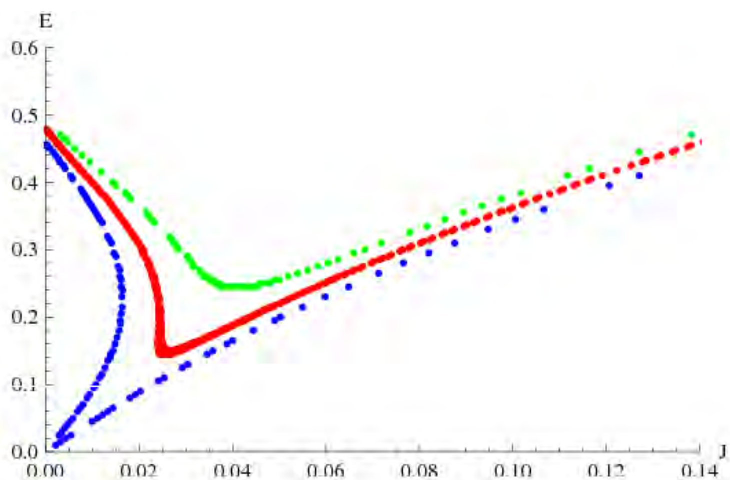


Figure 6: The green ($T = 0.446208$), the red ($T_c = 0.447958$) and the blue ($T = 0.450158$) curves corresponding to a crossover, second and first order phase transitions, respectively. We set $m = 1.66$ and $B = 0.3$.

Non-supersymmetric D3-D5

	t	x	y	w	z	S^3	θ	ψ
$D3$	$\times$	$\times$	$\times$	$\times$				
$D5$	$\times$	$\times$			$\times$	$\times$		

1+1 dimensional subspace on which the matter fields live.

Current $$J^2 = \mathcal{N}^2(2\pi\alpha')^2|g_{tt}|\cos^6\theta(z_*)$$

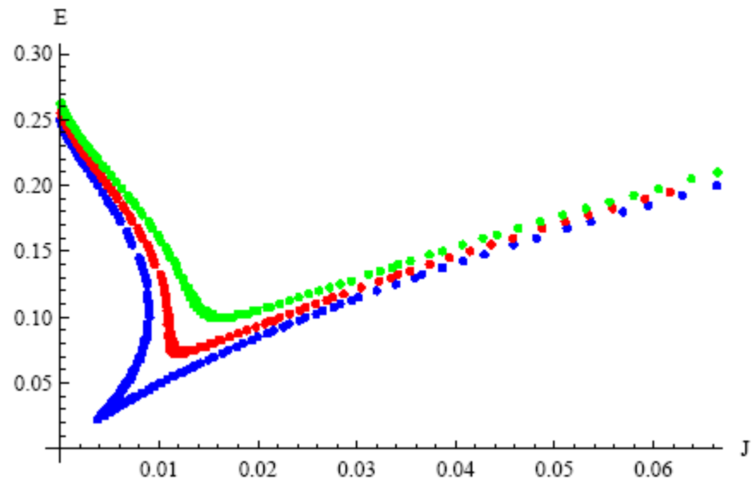


Figure 7: The green ($T = 0.446658$), the red ($T_c = 0.447158$) and the blue ($T = 0.447658$) curves corresponding to a crossover, second and first order phase transitions, respectively. We set $m = 0.905$.

Results

- Our numerical calculation shows that various types of transition occur even in the presence of magnetic field. But, opposite to the case of zero magnetic field, in this case the configuration with the larger value of electric field is energetically favorable.

- It seems that, at least for the systems we have considered, neither supersymmetry nor the dimension of the subspace on which fundamental matter lives alters the transitions, qualitatively.

- The type of transition depends on the value of magnetic field as well as the temperature. In fact by increasing the critical magnetic field, the critical temperature also rises.