Singleton deformation of higher-spin theory and the phase structure of the $3d$ $O(N)$ vector model

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[based on arXiv:1212.4421 with R. G. Leigh]

Outline

1. Motivation

2. The $O(N)$ vector model
   - A lightning review of the model
   - The $O(N) \to O(N-1)$ symmetry breaking
   - Anomalous dimensions

3. $O(N)$/HS holography
   - The HS/$O(N)$ conjecture
   - The gap equations from holography
   - The singleton deformation of higher-spin theory and boundary symmetry breaking
   - The calculation of boundary anomalous dimensions

4. Summary and outlook

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AdS/CFT provides a concrete relationship between gauge theories and gravity.

- Using supergravity (practically), we describe quite well the spectrum of gauge invariant operators in $\mathcal{N}=4$ SYM at strong coupling.
- Higher-spin (space-time or internal) single-trace operators correspond to string sigma-model configurations $\rightarrow$ the integrability of the spectrum of superconformal gauge theories.

Strings and gauge fields

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If holography is a larger concept than AdS/CFT, it need not be tied to gauge fields or strings.

- To test the idea, study holographically conformal vector models that stand alone without the need of gauge fields $\Rightarrow$ are not embedded in string theory.
- The best understood models are the $O(N)$ bosonic and fermionic (Gross-Neveu) vector models.
- Such theories have been extensively studied in using the $1/N$ expansion combined with CFT techniques. [e.g. Zinn-Justin (89)]
- There is a rather good understanding of the spectrum of anomalous dimensions of all of their operators i.e. the elementary scalar, $O(N)$-singlets and non-singlet composites.

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Motivation

An example:

- The $1/N$ anomalous dimensions of the $O(N)$-singlet higher-spin currents are:

$$J_{(s)} \sim \phi^a \partial_{\{\mu_1} \ldots \partial_{\mu_s\}} \phi^a \,, \quad a = 1,2,..,N$$

$$\Delta_s = s + 1 + 4\gamma_\phi \frac{s-2}{2s-1} + \cdots \,, \; s = 2k \,, \; k = 1,2,\ldots \,, \; \gamma_\phi \sim O(1/N)$$

- For $s \to \infty$ these tend to

$$\Delta_s \to 2\left(\frac{1}{2} + \gamma_\phi\right) + s$$

  i.e. the sum of the anomalous dimensions of the elementary fields ϕ^a.

- They are all determined by γ_ϕ, i.e. the anomalous dimension of the elementary field ϕ^a : $\to$ contrast with $\mathcal{N} = 4$ SYM.
- They do not exhibit the $\ln S$ growth of the corresponding higher-spin currents in gauge theories $\Rightarrow$ the model is not a gauge theory.
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The conjectures:

The bosonic vector model/HS conjecture

The holographic duality between the $O(N)$ *singlet sector* of the bosonic vector model and the simplest *higher-spin gauge theory* on AdS_4 was proposed in [Klebanov and Polyakov (02)].

The fermonic vector model/HS conjecture

The analogous conjecture regarding the $O(N)$ fermionic vector model - complicated due to parity - was made in [Leigh and T. P. (03)]

- The bosonic conjecture has been tested up to 3-pt couplings: the bulk scalar cubic vertex vanishes [T. P. (03)], and *most* higher-spin cubic couplings give correctly the corresponding boundary 3-pt functions [e.g. Giombi and Yin (09)].
- Important progress has also been made in the study of the Vasiliev higher-spin theories [Vasiliev et. al., Sundell, Sezgin, Sagnotti et.al, Boulanger, Bekaert, et. al.]. This is a rather technical subject.

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On the other hand:
Vector models exhibit the field theoretic manifestations of global and discrete symmetry breaking.

- The bosonic model exhibits the $O(N) \to O(N-1)$ global symmetry breaking pattern.
- The fermionic model exhibits parity symmetry breaking.

Holography without gauge fields?

- What is the bulk counterpart of the global $O(N)$ boundary symmetry? I.e. are there are non-perturbative objects (analogues of D3-branes) in higher-spin gauge theory on AdS_4 that give rise to the global $O(N)$ symmetry in the boundary?
- It is not known if only the $O(N)$ singlet sector can be described holographically. ⇒ the vector theories possess nontrivial and well-understood non-singlet sectors.

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In this talk:

The Vasiliev HS theory is incomplete:

I will argue that the bulk HS theory needs to be deformed by of by singletons to account for the $O(N) \to O(N-1)$ symmetry breaking pattern of the vector model.

Specifically:

- Using the singleton deformation I will reproduce the boundary gap equation.
- Using the singleton deformation I will reproduce the known anomalous dimension of the elementary scalars ⇒ this raises the issue whether $O(N)$ symmetry breaking is related to higher-spin symmetry breaking.

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4 Summary and outlook

- N elementary (Euclidean) scalar fields $\phi^a(x)$, $a = 1, 2, .., N$ with Lagrangian

$$L = \frac{1}{2}\int d^3x\, \partial_\mu\phi^a\partial_\mu\phi^a\,,$$

subject to the constraint

$$\phi^a\phi^a = \frac{1}{G}$$

- We introduce a Lagrange multiplier scalar field ρ as

$$Z = \int(\mathcal{D}\phi^a)(\mathcal{D}\rho)e^{-I(\phi^a,\rho)}$$

$$I(\phi^a,\rho) = \frac{1}{2}\int d^3x\,\phi^a(-\partial^2)\phi^a + \frac{1}{2}\int d^3x\,\rho\left(\phi^a\phi^a - \frac{N}{g}\right)\,,\ g = GN$$

- The dimensionful coupling $1/G$ sets the physical mass scale of the theory: $G \to 0$ is the free field theory limit which lies in the UV.

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- The partition function Z and the effective action $S_{eff}(\rho)$ are given by

$$Z = \int (\mathcal{D}\rho) e^{-NS_{eff}(\rho)}\,,\;\; S_{eff}(\rho) = \frac{1}{2}\mathrm{Tr}\ln(-\partial^2+\rho) - \int d^3x\frac{\rho}{2g}$$

- The saddle point at large-N, with constant $\rho_0 = m^2$, yields the gap equation

$$\frac{\partial S_{eff}(\rho)}{\partial\rho}\Big|_{\rho_0} = 0 \Rightarrow \frac{1}{g} = \int \frac{d^3p}{(2\pi)^3}\frac{1}{p^2+\rho_0}$$

- The large-N expansion is obtained setting

$$\rho(x) = \rho_0 + \frac{1}{\sqrt{N}}\sigma(x)\,,$$

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$$\rho(x) = \rho_0 + \frac{1}{\sqrt{N}}\sigma(x)\,,$$

- The partition function Z and the effective action $S_{eff}(\rho)$ are given by

$$Z = \int (\mathcal{D}\rho) e^{-NS_{eff}(\rho)} \,, \; S_{eff}(\rho) = \frac{1}{2}\mathrm{Tr}\ln(-\partial^2 + \rho) - \int d^3x \frac{\rho}{2g}$$

- The saddle point at large-N, with constant $\rho_0 = m^2$, yields the gap equation

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$$\rho(x) = \rho_0 + \frac{1}{\sqrt{N}}\sigma(x) \,,$$

- The effective action $\mathcal{S}^N_{eff}(\sigma,\rho_0)$ for the real fluctuations σ is

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\begin{aligned}
S_{eff}(\rho) &= V_{eff}(\rho_0,g)+\frac{1}{N}\mathcal{S}^N_{eff}(\sigma,\rho_0) \\
V_{eff}(\rho_0,g) &= \frac{1}{2}\mathrm{Tr}\ln(-\partial^2+\rho_0)-\frac{\rho_0}{2g}(Vol)_3 \\
\mathcal{S}^N_{eff}(\sigma,\rho_0) &= \frac{1}{2}\int \sigma(x)\Delta(x,y;\rho_0)\sigma(y) \\
&\quad +\frac{1}{3!\sqrt{N}}\int \sigma(x)\sigma(y)\sigma(z)P(x,y,z;\rho_0)+..
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- The generating functional $W[\eta]$ for connected correlation functions of σ is

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Outline

1. Motivation

2. The $O(N)$ vector model
   - A lightning review of the model
   - The $O(N) \to O(N-1)$ symmetry breaking
   - Anomalous dimensions

3. $O(N)$/HS holography
   - The HS/$O(N)$ conjecture
   - The gap equations from holography
   - The singleton deformation of higher-spin theory and boundary symmetry breaking
   - The calculation of boundary anomalous dimensions

4. Summary and outlook

- The gap equation determines the vacuum structure:
  With a UV cutoff Λ for the momentum integral, it is rewritten as

$$\begin{aligned}\frac{1}{g} &= \int^{\Lambda}\frac{d^3p}{(2\pi)^3}\frac{1}{p^2}-\int^{\Lambda}\frac{d^3p}{(2\pi)^3}\frac{\rho_0}{p^2(p^2+\rho_0)}\\ &= \frac{\Lambda}{2\pi^2}-\frac{\sqrt{|\rho_0|}}{2\pi^2}\arctan\frac{\Lambda}{\sqrt{|\rho_0|}}\end{aligned}$$

- We define a critical coupling g_* as

$$\frac{1}{g_*}=\frac{\Lambda}{2\pi^2}\,,$$

- The gap equation takes the suggestive form

$$\left(\frac{1}{g_*}-\frac{1}{g}\right)=\frac{\sqrt{|\rho_0|}}{2\pi^2}\arctan\frac{\Lambda}{\sqrt{|\rho_0|}}=\frac{\sqrt{|\rho_0|}}{4\pi}+O(\rho_0/\Lambda)$$

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The vacuum structure is found comparing g to g_*:

- For $g > g_*$; we find $m = \sqrt{|\rho_0|} \neq 0$ and the theory is massive.
- For $g = g_*$; there is no mass scale left in the theory $\rightarrow$ the generating functional of connected correlation functions of a scalar operator σ with dimension $\Delta = 2 + O(1/N)$ in a three-dimensional CFT - the *critical* $O(N)$ vector model.
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- Integrating over the remaining $N-1$ elementary scalars we obtain

$$Z = \int [\mathcal{D}\phi][\mathcal{D}\rho]\, e^{-(N-1)S_{eff}(\rho,\phi)}$$

- The effective action is now defined as

$$S_{eff}(\phi,\rho) = S_{eff}^{N-1}(\rho) + \frac{1}{2(N-1)}\int d^3x\, \phi(-\partial^2+\rho)\phi$$

$$S_{eff}^{N-1}(\rho) = \frac{1}{2}\mathrm{Tr}\ln(-\partial^2+\rho) - \frac{N}{(N-1)}\int d^3x\, \frac{\rho}{2g}$$

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$$\rho(x) = \rho_0 + \frac{1}{\sqrt{N-1}}\sigma(x)\,,\ \phi(x) = \phi_0 + \varphi(x)\,.$$

- ρ_0, ϕ_0 are determined by the

modified gap equations

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$O(N) \to O(N-1)$ symmetry breaking pattern

- The effective action for the $O(N)$ model $\leftarrow$ the effective action of the $O(N-1)$ model by *integrating in* φ with a marginal deformation $\int \sigma\varphi^2$ and linear interaction $\int \varphi\sigma$.
- At the critical point $\rho_0 = \phi_0 = 0$, one integrates in a massless elementary scalar $\varphi(x)$ with marginal interaction.
- The $O(N-1)$ model "eats" elementary scalars with $O(1/\sqrt{N})$ marginal interactions by enlarging its symmetry, i.e. shifting $N-1 \to N$.

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The modified gap equation is written

$$\frac{\phi_0^2}{N-1} = \left(\frac{N}{N-1}\frac{1}{g} - \frac{1}{g_*}\right) + \frac{|m|}{4\pi} + \cdots$$

and differs from the previous gap equation in two ways:

- Firstly, we notice the presence of an extra term on the left-hand side.
- Secondly, there is an extra $N/(N-1)$ factor in front of the coupling constant $1/g$.

These two differences are intimately related as we will see later..

We have an explicit manifestation of the Goldstone mechanism.

- Away from the critical point ϕ_0 and $|m|$ cannot be nonzero simultaneously, and $|m| < \Lambda$.
- When $g < Ng_*/(N-1)$ we are in the UV, the mass vanishes but we always have $\phi_0 \neq 0 \Rightarrow$ away from the UV fixed point, the $O(N)$ symmetry is always broken to $O(N-1)$. As usual we also have $N-1$ Goldstone bosons which are seen here as the massless elementary scalars that were integrated out.

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- When the coupling is tuned to

$$g = \frac{N}{N-1} g_*$$

we have $\phi_0 = m = 0$ and we arrive at the critical $O(N)$ vector model.

- The above critical point differs from the old critical point which required tuning the bare coupling constant exactly to $g = g_*$.
- By writing as

$$\frac{N}{N-1}\frac{1}{g} - \frac{1}{g_*} = \frac{1}{g} - \frac{1}{g_*} + \frac{1}{N-1}\frac{1}{g}$$

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- In this regime, the theory enters an $O(N)$-symmetric massive phase.
- The common mass for the elementary fields

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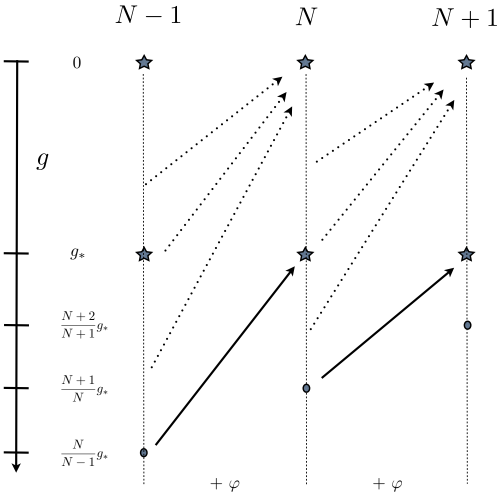

Figure : The phase diagram of the vector models. Stars denote the CFTs. The solid arrows denote marginal deformations towards the IR fixed point after the absorption of an elementary scalar φ. The dotted arrows denote irrelevant double-trace deformations leading to the UV fixed point of the symmetry enhanced theory.

- We note that the value of the critical coupling g_* is independent of N.
- Starting then from an $O(N-1)$ model, the absorption of the elementary scalar ϕ is done once we enter the *massive* phase of the theory, namely when $g = Ng_*/(N-1) > g_*$.
- Then it is possible to deform the theory by a marginal coupling and return to the universal fixed point at g_*, having however enlarged the symmetry to $O(N)$.
- Starting deeper in the massive phase with $g > Ng_*/(N-1)$ the model absorbs the elementary scalar and flows to the massive phase of the $O(N)$ model under the marginal deformation.
- Finally, when $g < Ng_*/(N-1)$ we assign the difference

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- To unveil the meaning of this term we can shift the scalar fluctuation as

$$\varphi = \hat{\varphi} + \frac{\phi_0}{\sqrt{N-1}}\frac{1}{-\partial^2}\sigma\,,$$

- A short calculation then gives

$$Z \sim \int e^{-\left[\mathcal{S}_{eff}^{N-1}(\sigma,0)+\frac{1}{2}\int\hat{\varphi}D_0\hat{\varphi}+\frac{1}{2\sqrt{N-1}}\int\sigma\hat{\varphi}^2-\frac{\phi_0^2}{2(N-1)}\int\frac{1}{-\partial^2}\sigma^2+..\right]}.$$

- The last term in the exponent is a nonlocal version of the irrelevant double-trace deformation $\int\sigma^2$ which drives the theory in the UV where we expect to find the free $O(N)$ model.
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Outline

1. Motivation

2. The $O(N)$ vector model
   - A lightning review of the model
   - The $O(N) \to O(N-1)$ symmetry breaking
   - Anomalous dimensions

3. $O(N)$/HS holography
   - The HS/$O(N)$ conjecture
   - The gap equations from holography
   - The singleton deformation of higher-spin theory and boundary symmetry breaking
   - The calculation of boundary anomalous dimensions

4. Summary and outlook

- To calculate correlation functions of ϕ^a and σ we couple the partition function to sources J^a and η as

$$Z \to Z[J^a, \eta] = \int [\mathcal{D}\phi^a][\mathcal{D}\rho]\; e^{-I(\phi^a,\rho)+\int \phi^a J^a + \int \eta\rho}\,.$$

- At $g = g_*$ this gives the generating functional for the critical $O(N)$ model

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- Using the above, one can perform a systematic $1/N$ expansion for all correlation functions of ϕ^a and σ. Using conformal "uniqueness" techniques, the anomalous dimensions of ϕ^a and σ up to $O(1/N^3)$ were calculated long time ago [A. Vasiliev et. al. (81-81)]. Similar results have been obtained in the fermionic and supersymmetric $O(N)$ cases [Gracey (91-92)].
- Soon afterwards [Rühl et. al. (92-93)] initiated the study of the operator spectrum of the bosonic $O(N)$ vector model.
- Finally, in [T. P. (94-96)] the conformal bootstrap of the bosonic and fermionic models was formulated, and it was argued that all the dynamical information is based on the cancellation of *shadow singularities*.

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The $O(N)$ Vector Model

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- The systematic $1/N$ expansion is easily obtained. From conformal invariance we have

$$\langle\phi^a(x)\phi^b(0)\rangle = \frac{C_\phi}{x^{2\Delta_\phi}}\delta^{ab}\,,\quad \langle\sigma(x)\sigma(0)\rangle = \frac{C_\sigma}{x^{2\Delta_\sigma}}$$

- We fix $d=3$ and define three critical indices γ_ϕ, κ and z of order $O(1/N)$ as

$$\Delta_\phi = \frac{1}{2}+\gamma_\phi\,,\quad \Delta_\sigma = 2-2\gamma_\phi-2\kappa\,,\quad C_\phi^2 C_\sigma = \frac{1}{\pi^4}+z$$

- The two-point function of ϕ^a is given by σ-exchange. One finds

$$\langle\phi^a(x)\phi^b(0)\rangle = \frac{1}{4\pi}\frac{1}{|x|}\left[1-\frac{1}{N}\frac{4}{3\pi^2}\ln|x|^2+\ldots\right]\delta^{ab}$$

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4. Summary and outlook

- The conserved higher-spin currents of a 3d CFT form unitary irreducible representations (UIR) of $SO(3,2)$, $D(\Delta, s)$, with dimensions $\Delta = s + 1$.
- When s is even, these arise in the parity-even tensor product of two singleton UIRs $D(1/2,0)$ as (Flato-Fronsdal theorem).

$$[D(1/2,0)\otimes D(1/2,0)]_S = D(1,0)\oplus\sum_{s=1}^{\infty} D(2s+1,2s)\,.$$

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- The suggested $O(N)$/HS correspondence proceeds by considering a bulk action, (although the full Lagrangian of HS theory is still elusive), with the schematic form

$$I_{HS} = \sum_{s=0,2,4,..}^{\infty} \int d^4x\sqrt{-g}\frac{1}{2}\Phi^{(s)}\left[\Box_s - \frac{1}{L^2}(s^2-2s-2)\right]\Phi^{(s)} + O(\frac{1}{\sqrt{N}})$$

- $\Phi^{(s)}$ denote symmetrized and double-traceless rank-s tensors, $\Box_s$ are generalized Pauli-Fierz operators on the fixed AdS_4 background metric $g_{\mu\nu}$, and $(s^2-2s-2)/L^2$ is a mass term that is necessary to maintain higher-spin gauge invariance on AdS_4.
- The quadratic part of I_{HS} yields the two-point functions of all free higher-spin currents normalized to $O(1)$.
- More precisely, since $\Phi^{(0)}$ is a conformally coupled scalar, in order to obtain the two-point function of $D(1,0)$ in the boundary one needs to quantize using the so-called *alternative quantization* AQ.

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- The cubic interaction terms in I_{HS} would then give rise to the three-point functions of the $O(N)$ model which scale as $1/\sqrt{N}$. Higher order interaction terms would give rise to higher-point correlation functions in the boundary.
- Upon introduction of interactions, the free $O(N)$ theory flows down to the IR critical point in which a dimension $\Delta = 2$ operator, namely the UIR $D(2,0)$, is present in the spectrum.
- There, higher-spin symmetry is broken since the HS currents acquire nonzero anomalous dimensions of order $1/N$. Nevertheless, higher-spin symmetry is restored at least at $N \to \infty$.
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$O(N)$/HS holography

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1. Motivation

2. The $O(N)$ vector model
   - A lightning review of the model
   - The $O(N) \to O(N-1)$ symmetry breaking
   - Anomalous dimensions

3. $O(N)$/HS holography
   - The HS/$O(N)$ conjecture
   - The gap equations from holography
   - The singleton deformation of higher-spin theory and boundary symmetry breaking
   - The calculation of boundary anomalous dimensions

4. Summary and outlook

Holography $\to W[J]$: J source for an operator $\mathcal{O}$ in the dual field theory.

- This on-shell action is in general supplemented by boundary terms that *a) renormalize* the theory, and *b) modify the boundary conditions* of the bulk fields
- If we know $W[J]$ we can Legendre transform it to get the quantum effective action $\Gamma[\langle\mathcal{O}\rangle]$ whose extrema determine the vacuum structure of the theory.
- A Lagrangian deformation of the *boundary field theory action* by a functional $f(\mathcal{O})$ of an operator $\mathcal{O}$, corresponds - at least at large N - to a simple deformation of the quantum effective action

$$\Gamma_f[\sigma] = \Gamma_0[\sigma] + f(\sigma)\,, \quad \sigma = \langle\mathcal{O}\rangle\,.$$

- Thus, given such a deformation, the gap equation will be obtained as

$$\frac{\delta\Gamma_f}{\delta\sigma}\bigg|_{\sigma=\sigma_*} = 0$$

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- The higher spin theory action on AdS_4 includes the bulk scalar field $\Phi^{(0)} \equiv \Phi$ of mass $m^2L^2 = -2$ with asymptotic behaviour

$$\Phi \sim \alpha z + \beta z^2$$

- In this particular case, we have a choice: standard quantization (SQ) assigns α as the source for a $\Delta = 2$ operator with vev β. Alternative quantization (AQ) instead interprets β as the source for a $\Delta = 1$ operator with vev α.
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- To mimic the field theory analysis, we propose extending the bulk theory to contain two fields with $m^2L^2 = -2$, namely

$$I_{extHS} = I_{HS} + \int d^4x\sqrt{-g}\frac{1}{2}\Sigma\left[\Box + \frac{2}{L^2}\right]\Sigma\,.$$

- We take Φ in AQ, and Σ in SQ. Asymptotically, we have

$$\begin{aligned}\Phi &\sim \alpha z + \beta z^2\\ \Sigma &\sim \eta z + \sigma z^2\end{aligned}$$

so that Φ gives rise to a $\Delta = 1$ operator with vev α, while Σ gives rise to a $\Delta = 2$ operator with vev σ.

- We assume that these fields do not mix in the bulk. This means that the regularity conditions of the bulk equations yield $\alpha = \alpha(\beta)$ and $\sigma = \sigma(\eta)$, and determine the boundary generating functional as

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- Also note that starting from the two-point functions of both the operators with $\Delta = 1$ and $\Delta = 2$ are normalized to $O(1)$. This means, for example, that in terms of the elementary fields $\alpha \sim (\phi^a \phi^a)/\sqrt{N}$.

If this were the full story, constructing $\Gamma[\alpha, \sigma]$ would give no sign of a gap equation for the $O(N)$ model, as Σ is decoupled from Φ (as well as the rest of the higher spin fields).

- To rectify that, we introduce boundary terms that couple the two fields together i.e. a Lagrangian deformation of the form

$$f(\alpha, \sigma) = \int \left(\alpha\sigma + V(\sigma) - \frac{1}{3}\lambda(\alpha - h)^3 \right), \quad V(\sigma) = -\frac{\lambda'}{g}\sigma.$$

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$$f(\alpha, \sigma) = \int \left(\alpha\sigma + V(\sigma) - \frac{1}{3}\lambda(\alpha - h)^3 \right), \quad V(\sigma) = -\frac{\lambda'}{g}\sigma.$$

with λ and λ' dimensionless and h is a parameter with dimensions of mass.

- The different relative signs in which arise because of the opposite quantizations used for the bulk fields $\Leftarrow$ the on-shell bulk action equals *minus* the boundary generating functional if one uses SQ.
- Also note that starting from the two-point functions of both the operators with $\Delta = 1$ and $\Delta = 2$ are normalized to $O(1)$. This means, for example, that in terms of the elementary fields $\alpha \sim (\phi^a \phi^a)/\sqrt{N}$.

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$$\Gamma[\alpha,\sigma] = \int \left(\frac{1}{2}\alpha K_1\alpha - \frac{1}{2}\sigma K_1^{-1}\sigma + \sigma(\alpha - \frac{\lambda'}{g}) - \frac{1}{3}\lambda(\alpha - h)^3\right)$$

where K_1 is an appropriate kernel.

- The different signs arising from the different quantizations ensure the positivity of the quadratic kernels.
- For constant α and σ, we obtain the gap equations

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$$\frac{\sqrt{N}}{g} = h \pm \sqrt{\frac{1}{\lambda}}\sqrt{\sigma}$$

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- On the other hand, the nontrivial IR fixed point arises when $g \to g_*$. In this case, the introduction of the operator α is equivalent to a finite shift of the operator $\sigma \Rightarrow$ the operator α becomes ***redundant***.
- The $(\alpha - h)^3$ term has an interpretation in terms of the classically marginal term $(\phi^a\phi^a)^3$.
- h introduces relevant terms in order that the non-trivial fixed point is properly described and appears at a finite value of g. This is equivalent to the well-known property that *any* relevant deformation of the UV free fixed point will lead to the nontrivial IR theory.

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4. Summary and outlook

- Next, we deform the higher-spin action by a singleton field S as

$$I_{dHS} = I_{extHS} + \int d^4x\sqrt{-g}\frac{1}{2}S\left[\Box + \frac{5}{4L^2}\right]S\,,$$

- The singleton is a scalar field with bulk mass $m^2L^2 = -\frac{5}{4}$ with asymptotic behaviour

$$S \sim \xi z^{1/2} + \phi z^{5/2}.$$

- For such a field, the *only unitary quantisation* possibility is to do AQ [Andrande and Marolf (11)] giving an operator of $\Delta = 1/2$. This is a free field that consequently decouples from the rest of the CFT.
- However, it can be forced to have a non-trivial effect by coupling it to the other fields through an explicit boundary interaction, namely $f(\phi,\alpha,\sigma) = \tilde{\lambda}\sigma\phi^2$.

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- That this interaction is needed could have been anticipated from our calculatations of the effective action of the $O(N)$ model $\Rightarrow$ a $\sigma\varphi^2$ term was crucial for the symmetry breaking structure of the theory.
- Explicitly, we add to the deformed action the following boundary term

$$f_d(\alpha,\sigma,\phi) = \int \left[\alpha\sigma - \tilde{V}(\sigma) - \lambda\frac{1}{3}(\alpha - h)^3 + \tilde{\lambda}\sigma\phi^2\right], \quad \tilde{V}(\sigma) = \frac{\tilde{\lambda}'}{g}\sigma\,,$$

where using the results of the previous section we have set $h = \frac{\sqrt{N}}{g_*}$ and $\lambda = \frac{16\pi^2}{N}$.
- Other than the presence of the marginal term, a crucial difference between the above and the previous gap equation is in the linear deformation $\tilde{V}(\sigma)$ where $\lambda' \rightarrow \tilde{\lambda}' = \frac{N+1}{\sqrt{N}}$, as it is required to to be able to absorb the singleton field ϕ by suitably adjusting the coupling $1/g$ in the massive phase of the theory.

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- The gap equations are then

$$\alpha + \tilde{\lambda}\phi^2 = \frac{N+1}{\sqrt{N}}\frac{1}{g}$$
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- The third equation is familiar from the σ-model: there are two phases, one in which $\phi = 0$ (massive phase) and the other in which $\sigma = 0$ (broken phase).
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$$1: \quad \phi = 0, \qquad \alpha = \frac{N+1}{\sqrt{N}}\frac{1}{g}, \qquad \sigma = 16\pi^2\left(\frac{N+1}{N}\frac{1}{g} - \frac{1}{g_*}\right)^2$$

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- $\alpha \neq 0$ does *not* signal $O(N)$ since it is properly interpreted as the vev of an $O(N)$-invariant operator. Rather $\phi \neq 0$ implies $O(N+1) \to O(N)$.
- As before, there is a critical point when $g/g_* = (N+1)/N$. We can have $O(N+1)$ breaking only when $g/g_* < (N+1)/N$. For $g/g_* > (N+1)/N$, the only solution to the gap equations is of the first type, namely the massive phase.

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- At the critical point the operator α becomes redundant and the boundary term becomes

$$f_d(\sigma,\phi^2)=\frac{1}{\sqrt{N}}\int\sigma\phi^2\,.$$

- This is a simple marginal deformation of the extended higher-spin action and leads to a $1/N$ expansion for the boundary two-point functions of ϕ and σ. For example, we obtain

$$\begin{aligned}\langle\phi(x_1)\phi(x_2)\rangle_{def} &= \langle\phi(x_1)\phi(x_2)\rangle_0\\ &+\frac{1}{2N}\int\langle\phi(x_1)\phi(x_2)\sigma(x)\phi^2(x)\sigma(y)\phi^2(y)\rangle_0+\cdots\end{aligned}$$

where we have dropped the $O(1/\sqrt{N})$ term whose contribution vanishes, as do all other fractional powers of $1/N$.
- The above gives *the same* expansion as in the field theory analysis, at least to leading order in $1/N$. Hence, the singleton deformation gives for the boundary singleton field ϕ *the same* anomalous dimension as those for the UV dimensions of the elementary fields ϕ^a

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- A complete holographic description of the $O(N)$ vector model should account for its rich vacuum structure and in particular for its $O(N) \to O(N-1)$ symmetry breaking pattern.
- We have shown that this is possible if one deforms the AdS_4 higher-spin theory by a singleton field coupled to higher-spin multiplet only through a boundary marginal coupling. Then, *designing* the appropriate boundary conditions for the extended bulk action we were able to exactly reproduce the gap equations of the $O(N)$ vector model.
- We have argued that the bulk higher-spin theory absorbs the singleton field by shifting its parameter $N \to N+1$. This is the bulk dual of the global symmetry breaking/enhancement mechanism in the boundary.
- The boundary singleton interaction generates the same $1/N$ graphical expansion for the elementary scalar and "spin-zero current" as in the standard field theoretic treatment of the $O(N)$ model. Hence, the singleton deformation breaks higher-spin symmetry and yields the well-known anomalous dimensions for the elementary and "spin-zero" scalars of the $O(N)$ model, at least to leading order in $1/N$.

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- Is it important to understand better the boundary marginal coupling of the singleton to higher-spin currents. For example, given the singleton field ϕ, one may consider boundary couplings of the form

$$S_{HS} \sim \lambda' \int t^{\mu_1 \cdots \mu_s} \phi \partial_{\mu_1} ... \partial_{\mu_s} \phi \,,$$

where $t^{\mu_1 \cdot\cdot \mu_s}$ is the *leading* coefficient in the asymptotic behaviour of a bulk spin-s gauge field $\rightarrow$ *higher-spin dressing of the $O(N)$ model*.

- For $s \geq 2$ there are more than one possible terms. Generally, this has no effect on the vacuum structure, if that is determined by space-time constant configurations.

- It is expected that such couplings would lead to a graphical expansion for the 2-pt functions of the boundary higher-spin currents which would enable one to calculate their $1/N$ anomalous dimensions. Reproducing the result would then be a crucial test for our proposal.

- Is it important to understand better the boundary marginal coupling of the singleton to higher-spin currents. For example, given the singleton field ϕ, one may consider boundary couplings of the form

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- Our results can also be applied to the holographic description of three-dimensional fermionic and supersymmetric models with higher-spin duals. Notice that such models describe parity symmetry breaking, and it would be interesting to understand the bulk counterpart of it.
- In AdS_5/CFT_4 correspondence adding a probe D3-brane to IIB sugra on $AdS_5 \times S^5$ shifts by one unit $N \to N+1$ the fiveform flux. The singleton deformation is the analog process of the above in higher-spin gauge theory and its study might lead to a better geometric description for the dimensionless parameter N.
- The singleton deformation could also play an important role in the study of possible black-hole solutions for higher-spin theory on AdS_4. For example, since a continuous symmetry cannot be broken at finite temperature in 2+1 dimensions, we expect that bosonic singleton absorption would not be possible for higher-spin theories in black-hole backgrounds, while fermionic singleton absorption would be allowed.

Summary and outlook

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