Non-Fermi Liquids in Strongly Coupled Field Theories

From calculations with Oliver DeWolfe and Steve Gubser

Christopher Rosen
Crete Center for Theoretical Physics

Regional Meeting...6/13...Kolymbari

Where We're Going

About Fermi Surfaces

Defining characteristics and important properties

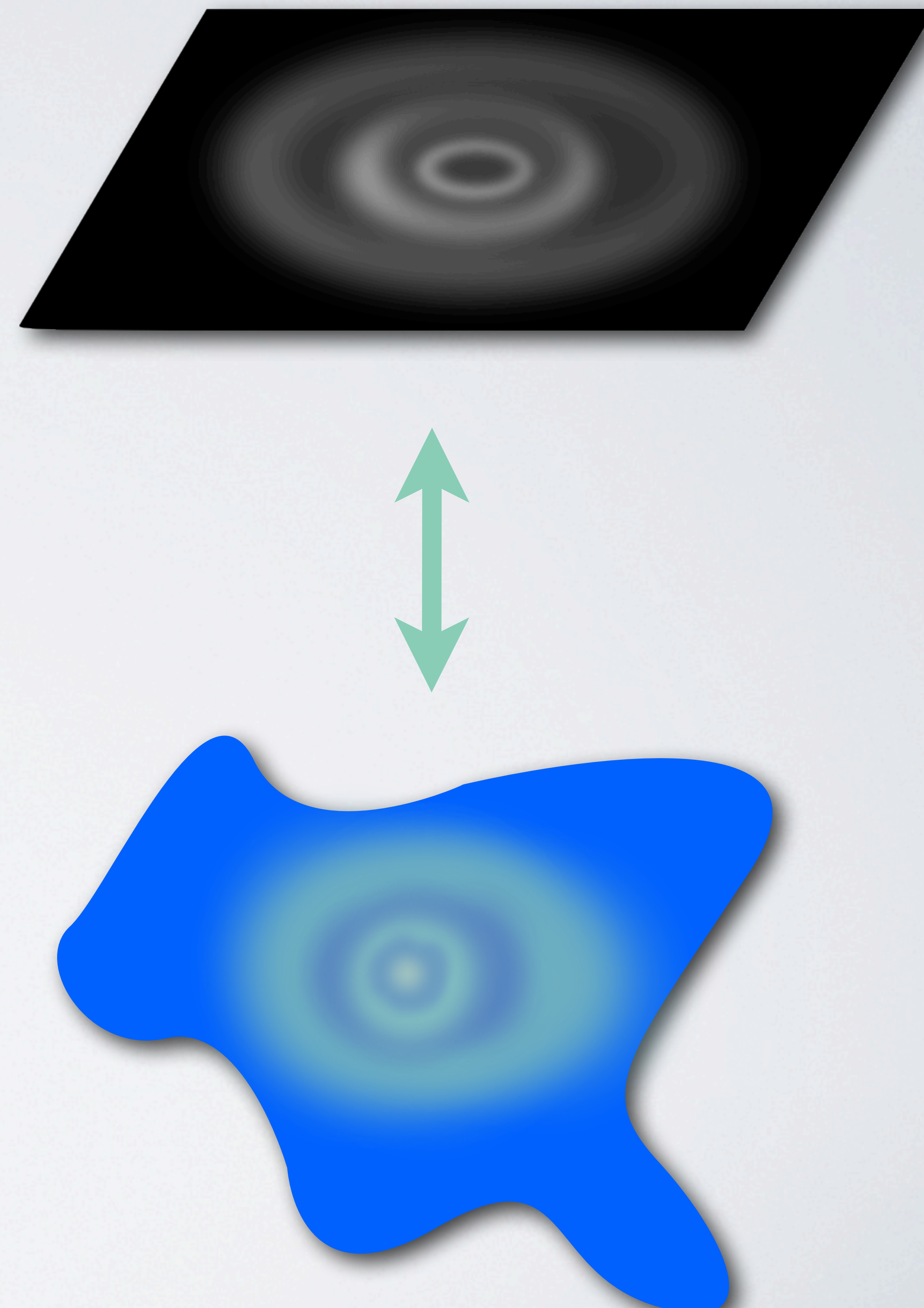
Fermi Surface Embeddings

Fermi surface physics in holography

Dirac equations sent from above

Survey of selected results

Points to Ponder

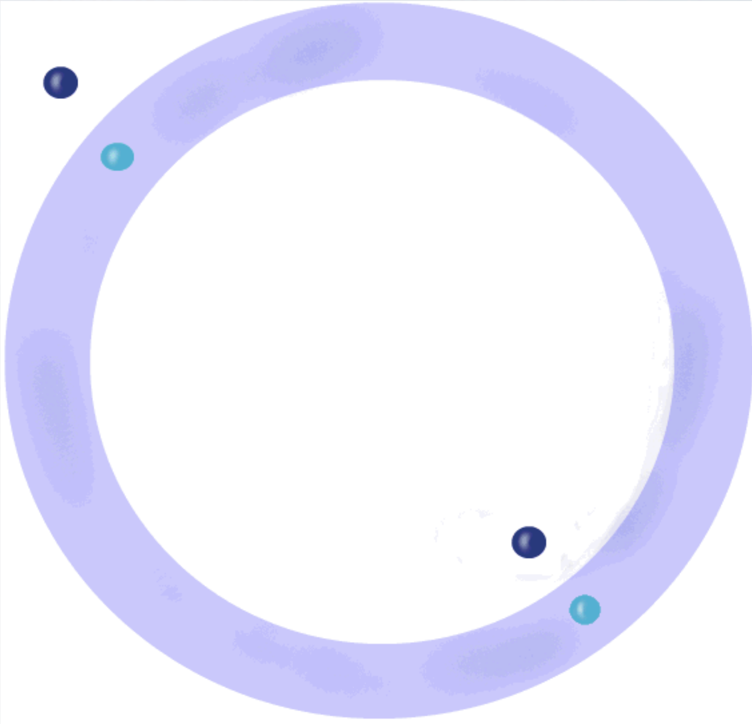


About Fermi Surfaces:

In Field Theory

A Fermi surface is the place, kF where

$$G_R{}^{-1}(\omega = 0, k = k_F) = 0$$



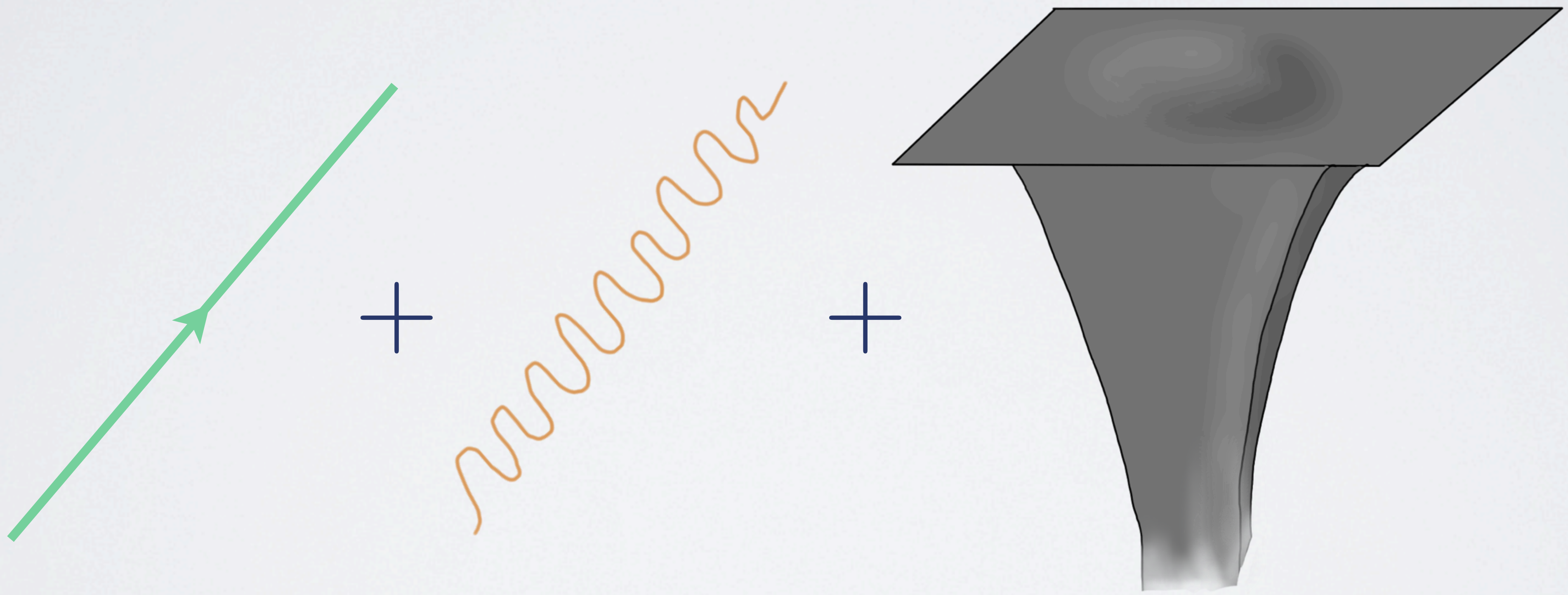
Low energy excitations about this surface can be parametrized by

$$G_R = \frac{Z}{\omega - v_F(k - k_F) + \Sigma(\omega, k)}$$

About Fermi Surfaces:

In Gravity Theories

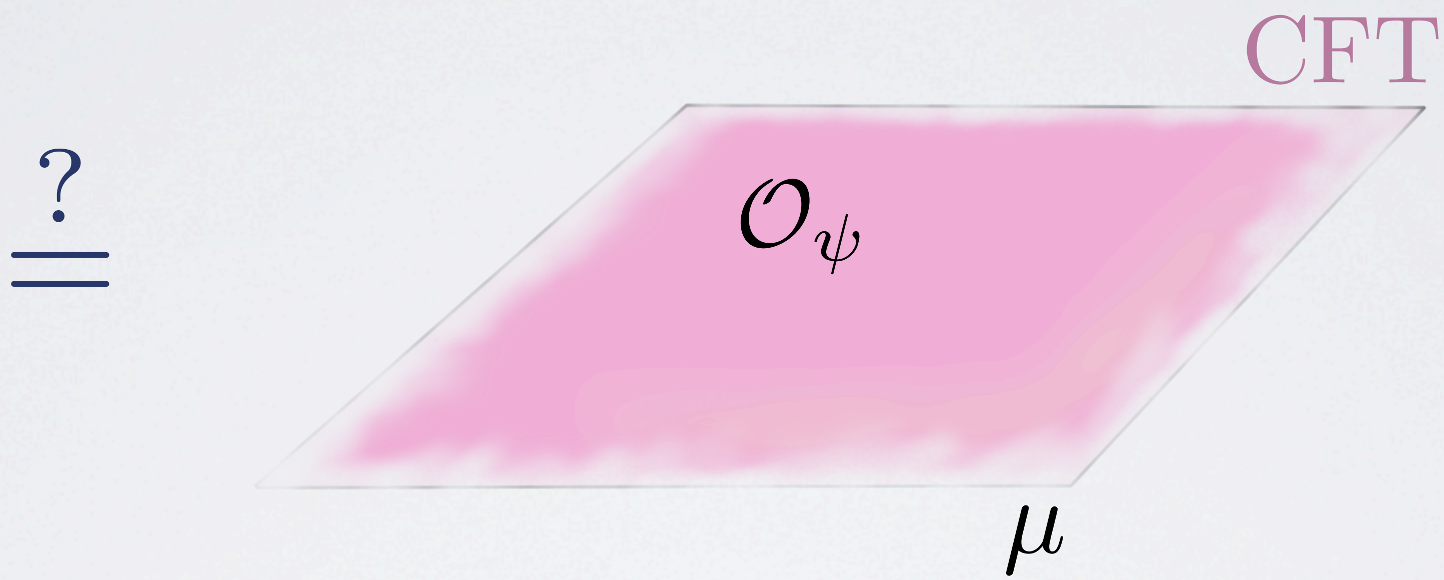
Bottom up finite density physics



About Fermi Surfaces:

In Gravity Theories

Bottom up finite density physics

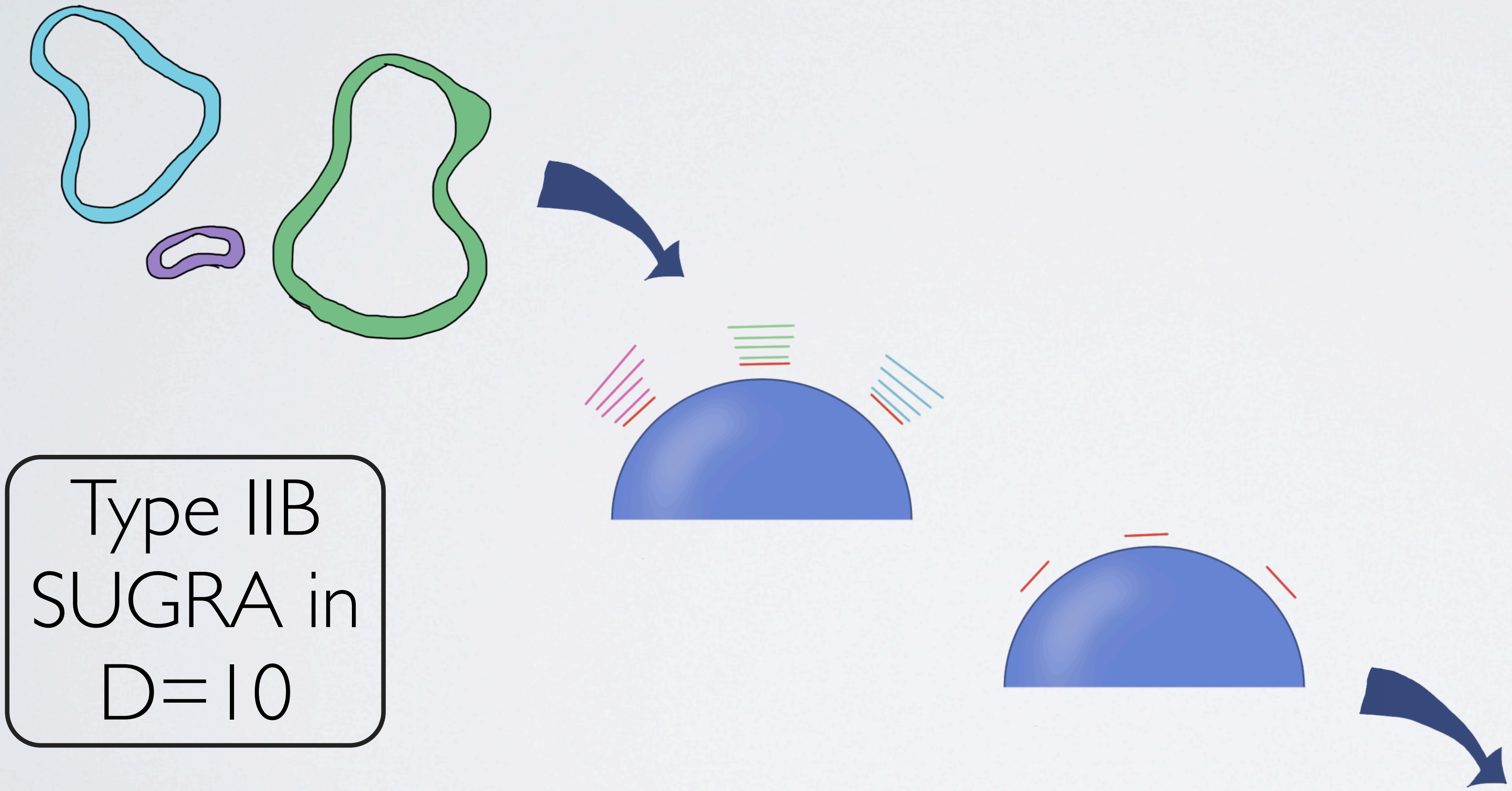


$$(i\Gamma^\mu \nabla_\mu + q\,\Gamma^\mu \mathcal{A}_\mu - m + \ldots)\,\psi = 0$$

About Fermi Surfaces:

In Gravity Theories

The top down alternative



Type IIB SUGRA in D=10

N=8 gauged SUGRA in D=5

$$\mathcal{L}_B\left[e, \mathcal{A}_i, \varphi_j\right]$$

Fermi Surface Embeddings

In Gravity Theories

Which background?

Try "2+1Q" BB's ⟷ N=4 SYM at (2x) Finite density, T

$$ds^2 = e^{2A(r)}\left(-h(r)\,dt^2 + d\vec{x}^2\right) - \frac{e^{2B(r)}}{h(r)}dr^2$$

$$a = \Phi_1(r)dt \qquad \mathcal{A} = \Phi_2(r)dt$$

$$\varphi = \phi(r)$$

The functions A, h, B, **Φ**1, **Φ**2, **φ**, are cumbersome but explicitly known

Fermi Surface Embeddings

In Gravity Theories

What about the fermions?

Fix background, study spin-1/2 fluctuations about it in SUGRA...

$$\left(i\Gamma^\mu \nabla_\mu + q_j \Gamma^\mu \mathcal{A}^j_\mu - m(\varphi) + i p_j(\varphi) \mathcal{F}^j_{\mu\nu} \Gamma^{\mu\nu}\right)\psi = 0$$

Important: fermion properties are no longer arbitrary

Fermi Surface Embeddings

In Gravity Theories

Bulk fermion fields of interest and their dual operators:

SUGRA fermion	AdS Mass	SO(6) Rep	SYM Op
ψ	$\frac{1}{2L}$	$\mathbf{20}$	$\mathrm{Tr}\lambda Z$
Ψ	$\frac{3}{2L}$	$\mathbf{4}$	$\mathrm{Tr}F_{+}\lambda$

We study fermions that don't mix with the gravitini

Fermi Surface Embeddings

In Gravity Theories

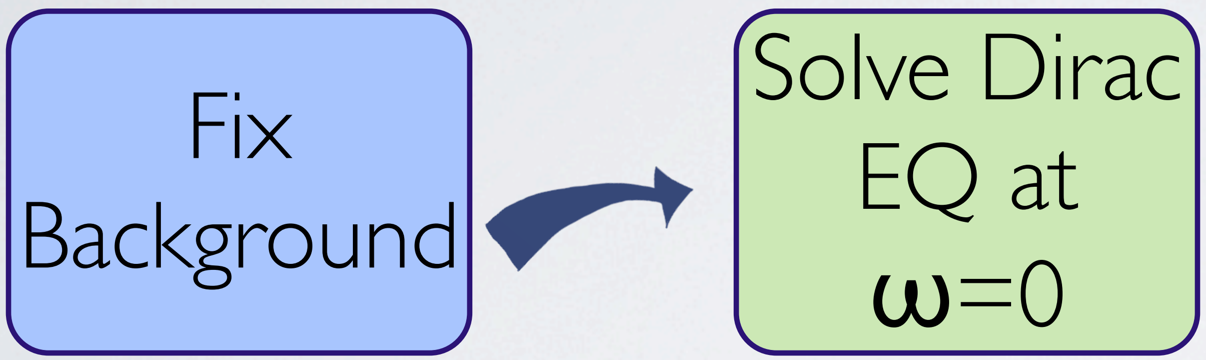
Workflow

Fix Background

Fermi Surface Embeddings

In Gravity Theories

Workflow



$$\psi_{r\to\infty} \sim A(k)\sqrt{r} + B(k)r^{-3/2}$$

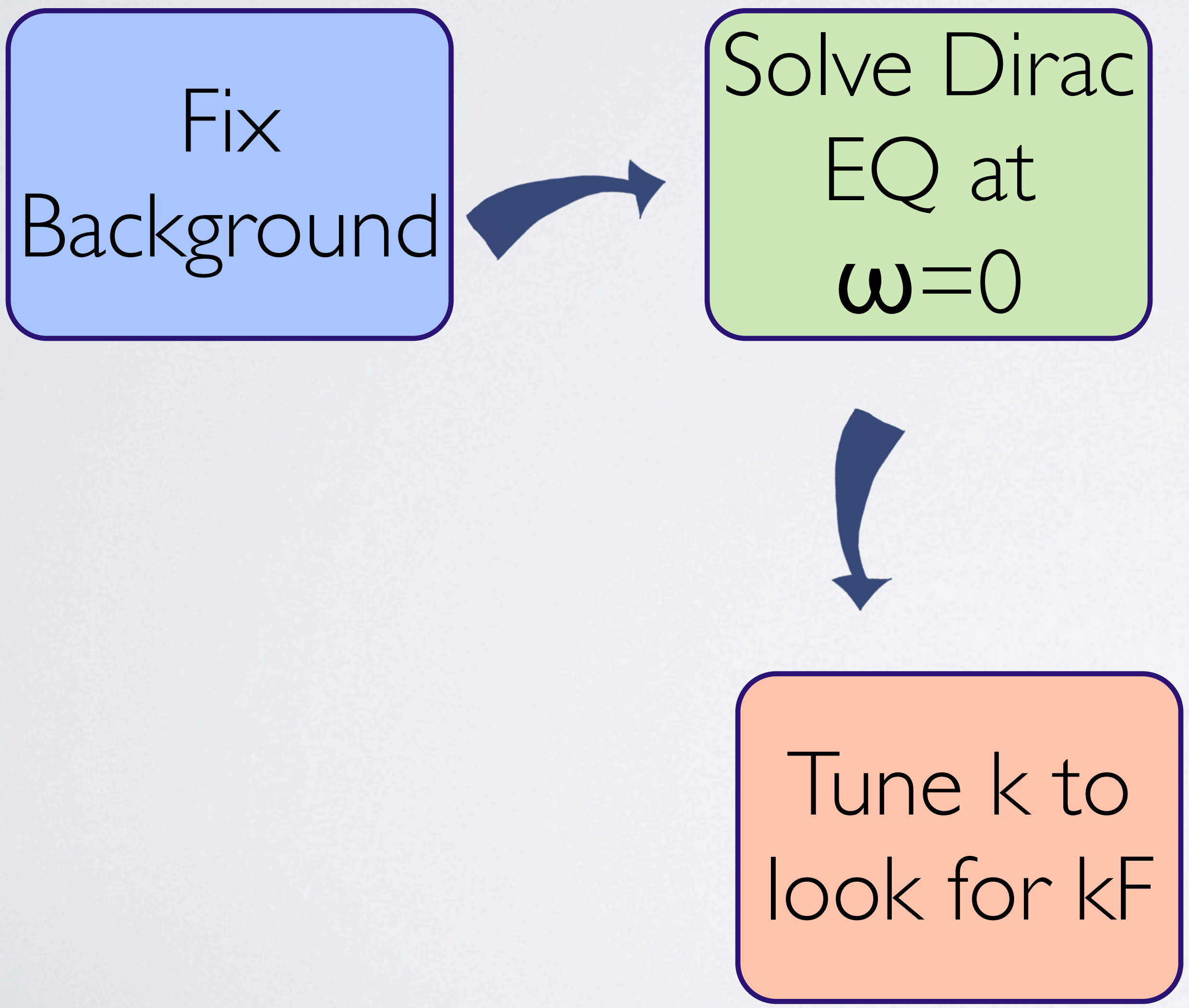
where

$$\delta S_{\rm CFT} = \int \mathrm{d}^4 x\, A(x)\mathcal{O}_\psi(x)$$

Fermi Surface Embeddings

In Gravity Theories

Workflow



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$$G_R(\omega = 0, k) \sim \frac{B(k)}{A(k)}$$

so

$$A(k_F) = 0$$

Fermi Surface Embeddings

In Gravity Theories

Finite frequency fluctuations

$$G_R = \frac{Z}{\omega - v_F(k - k_F) + \Sigma(\omega, k)}$$

Fermi Surface Embeddings

In Gravity Theories

Finite frequency fluctuations

$$G_R = \frac{Z}{\omega - v_F(k-k_F) + \Sigma(\omega, k)}$$

IR dynamics in 2+1Q systems is controlled by IR AdS2:

$$\Sigma(\omega, k) \sim e^{i\gamma_{k_F}}\omega^{2\nu_{k_F}} \quad \text{with} \quad [\mathcal{O}]_{\mathrm{IR}} = \frac{1}{2} + \nu_k$$

IR dimension governs dispersion relation, characterizes medium

if:

$\nu_{k_F} < \frac{1}{2}$ IR CFT O is relevant, Non-Fermi Liquid

$\nu_{k_F} > \frac{1}{2}$ IR CFT O is irrelevant, stable QP's

$\nu_{k_F} = \frac{1}{2}$ IR CFT O is marginal, like some cuprates

Fermi Surface Embeddings

In Gravity Theories

Finite frequency fluctuations

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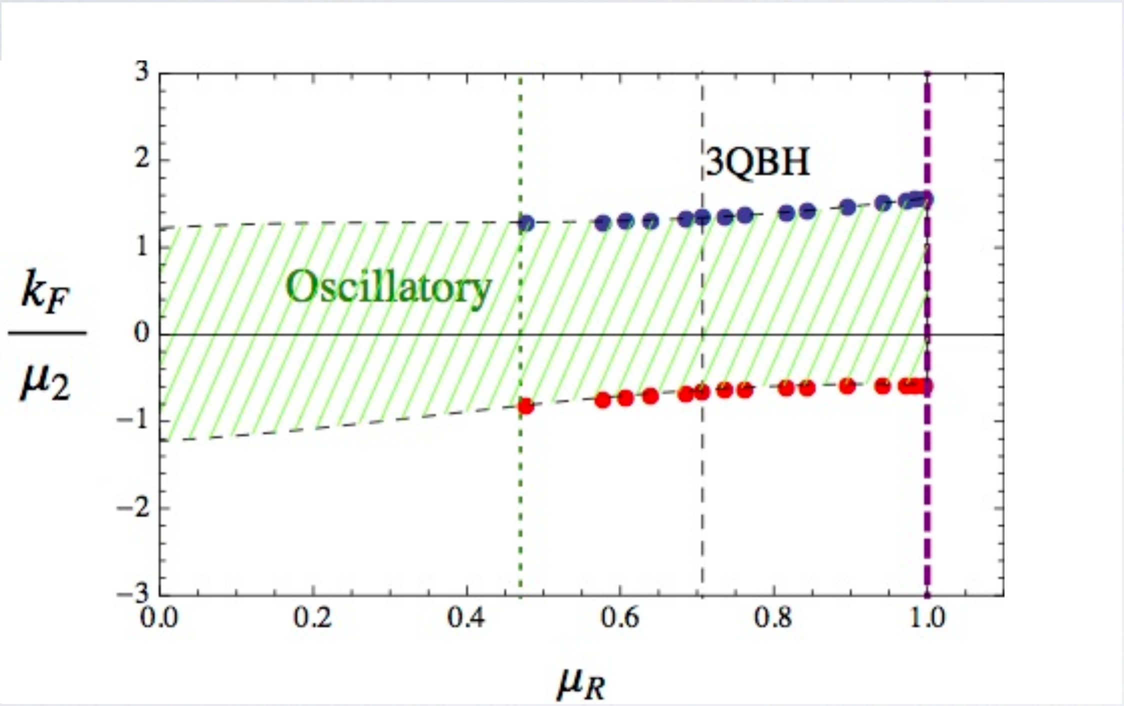
IR dimension governs dispersion relation, characterizes medium

In these embeddings, IR O is relevant--self energy dominates:

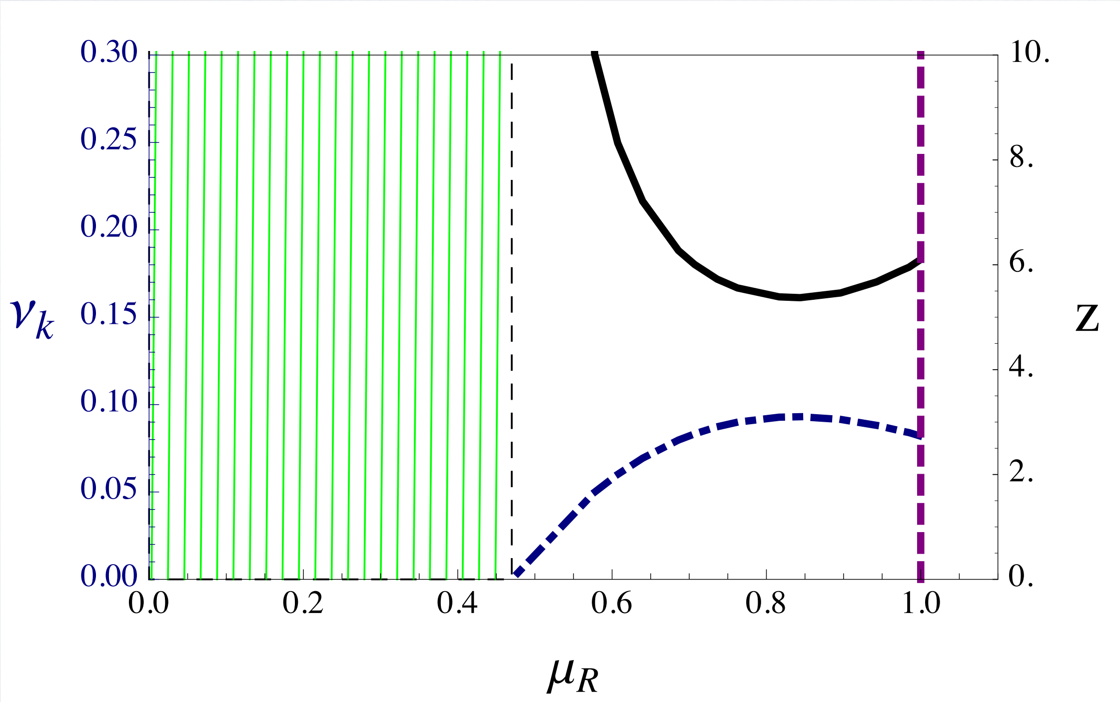
$$\omega_* \sim (k - k_F)^z \qquad \text{where} \qquad z \equiv \frac{1}{2\nu_{k_F}}$$

Fermi Surface Embeddings

In Gravity Theories



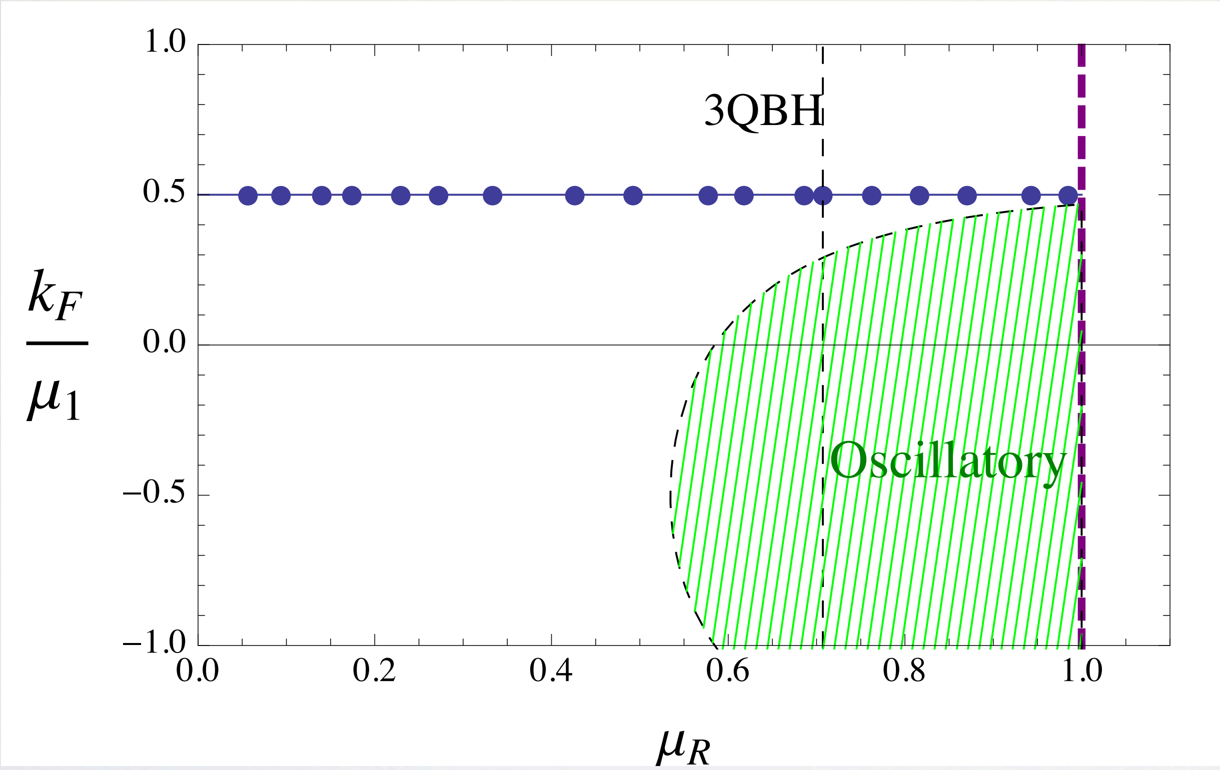
Fermion modes dual to Tr λZ may have 0 or 1 or 2 Fermi surfaces, depending on their charge

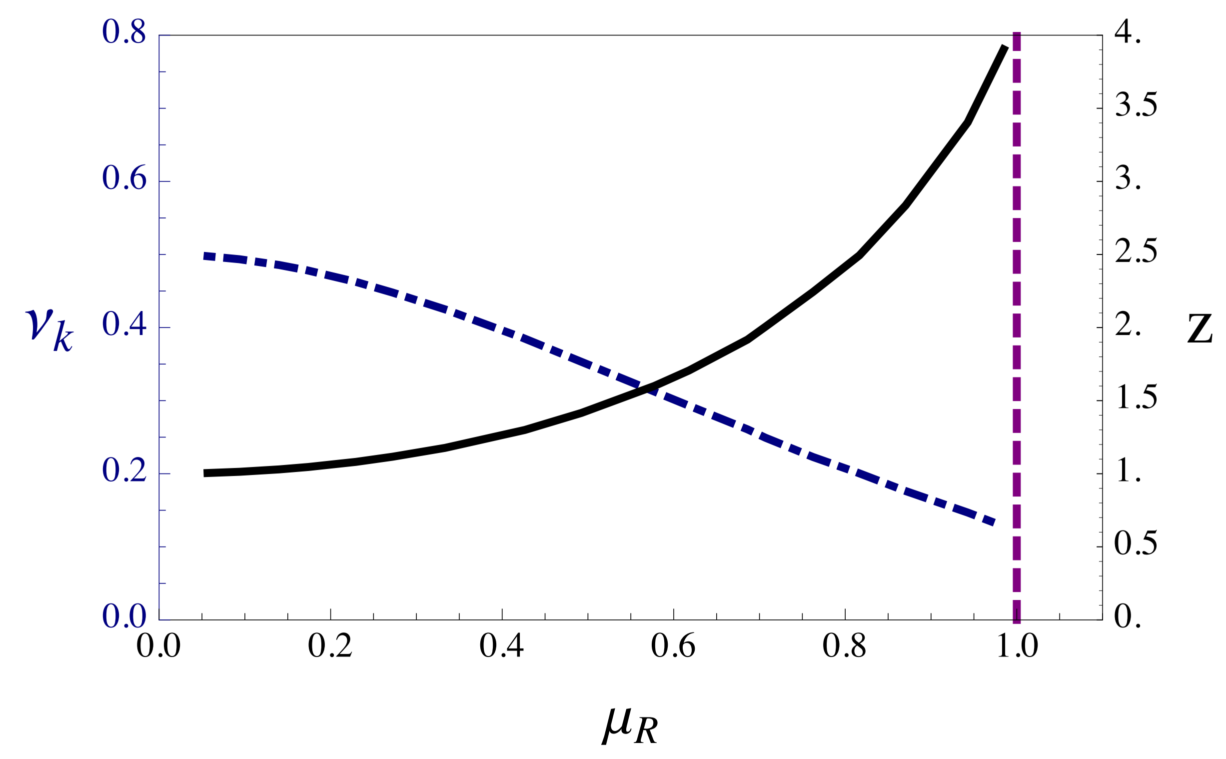


Fermion modes dual to Tr Fλ have no Fermi surface

Fermi Surface Embeddings

In Gravity Theories





These systems are almost always NFL's--but there is one mode that resembles a MFL

Fermi Surface Embeddings

Limits of the 2+1 charge solutions

2+1Q black branes have finite entropy density at zero temperature...

...Can we do a similar analysis in a more phenomenologically favorable background?

Fermi Surface Embeddings

The Extremal Two-Charge Solution

Background

$$A(r) = \log \frac{r}{L} + \frac{1}{3} \log \left(1 + \frac{Q^2}{r^2}\right)$$

$$B(r) = -\log \frac{r}{L} - \frac{2}{3} \log \left(1 + \frac{Q^2}{r^2}\right)$$

$$h(r) = 1 - \frac{Q^4}{(r^2 + Q^2)^2}$$

$$\boxed{\phi(r) = \sqrt{\frac{2}{3}} \log \left(1 + \frac{Q^2}{r^2}\right)}$$

Important!

$$\Phi(r) = \frac{Q}{2L} \left(1 - \frac{Q^2}{r^2 + Q^2}\right)$$

Fermi Surface Embeddings

The Extremal Two-Charge Solution

Fermi surfaces exist

So do novel features at finite **ω**...

Near the horizon

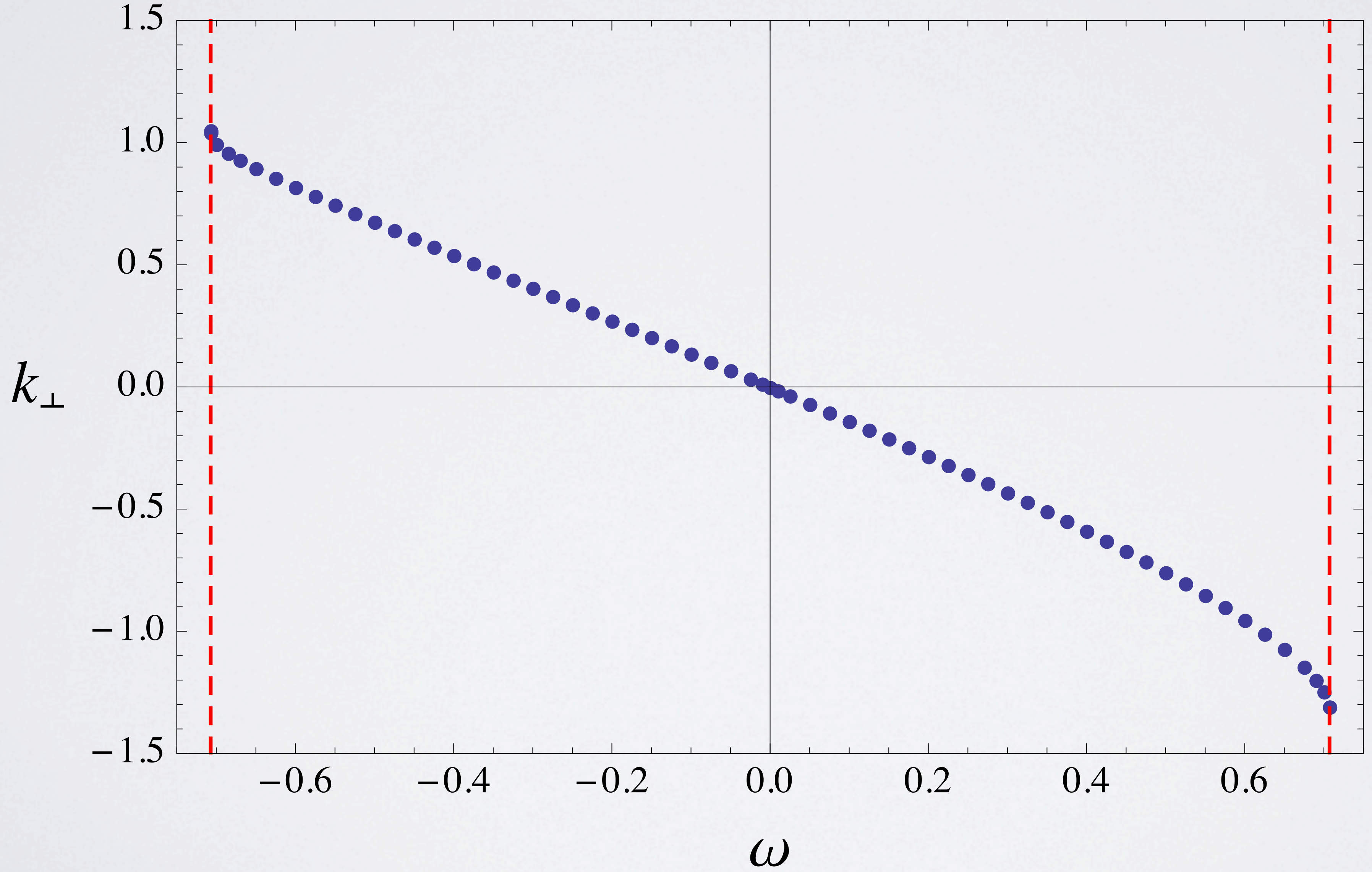
$$\psi_{r\to 0} \sim e^{-\frac{1}{2r}\sqrt{\Delta^2-\omega^2}}$$

So modes with **ω** < **Δ** are candidates for stable qp's in the field theory liquid

Fermi Surface Embeddings

The Extremal Two-Charge Solution

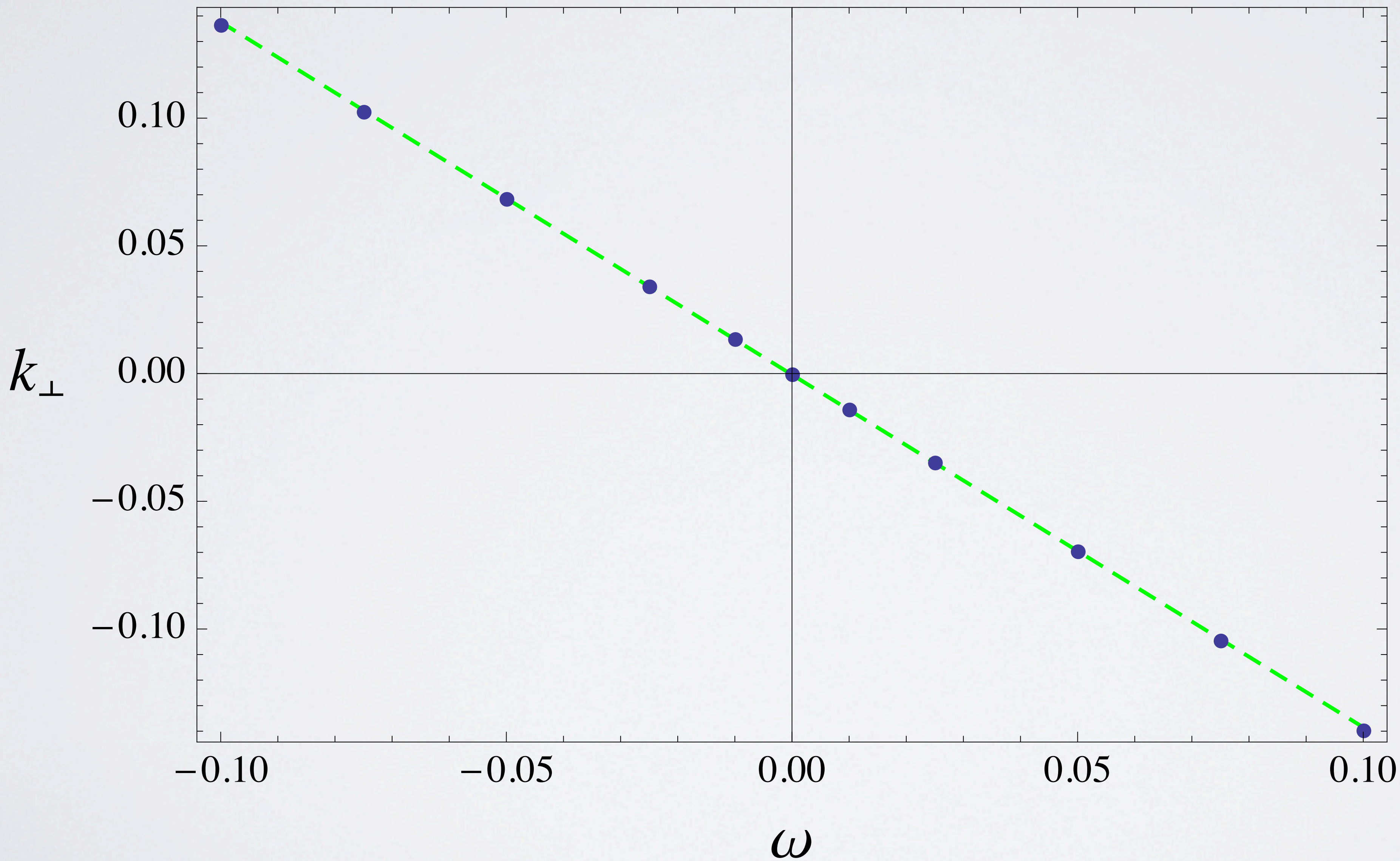
Low energy excitations



Fermi Surface Embeddings

The Extremal Two-Charge Solution

Low energy excitations



more Fermi-liquid-like?

Things to Think About

Fermi surface behavior is ubiquitous in N=4 SYM

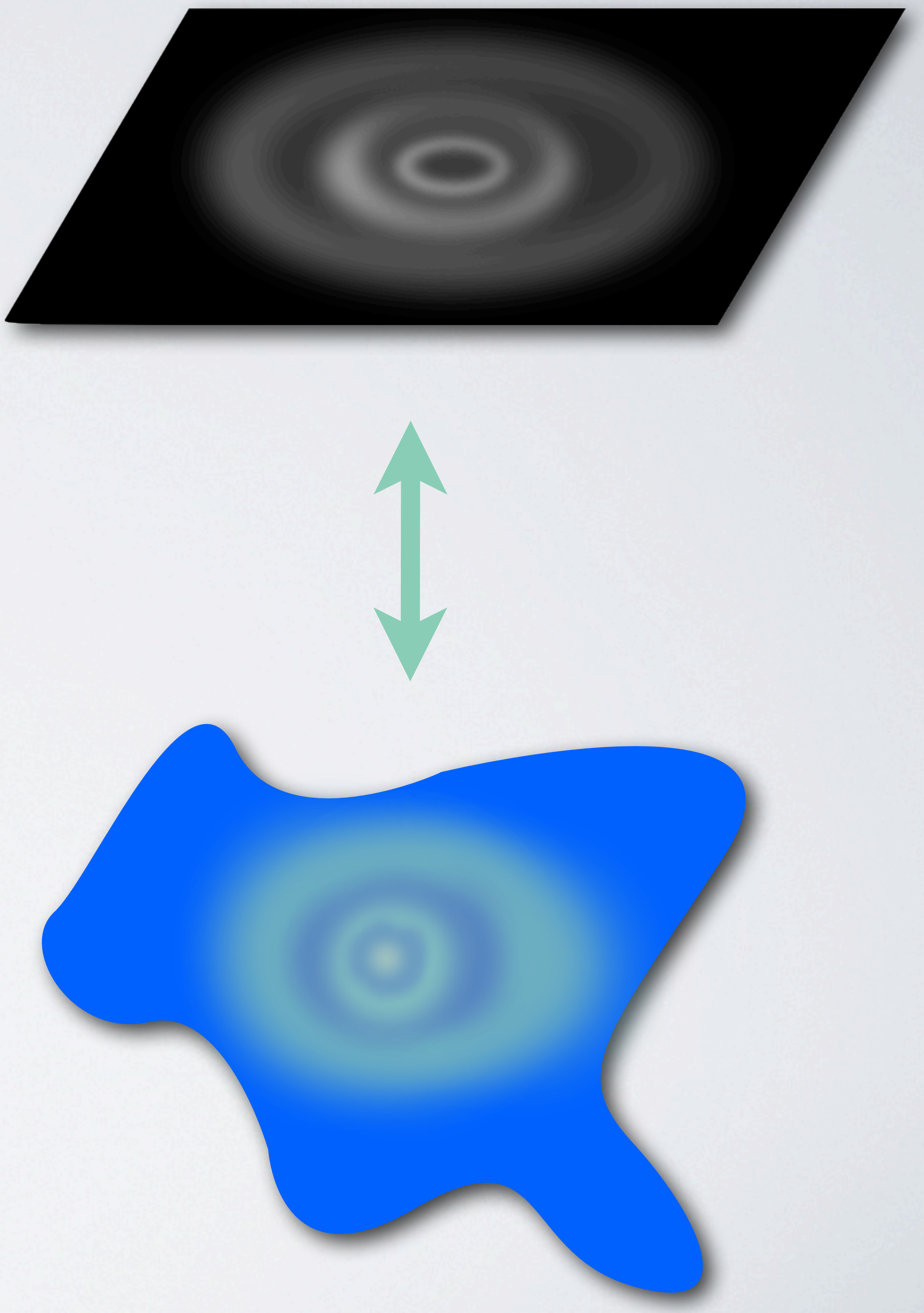
It also appears in ABJM

Top down methods provide concrete field theory infos

Interesting holographic physics awaits explanation near the limits of the 2+1 charge solutions

Many of these SUGRA solutions may have instabilities coming from the scalar sector

What role do they play?



More Info

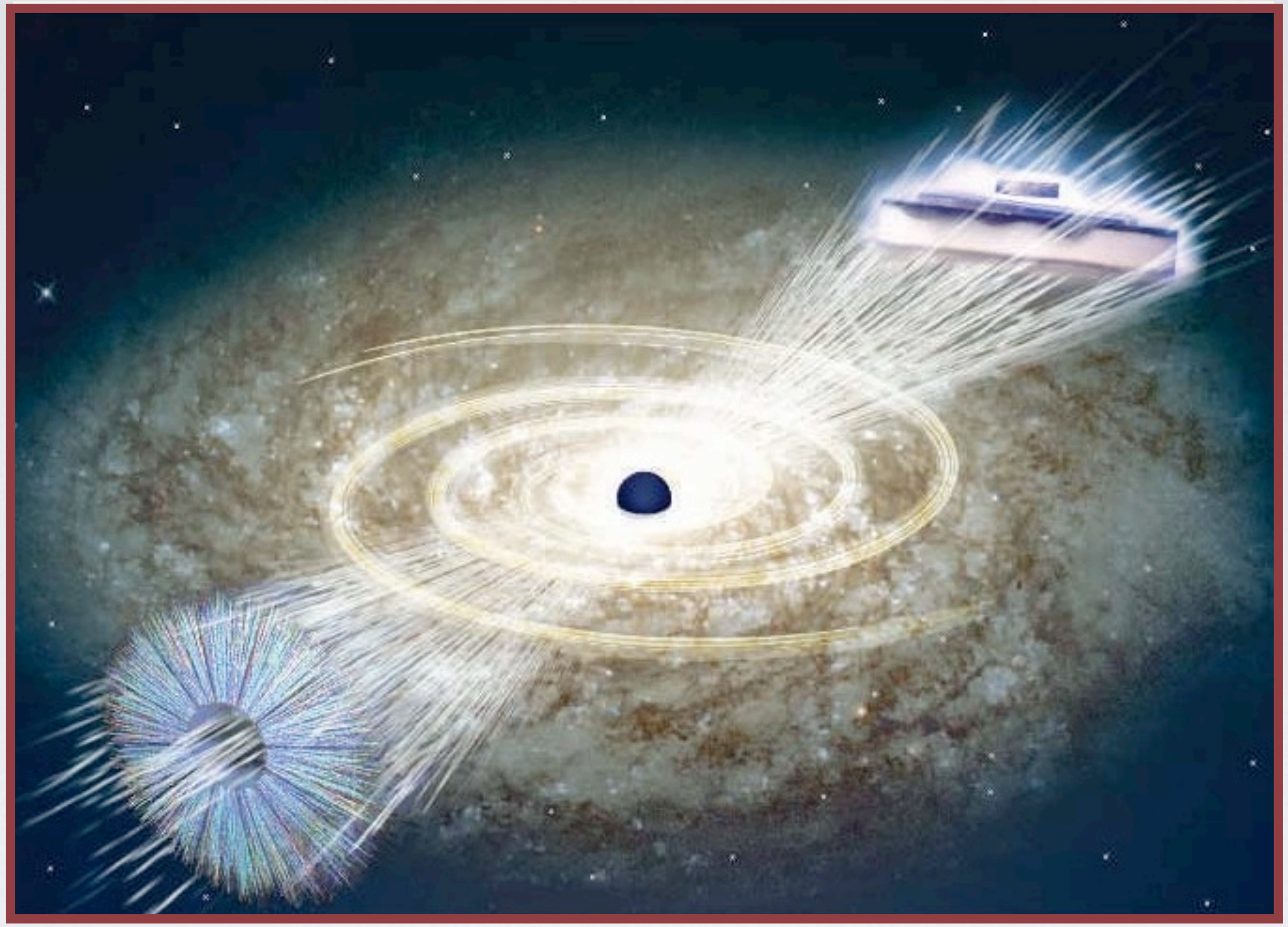
Bottom up Fermi Surfaces

[0907.2694], [1101.0597]

Fermi Surfaces in N=4 SYM

[1112.303], [1207.3352]

Thanks To:



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