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Neutrino mass textures from String symmetries

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GREECE

based on:

I. Antoniadis & GKL, 1205.6930

and work in progress...

Outline of the Talk

- Neutrino Data
- Basic ingredients of F-theory model building
- Mechanisms for fermion mass hierarchy
- Spectral cover description
- Classification of related discrete symmetries
- Examples of $SU(5)$ constructions
- Applications to neutrino physics

▲ Neutrino data

$$\left.\begin{array}{ll} V_\ell : & V_\ell^\dagger m_\ell V_\ell = m_\ell^{diag.} \\ V_\nu : & V_\nu^\dagger m_\nu V_\nu = m_\nu^{diag.} \end{array}\right\} \Rightarrow V = V_\ell^\dagger V_\nu \tag{1}$$

▲ circa 2000: *Tri-Bi maximal mixing:*

$$\sin^2\theta_{12} = \frac{1}{3},\ \sin^2\theta_{23} = \frac{1}{2},\ \theta_{13} = 0$$

$$V_{TB} = V_l^\dagger V_\nu = \begin{pmatrix} -\sqrt{\frac{2}{3}} & \frac{1}{\sqrt{3}} & 0 \\ \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{pmatrix}$$

▲ → *Theory Invariant under* Finite Symmetries $\mathcal{S}_4,\ \mathcal{A}_4\ \cdots$

▲ ~2010 *data suggest:* *TB-mixing not exact!*

$$\theta_{23} \neq \frac{\pi}{4} \quad \theta_{13} \approx \frac{\pi}{20} \neq 0$$

▲ *Discrete Anatomy of the Neutrino Mass Textures:* ▲

→ Expressing m_ν in terms of Finite Group Elements:

$$m_\nu = \sum_i c_i U_i$$

→ unique solution compatible with experimental data:

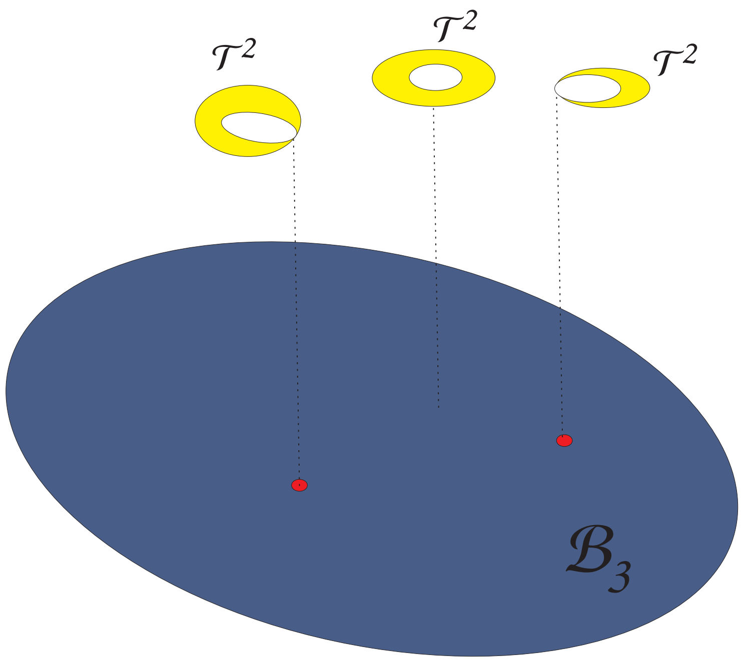
(... *N.D. Vlachos*)

$$V = \begin{pmatrix} \sqrt{\frac{2}{3} - s^2} & -\frac{1}{\sqrt{3}} & s \\ \sqrt{\frac{1}{6} - \frac{s^4}{2} + \frac{\sqrt{3}s}{2}} & \frac{1}{\sqrt{3}} & -\frac{\sqrt{1-\frac{3s^2}{2} - \frac{s}{\sqrt{2}}}}{\sqrt{2}} \\ \sqrt{\frac{1}{6} - \frac{s^4}{2} - \frac{\sqrt{3}s}{2}} & \frac{1}{\sqrt{3}} & \frac{\sqrt{1-\frac{3s^2}{2} + \frac{s}{\sqrt{2}}}}{\sqrt{2}} \end{pmatrix}$$

with $s = \sin\theta_{13}$.

★ **F-theory** (*C. Vafa hep-th/9602022*)

- Defined on a background $\mathcal{R}^{3,1} \times \mathcal{X}$
- $\mathcal{X}$ elliptically fibered **CY** 4-fold over B_3
- B_3 complex 3-fold base.



CY 4-fold: Points of B_3 represented by torus $\tau = C_0 + \imath/g_s$. Red points: 7-branes, $\perp$ to B_3

Fibration is described by the $\mathcal{W}$eierstraß $\mathcal{E}$quation ($\mathcal{WE}$)

$$y^2 = x^3 + f(z)\,x + g(z) \tag{2}$$

x, y parameters of the fibration

$f(z)$, $g(z) \rightarrow 8\,\&\,12$ degree polynomials in z.

For each point of B_3, eq(2) describes a torus labeled by z

The fiber degenerates at the zeros of the discriminant

$$\Delta = 4\,f^3 + 27\,g^2 \tag{3}$$

$$\Downarrow$$

$\Delta = 0 \;\Rightarrow\;$ singularity of internal manifold

Interpretation of geometric singularities

(*Witten, hep-th/9507121, Bershadsky et al, hep-th/9510225;*)

- **Singularities** of Internal Manifold $\rightleftarrows$ **gauge symmetries**

  ... encoded in the structure of $f(z)$, $g(z)$

- Types of **singularities** : $\mathcal{ADE}$ (*Kodaira classif.*)

  ... they determine:

  A) **gauge symmetries**

$$\rightarrow \begin{cases} SU(n) \\ SO(m) \\ \mathcal{E}_n \end{cases}$$

  B) **matter content**

ord($f(z)$)	ord$g(z)$)	ord($\Delta(z)$)	fiber type	Singularity
0	0	n	I_n	A_{n-1}
≥ 1	1	2	II	none
1	≥ 2	3	III	A_1
≥ 2	2	4	IV	A_2
2	≥ 3	$n+6$	I_n^*	D_{n+4}
≥ 2	3	$n+6$	I_n^*	D_{n+4}
≥ 3	4	8	IV^*	$\mathcal{E}_6$
3	≥ 5	9	III^*	$\mathcal{E}_7$
≥ 4	5	10	II^*	$\mathcal{E}_8$

Table 1: Kodaira's classification of Elliptic Singularities with respect to the vanishing order of f, g, Δ.

Useful algorithm for local description: Tate's form

Procedure: (*see Katz et al 1106:3854*) Expand f, g

$$f(z) = \sum_n f_n z^n, \; g(z) = \sum_m g_m z^m$$

Then

$$\Delta = 4\left[f_0 + f_1 z + \cdots)\right]^3 + 27\left[g_0 + g_1 z + \cdots\right]^2$$

Demand $z/\Delta \;\Rightarrow$

$$f_0 = -\frac{1}{3}\, t^2, \;\; g_0 = \frac{2}{27}\, t^3$$

while $\mathcal{WE}$ obtains Tate's $\mathbf{I_1}$ form:

$$y^2 = x^3 + t\, x^2 + (f_1 + f_2 z + \cdots) z\, x + (\tilde{g}_1 + \tilde{g}_2 z + \cdots) z$$

Tate's Form

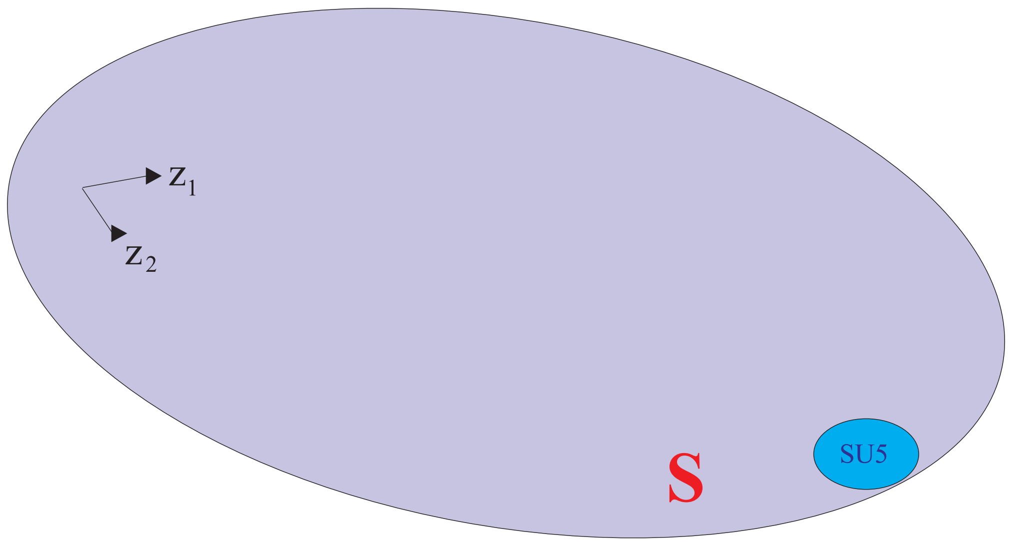
$$y^2 + a_1 x\, y + a_3 y = x^3 + a_2\, x^2 + a_4 x + a_6$$

The algorithm (*Partial results*)

Group	a_1	a_2	a_3	a_4	a_6	Δ
$SU(2n)$	0	1	n	n	$2n$	$2n$
$SU(2n+1)$	0	1	n	$n+1$	$2n+1$	$2n+1$
$SO(10)$	1	1	2	3	5	7
$\mathcal{E}_6$	1	2	3	3	5	8
$\mathcal{E}_7$	1	2	3	3	5	9
$\mathcal{E}_8$	1	2	3	4	5	10

F-theory: Model Building

GUTs associated to 7-branes wrapping certain class of *'internal'* **2-complex dim.** surface **S**



▲ The precise gauge group is determined by the singular fibers over the surface **S**.

▲ Elliptic Fibration: Highest singularity is E_8

▲ Gauge symmetry: any E_8 subgroup:

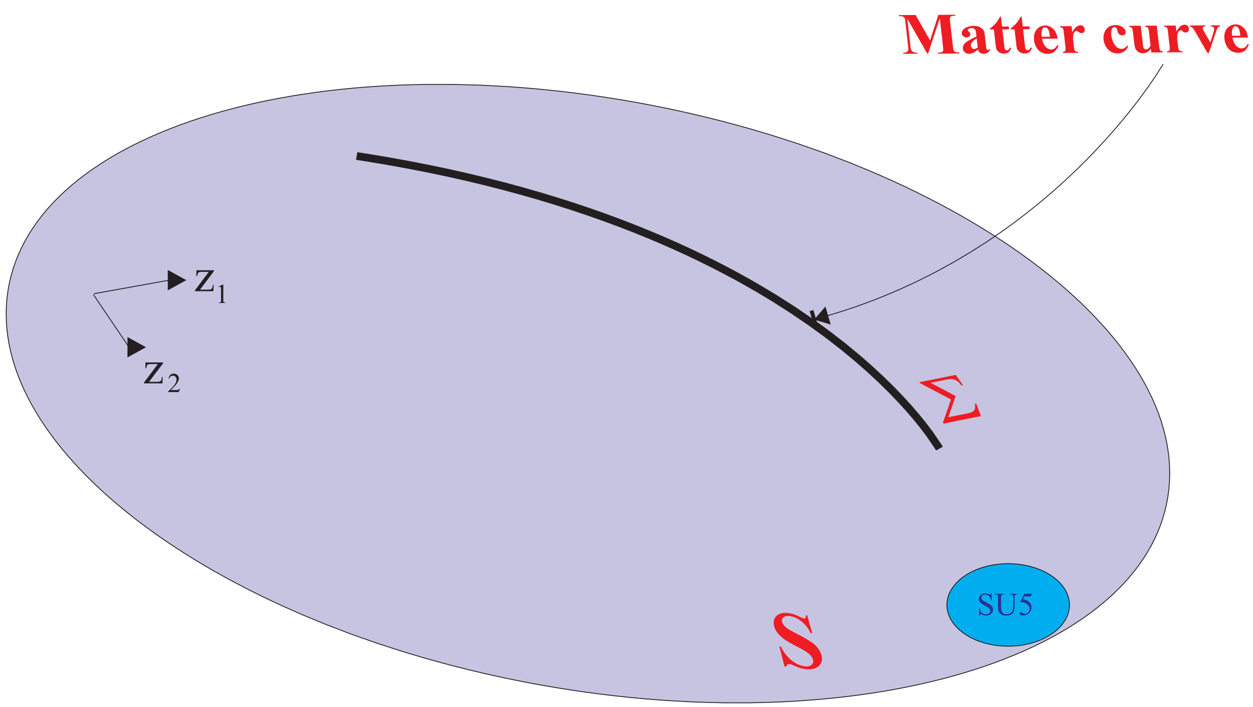
$$\mathcal{E}_8 \to \mathbf{G_{GUT}} \times \mathcal{C}_{\text{spectral cover}}$$

★ **Spectral Cover** $\Rightarrow$ useful local properties of G_{GUT}

▲ Sensible choice: $G_{GUT} = SU(5)$

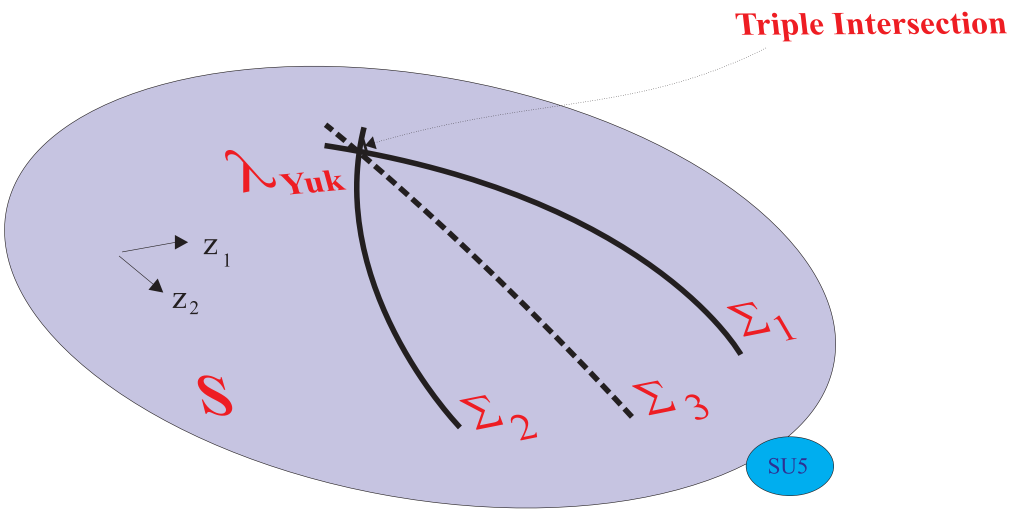
(*a single condition* $c_1(\mathcal{L}) \cdot c_1(\mathcal{L}) = -2$ *ensures absence of exotics*)

Matter is localised along intersections with other 7-branes...



Along a matter curve Σ gauge symmetry is enhanced...

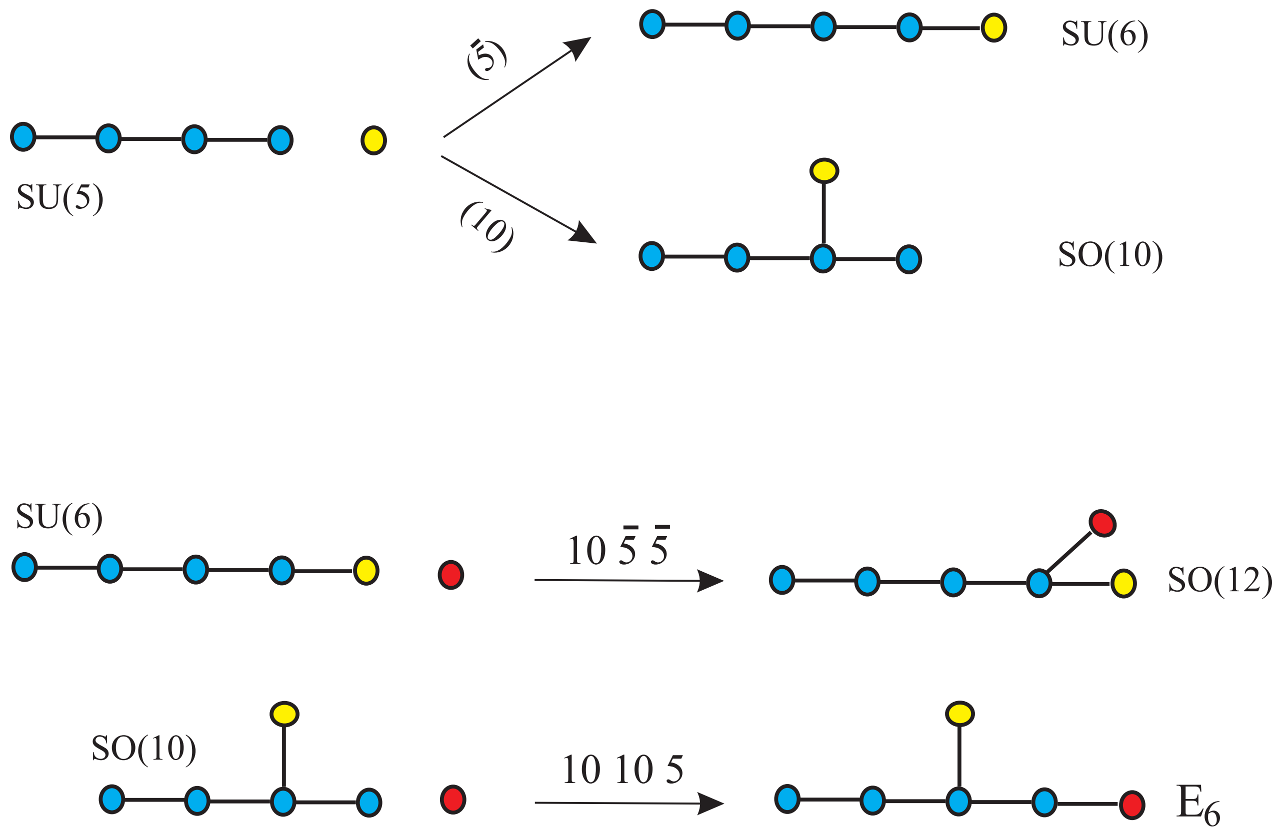
Yukawa couplings are formed at triple intersections...



where **gauge symmetry** is further enhanced:

$$\lambda_b \, 10 \cdot \bar{5} \cdot \bar{5} \ \in SO(12)$$
$$\lambda_t \, 10 \cdot 10 \cdot 5 \in E_6$$

Symmetry enhancements for $SU(5)$.



$G_S = SU(5)$: Singularity enhancement:

- Matter curves accommodating $\bar{\mathbf{5}}$ are associated with $SU(6)$

$$\Sigma_{\bar{5}} = S \cap S_{\bar{5}} \quad \Rightarrow \quad SU(5) \to SU(6)$$

$$\mathrm{ad}_{SU_6} = 35 \quad \to \quad 24_0 + 1_0 + 5_6 + \bar{5}_{-6}$$

- Matter curves accommodating $\mathbf{10}$ are associated with $SO(10)$

$$\Sigma_{10} = S \cap S_{10} \quad \Rightarrow \quad SU(5) \to SO(10)$$

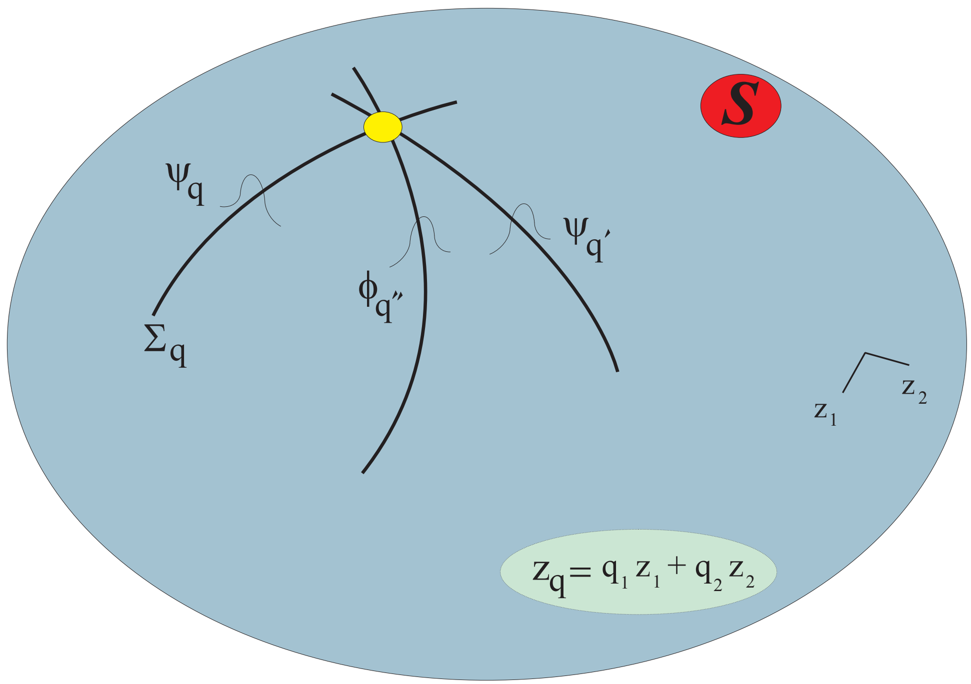
$$\mathrm{ad}_{SO_{10}} = 45 \quad \to \quad 24_0 + 1_0 + 10_4 + \overline{10}_{-4}$$

- Further enhancement in triple intersections→ **Yukawas**:

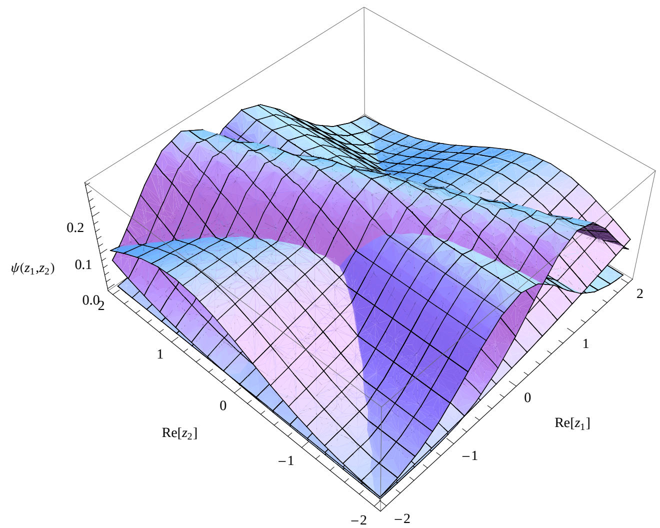
$$SO(10) \equiv E_5 \quad \Rrightarrow \quad E_6 \to \mathbf{10} \cdot \mathbf{10} \cdot \mathbf{5}$$

$$SU(6) \quad \Rrightarrow \quad SO(12) \to \mathbf{10} \cdot \bar{\mathbf{5}} \cdot \bar{\mathbf{5}}$$

Matter fields are represented by wavefunctions ψ_i, ϕ on the intersections of 7-branes with **S**.



Matter Wavefunctions fall off rapidly away from the curves



Yukawa coupling $\propto$ integral of overlapping wavefunctions at the intersection

$$\lambda_{ij} \sim \int_S \psi_U^j \psi_Q^i \psi_H$$

Integral's main dependence is on local details near the intersection

$\Rightarrow$ reliable λ_{ij}-estimation without knowing global geometry!

Mechanisms for Fermion mass hierarchy

▼ If all three families are on the same matter curve, masses to lighter families can be generated by:
i) non-commutative fluxes *Cecotti et al, 0910.0477*
ii) non-perturbative effects, *Aparicio et al, 1104.2609*

▼ If families are distributed on different matter curves:
Implementation of Froggatt-Nielsen mechanism,
Dudas and Palti, 0912.0853
GKL and G.G. Ross, 1009.6000

▲▲ Combined mechanism:

Only two families on the same matter curve

⋆ **Origin** and **Nature** of **Family Symmetries** ⋆

In F-theory all matter descends from the E_8-adjoint decomposition
We already assumed

$$E_8 \to SU(5)_{GUT} \times SU(5)_\perp$$

therefore

$$248 = (24,1) + (1,24_\perp) + (10,5_\perp) + (\bar{5},10_\perp) + (5,\overline{10}_\perp) + (\overline{10},\bar{5})_\perp$$

Interpretation from geometric point of view:

$SU(5)_{GUT}$ fields reside on matter curves:

$$\Sigma_{10_{t_i}} \quad : \quad n_{10} \times 10_{t_i} + \bar{n}_{\bar{10}} \times \overline{10}_{-t_i} \tag{4}$$

$$\Sigma_{5_{t_i+t_j}} \quad : \quad n_5 \times \bar{5}_{t_i+t_j} + \bar{n}_{\bar{5}} \times 5_{-t_i-t_j} \tag{5}$$

Families on different curves distinguished by roots $t_i, t_j \in SU(5)_\perp$

★ Monodromies reduce $SU(5)_\perp$ symmetry ★

Geometric equivalent description useful to local F-theory:

Spectral Cover Description:

★ *local patch around GUT singularity described by*

$$\mathcal{C}_5 = \prod_{i=1}^{5}(s - t_i) = b_0 s^5 + b_2 s^3 + b_3 s^2 + b_4 s + b_5 = 0, \qquad b_1 = 0$$

coefficients b_k ($\in \mathcal{F}$) carry information of geometry...
★ $SU(5)_\perp$ breaking corresponds to any of the possible spittings of the *Spectral Cover*:

$$\begin{array}{ccc} \mathcal{C}_5 & \to & \mathcal{C}_4 \times \mathcal{C}_1 \\ \mathcal{C}_5 & \to & \mathcal{C}_3 \times \mathcal{C}_2 \\ \cdots & & \cdots \end{array}$$

★ Examples ★

▲ Application: The $\mathcal{C}_4 \times \mathcal{C}_1$ case

▲ Motivation: The neutrino sector

▲ $\mathcal{C}_4 \times \mathcal{C}_1$ implies the splitting of the polynomial in two factors

$$\sum b_k s^{5-k} = (\underbrace{a_1 + a_2 s + a_3 s^2 + a_4 s^3 + a_5 s^4}_{\mathcal{C}_4})(\underbrace{a_6 + a_7 s}_{\mathcal{C}_1})$$

Topological properties of a_i are fixed in terms of those of b_k, by equating coefficients of same powers of s

$$b_0 = a_5 a_7, \ b_5 = a_1 a_6, \ etc...$$

Moreover:

▲ $\mathcal{C}_1$: associated to a $\mathcal{U}(1)$

▲ $\mathcal{C}_4$: reduction to (i) continuous $SU(4)$ subgroup, or (ii) to Galois group $\in S_4$ (*see also Marsano et al 0906.4672*)

Properties and Residual Spectral Cover Symmetry

- If $\mathcal{H} \in S_4$ the **Galois** group, final symmetry of the model is:

$$SU(5)_{GUT} \times \underbrace{\mathcal{H} \times \mathcal{U}(1)}_{family\ symmetry}$$

- The final subgroup $\mathcal{H} \in S_4$ is linked to specific topological properties of the polynomial coefficients a_i.

- a_i coefficients determine useful properties of the model, such as

$i)$ **Geometric** symmetries $\rightarrow$ $\mathcal{R}$-parity
$ii)$ **Flux** restrictions on the matter curves

- Fluxes determine useful properties on the matter curves including :

Multiplicities and **Chirality** of matter/Higgs representations

Determining the **Galois** group in $\mathcal{C}_4$-spectral cover

In order to find out which is the **Galois** group, we examine *partially symmetric* functions of roots t_i (*Lagrange method*)

1.) The Discriminant Δ

$$\Delta = \delta^2 \text{ where } \delta = \prod_{i<j}(t_i - t_j)$$

δ *is invariant under* S_4**-even** *permutations* $\Rightarrow \mathcal{A}_4$

Δ symmetric $\rightarrow$ can be expressed in terms of coefficients $a_i \in \mathcal{F}$

$$\Delta(t_i) \rightarrow \Delta(a_i)$$

If $\Delta = \delta^2$, such that $\delta(a_i) \in \mathcal{F}$, then

$$\mathcal{H} \subseteq \mathcal{A}_4 \text{ or } V_4 \ \ (= \textit{Klein group})$$

If $\Delta \neq \delta^2$, (i.e. $\delta(a_i) \notin \mathcal{F}$), then

$$\mathcal{H} \subseteq \mathcal{S}_4 \text{ or } \mathcal{D}_4$$

2.) *To study possible reductions of* S_4, A_4 *to their subgroups, another partially symmetric function should be examined:*

$$f(x) = (x - x_1)(x - x_2)(x - x_3)$$

$$x_1 = t_1 t_2 + t_3 t_4, \quad x_2 = t_1 t_3 + t_2 t_4, \quad x_3 = t_2 t_3 + t_1 t_4$$

$x_{1,2,3}$ are invariant under the three *Dihedral groups* $D_4 \in S_4$.

Combined results of Δ and $f(x)$:

	$\Delta \neq \delta^2$	$\Delta = \delta^2$
$f(x)$ irreducible	S_4	A_4
$f(x)$ reducible	D_4, Z_4	V_4

The induced restrictions on the coefficients a_i

1. Tracelessness condition $b_1 = 0$ demands

$$a_4 = a_0 a_6, \quad a_5 = -a_0 a_7$$

2. The requirement that the discriminant is a square $\Delta = \delta^2$ imposes the following relations among a_i:

$$a_1 = {e_1}^2, \; {a_2}^2 = \mu a_1 \, a_3, \;\; a_3^2 \rightarrow \lambda a_0 a_1 a_7$$

3. Reducibility of the function $f(x)$ is achieved if

$$f(0) = 4 a_5 a_3 a_1 - a_1 {a_4}^2 - a_5 {a_2}^2 = 0$$

▲ 1^{st}Example : $\mathcal{A}_4$

Gauge Symmetry: $\mathbf{SU(5)_{GUT}} \times \mathbf{A_4} \times \mathbf{U(1)}$

$F = (10, 3)$	t_a	$F = (Q, u^c, e^c)$
$F_x = (10, 1)$	t_s	$F_x = (Q, u^c, e^c)$
$\bar{F}_y = (\overline{10}, 1)_{-t_5}$	$-t_5$	$\bar{F} = (\bar{Q}, \bar{u}^c, \bar{e}^c)$
$H = \bar{(}5, 3)$	$t_s \pm t_a$	h_u
$\bar{f} = (\bar{5}, 3)_{+t_5}$	$\frac{1}{4}(t_s - t_a) + t_5$	$\bar{f}_i = (\ell, d^c)_i$
$\bar{H} = (\bar{5}, 1)_{+t_5}$	$\frac{1}{4}(t_s + 3t_a) + t_5$	$\bar{h}_d$
$\theta_a = (1, 3)$	0	θ_{ij}
$\theta_b = (1, 3)$	t_a	θ_{i4}
$\theta_c = (1, 3)_{-t_5}$	$\frac{1}{4}(t_s - t_a) - t_5$	θ_{i5}
$\theta' = (1, 1')_{-t_5}$		θ_{45}
$\theta'' = (1, 1'')_{+t_5}$		θ_{54}

Yukawa terms

1^{st} choice: $3 \times (Q, u^c, e^c) \in F = (10, 3) \rightarrow$ tree-level coupling:

$$\mathcal{W}_u \supset (10, 3)_{t_i}\,(10, 3)_{t_i}\,(5, 3)_{-2t_i}$$

$\rightarrow$ **Wrong** quark mass relations!

2^{nd} choice: $3 \times (Q, u^c, e^c) \in F_x = (10, 1) \rightarrow$ fourth-order coupling:

$$\frac{1}{\Lambda}\,(10, 1)\,(10, 1)\,(5, 3)\,(1, 3) \leftrightarrow \lambda_t\ F_x F_x H \theta_b$$

In this case, lighter generations receive masses from non-commutative fluxes and/or non-perturbative effects

Neutrinos

$$\mathcal{W}_\nu \supset \frac{1}{\Lambda^3}(\bar{5}, 3)_{t_i+t_5}\,(\bar{5}, 3)_{t_i+t_5}\,(5, 3)_{-2t_i}\,(5, 3)_{-2t_i}\theta_{i5}\theta_{i5}$$

$\mathcal{F}-\mathcal{A}_4$ has a rich neutrino sector

Example

Take the vevs: $\langle\theta_{(1,3)}\rangle \sim a_i$, $\langle\, h_{(5,3)}\rangle \sim v_i$

$$\{a_1 \to 1, a_2 \to 0, a_3 \to 0, v_1 \to 0, v_3 \to v_2\}$$

$$m_\nu \propto \begin{pmatrix} 2 & 1\,c & 1\,c \\ 1\,c & 13 & -4\,c \\ 1\,c & -4\,c & 13 \end{pmatrix}$$

with c accounting for corrections (charged leptons, etc).
For $c = 2$ we get the right mixing, and the mass ratio $\Delta m^2_{23}/\Delta m^2_{13} \sim 10$ close to the expected value.

▲ $2^{nd} Example : SU(5)_{GUT} \times Z_2 \times Z_2 \times U(1)$

Spectral cover equation and field content:

$$\mathcal{C}_5(s) = \left(a_3 s^2 + a_2 s + a_1\right)\left(a_6 s^2 + a_5 s + a_4\right)(a_7 + a_8 s)$$

$SU(5)$	$U(1)_Y$-flux	$U(1)_X$	SM spectrum
$10^{(1)}_{t_{1,2}}$	0	2	$2 \times (Q, u^c, e^c)$
$10^{(2)}_{t_3}$	1	1	$(1 \times Q, -, 2 \times e^c)$
$10^{(3)}_{t_5}$	-1	0	$(-, 1 \times u^c, 1 \times \bar{e}^c)$
$5^{(0)}_{-t_1-t_2}$	0	1	$1 \times (d, h_u)$
$5^{(1)}_{-t_{1,2}-t_3}$	0	-1	$1 \times (d^c, \ell)$
$5^{(2)}_{-t_{1,2}-t_5}$	0	-1	$1 \times (d^c, \ell)$
$5^{(3)}_{-t_{3,4}-t_5}$	-1	0	$1 \times (h_d, -)$
$5^{(4)}_{-t_3-t_4}$	1	-2	$(2 \times d^c, 1 \times \ell)$

The Neutrino Sector

Left handed neutrinos are in the following fiveplets

$$\nu_1 \in \bar{5}_{t_3+t_4},\ \nu_2 \in \bar{5}_{t_1+t_5},\ \nu_3 \in \bar{5}_{t_1+t_3}$$

Their Right Handed partners can be sought among KK-modes of the singlet fields θ_{ij}^{KK} (*Antoniadis et al hep-th/0210263*)
In $F - SU(5)$ however,

$$\theta_{ij}^{KK} \to \nu^c;\ \theta_{ji}^{KK} \to \bar{\nu}^c, \Rightarrow \nu^c \neq \bar{\nu}^c$$

Remarkably, due to the monodromies (*Vafa et al 0904.1419*)

$$\theta_{12}^{KK} \equiv \theta_{21}^{KK} \to \nu_a^c = \bar{\nu}_a^c,\ \theta_{34}^{KK} \equiv \theta_{43}^{KK} \to \bar{\nu}_b^c = \nu_b^c$$

A convenient arrangement on matter curves:

$$\nu_1^c = \nu_a^c,\ \nu_2^c = \nu_b^c,\ \nu_3^c = \nu_b^c$$

★ The effective neutrino mass matrix

$$m_\nu^{eff} = m_{\nu_D} M_R^{-1} m_{\nu_D}^T$$

Assumptions:

▲ Kaluza-Klein scale ∼ GUT scale, $M_{KK} \sim M_X$

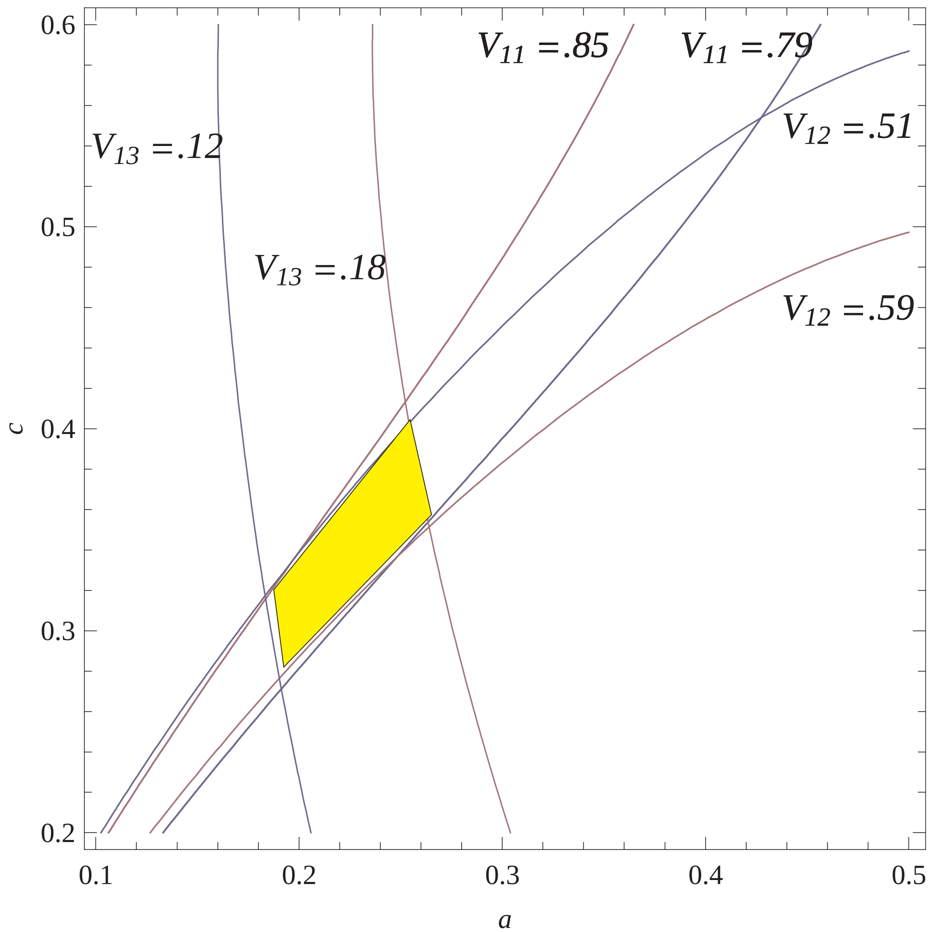
▲ Singlet vevs $\frac{\langle \theta_{ij} \rangle}{M_X} \to a, b, c$, such that $r = \frac{b}{a} < a, c < 1$

$$m_\nu^{eff} \approx \begin{pmatrix} 2a^2 & a(c+r) & a \\ a(c+r) & c^2 + r^2 & r \\ a & r & 1 \end{pmatrix} \frac{m_0^2}{M_X}$$

- To leading order, Mixing effects are linked to singlet vevs a, b, c

▲ Consistency check: mixing $(V_\nu)_{ij}$ should be derived for $a, b, c < 1$

(a, c) restricted region, from all V_{ij} elements



A few remarks

★ Current **F-Theory** Models provide a dictionary between:

Manifold Singularity $\leftrightarrows$ **Gauge Symmetry** G_{GUT}

$$\mathcal{E}_8 \to \mathbf{G_{GUT}} \times \mathcal{C}_{\text{spectral cover}}$$

Spectral Cover provides **Additional Structure beyond GUTs**

$$\Downarrow$$

★ $\mathcal{C}_{\text{spectral cover}} \to$ *Finite Groups such as* $A_4, D_4, V_4 = Z_2 \times Z_2$

$$\Downarrow$$

★ A natural way to interpret the peculiar Neutrino properties

This way, the neutrino physics is linked directly to the topological properties of the internal manifold

Additional Material...

Matter Parity from Geometry?

topological properties are encoded in b_k coefficients

Consider the phase transformation (*Hayashi et al, 0910:2762*)

$$s \to s\, e^{i\phi}, \; b_k \to b_k\, e^{i(\xi-(6-k)\phi)}$$

Let's apply to $SU(5)$ case:
...spectral cover equation picks up an overall phase

$$\mathcal{C}_5 : \sum_k b_k s^{5-k} \to e^{i(\xi-\phi)} \sum_k b_k s^{5-k}$$

★ Z_2-parity: $\phi = \pi$:

$$s \to -s, \; b_k \to (-1)^k e^{i\xi} b_k$$

★ Communicating Matter Parity to Matter Curves ★

Example: *Consider relations in $\mathcal{Z}_2$ monodromy:*

$$b_k = \sum a_l a_m a_n, \qquad l + m + n = N - k,\ N = 17 \tag{6}$$

Choose a_n to transform as

$$a_n \to a_n\, e^{i(\zeta - n\,\phi)}$$

$$\to\ b_k \propto a_l a_m a_n \to a_l a_m a_n\, e^{3\zeta-(N-k)\phi} \tag{7}$$

(6) & (7) consistent for $\xi = \phi = \pi, \zeta = 0$,

$$\boxed{a_n \to (-1)^n\, a_n}$$

★ Implications on matter curves:

$$5^{(0)} \sim a_6 a_7 \to (-1)^{(6+7)} = (-)$$

... associate this to matter parity!

$$\begin{pmatrix} 2a^4 & a^2(b+ac) & a^3 \\ a^2(b+ac) & \frac{-2bcM_X a^3 + c^2 M a^2 + b^2 M}{M - a^2 M_X} & \frac{abM - a^4 c M_X}{M - a^2 M_X} \\ a^3 & \frac{abM - a^4 c M_X}{M - a^2 M_X} & \frac{a^2 M}{M - a^2 M_X} \end{pmatrix} m_{\nu_0}$$

roots $\sum_i s_i = 0$ identified with $SU(5)_\perp$ Cartan subalgebra:

$$Q_t = \mathrm{diag}\{t_1, t_2, t_3, t_4, t_5\}$$

★ *Matter curves* characterised by t_i's

Polynomial coefficients depend on t_i

$$b_k = b_k(t_i)$$

Inversion implies **branchcuts**! ⇒ ..Simplest monodromy Z_2 : :

$$a_1 + a_2 s + a_3 s^2 = 0 \to s_{1,2} = \frac{-a_2 \pm \sqrt{w}}{2a_3}$$

Under $\theta \to \theta + 2\pi \to \sqrt{w} \to -\sqrt{w}$ branes interchange locations

$$s_1 \leftrightarrow s_2 \text{ or } t_1 \leftrightarrow t_2$$

2 $\mathbf{U(1)}$'s related by monodromies ... gauge symmetry reduces to:

$$SU(5) \times U(1)^4 \to \mathbf{SU(5)} \times \mathbf{U(1)^3}$$

Weierstrass' equation for the $SU(5)$ singularity

$$y^2 = x^3 + b_0 z^5 + b_2 x z^3 + b_3 y z^2 + b_4 x^2 z + b_5 x y$$

$\rightarrow$ spectral cover obtained by defining homogeneous coordinates

$$z \rightarrow U,\ x \rightarrow V^2,\ y \rightarrow V^3,\ s = U/V$$

so Weierstrass becomes

$$0 = b_0 s^5 + b_2 s^3 + b_3 s^2 + b_4 s + b_5$$