Logarithmic representations of non-relativistic conformal algebras

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Contents:

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M Henkel and S Rouhani
Logarithmic correlators or responses in non-relativistic analogues of conformal invariance
arXiv preprint arXiv:1302.7136
To appear in J. Phys. A

Conformal Invariance in d=2

- In Two dimensions Conformal Invariance = all holomorphic transformations of the plane

$$z = x + i y \qquad z \longrightarrow f(z)$$

⇒ infinite dimensional symmetry

This treats both dimensions on the same footing , for a Galilean symmetry we need to set one of them apart

Non-relativistic conformal symmetries

- The standard conformal algebra is obtained by adding scale invariance and special conformal transformations to the Poincare group.
- We wish to do the same with "Galilean" symmetry
- Except that we have different versions of "Galilean" symmetries leading to different

NR-CFTs

Motivation

- The AdS_{d+1}/CFT_d duality can be modified to correspond to a conformal theory on the boundary having non-relativistic symmetry e.g. Schrodinger-Virasoro. In this case you go from d+2 dimensions to d dimensions. This is done by choosing a metric which has Schrodinger symmetry asymptotically

- Low energy, non-equilibrium phenomena, near critical points is scale invariant and non-relativistic, perhaps also conformally invariant.
- eg Prof. Son's talk in this meeting

- D. T. Son, "Toward an AdS/cold atoms correspondence: a geometric realization of the Schroedinger symmetry," Phys. Rev. D 78, 046003 (2008) [arXiv:0804.3972 [hep-th]].
- K. Balasubramanian and J. McGreevy, "Gravity duals for non-relativistic CFTs," Phys.Rev. Lett. 101, 061601 (2008) [arXiv: 0804.4053 [hep-th]].

Motivation

- Use light cone coordinates and compactify in one direction.

$$ds^2 = \frac{dx_i dx_i + 2dx^+ dx^-}{z^2} + \frac{dz^2}{z^2} - \frac{(dx^-)^2}{z^4}$$

•Asymptotic symmetry of geometries with Schrodinger isometry

Mohsen Alishahiha, Reza Fareghbal, Amir E. Mosaffa, Shahin Rouhani Phys. Lett. B 675: 133-136,2009

•Asymptotic symmetries of Schrodinger spacetimes

Geoffrey Compere, Sophie de Buyl, Stephane Detournay, and Kentaroh Yoshida, Journal of High Energy Physics, Issue 10, pp. 032 (2009).

Motivation

- The Schrodinger algebra is a sub-algebra of the relativistic conformal algebra in one higher space-time dimension
- To show this you have to use Bargmann constructionwhich is how you get a non-relativistic symmetry out of a totally relativistic setting

Comments on Galilean conformalfield theories and their geometric realization
Dario Martelli and Yuji Tachikawa, arxiv 0903.5184

Galilean symmetries

1. Symmetries of Schrodinger equation: Sch(d)
2. Contraction of Poincare Algebra; or contraction of O(d,2): Conformal Galilean Algebra : CGA.
3. Construct consistent algebras, around the known operators of Galilean symmetry:

   l-Galilei Algebras: $l=1/2$ Sch(d), $l=1$ CGA

l-Galilei algebra

We ask for a few desirable properties:

1. Conformal symmetry in some sense
2. We would like Galilean causality so $f(r)\partial_t$ operator is not permitted.
3. We would like global conformal transformation in time:

$$t \to t' = \frac{\alpha t+\beta}{\gamma t+\delta} \qquad \alpha\delta - \beta\gamma = 1$$

We add up these operators and try to make a finite closed algebra

We end up with the class of algebras: *l_Galilei* algebra sometimes called *spin-l Galilei* algebra

l-Galilei algebra

- Action on space and time coordinates is given by:

$$\vec{x} \to \frac{R\vec{x} + t^{2l}\vec{c}_{2l} + \cdots + t\vec{c}_1 + \vec{c}_0}{(gt+f)^{2l}}, \quad t \to \frac{at+b}{gt+f}$$

$$af - bg = 1$$

- Scaling of time direction is anisotropic to space direction (f=1/a set all other parameters to zero):

$$t \to a^2 t, \quad \vec{x} \to a^{2l}\vec{x} \qquad z = \frac{1}{l}$$

l-Galilei algebra

Infinitesimal operators are :

$$H = -\partial_t \qquad D = -(t\partial_t + l x_i \partial_i)$$

$$K = t^2\partial_t + 2l x_i \partial_i \qquad J_{ij} = -(x_i\partial_j - x_j\partial_i)$$

$$P_i^n = (-t)^n \partial_i \qquad n = 1..2l$$

Commutators:

$$[D,H] = H \qquad [D,K] = -K \qquad [H,P_i^n] = -nP_i^{n-1}$$

$$[K,H] = 2D \qquad [D,P_i^n] = (l-n)P_i^n$$

$$[J_{ij},P_k^n] = -(P_i^n\delta_{jl} - P_j^n\delta_{il}) \qquad [J_{ij},J_{kl}] = SO(d),$$

$$[K,P_i^n] = (2l-n)P_i^{n+1}$$

M. Henkel, Phys. Rev. Lett. 78, 1940 1997; e-print arXiv:cond-mat/9610174.
J. Negro, M. A. del Olmo, and A. Rodríguez-Marco, J. Math. Phys. 38, 3786 1997.

l-Galilei algebra

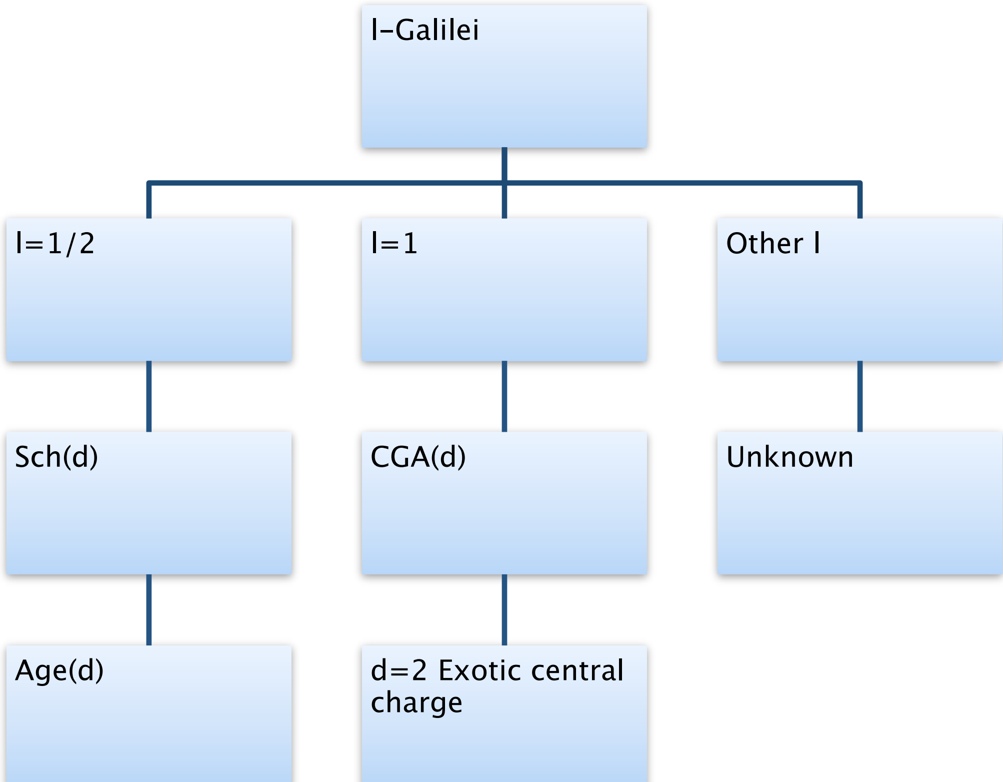
Due to the last commutation relation the algebra closes if :

$$l = \frac{N}{2} \qquad N \in \mathbb{N}$$

So, dynamical scaling takes special values for this class of nonrelativistic conformal algebras. Scaling operator now scales space and time as

$$t \to \lambda^z t \qquad x \to \lambda x \qquad z = \frac{1}{l}$$

l=1/2 gives Schrodinger algebra and l= 1 gives CGA.


l-Galilei
l=1/2
l=1
Other l
Sch(d)
CGA(d)
Unknown
Age(d)
d=2 Exotic central charge

Schrodinger Symmetry Sch(d)
l=1/2

The algebra of symmetries of the Schrodinger equation:

$$\left(i\partial_t + \frac{1}{2m}\partial_i\partial_i\right)\psi = 0$$

It consists of Galilean transformations:

$$H = -\partial_t \qquad P_i = \partial_i$$

$$J_{ij} = -(x_i\partial_j - x_j\partial_i) \qquad B_i = -t\partial_i - M x_i$$

Continued……

Schrodinger symmetry Sch(d)

Special Schrodinger transformation

$$K_i = t\, x_i \partial_i + t^2 \partial_t + \frac{M}{2} r^2 + h\, t$$

which produces

$$x_i \to \frac{x_i}{(1+\mu t)} \qquad t \to \frac{t}{(1+\mu t)}$$

Dilation

$$D = 2t\partial_t + x_i \partial_i + h$$

which scales space and time anisotropically

$$x_i \to \lambda\, x_i \qquad t \to \lambda^2\, t$$

Schrodinger symmetry Sch(d)

These operators together produce the following coordinate transformations:

$$\vec{r} \to \overrightarrow{r}' = \frac{\mathcal{R}\vec{r}+\vec{v}t+\vec{a}}{\gamma t+\delta} \qquad t \to t' = \frac{\alpha t+\beta}{\gamma t+\delta}$$

$$\alpha\delta - \beta\gamma = 1$$

And this algebra admits a central charge which is related to the physical mass:

$$[B_i, P_j] = \mathcal{M}\delta_{ij}$$

Schrodinger symmetry: *infinite extension*

There exists a Virasoro like infinite extension to the Schrodinger algebra called Schrodinger-Virasoro algebra (SV) :

$$T^n = -t^{n+1}\partial_t - \frac{1}{2}(n+1)t^n x_i \partial_i - \frac{1}{4}n(n+1)\mathcal{M}t^{n-1}x^2$$

$$P^m{}_i = -t^{m+\frac{1}{2}}\partial_i - (m+\frac{1}{2})t^{m-1/2}x_i\mathcal{M}$$

$$M^n = -\mathcal{M}t^n \qquad\qquad m+\frac{1}{2}, n \in \mathbb{Z}$$

M. Henkel, "Schrodinger Invariance in Strongly Anisotropic Critical Systems," J. Stat. Phys. 75 (1994) 1023 [arXiv:hep-th/9310081].

Schrodinger symmetry: *infinite extension*

Some of the commutators are:

$$[T^n, T^m] = (n-m)T^{n+m} \quad [T^n, P_i^m] = (\frac{n}{2} - m)P_i^{n+m}$$

$$[T^n, M^m] = -mM^{n+m} \quad [P_i^n, P_j^m] = (n-m)\delta_{ij}M^{n+m}$$

$$[P_i^n, M^m] = [M^n, M^m] = 0$$

Schrodinger symmetry:
infinite extension

Schrodinger algebra is recovered as:

$$T^{-1} = H \qquad T^0 = D/2 \qquad T^1 = K$$

$$P_i^{-\frac{1}{2}} = P_i \qquad P_i^{\frac{1}{2}} = B_i \qquad M^0 = -\mathcal{M}$$

Conformal Galilean Algebra CGA l=1

Conformal Algebra upon contraction leads to a non-relativistic algebra:

$$x \to \frac{x}{c} \qquad t \to t \qquad c \to \infty$$

Barut 1972

Arjun Bagchi, Rajesh Gopakumar, Ipsita Mandal, Akitsugu Miwa, "GCA in 2d ", JHEP 1008:004,2010

Conformal Galilean Algebra CGA

Some familiar operators are recovered:

$$P_0 \to P_0$$

$$J_{0i} = tc\partial_i - \frac{1}{c}x_i\partial_t$$

$$\frac{1}{c}J_{oi} \to B_i = t\partial_i$$

Conformal Galilean Algebra CGA

And we end up with CGA :

$$P_i = \partial_i \qquad H = -\partial_t$$

$$B_i = t\partial_i \qquad J_{ij} = -(x_i\partial_j - x_j\partial_i)$$

$$D = -t\partial_t - x_i\partial_i \qquad K_i = t^2\partial_i$$

$$K = K_0 = -(2tx_i\partial_i + t^2\partial_t)$$

Note that scaling operator D, scales space and time isotropically in this non-relativistic algebra. Also there is no “mass’ central charge thus this symmetry describes massless non-relativistic particles !

Exotic Galilean algebra

CGA in 2+1 dimensions admits a central charge which is called Exotic.

$$[B_i, B_j] = \Theta \epsilon_{ij}$$

$$[P_i\,, K_j] = -2\Theta \epsilon_{ij}$$

Physical interpretation of this charge has been of interest. For example see:

Lukierski, J., Stichel, P. C., and Zakrzewski,W. J., “Exotic Galilean conformal symmetry and its dynamical realisations,” 290 Phys. Lett. A 357, 1 (2006); e-print [arXiv:0511259 [hep-th]].

M. A. del Olmo and M. S. Plyushchay, “Electric Chern-Simons Term, Enlarged Exotic Galilei Symmetry and Noncommutative Plane,” Annals Phys. 321 (2006) 2830 [arXiv:hep-th/0508020].

J.-M. L´evy-Leblond, “Nonrelativistic Particles and Wave Equations,” Comm. Math. Phys. 6, 4 (1967), 286-311.

Galilean Conformal Algebra: *Infinite Extension*

Similar to Schrodinger algebra CGA does have an infinite extension which is called *Full CGA*

$$T^n = -(n+1)t^n x_i \partial_i - t^{n+1}\partial_t$$

$$M_i^n = t^{n+1}\partial_i$$

$$J_{ij}^n = -t^n(x_i\partial_j - x_j\partial_i)$$

which in 1+1 dimensions simplifies to :

$$[T^m, M_i^n] = (m-n)M_i^{m+n} \qquad [M_i^m, M_j^n] = 0$$

$$[T^m, T^n] = (m-n)T^{m+n}$$

Galilean Conformal Algebra: *Infinite Extension*

CGA is recovered in terms of Full CGA generators:

$$T^{-1} = H \qquad T^{0} = D \qquad T^{1} = K$$

$$M_i^{-1} = P_i \qquad M_i^{0} = B_i \qquad M_i^{1} = K_i$$

Similar to the Schrodinger symmetry this symmetry can be also be realized within the AdS/CFT correspondence:

A. Bagchi and R. Gopakumar, J. High Energy Phys. 07 (2009)037; e-print arXiv:hep-th/0902.1385. JHEP 0907:037,2009

CGA from contraction…(d=2)

we impose contraction limit on Virasoro operators and observe:

$$L^n = -\frac{1}{2}(t + i\frac{x}{c})^{n+1}(\partial_t - ic\partial_x)$$
$$= -t^{n+1}(-ic\partial_x + \partial_t + (n+1)\frac{x}{t}\partial_x + O(\frac{1}{c})$$
$$\bar{L}^n = -t^{n+1}(ic\partial_x + \partial_t + (n+1)\frac{x}{t}\partial_x + O(\frac{1}{c})$$

$$T^n = L^n + \bar{L}^n + O(\frac{1}{c})$$

$$M^n = -i\frac{L^n - \bar{L}^n}{c} + O(\frac{1}{c})$$

by contraction the Full CGA is obtained from Virasoro algebra

Other Non-relativistic conformal symmetries:

Other symmetries can be generated by letting go of certain operators for example time translation

In Physical systems where you have aging you loose time translation invariance; and obtain a different symmetry algebra: Age(d)

Possible applications:
contact process,
spin glasses,
colloidal fluids
…

Malte Henkel, Nucl. Phys. B869 [FS], 282-302 (2013)arxiv: 1009.4139

Age(d)

Translations $$P_i = \partial_i$$

Dilations $$D = 2t\partial_t + x_i\partial_i + h$$

Boosts $$B_i = -t\partial_i - M x_i$$

Special Conformal $$K_i = t x_i\partial_i + t^2\partial_t + \frac{M}{2}r^2 + h' t$$

In general the two conformal weights h and h'
need not be the same.

Age(1)

Consider the Schrodinger operator:

$$S = 2M\partial_t - \partial_x^2 + (2M + h + h' - 1)\frac{1}{t}$$

then solutions of the equation;

$$S\psi = 0$$

Are mapped into each other by elements of Age(1).

Logarithmic Representations

LCFT's arise out of representations which are reducible but not decomposable:

$$L^0 \phi_h(z)|0\rangle = h\phi_h(z)|0\rangle$$

$$L^0 \psi_h(z)|0\rangle = h\psi_h(z)|0\rangle + \phi_h(z)|0\rangle$$

Reviews : M. Flohr, *Bits and pieces in logarithmic conformal field theory, Int. J. Mod. Phys. A 18* (2003) 4497 [arXiv:hep-th/0111228].

M.R. Gaberdiel, *An algebraic approach to logarithmic conformal field theory, Int. J. Mod.* Phys. A 18 (2003) 4593 [arXiv:hep-th/0111260

Logarithmic Correlators

Take a nilpotent variable:

$$\theta^2 = 0$$

and use this to derive the properties of the LCFT:

$$L^0|h+\theta\rangle = (h+\theta)|h+\theta\rangle$$

Expansion in powers of θ yields back the original expressions

Logarithmic conformal field theory through nilpotent conformal dimensions
S Moghimi-Araghi, S Rouhani, M Saadat
Nuclear Physics B 599 (3), 531-546

Schrodinger Symmetry: Logarithmic Correlators

We ask if logarithmic representations exist for Sch(d)?

Construct a "super field" using the nilpotent variable θ :

$$\mathbf{\Phi}(z,\theta) = \phi(z) + \theta\psi(z)$$

$$\mathbf{\Phi}(z,\theta)|0\rangle = |h+\theta\rangle$$

$$T^0|h+\theta\rangle = (h+\theta)|h+\theta\rangle$$

Logarithmic Schrodinger-Virasoro (LSV):

We can impose symmetries via Ward identity on quasi-primary fields and obtain two-point functions:

$$\langle \phi_1(x_1,t_1)\phi_2^*(x_2,t_2)\rangle = 0$$

$$\langle \phi_1(x_1,t_1)\psi_2^*(x_2,t_2)\rangle = bt^{-2h_1}\delta_{\mathcal{M}_1,\mathcal{M}_2}\exp\left(-\frac{\mathcal{M}_1 x^2}{2t}\right)$$

$$\langle \psi_1(x_1,t_1)\psi_2^*(x_2,t_2)\rangle = t^{-2h_1}\delta_{h_1,h_2}\delta_{\mathcal{M}_1,\mathcal{M}_2}\exp\left(-\frac{\mathcal{M}_1 x^2}{2t}\right)(c - 2b\,log(t))$$

A. Hosseiny and S. Rouhani, "Logarithmic correlators in nonrelativistic conformal field theory" J. Math. Phys. 51, 102303 (2010);e-print arXiv:hep-th/1001.1036

Logarithmic CGA (LCGA):

We can obtain LCGA by contraction of Logarithmic Virasoro Representation

Consider the most general logarithmic representation in which both left and right scaling weights have Jordan cell structure:

$$L^0|h,\bar{h},1\rangle = h|h,\bar{h},1\rangle + \acute{h}|h,\bar{h},0\rangle$$

$$\bar{L}^0|h,\bar{h},1\rangle = \bar{h}|h,\bar{h},1\rangle + \acute{\bar{h}}|h,\bar{h},0\rangle\,.$$

$$M^0|\Delta,\xi,1\rangle = M^0|h,\bar{h},1\rangle = -i\frac{h}{c}|h,\bar{h},1\rangle + i\frac{\bar{h}}{c}|h,\bar{h},0\rangle - \frac{i}{c}(\acute{h} - \acute{\bar{h}}|h,\bar{h},0\rangle$$

$$= \xi|\Delta,\xi,1\rangle + \acute{\xi}|\Delta,\xi,1\rangle$$

Logarithmic CGA: LCGA

So, we have

$$\acute{\Delta} = \acute{h} + \acute{\bar{h}} \qquad \acute{\xi} = \frac{\acute{h} - \acute{\bar{h}}}{c}$$

Now, we can follow on and find two point functions

Logarithmic CGA (LCGA):

For $\psi\psi$ two-point function where logarithmic term appears we have:

$$< \psi_1(z_1,\bar{z}_1)\psi_2(z_2,\bar{z}_2) >=$$

$$\left(-2a\left[\acute{h}_1\log(z)+\acute{\bar{h}}_1\log(\bar{z})\right]+b\right)z^{-2h_1}\bar{z}^{-2\bar{h}_1}\,\delta_{h_1,h_2}\delta_{\bar{h}_1,\bar{h}_2}$$

Logarithmic CGA: LCGA

If we follow contraction limit for logarithmic CGA we obtain

$$\langle \psi_1(x_1, t_1)\psi_2(x_2, t_2)\rangle_{GCA} =$$

$$\delta_{\Delta_1,\Delta_2}\delta_{\xi_1,\xi_2} t^{-2\Delta_1}\, exp\left(\frac{2\xi_1 x}{t}\right)(-2a\acute{\Delta} log(t) - 2a\acute{\xi}\,\frac{x}{t} + b)$$

We could have followed the same approach as used for LSV and we arrive at exactly the same correlators.

Logarithmic Age

2pt correlation function for logarithmic Age(1)

$$h_0(y) = h_0 - \left[\left(\frac{x_1'}{2}+\xi_1'\right)g_{21,0} + \left(\frac{x_2'}{2}+\xi_2'\right)g_{12,0}\right]\ln|y-1| - \left[\frac{x_1'}{2}g_{21,0} - \left(\frac{x_2'}{2}+\xi_2'\right)g_{12,0}\right]\ln|y|$$
$$+\frac{1}{2}f_0\left[\left(\left(\frac{x_1'}{2}+\xi_1'\right)\ln|y-1| + \frac{x_1'}{2}\ln|y|\right)^2 - \left(\frac{x_2'}{2}+\xi_2'\right)^2\ln^2\left|\frac{y}{y-1}\right|\right] \qquad (3.15)$$

$$\langle \psi(t_1)\psi(t_2)\rangle \approx h_0(y) \qquad y = \frac{t_1}{t_2}$$

Malte Henkel, Nucl. Phys. B869 [FS], 282-302 (2013)arxiv: 1009.4139

Contact process

Aging happens due to formation of clusters of typical size : $l(t) \approx t^{1/z}$

$at\ site\ x:$

1 ® 0 $rate\ 1$

0 ® 1 $rate\ \lambda \sum_{y \in nbhd(x)} \eta(y)$

For a given graph G, there exists a critical value of λ for which sites 1 survives.

The long-time dynamics of the critical contact process which is brought suddenly out of an uncorrelated initial state undergoes ageing in close analogy with quenched magnetic systems.

Jose J Ramasco , Malte Henkel, Maria Augusta Santos and Constantino A da Silva Santos, J. Phys. A: Math. Gen. 37 (2004) 10497–10512

Contact process

Order parameter $\phi(t,x)$

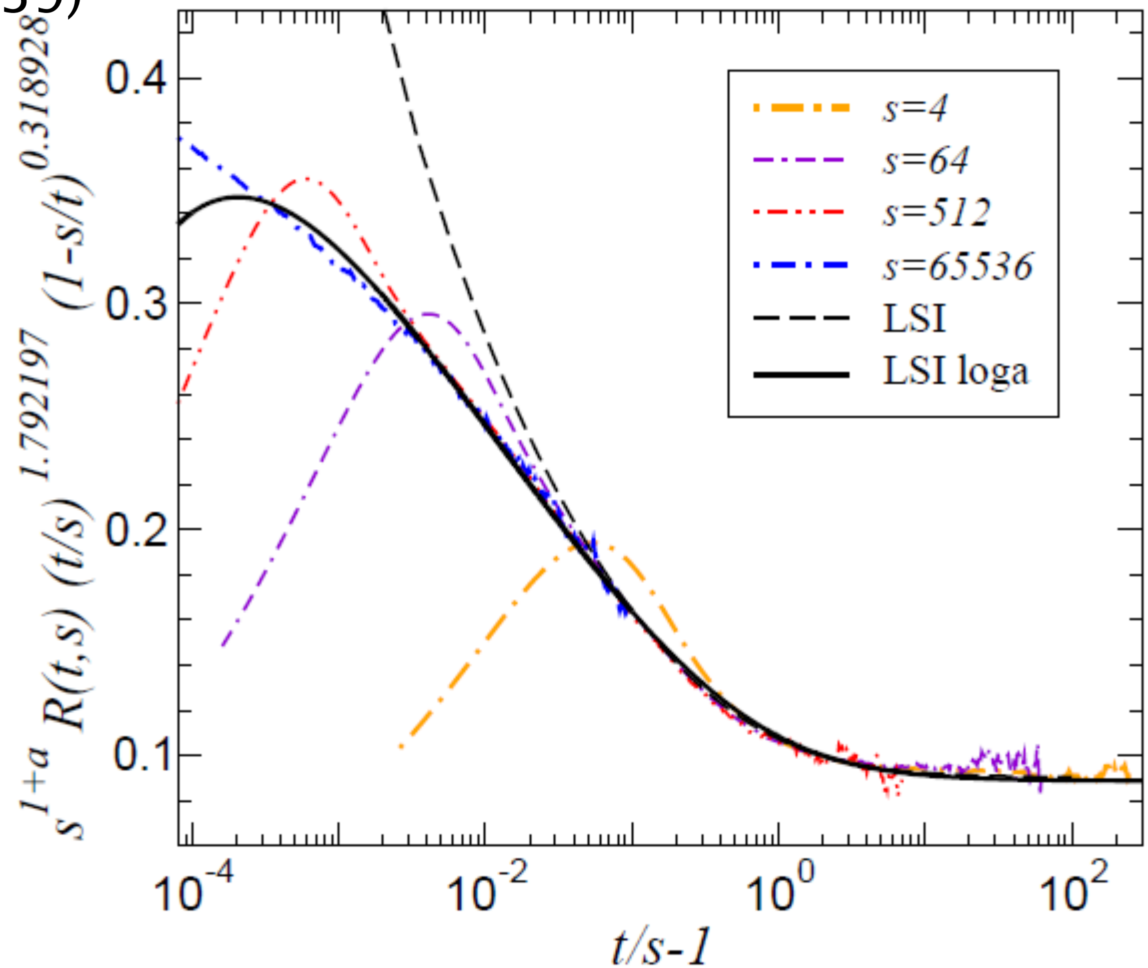
Auto-correlator $\Gamma(t,s) = \left\langle \phi(t,x)\phi(s,x) \right\rangle$

Auto-response function $R(t,s) = \left. \frac{\delta \left\langle \phi(t,x) \right\rangle}{\delta h(s,x)} \right|_{h=0}$

Dynamic Scaling: $\Gamma(t,s) \approx s^{-b} f(t/s), \quad R(t,s) \approx s^{-1-a} g(t/s)$

$$y = \frac{t}{s}$$

Scaling of the auto-response function of the 1D critical contact process, as a function of y = t/s, for several values of the waiting time s
(from Malte Henkel, Nucl. Phys. B869 [FS], 282-302 (2013)arxiv: 1009.4139)



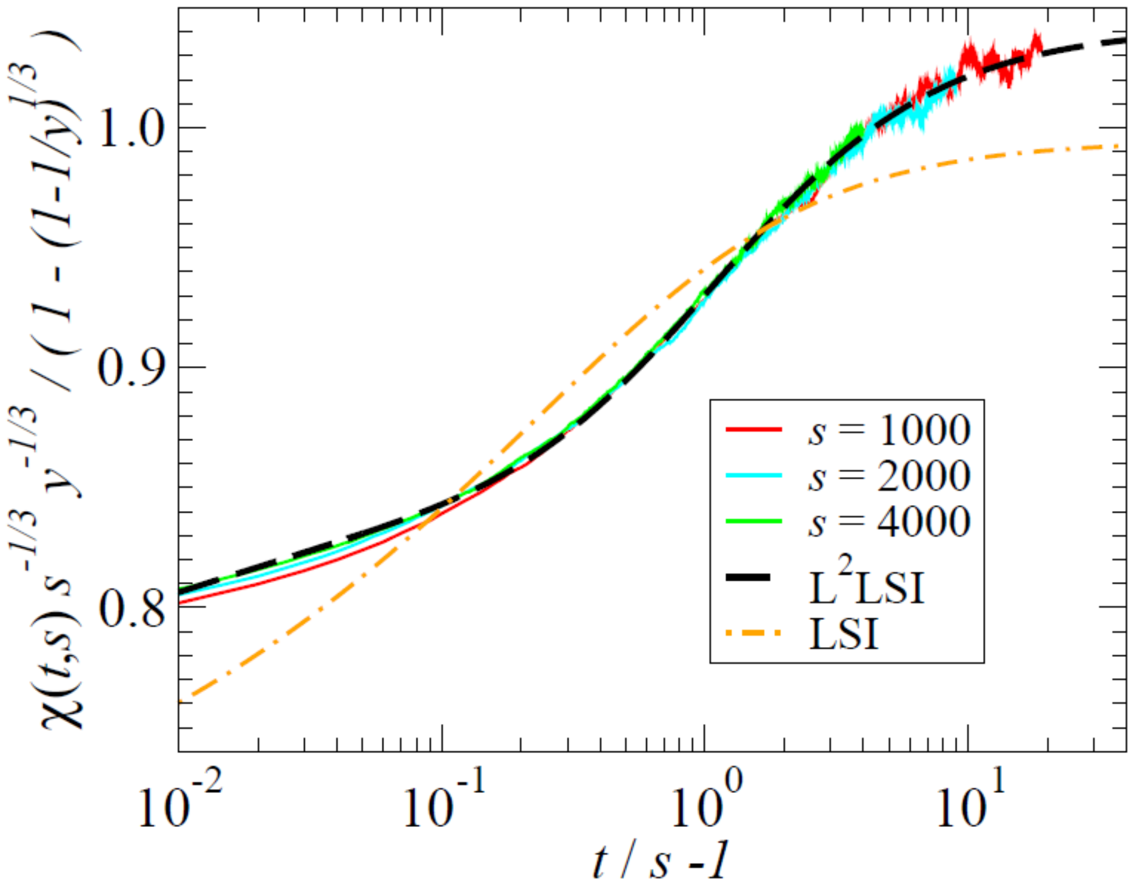
Kardar–Parisi–Zhang (KPZ)

$$\frac{\partial h}{\partial t} = \nabla^2 h + \lambda(\nabla h)^2 + \eta$$

h height of a growing surface

η white noise

Also there is evidence that in d=1 KPZ the response function fits the logarithmic



(Malte Henkel, Jae Dong Noh and Michel Pleimling, Phys. Rev. E85, 030102(R) (2012) arXiv:1109.5022

Representations of the Exotic Algebra

We augment the fields with internal degrees of freedom, in order t get the central charge right:

$$\psi_{\gamma}(t, \vec{x}, \Xi, \vec{v})$$

Comments on Galilean conformal field theories and their geometric realization
Dario Martelli and Yuji Tachikawa, arxiv 0903.5184

Log Exotic

We can now go through a similar process for a pair of quasi primaries in the case of the Exotic Algebra (ϕ,ψ) and find:

$$<\phi_1\phi_2> = 0$$

$$<\psi_1\phi_2> = \delta_{x1,x2}\,\delta_{\gamma_1,\gamma_2}\delta_{\Xi_1+\Xi_2,0}\,t^{-x^+}e^{\frac{\Xi}{2}\epsilon_{ij}\left(\frac{v_j}{4}-u_j\right)v_i}e^{-\gamma_i\,u_i}\mathcal{O}_1(u_i+\frac{v_i}{2})$$

$$<\psi_1\psi_2> = \delta_{x1,x2}\,\delta_{\gamma_1,\gamma_2}\delta_{\Xi_1+\Xi_2,0}\,t^{-x^+}e^{\frac{\Xi}{2}\epsilon_{ij}\left(\frac{v_j}{4}-u_j\right)v_i}e^{-\gamma_i\,u_i}[\mathcal{O}_1\left(u_i+\frac{v_i}{2}\right)(-2x^+\log t - 2\gamma_i\,u_i) + \mathcal{O}_2\left(u_i+\frac{v_i}{2}\right)]$$

Functions O_1 and O_2 are arbitrary

Henkel ,Hosseiny, Rouhani, in prep

Outlook:

- Representation Theory (SV, LSV, Full CGA, LCGA, l–Galilei, AdS/Age…?
- Staggered Modules ..?
- More Physical applications…?

Thank you

π

Logarithmic CGA; an Algebraic Approach

Recall that scaling fields in CGA are identified by their scaling weight and rapidity

$$[T^0, \phi] = \Delta\phi \qquad [M^0, \phi] = \xi\phi$$

Under infinitesimal changes , primary fields are transformed as:

$$[T^n, \phi] = (-1)^n$$
$$[(n+1)t^n x_i \partial_i + t^{n+1}\partial_t + (n+1)(t^h\Delta - nt^{n-1}x\xi)]\phi$$

$$[M^n, \phi] = (-1)^n[-t^{n+1}\partial_i + (n+1)t^n\xi]\phi$$

Logarithmic CGA; an Algebraic Approach

As far as we are concerned with quasi-primary fields it is easy to check that a Jordan form is possible. We can utilize nilpotent variables and observe:

$$\mathbf{\Phi} = \psi + \theta\phi \qquad \tilde{\Delta} = \Delta + \acute{\Delta}\theta \qquad \tilde{\xi} = \xi + \acute{\xi}\theta$$

$$[T^0, \Phi] = \tilde{\Delta}\Phi \qquad [M^0, \Phi] = \tilde{\xi}\Phi$$

Logarithmic CGA; an Algebraic Approach

Now, we can impose CGA symmetry on quasi-primary fields and calculate two-point functions 0f "superfields"

$$< \Phi(x_1,t_1,\theta_1)\Phi(x_2,t_2,\theta_2) >= e^{-\frac{\left(2\xi_1+\xi'(\theta_1+\theta_2)\right)x}{t}}$$

$$[at^{-2\Delta_1}(\theta_1+\theta_2) - 2a\Delta' \log t\, t^{-2\Delta_1}\,\theta_1\theta_2 + bt^{-2\Delta_1}\theta_1\theta_2]$$

$$\delta_{\xi_1,\xi_2}\delta_{\Delta_1,\Delta_2}$$

Logarithmic CGA; an Algebraic Approach

Expanding in nilpotent variables, we obtain:

$$< \phi_1 \phi_2 >= 0$$

$$< \phi_1 \psi_2 >= a e^{-2\xi_1 \frac{x}{t}} t^{-2\Delta_1} \delta_{\xi_1,\xi_2} \delta_{\Delta_1,\Delta_2}$$

$$< \psi_1 \psi_2 >= e^{-2\xi_1 \frac{x}{t}} t^{-2\Delta_1} \delta_{\xi_1,\xi_2} \delta_{\Delta_1,\Delta_2} \left[-2a\Delta' \log t - 2a\xi' \frac{x}{t} + b\right]$$

Representations of Schrodinger-Virasoro

To build representaitons of Schrodinger-Virasoro algebra; inspired by relativistic CFT; we assume existence of fields which are eigenstates of scaling operator:

$$[T^0, \phi] = h\phi$$

Assuming a vacuum now gives rise to eigenstates of T^0

$$\phi|0\rangle = |h\rangle$$

Representations of Schrodinger-Virasoro

Since M commutes with every thing each state is also labeled by eigenvalue of M as well:

$$M^{0}\left|h,M\right\rangle = M\left|h,M\right\rangle$$

Representations of Schrodinger-Virasoro

Other operators now work as ladder operators:

$$[T^0,[T^n,\phi]] = (h-n)[T^n,\phi]$$

$$T^{-n}|h\rangle \rightarrow |h+n\rangle$$

$$P^{-m}|h\rangle \rightarrow |h+m\rangle$$

$$M^{-n}|h\rangle \rightarrow |h+n\rangle$$

Representations of Schrodinger-Virasoro

Now, we define the vacuum state, annihilated by

$$M^n,\ P^m,\ T^n\ \left|0\right\rangle = 0 \qquad n, m > 0$$

As in CFT Null states exist and it is interesting to note that the first null state which appears at the second level is the Schrodinger equation.

$$|\chi\rangle = ((P^{-\frac{1}{2}})^2 - 2MT^{-1})|h\rangle$$

Higher Null states give rise to other differential equations.

Full CGA: Representations in 1+1dimensions

One observes that T^0 and M^0 commute and representations can be simultaneous eigenstates of both:

$$T^0|\Delta,\xi\rangle = \Delta|\Delta,\xi\rangle$$

$$M^0|\Delta,\xi\rangle = \xi|\Delta,\xi\rangle$$

Full CGA: Representations in 1+1dimensions Utilizing Contraction

- We observe that full CGA can be obtained directly from contracting the conformal algebra

- While it is not necessarily the case that contraction on Representations should work but in this case we can derive the representation of the full CGA by contraction,

Full CGA: Representations in 1+1dimensions Utilizing Contraction

Note that conformal symmetry in 2 dimensions is composed of two Virasoro algebras:

$$L^n = -z^{n+1}\partial_z$$

$$\bar{L}^n = -\bar{z}^{n+1}\partial_{\bar{z}}$$

CGA From Contraction …

Consider the usual eigensates of the scaling operator

$$L^0|h,\bar{h}\rangle = h|h,\bar{h}\rangle \qquad \bar{L}^0|h,\bar{h}\rangle = \bar{h}|h,\bar{h}\rangle$$

In the contraction limit we have

$$T^0|h,\bar{h}\rangle = (L^0+\bar{L}^0)|h,\bar{h}\rangle = (h+\bar{h})|h,\bar{h}\rangle$$
$$M^0|h,\bar{h}\rangle = -\frac{i}{c}(L^0-\bar{L}^0)|h,\bar{h}\rangle = \frac{i}{c}(\bar{h}-h)|h,\bar{h}\rangle$$

In other words

$$T^0|\Delta,\xi\rangle = \Delta|\Delta,\xi\rangle$$
$$M^0|\Delta,\xi\rangle = \xi|\Delta,\xi\rangle$$

$$\Delta = h+\bar{h}$$
$$\xi = \frac{i}{c}(\bar{h}-h)$$

Nonrelativistic conformal algebras in 2+1 dimensions

While relativistic conformal symmetry is infinite dimensional only in d=2 we have Schrodinger-Virasoro symmetry and full CGA in any d!

We notice that we can have this extension in any d in non-relativistic symmetries since time decouples from space and we can have Mobius transformations in the time direction.

So 2+1 is a special case ! We can have a very large algebra which has an infinite extent in space direction as well as time !

Nonrelativistic conformal algebras in 2+1 dimensions

We first notice that we have a conformal symmetry in space

$$L_n = -z^{n+1}\partial_z \quad \bar{L}_n = -\bar{z}^{n+1}\partial_{\bar{z}}$$

We want Galilean causality i.e. no $f(r)\partial_t$

Global conformal transformation in time

$$t \to t' = \frac{\alpha t+\beta}{\gamma t+\delta} \quad \alpha\delta - \beta\gamma = 1$$

Non-relativistic conformal algebras in 2+1 dimensions

Consider the following operators (free index *l*)

$$L_m^n = -t^n z^{m+1} \partial_z$$
$$\bar{L}_m^n = -t^n \bar{z}^{m+1} \partial_{\bar{z}}$$
$$T^n = -(t^{n+1}\partial_t + l(n+1)t^n(z\partial_z + \bar{z}\partial_{\bar{z}}))$$

Commutators:

$$[L_m^k, L_n^l] = (m-n)L_{m+n}^{k+l} \qquad [\bar{L}_m^k, \bar{L}_n^l] = (m-n)\bar{L}_{m+n}^{k+l}$$

$$[L_m^k, \bar{L}_n^l] = 0 \qquad [T^m, T^n] = (m-n)T^{m+n}$$

$$[L_m^k, T^n] = (k + mln + ml)L_m^{k+n} \qquad [\bar{L}_m^k, T^n] = (k + mln + ml)\bar{L}_m^{k+n}$$

A. Hosseiny, S. Rouhani, “Affine extension of Galilean conformal algebra in 2+1 dimensions”, J. Math. Phys. 51, (2010) 052307 [hep-th/0909.1203]

Nonrelativistic conformal algebras in 2+1 dimensions

This algebra admits different central charges

$$[T^m, T^n] = (m-n)T^{m+n} + \frac{c}{12}m(m^2-1)\delta_{m+n,0}$$

$$\left[L^i_m, L^j_n\right] = (m-n)L^{i+j}_{m+n} + mC_s\delta_{m+n}\delta_{i+j}$$

$$\left[L^j_m, T^i\right] = (j + mil + ml)L^{i+j}_m + \frac{1}{2}jC_s\delta_{m,0}\delta_{i+j}$$

Representations of this algebra has not been worked out yet

Nonrelativistic conformal algebras in 2+1 dimensions

Note that the class of l-Galilei algebras in 2+1 dimensions is a subset of this class

$$K = -t^2\partial_t - 2lt(z\partial_z + \bar{z}\partial_{\bar{z}}) = T^1$$

$$\{P_i^n\} = \{t^n\partial_z, t^n\partial_{\bar{z}}\} = \{L_{-1}^n, \bar{L}_{-1}^n\}$$

$$J = i(z\partial_z - \bar{z}\partial_{\bar{z}}) = -i(L_0^0 - \bar{L}_0^0)$$

$$H = -\partial_t = T^{-1} \qquad D = -t\partial_t - l(z\partial_z + \bar{z}\partial_{\bar{z}}) = T^0$$