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Non-singular String Cosmology

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Based on:

C. Kounnas, H. Partouche, N. Toumbas [arXiv:1111.5816](#) Class.Quant.Grav.

C. Kounnas, H. Partouche, N. Toumbas [arXiv:1106.0946](#) Nucl.Phys.

I. Florakis, C. Kounnas, H. Partouche, N. Toumbas [arXiv:1008.5129](#) Nucl.Phys.

# 1. Introduction

In stringy gravity and cosmology new interesting phenomena occur.

Conventional notions from general relativity like :

Geometry and Topology

are well defined *only as low energy and/or small curvature approximations* of the stringy setup.

- At very small distances and at strong curvature scales, purely stringy phenomena imply that the physics can be quite different from what one might expect from the “naive” field theory approximation.
- New possibilities in the context of quantum cosmology and especially in the context of the “Stringy Big-Bang” picture versus “the initial singularity of the Big-Bang picture in General Relativity” .

Assuming for instance a compact space and sufficiently close to the singularity, the typical scale of the universe reaches at these early times the gravitational scale  $M_{string}$ .

At this early epoch classical gravity is no longer valid and has to be replaced by a more fundamental singularity-free theory such as (super-)string theory.

- The main obstruction in the stringy cosmological framework is the Hagedorn temperature limitation  $T < T_H$ .

It is well known that for high temperatures,  $T > T_H$ ,  
the string partition function diverges

*A thermal winding state becomes tachyonic.*

However, this is not a pathology in string theory.

*It is a signal of a phase transition towards to a new vacuum.*

There are many proposals about the “*High Temperature Phase of the Universe*”.

The Hagedorn-like singularities have to be resolved :

- by a stringy phase transition

OR

- by choosing Hagedorn-free string vacua in the early stage of the universe

- A noticeable progress has been made in constructing Hagedorn-free string vacua by turning on special gravito-magnetic fluxes injecting into the thermal vacuum non-trivial winding and momentum charges.

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The fundamental property of the new thermal vacuum is the restoration of thermal T-duality symmetry, implying a maximal critical temperature  $T_c$ ,

$$T = T_c e^{-|\sigma|} , \quad R = R_c e^{\sigma} , \quad Z(\sigma = -\sigma).$$

The thermal modulus  $\sigma$  parametrizes the radius of the Euclidean time circle,  $R$ .

- The *non-singular string cosmological solutions* are based on a mechanism which *resolves the Hagedorn instabilities* of strings at finite  $T$ . This mechanism was shown to be generic in a large class of initially  $N_4 = (4, 0)$  superstring models.

- The special “*gravito-magnetic fluxes*” not only *restore the thermal T-duality* but also *avoid the thermal tachyonic instabilities*, thanks to the *asymptotically supersymmetric structure* appearing in the right-moving sector.

- In all such thermal stringy systems there exist three characteristic regimes, each with a distinct effective field theory description:

- (i) An asymptotically cold regime associated with light thermal momenta:

$$\{\mathcal{M}(\sigma > 0)\}$$

- (ii) A dual asymptotically cold regime associated with light thermal windings:

$$\{\mathcal{W}(\sigma < 0)\}$$

- (iii) An intermediate “Brane” regime with additional massless thermal states leading to an enhanced Euclidean gauge symmetry:  $U(1) \rightarrow SU(2)$

$$\{\mathcal{B}(\sigma = 0)\}$$

- The extra massless states, localized at  $T = T_c$ , source Euclidean branes. They have non-vanishing momentum and winding charges,  $m = -n = \pm 1$  gluing together the “momentum” and “winding” regimes via a “Brane regime” at  $T = T_c$ .

- Thanks to the *asymptotically right-moving supersymmetric structure*, in both the momentum  $\{\mathcal{M}(\sigma > 0)\}$  and winding  $\{\mathcal{W}(\sigma < 0)\}$  regimes, the energy density and pressure are well-approximated by the energy density and pressure of massless thermal radiation up to the critical temperature  $T_c$ :

$$\rho = (d - 1)P, \quad P = n^* \Sigma_d T^d, \quad T = T_c e^{-|\sigma|}.$$

$\Sigma_d$  is the Stefan-Boltzmann constant and  $n^*$  is the number of massless states (at  $T = 0$ ), modulo a spin-statistics factor for fermions:

$$\Sigma_d = \frac{\Gamma(d/2)}{\pi^{d/2}} \zeta(d), \quad n^* = n^B + n^F \left( \frac{2^{d-1} - 1}{2^{d-1}} \right).$$

- The conical singularity in  $|\sigma|$  is resolved by *spacelike branes* providing localized (in time with  $\sigma(t) = 0$ ) negative pressure contributions which turn out to be crucial in order to evade the constraints on realizing singularity-free bouncing cosmologies.

One of the non-singular string cosmologies can be seen as gluing together the three effective field theory regimes at a given time  $\tau = \tau_c$  :

- i) *Winding regime*  $\equiv \{\mathcal{W}(\sigma < 0; \tau < \tau_c)\}$ ,
- ii) *Brane regime*  $\equiv \{\mathcal{B}(\sigma = 0; \tau = \tau_c)\}$ ,
- iii) *Momentum regime*  $\equiv \{\mathcal{M}(\sigma > 0; \tau > \tau_c)\}$

In terms of the above, the non-singular *perturbative* string cosmology is obtained by gluing the above three mentioned regimes at  $\tau_c$ :

$$\{\mathcal{C}_{\text{String}}(\tau)\} \equiv \{\mathcal{W}(\tau < \tau_c)\} \oplus \{\mathcal{B}(\tau = \tau_c)\} \oplus \{\mathcal{M}(\tau > \tau_c)\}.$$

- The existence of a stringy mechanism gluing distinct effective field theories was conjectured in the past by several authors and in several related contexts; for instance the gluing of  $N = 2$  Calabi-Yau theories with  $N = 2$  Landau-Ginsburg theories (Witten 1993), or even the *gluing of theories related by  $S$  and/or  $T$  dualities*. In the majority of examples, the precise gluing mechanism is difficult to establish, since *the intermediate region is non-perturbative*.

- Another interesting possibility of  $d$ -dimensional string cosmologies is obtained by gluing at  $\tau_c$  a Brane regime  $\{\mathcal{B}(\tau \leq \tau_c)\}$  with the momentum  $\{\mathcal{M}(\tau > \tau_c)\}$  regime:

$$\{\mathcal{C}_{String}(\tau)\} \equiv \{\mathcal{B}(\tau \leq \tau_c)\} \oplus \{\mathcal{M}(\tau > \tau_c)\}.$$

Initially the Universe is described by the Brane regime with constant  $\sigma$ -model temperature and scale factor,  $T = T_c$  and  $a = a_c$ .

The string coupling grows from super-weak values in the very early past, reaching a maximal value  $g_{str}^*$  at  $\tau_c$ .

For  $(\tau > \tau_c)$ , the Universe exits into the radiation dominated momentum regime.

The microscopic origin of the effective action in the Brane regime follows from the underlying stringy description of the thermal system at the extended symmetry point  $\sigma = 0$ , incorporating various fluxes in the effective gauged supergravity theory.

The entire cosmological evolution can be treated perturbatively provided that the critical value of the string coupling  $g_{str}^*$  is sufficiently small.



- Other interesting  $d$ -dimensional string cosmologies are obtained by gluing:

$$\{\mathcal{C}_{String}(\tau)\} \equiv \{\mathcal{W}(\tau < \tau_c^-)\} \oplus \{\mathcal{B}(\tau_c^- \leq \tau \leq \tau_c^+)\} \oplus \{\mathcal{M}(\tau > \tau_c^+)\}.$$

- (i) Contracting winding phase for  $\tau < \tau_c^-$  with **growing dilaton**
- (ii) Brane phase with constant temperature and **bouncing dilaton** for  $\tau_c^- \leq \tau \leq \tau_c^+$
- (iii) Expanding momentum phase for  $\tau > \tau_c^+$  with a **decreasing dilaton**

The cosmological evolution is **perturbative** provided the string coupling at the end points of the brane, is sufficiently small.

$$g_{\text{str}}(\tau_c^-), g_{\text{str}}(\tau_c^+) \ll 1.$$

## 2. The three effective field theory actions up to genus-1

During the cosmological evolution, the universe may enter into these three regimes, since the thermal modulus  $\sigma$  acquires non-trivial time-dependence,  $\sigma(\tau)$ .

The transition from the Winding to the Momentum regime (and vice versa), necessarily crosses the Brane regime, where, in principle, the Universe may stay for a certain amount of time.

In order to study the stringy cosmological evolution, it is necessary to derive the three effective actions and specify the gluing mechanism of the Winding with the Brane regime as well as the Brane with the Momentum regime.

The Momentum and Winding regimes are described by an effective  $d$ -dimensional dilaton-gravity string frame action up to genus-1 :

$$\mathbf{S}_M = \int d^d x \sqrt{|g|} \Theta(\sigma) \left[ e^{-2\phi} \left( \frac{1}{2} \mathcal{R} + 2(\nabla\phi)^2 + \frac{1}{12} H^2 + \dots \right) + P(|\sigma|) \right]$$

$$\mathbf{S}_W = \int d^d x \sqrt{|g|} \Theta(-\sigma) \left[ e^{-2\phi} \left( \frac{1}{2} \mathcal{R} + 2(\nabla\phi)^2 + \frac{1}{12} H^2 + \dots \right) + P(|\sigma|) \right]$$

The term proportional to the pressure  $P$  is the genus-1 contribution.

The ... denote the contributions of other moduli, fermions and gauge-bosons.

The two actions, modulo the  $\Theta$ -constraints, look identical. This is a consequence of thermal duality which implies that both the Momentum and Winding regimes can be *simultaneously described by a unique expression in terms of the duality invariant temperature*  $T = T_c e^{-|\sigma|}$ .

*The Brane regime at  $\sigma = 0$  is purely stringy with extra massless thermal states carrying non-trivial winding and momentum charges.*

The existence of such states is crucial for realizing the gluing mechanism between the Winding-Brane-Momentum regimes.

The brane regime is well described in terms of an exact 2d-conformal field theory based on an  $[SU(2)_L]_{k=2}$  associated with the fermionic extended symmetry point.

The genus-0 contributions admit a brane interpretation with a well-defined brane tension in terms of non-trivial backgrounds of the extra massless thermal scalars with non-trivial gradients along the directions transverse to the (Euclidean) time:

$$h_{\hat{\mu}\hat{\nu}} \equiv G_{IJ} \nabla_{\hat{\mu}} \varphi^I \nabla_{\hat{\nu}} \varphi^J = \frac{\kappa}{d-1} g_{\hat{\mu}\hat{\nu}} \neq 0 \quad \hat{\mu}, \hat{\nu} \neq 0, \quad I, J = \text{Extra States}.$$

.

The action at  $\tau_i$  with  $\sigma(\tau_i) = 0$  takes the familiar form of the Nambu-Goto action for branes.

$$\mathbf{S}_B(\tau_i) = - \sum_i \int d\tau d^{d-1}x \kappa \delta(\tau - \tau_i) \sqrt{g_\perp} e^{-2\phi}$$

Another interesting possibility is to consider a continuous brane distribution in an interval of time:  $\tau_- \leq \tau \leq \tau_+$  with  $\sigma(\tau) = 0$

$$\mathbf{S}_B = - \int d\tau' |g_{00}| d\tau \delta(\tau - \tau') \Theta(\tau' - \tau_-) \Theta(\tau_+ - \tau') d^{d-1}x \sqrt{g_\perp} V(\phi) + \mathbf{S}_{\tau_-} + \mathbf{S}_{\tau_+}$$

$$\longrightarrow \mathbf{S}_B = - \int d^d x \Theta(\tau - \tau_-) \Theta(\tau_+ - \tau) \sqrt{|g|} V(\phi) + \mathbf{S}_{\tau_-} + \mathbf{S}_{\tau_+}$$

$\mathbf{S}_{\tau_\pm}$  are localized terms at the end points of the brane:

$$\mathbf{S}_{\tau_\pm} = - \int d\tau d^{d-1}x \kappa_\pm \delta(\tau - \tau_\pm) \sqrt{g_\perp} V_\pm(\phi)$$

Up to now, the brane effective potential is:  $V(\phi) = \kappa e^{-2\phi}$

The most general brane effective potential  $V(\phi)$  is obtained by considering non-trivial gauge fluxes with  $\langle F_{\mu\nu}^a \rangle \neq 0$ .

At the extended symmetry point,  $\sigma = 0$ , the  $U(1)$  Euclidean time-manifold is extended to an  $SU(2)_{k=2}$  manifold. At this point the target space is naturally described by a  $n = d + 2$ -dimensional space, rather than the “naive” field theory  $d$ -dimensional one. It is based on an effective action at the time-interval  $D \equiv (\tau_-, \tau_+)$  :

$$S_D = - \int d^n x \sqrt{g} \{ \mathcal{A}(\phi ; \varphi^I) (\partial\phi^I)^2 + \mathcal{B}_L(\phi ; \varphi^I) (F_{\mu\nu})_L^2 + \mathcal{C}_R(\phi ; \varphi^I) (F_{\mu\nu})_R^2 + \dots \}$$

- $\varphi^I$  are the massless scalars,
- $(F_{\mu\nu})_L$  is the field strength for the graviphotons,
- $(F_{\mu\nu})_R$  is the field strength for the matter gauge bosons
- $\dots$  denote the contributions of the fermionic fields and other fields of the effective *gauged supergravity theory* in  $n$ -dimensions.

Thus, in the presence of non-trivial gauge fluxes and scalar field gradients an effective potential for the dilaton field is generated :

$$V(\phi) = Ae^{-2\phi} + B(\mathcal{U}) + \tilde{C}e^{-2\phi}$$

This form persists when the theory is reduced to  $d$ -dimensions since the two extra compact dimensions of the  $SU(2)_{k=2}/U(1)$  manifold have finite constant volume.

The dilaton dependence of each individual term is dictated by the structure of the  $N_4 = 4$  supergravity theory, (gauged or not), in four spacetime dimensions.

Finally, the Brane-dilaton effective potential in the interval  $(\tau_-, \tau_+)$  becomes:

$$V(\phi) = B(\mathcal{U}) + C e^{-2\phi} - P_c$$

- $C = A + \tilde{C}$  is a constant
- $\phi$  is the  $d$ -dimensional dilaton
- $\mathcal{U}$  is the volume of the internal  $10 - d$  compact space
- $P_c$  is the is the genus-1 contribution defining the pressure at  $T_c$ ,  $\sigma = 0$

### 3. The String cosmological solutions

The string effective action and the resulting cosmological solutions are defined for all times and are obtained once we combine consistently the three regimes.

$$\mathbf{S} = \mathbf{S}_W + \mathbf{S}_B + \mathbf{S}_M ,$$

The thermal modulus  $\sigma$  is then unrestricted in  $\mathbf{S}$ , even though it is restricted in the individual effective actions  $S_W$ ,  $S_M$  and  $S_B$ .

We are interested in homogenous and isotropic backgrounds

$$ds^2 = -N(\tau)^2 d\tau^2 + a(\tau)^2 d\Omega_k^2$$

$\Omega_k$  is a  $(d - 1)$ -dimensional Einstein space with curvature  $k \leq 0$ .

The equations of motion for the lapse function  $N$ , the scale factor  $a$  and the dilaton  $\phi$  are: ( $H \equiv \dot{a}/a$ ):



$$(N) : \quad \frac{1}{2}(d-1)(d-2) \left( H^2 + k \frac{N^2}{a^2} \right) = 2(d-1)H\dot{\phi} - 2\dot{\phi}^2 + e^{2\phi} N^2(\rho + V),$$

$$(a) : \quad (d-2) \left( \frac{\ddot{a}}{a} - H \frac{\dot{N}}{N} \right) + \frac{1}{2}(d-2)(d-3) \left( H^2 + k \frac{N^2}{a^2} \right) \\ = 2\ddot{\phi} + 2(d-2)H\dot{\phi} - 2\dot{\phi}^2 - 2\frac{\dot{N}}{N}\dot{\phi} - e^{2\phi} N^2(P - V)$$

$\phi$ -equation

$$\ddot{\phi} + (d-1)H\dot{\phi} - \dot{\phi}^2 - \frac{\dot{N}}{N}\dot{\phi} - \frac{d-1}{2} \left( \frac{\ddot{a}}{a} - H \frac{\dot{N}}{N} \right) - \frac{1}{4}(d-1)(d-2) \left( H^2 + k \frac{N^2}{a^2} \right) = \frac{1}{4} e^{2\phi} N^2 \frac{dV}{d\phi}$$

Using the above equations and that  $\rho = -P - \frac{\partial}{\partial|\sigma|}P \rightarrow$  the entropy conservation:

$$(\dot{\rho} + \dot{P}) + ((d-1)H + |\dot{\sigma}|)(\rho + P) = 0$$

*$\phi$ -equation modulo trace equation*

$$2\ddot{\phi} - 4\dot{\phi}^2 + 2(d-1)H\dot{\phi} - 2\dot{\phi}\frac{\dot{N}}{N} = e^{2\phi}N^2[(d-1)P - \rho] - \sum_i \kappa_i N \delta(\tau - \tau_i)$$

→ the first time-derivative of the dilaton is discontinuous across  $\tau_i$

This discontinuity is resolved by the localized terms at the end points of the brane.

The gluing mechanism  $\{W \oplus B\}$  and  $\{B \oplus M\}$  is automatically valid !

3.1 Bouncing cosmology with vanishing curvature,  $k = 0$  and  $\tau_{\pm}=0$   
 In the  $\sigma$ -model frame and in the conformal gauge

$$\ln \frac{N}{a_c} = \ln \frac{a}{a_c} = \ln \frac{T_c}{T} = |\sigma|,$$

$$\sigma = \frac{\text{sign}(\sigma)}{d-2} \left[ \eta_+ \ln \left( 1 + \frac{\omega a_c |\tau|}{\eta_+} \right) - \eta_- \ln \left( 1 + \frac{\omega a_c |\tau|}{\eta_-} \right) \right]$$

$$\phi = \phi_c + \frac{\sqrt{d-1}}{2} \left[ \ln \left( 1 + \frac{\omega a_c |\tau|}{\eta_+} \right) - \ln \left( 1 + \frac{\omega a_c |\tau|}{\eta_-} \right) \right]$$

$\omega$  is proportional to the brane tension  $\kappa$ , which is responsible for the gluing:

$$\eta_{\pm} = \sqrt{d-1} \pm 1, \quad \omega = \kappa \frac{d-2}{4\sqrt{d-1}}, \quad \kappa = 2\sqrt{2(d-1)} \sqrt{n^* \Sigma_d} T_c^{d/2} e^{\phi_c}$$

Note that  $\kappa$  is of order  $\mathcal{O}(e^{\phi_c} = g_c)$  rather than the naive  $\mathcal{O}(e^{2\phi_c} = g_c^2)$

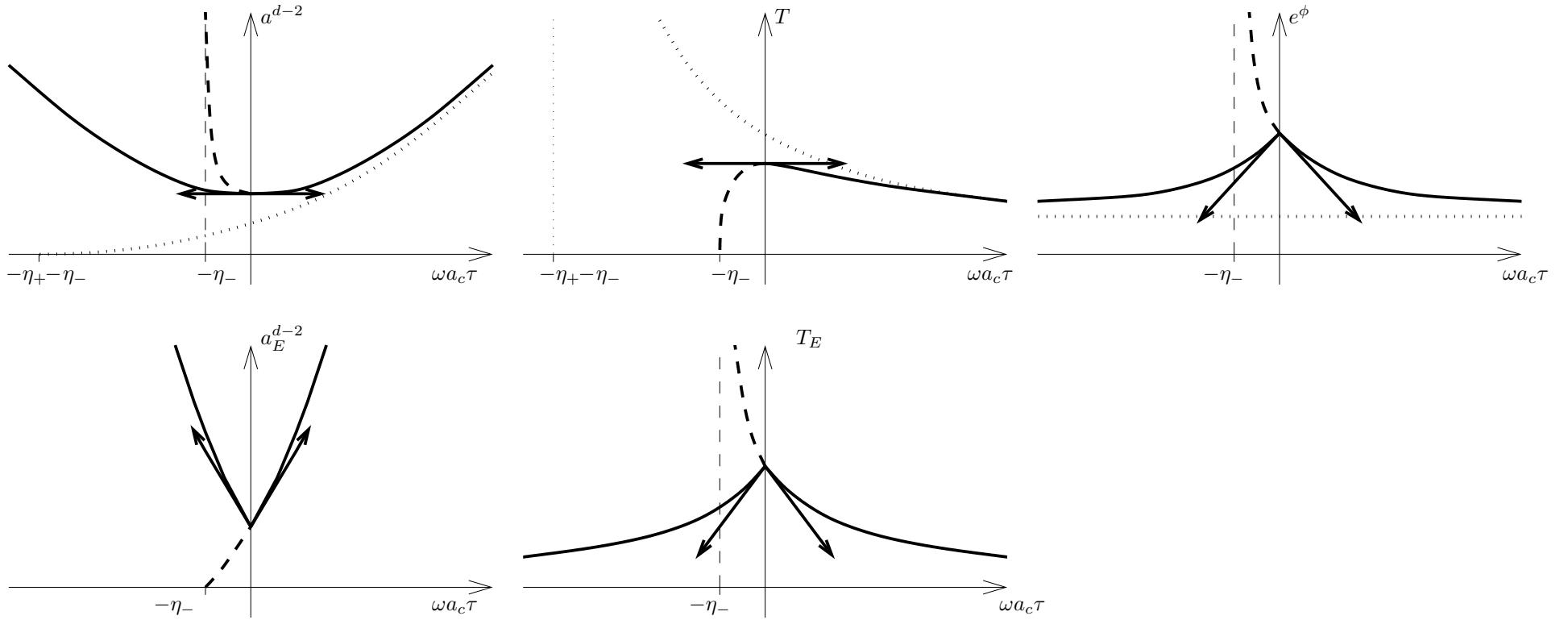
In the neighbourhood of the brane  $|\kappa a_c \tau| \ll 1$   
the metric is regular while the dilaton field exhibits a conical singularity

$$\sigma = \frac{\text{sign}(\sigma)}{16(d-1)} (\kappa_R a_c \tau)^2 + \mathcal{O}(|\kappa_R a_c \tau|^3)$$

$$\phi = \phi_c - \frac{|\kappa_R a_c \tau|}{4} + \mathcal{O}((\kappa_R a_c \tau)^2)$$

Far from the brane  $|\kappa a_c \tau| \gg 1$ ,  
the dilaton is asymptotically constant,  
the temperature drops and the scale factor tends to infinity with  $aT = a_c T_c = \text{constant}$ .

The whole evolution in  $\sigma$ -model frame describes a bounce, where the scale factor and temperature are smooth. In the Einstein frame,  $(N_E, a_E, 1/T_E) = e^{-\frac{2\phi}{d-2}} (N, a, 1/T)$ , they develop conical singularities inherited from the rescaling with the string coupling.



To complete our analysis we also display the cosmological evolution in  $d = 2$ :

$$\sigma = \frac{\text{sign}(\sigma)}{2} \left[ \frac{\kappa_R a_c |\tau|}{2} - \ln \left( 1 + \frac{\kappa_R a_c |\tau|}{2} \right) \right], \quad \phi = \phi_c - \frac{1}{2} \ln \left( 1 + \frac{\kappa_R a_c |\tau|}{2} \right).$$

This solution was found in the context of the 2d Hybrid models  $MSDS$ -SUSY but it is also valid in the more general tachyon-free two-dimensional thermal models.

The left-moving sector of the Type II  $MSDS$  models is described in terms of  $SO(8)$  characters as in the conventional superstring and in terms of the chiral  $E_8$  lattice.

The right moving sector is described in terms of  $SO(24)$  characters giving rise to Massive Spectrum Degeneracy Symmetry  $MSDS$

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The partition function is given by:

$$Z = \frac{V_2}{(2\pi)^2} \int_{\mathcal{F}} \frac{d^2\tau}{4(\text{Im}\tau)^2} \frac{1}{\eta^8} \Gamma_{E_8}(\tau) (V_8 - S_8) (\bar{V}_{24} - \bar{S}_{24}), \quad (\bar{V}_{24} - \bar{S}_{24}) \equiv 24$$

This is an  $N = (4, 0)$  supersymmetric model with respect to ordinary supersymmetry, with an  $MSDS$ -symmetric anti-holomorphic sector.

The thermal (singularity-free) Hybrid-model is obtained by inserting the Euclidean time  $\Gamma_{(1,1)}$  lattice, thermally coupled to the left-moving fermions.

$$\frac{Z_{\text{thermal}}}{V_1} = \int_{\mathcal{F}} \frac{d^2\tau}{8\pi(\text{Im}\tau)^{3/2}} (\bar{V}_{24} - \bar{S}_{24}) \frac{\Gamma_{E_8}}{\eta^8} \\ \times \sum_{m,n} \left( V_8 \Gamma_{m,2n} + O_8 \Gamma_{m+\frac{1}{2},2n+1} - S_8 \Gamma_{m+\frac{1}{2},2n} - C_8 \Gamma_{m,2n+1} \right).$$

The thermal model remains tachyon-free for all values of  $R$

The correct definition of the physical temperature (in duality invariant manner) is:

$$T = T_c e^{-|\sigma|}, \quad T_c = \frac{\sqrt{2}}{2\pi}, \quad \longrightarrow \quad T \leq T_c$$

An effective action description EXISTS, provided we take in to account the extra states which are localized at the extended symmetry point  $\sqrt{2}R = 1$  (in the Euclidean).

### 3.2 The Brane regime

In this regime the temperature  $T = T_c$  and the scale factor  $a = a_c$  are fixed (in the  $\sigma$ -model frame). The only non-trivial evolution is that of the dilaton which depends on the flux parameter  $C$  of the effective potential.

$$C = 0 : \quad e^{-\phi} = a_c \sqrt{\frac{\rho_c + P_c}{2}} (c^2 + |\tau|), \quad a = a_c, \quad T = T_c, \quad \forall \tau \leq \tau_+ = 0$$

$$C > 0 : \quad e^{-\phi} = \sqrt{\frac{\rho_c + P_c}{C}} \sinh \left[ a_c \sqrt{\frac{C}{2}} (c^2 + |\tau|) \right], \quad a = a_c, \quad T = T_c, \quad \forall \tau \leq \tau_+ = 0$$

In both cases  $\tau_- = -\infty$ . The choice of  $c^2$  determines the string coupling at the transition towards the Momentum phase,  $\phi_+ = \phi(\tau = \tau_+ = 0)$ .

$$C < 0 : \quad e^{-\phi} = \sqrt{\frac{\rho_c + P_c}{|C|}} \sin \left[ a_c \sqrt{\frac{|C|}{2}} (c^2 + |\tau|) \right], \quad a = a_c, \quad T = T_c, \quad \forall \tau_- \leq \tau \leq \tau_+ = 0$$



In order to avoid a non-perturbative regime, we need to impose a bound for  $\tau_-$  :

$$\tau_- > \tau_{\min} = -\frac{\pi}{a_c} \sqrt{\frac{2}{|C|}} + c^2$$

In this case ( $C < 0$ ) the Brane regime lasts for a finite time interval with  $\phi_{\pm} \equiv \phi(\tau_{\pm})$ .  $\tau_-$  determines the string coupling when the Universe enters the Brane regime from the Winding phase, while  $\tau_+ = 0$  controls the dilaton at the exit of the Brane regime towards the Momentum phase.

Although the metric is flat in the  $\sigma$ -model frame, it acquires a non-trivial time-dependence in the Einstein frame due to the rescaling:

$$g_{\mu\nu}^E \equiv e^{-\frac{4\phi}{d-2}} g_{\mu\nu}.$$

In the cosmological frame:  $ds_E^2 = -dt^2 + a_E^2(t) dx_i^2$ .  $T_E$ ,  $a_E$  and  $\phi$  have very simple expressions as functions of  $t$  when  $d = 4$ .

$$e^{-2\phi} = \left(\frac{T_c}{T_E}\right)^2 = \left(\frac{a_E}{a_c}\right)^2 = \sqrt{2(\rho_c + P_c)} (\delta + |t|) + \frac{C}{2} (\delta + |t|)^2, \quad \forall t : t_- < t < t_+ = 0$$

$$C \geq 0 \quad t_- = -\infty, \quad t_+ = 0$$

$$C < 0 \quad t_+ > t_- > -\frac{2\sqrt{2(\rho_c + P_c)}}{|C|} + \delta, \quad t_+ = 0$$

- Having at our disposal non-singular string cosmological solutions

$$\{\mathcal{C}_{\text{String}}(\tau)\} \equiv \{\mathcal{W}(\tau < \tau_c)\} \oplus \{\mathcal{B}(\tau = \tau_c)\} \oplus \{\mathcal{M}(\tau_c < \tau)\}.$$

$$\{\mathcal{C}_{\text{String}}(\tau)\} \equiv \{\mathcal{B}(\tau < \tau_+)\} \oplus \{\mathcal{M}(\tau_+ < \tau)\}.$$

$$\{\mathcal{C}_{\text{String}}(\tau)\} \equiv \{\mathcal{W}(\tau < \tau_-)\} \oplus \{\mathcal{B}(\tau_- < \tau < \tau_+)\} \oplus \{\mathcal{M}(\tau_+ > \tau)\}.$$

- We explicitly calculated the spectrum of fluctuations at early times
- We have determined their propagation at later cosmological times and
- We are comparing them to the current observational data (preliminary results).

This is possible since we have analytical control on the theory describing the brane.

$$\Phi = -\hat{C}_1 \frac{\xi^3 \sqrt{3(x^2 + \xi^2)}}{x^4} + \frac{(8c_\sigma - 3c_\varphi)}{3} \left( \frac{\xi^2(x^2 + 2\xi^2)}{x^4} \right) - \frac{2}{3}c_\sigma$$

$$\varphi = -\hat{C}_1 \frac{3\xi^4}{x^4} - \frac{(8c_\sigma - 3c_\varphi)}{3} \left( \frac{\xi \sqrt{3(x^2 + \xi^2)}(x^2 - 2\xi^2)}{x^4} \right) + (3c_\sigma - c_\varphi).$$

$\Phi$  describes the metric fluctuations :

$$ds^2 = -a^2(1 + 2\Phi)d\tau^2 + a^2(1 - 2\Phi)dx^2 .$$

$$x = x_c \frac{a}{a_c}, \quad x_c = \sqrt{2}, \quad x \geq \sqrt{2}$$

$\varphi$  describes the dilaton fluctuation

$\xi$  is the time-scale of the problem

$$\xi = \frac{1}{\sqrt{2n^*} \tilde{T}_c^2 \tilde{a}_c e^{\phi_c}} .$$

This work is currently under progress

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The important output of our analysis is:

The relevant primordial perturbation for  $\Phi$  and  $\varphi$  in the contracting phase that persist in the expanding phase after crossing the S-brane are time independent constants:

$$\Phi_k = -\frac{2}{3}c_\sigma(k), \quad \varphi_k = -c_\varphi(k) + 3c_\sigma(k)$$

—→ *leading to a scale invariant spectrum of fluctuations*

## 4. Conclusions

- Cosmological consequences of the **stringy gluing mechanism** between different string effective field theories.
- The gluing mechanism is induced by the appearance of extra massless string states when the temperature reaches its maximal critical value.
- The region  $T = T_c$ , admits a natural “brane interpretation”, with tensions  $\kappa_{\pm}$  given by the non-trivial gradients  $\langle \partial_{\hat{\mu}} \varphi^I \rangle \neq 0$ , of the extra massless scalars.
- Non-trivial gauge fluxes on the Brane give rise to an effective dilaton potential,

$$V(\phi) = B(\mathcal{U}) - P_c + C e^{-2\phi}$$

- In the Einstein cosmological frame, all solutions correspond to bouncing Universes.

- The cosmological solutions remain perturbative, provided that the critical value of the string coupling at the endpoints of the brane is sufficiently small.
- This class of bouncing cosmologies provides the first  $d$ -dimensional examples, where *both the Hagedorn instability as well as the classical Big Bang singularity* are successfully resolved, remaining in a perturbative regime throughout the evolution.