# One-loop Kähler metrics of Calabi-Yau orientifolds

Michael Haack (Arnold-Sommerfeld Center, LMU Munich)
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1112.5156 (with Marcus Berg and Jin U Kang)
work in progress (with Marcus Berg, Jin U Kang and Stefan Sjörs)

# Overview

- Motivation
- Some known results on quantum corrections to $K$
- Recent work on 1-loop contributions to moduli space metric (focus on poorly understood contributions including those from massive string states)
- Conclusion

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(Perturbative) quantum corrections to effective action can be important

- if "zero effect" at tree level
  (e.g. no-scale structure of potential)
- for "precision phenomenology"
  (e.g. structure of soft SUSY breaking terms)
- for breaking of symmetries
  (e.g. shift symmetry for models of inflation)

# $\mathcal{N} = 1,\ d = 4$ Supergravity

$$\frac{\mathcal{L}_{\rm bos}}{(-G)^{1/2}} = \frac{1}{2\kappa^2}R - K_{,\bar{I}J}D_\mu\bar{\Phi}^{\bar{I}}D^\mu\Phi^J - \frac{1}{4}{\rm Re}(f_{ab}(\Phi))F^a_{\mu\nu}F^{b\mu\nu}$$
$$-\frac{1}{8}{\rm Im}(f_{ab}(\Phi))\epsilon^{\mu\nu\rho\sigma}F^a_{\mu\nu}F^b_{\rho\sigma} - V(\Phi,\bar{\Phi})$$

with $V(\Phi,\bar{\Phi}) = e^K(G^{\bar{I}J}D_{\bar{I}}\overline{W}D_JW - 3|W|^2) + {\rm Re}(f_{ab})\mathcal{D}^a\mathcal{D}^b$

$$D_JW \equiv \partial_{\phi^J}W + \partial_{\phi^J}K\,W$$

- Superpotential $W$
- Gauge kinetic function $f_{ab}$
- Kähler potential $K$

# No-scale structure

For concreteness type IIB

Assume: $(i)$ $W$ independent of Kähler moduli $T_i$

$(ii)$ $K = -2\ln(\mathcal{V}(T_i)) + K_{\rm rest}(\Phi_\alpha)$

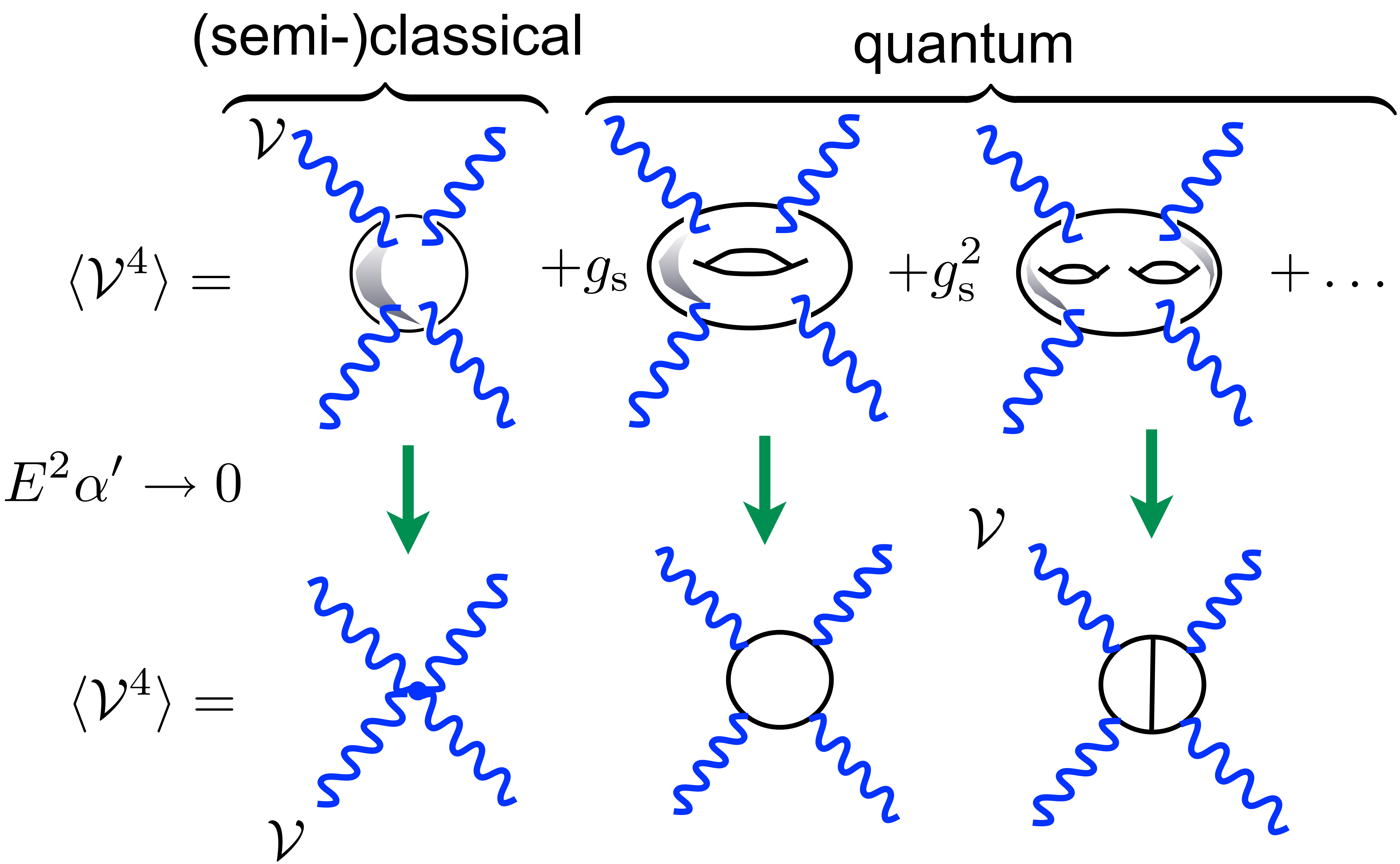
$$(i) \Longrightarrow V = e^K (G^{\bar{I}J} D_{\bar{I}}\bar{W} D_J W - 3|W|^2)$$
$$= e^K (G^{\bar{\alpha}\beta} D_{\bar{\alpha}}\bar{W} D_\beta W + (G^{\bar{i}j} K_{\bar{i}} K_j - 3)|W|^2)$$

$$(ii) \Longrightarrow G^{\bar{i}j} K_{\bar{i}} K_j = 3$$

I.e. $V$ only depends trivially on Kähler moduli

$\Longrightarrow$ Quantum corrections important

# String perturbation theory



# Quantum Corrections

- Superpotential $W = W^{\mathrm{tree}} + W^{\mathrm{non-pert}}$

- Gauge kinetic function $f = f^{\mathrm{tree}} + f^{\mathrm{1-loop}} + f^{\mathrm{non-pert}}$

- Kähler potential $K = K^{\mathrm{tree}} + \sum_{n=1}^{\infty} K^{n-\mathrm{loop}} + K^{\mathrm{non-pert}}$

# Example: Large Volume Scenario

[Balasubramanian, Berglund, Conlon, Quevedo]

- Interplay between $W_{np} \sim e^{-aT_s}$ and $\delta K_{\alpha'}$

$$K = -2\ln(\mathcal{V}) + \ldots \rightarrow -2\ln(\mathcal{V} + \tfrac{1}{2}\xi S_1^{3/2}) + \ldots$$

$$\xi = -\zeta(3)\chi/(2(2\pi)^3)$$

[Becker, Becker, Haack, Louis]

- Schematically: $V \approx c_1 \frac{e^{-2a\tau_s}}{\mathcal{V}} + c_2 \frac{e^{-a\tau_s}}{\mathcal{V}^2} + c_3 \xi \frac{1}{\mathcal{V}^3}$

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What about stability to further perturbative corrections?

# Phenomenology

- In Large Volume Scenario, soft terms due to fluxes (e.g. gaugino masses) display cancellation leading to hierarchy [Balasubramanian, Berglund, Conlon, Quevedo]

$$M_a << m_{3/2}$$

- Soft supersymmetry parameters depend on $K$ and $f$

E.g. gaugino masses:

$$M_a = \frac{1}{2}\frac{1}{\mathrm{Re} f_a}\sum_I F^I \partial_I f_a$$

with

$$F^I = e^{K/2} G^{\bar{J} I} D_{\bar{J}} \bar{W}$$

- Both, existence of LVS minima and structure of soft terms, largely insensitive to perturbative corrections to Kähler potential

[Berg, Haack, Pajer]

- "Extended no-scale structure"

[Cicoli, Conlon, Quevedo]

# Some known results on quantum corrections to $K$

- Focus on type I theory and $K$ for moduli
- Methods:
  - ★ Truncation of type II results
  - ★ Duality (heterotic theory / F-theory)
  - ★ Scattering amplitudes in type I ← focus of this talk

- Truncation from type II, for instance:

$$K = -2\ln(\mathcal{V}) + \ldots \to -2\ln(\mathcal{V} + \tfrac{1}{2}\xi S_1^{3/2}) + \ldots$$

$$\xi = -\zeta(3)\chi/(2(2\pi)^3)$$

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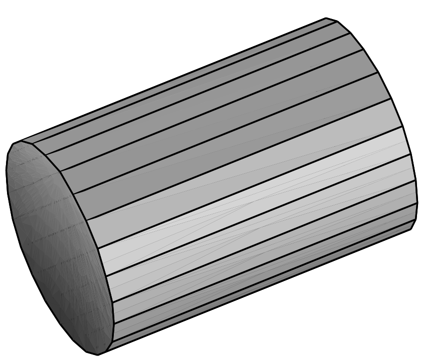
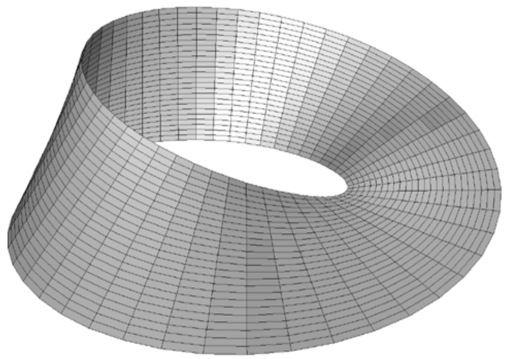
1-loop misses contributions from

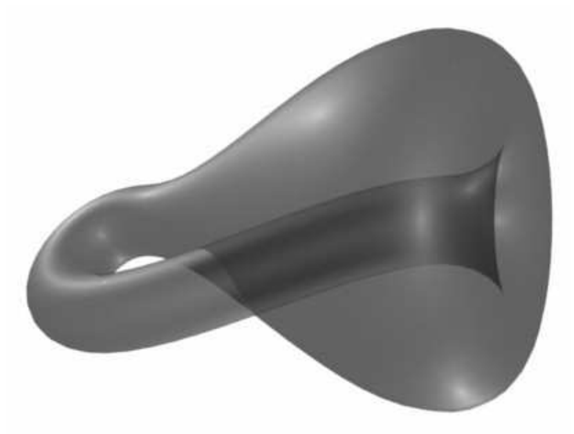
Annulus:



Möbius strip:



Klein bottle:



- Indirect approaches:

  - ★ S-duality to heterotic string [e.g. Camara, Dudas]

  - ★ F-theory [e.g. Garcia-Etxebarria, Hayashi, Savelli, Shiu; Grimm, Savelli, Weißenbacher]

Mainly used in $\mathcal{N} = 2$ context so far

- Type I amplitudes: Feasible for toroidal orientifold models

- Result for orientifold models of type IIB (e.g. $\mathbb{T}^2 \times (\mathbb{T}^4/\mathbb{Z}_2)$):

[Antoniadis, Bachas, Fabre, Partouche, Taylor; Antoniadis, Partouche, Taylor; Berg, Haack, Körs]

$$
\begin{aligned}
K \quad = \quad & -\ln\left((S+\bar{S})(T+\bar{T})(U+\bar{U})\right) \\
& -\ln\left(1-\frac{1}{8\pi}\sum_i \frac{N_i(\phi_i+\bar{\phi}_i)^2}{(T+\bar{T})(U+\bar{U})} - \frac{1}{128\pi^6}\sum_i \frac{\mathcal{E}_2(\phi_i,U)}{(S+\bar{S})(T+\bar{T})}\right)
\end{aligned}
$$

sum over terms containing $E_2$

$$
E_2(\phi,U) = \sum_{(n,m)\neq(0,0)} \frac{U_2}{|n+mU|^2} \exp\left[2\pi i \frac{\phi(n+m\bar{U})-\bar{\phi}(n+mU)}{U-\bar{U}}\right]
$$

- Similar contributions arise from $\mathcal{N}=2$ sectors of type I orientifolds with $\mathcal{N}=1$

  (no contributions from string oscillators; modular functions)

- Usual lore: $\mathcal{N}=1$ sectors less interesting, because they do not lead to moduli dependent results

# Caveats

- Moduli dependence in $\mathcal{N}=1$ sectors via:
  - ★ Normalization of vertex operators
  - ★ Weyl rescaling to Einstein frame

$$\left.\begin{array}{l}\text{Normalization of vertex operators}\\ \text{Weyl rescaling to Einstein frame}\end{array}\right\} \Longrightarrow \delta K \sim \frac{1}{\sqrt{S_1}\mathcal{V}}$$

[Berg,Haack,Körs]

- Expect further moduli dependence in $\mathcal{N}=1$ sectors in presence of world volume fluxes or for branes at angles

# Gauge coupling thresholds

- $$\frac{1}{g_a^2(\mu)} = \frac{1}{g^2_{a,\mathrm{string}}} + \frac{b_a}{16\pi^2} \ln\left(\frac{M_s^2}{\mu^2}\right) + \Delta_a$$

- Branes at angles: [Lüst, Stieberger; Akerblom, Blumenhagen, Lüst, Schmidt-Sommerfeld]

$$\Delta_a \sim \sum_b \ln\left(\frac{\Gamma(\varphi^1_{ab})\Gamma(\varphi^2_{ab})\Gamma(1+\varphi^3_{ab})}{\Gamma(1-\varphi^1_{ab})\Gamma(1-\varphi^2_{ab})\Gamma(-\varphi^3_{ab})}\right)$$

- $\varphi_{ab} = \varphi_a - \varphi_b$ depend on torus complex structure

# Some generalities of the amplitude calculations

- Aim: read off Kähler metric from scalar 2-pt fct.
- 2-pt fct. $= 0$ on-shell
- Trick: use $\delta \equiv p_1 \cdot p_2 \neq 0$ in intermediate steps

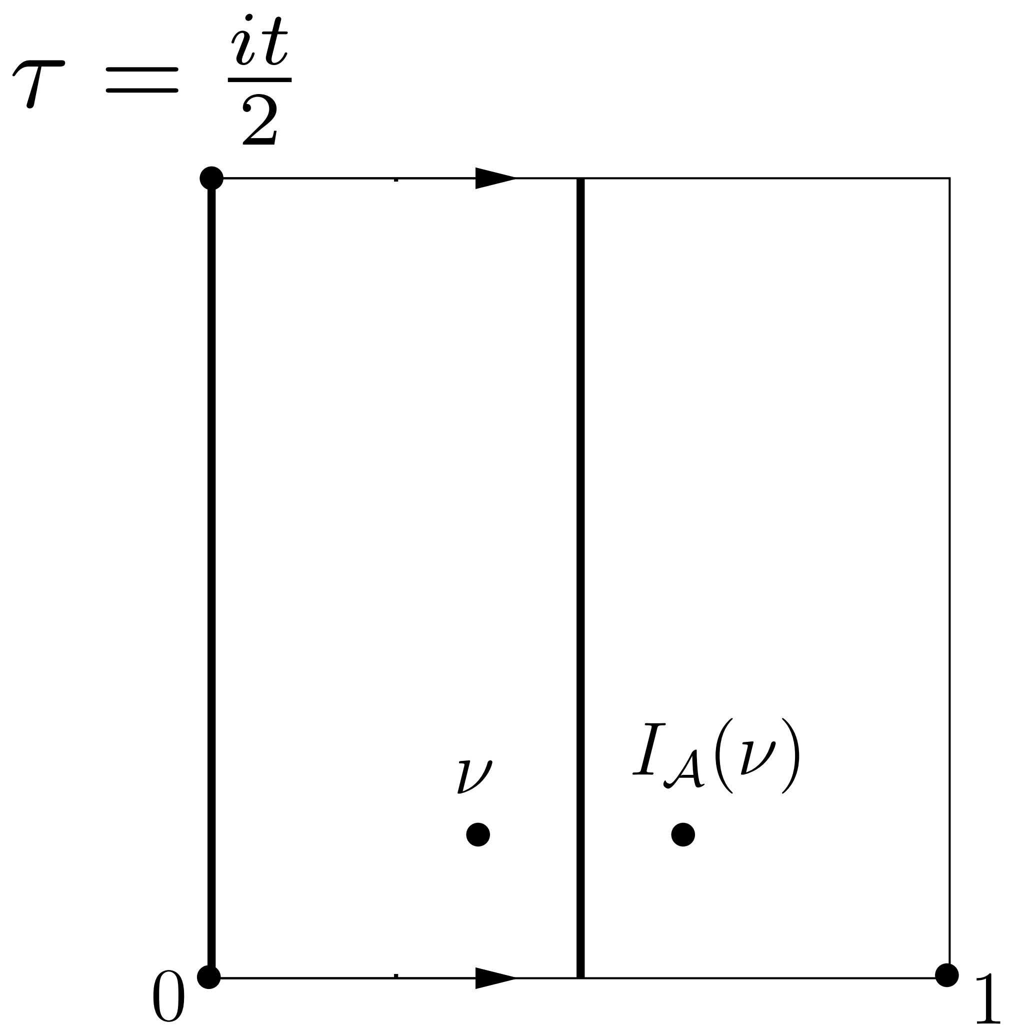
[Atick, Dixon, Sen; Minahan; Antoniadis, Bachas, Fabre, Partouche, Taylor; Antoniadis, Kirtsis, Rizos; cf. also Kiritsis, Kounnas, ...]

- $\langle \Phi_i \Phi_j \rangle = \delta\, G_{ij} + \mathcal{O}(\delta^2)$

# Method of images

[Burgess, Morris; Antoniadis, Bachas, Fabre, Partouche, Taylor]

Example annulus: $I_{\mathcal{A}}(\nu) = 1 - \bar{\nu}$



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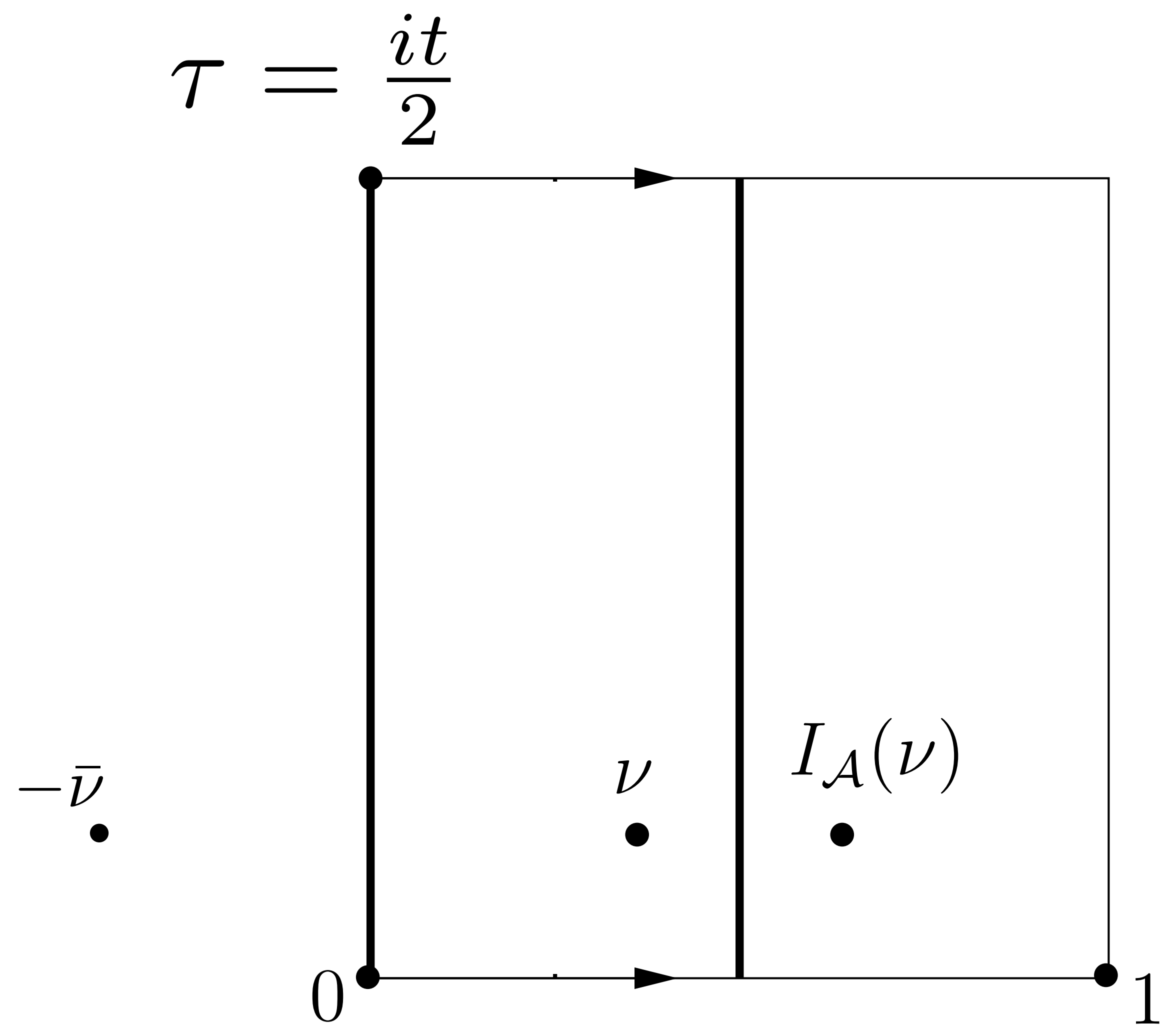

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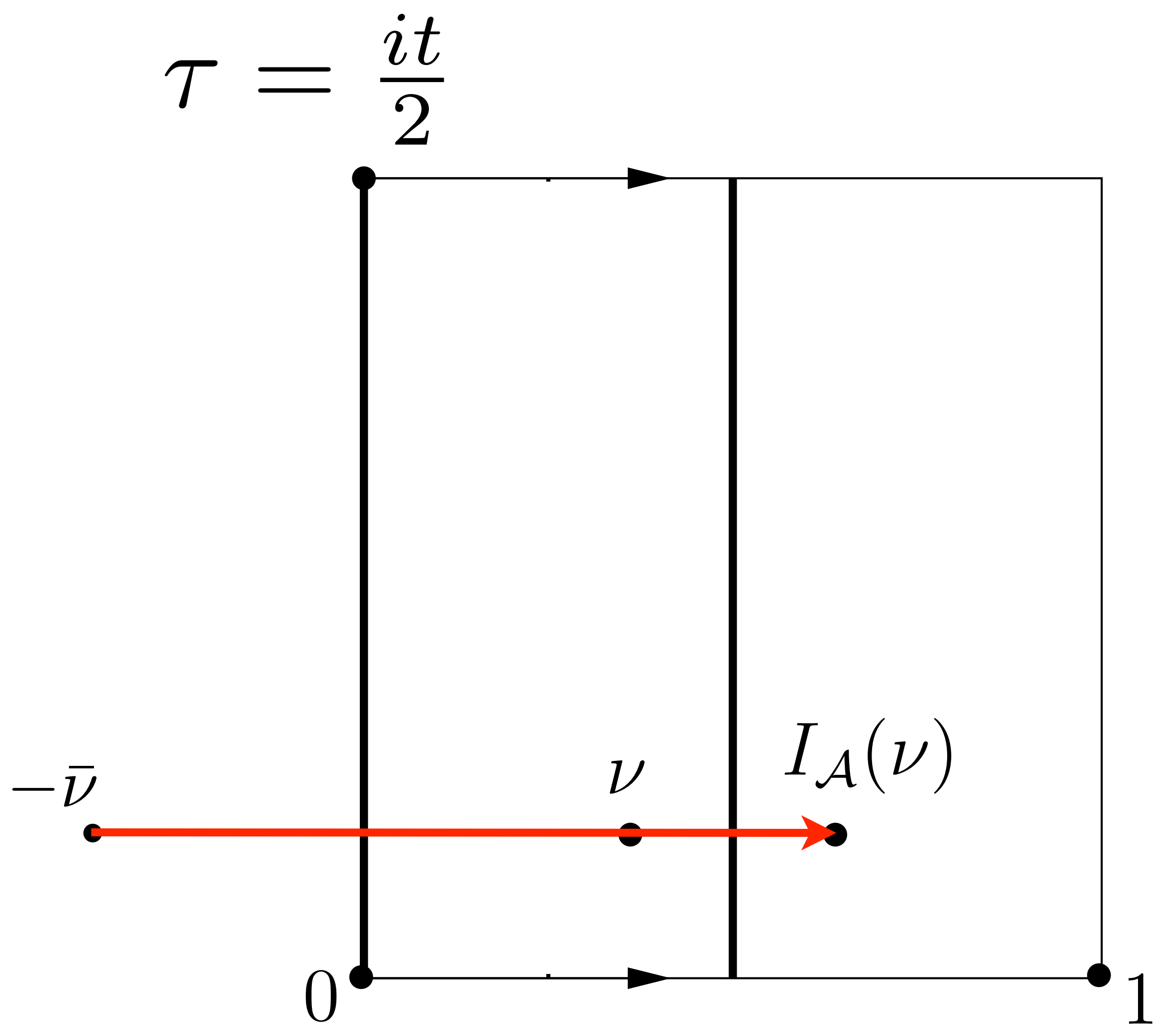

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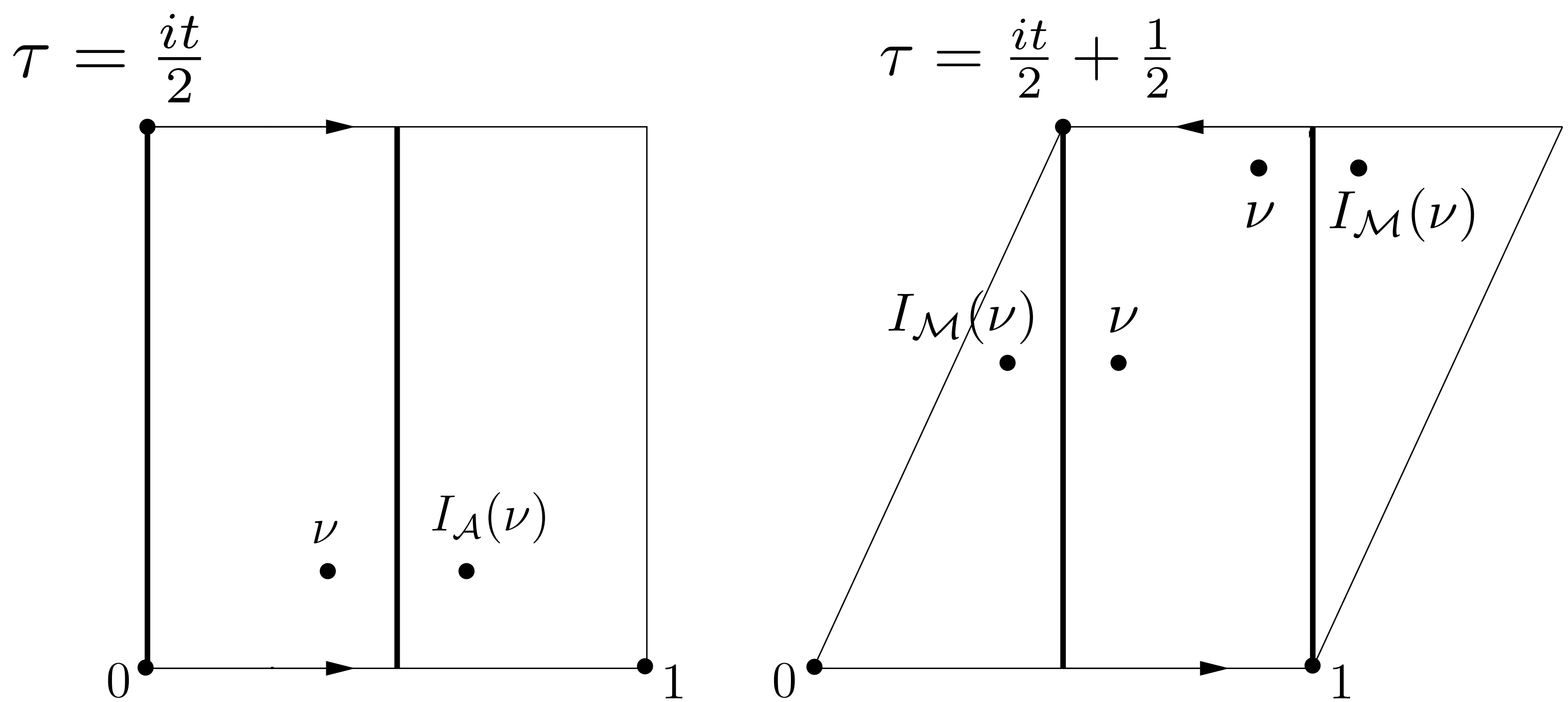
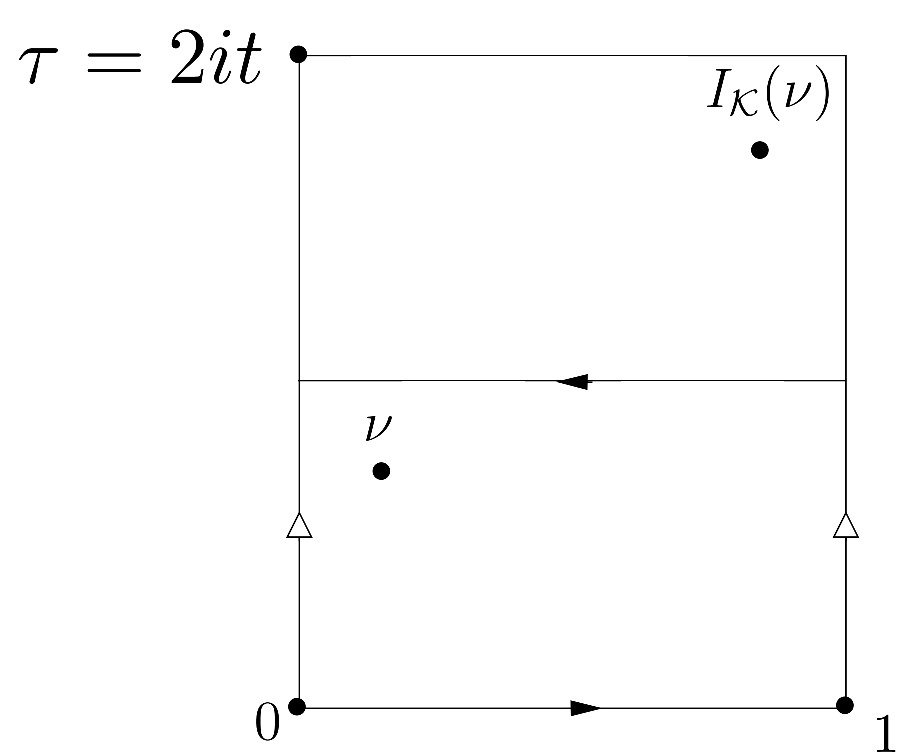



$$I_{\mathcal{A}}(\nu) = 1 - \bar{\nu}$$

$$I_{\mathcal{M}}(\nu) = 1 - \bar{\nu}$$

$$I_{\mathcal{K}}(\nu) = 1 - \bar{\nu} + \tau/2$$

- Propagator on $\mathcal{A}, \mathcal{M}, \mathcal{K}$ via symmetrizing of propagator on covering torus under involution (e.g. annulus):

$$\begin{aligned}\langle X(\nu_1)X(\nu_2)\rangle_{\mathcal{A}} \quad &\sim \quad \langle X(\nu_1)X(\nu_2)\rangle_{\mathcal{T}} + \langle X(I_{\mathcal{A}}(\nu_1))X(\nu_2)\rangle_{\mathcal{T}} \\ &+\langle X(\nu_1)X(I_{\mathcal{A}}(\nu_2))\rangle_{\mathcal{T}} + \langle X(I_{\mathcal{A}}(\nu_1))X(I_{\mathcal{A}}(\nu_2))\rangle_{\mathcal{T}}\end{aligned}$$

- Lifting to covering torus (e.g. annulus): [Antoniadis, Bachas, Fabre, Partouche, Taylor]

$$\int_{\mathcal{A}} d^2\nu \, (f(\nu) + f(I_{\mathcal{A}}(\nu)) = \int_{\mathcal{T}} d^2\nu \, f(\nu)$$

# 2 concrete examples

- Open string scalars for D6-branes at angles (i.e. type IIA orientifold)

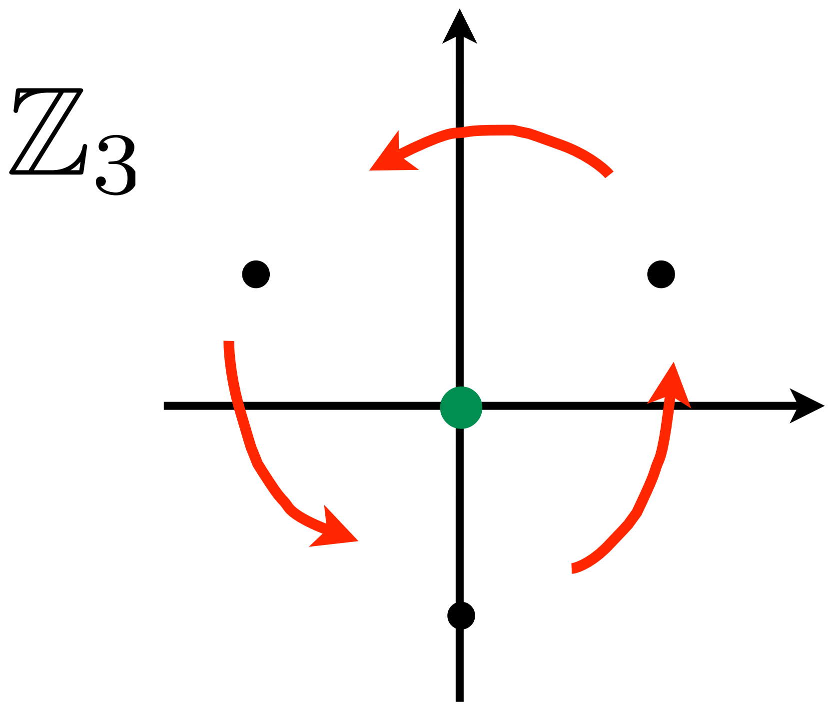
$$\Phi_i \equiv T_i \phi^i - A_i$$

position → $\phi^i$; Wilson line → $A_i$

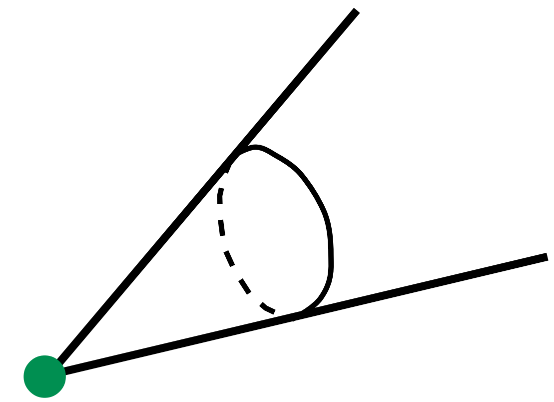
- Closed string Kähler moduli in type IIB orientifold

# Simple models for extra dimensions

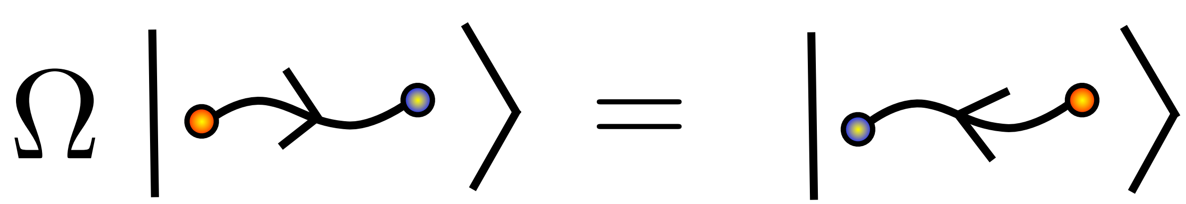
*Orbifold*: Identify under spatial rotation

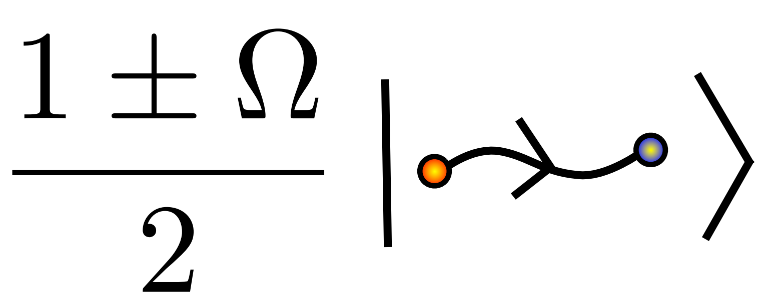



makes cone



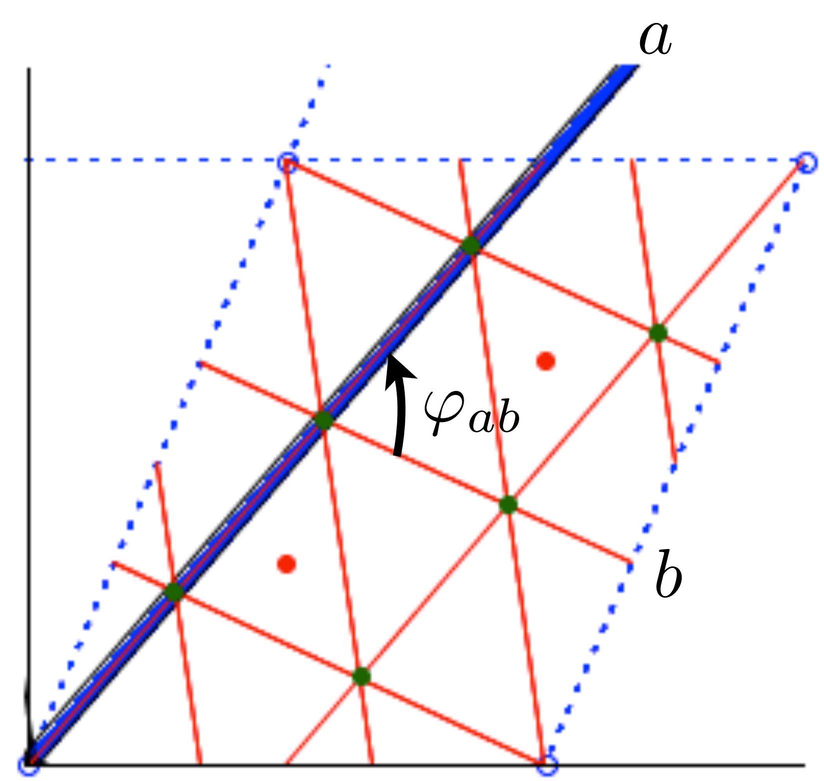
*Orientifold*: Identify under worldsheet reflection




unoriented eigenstate

# Open string scalars

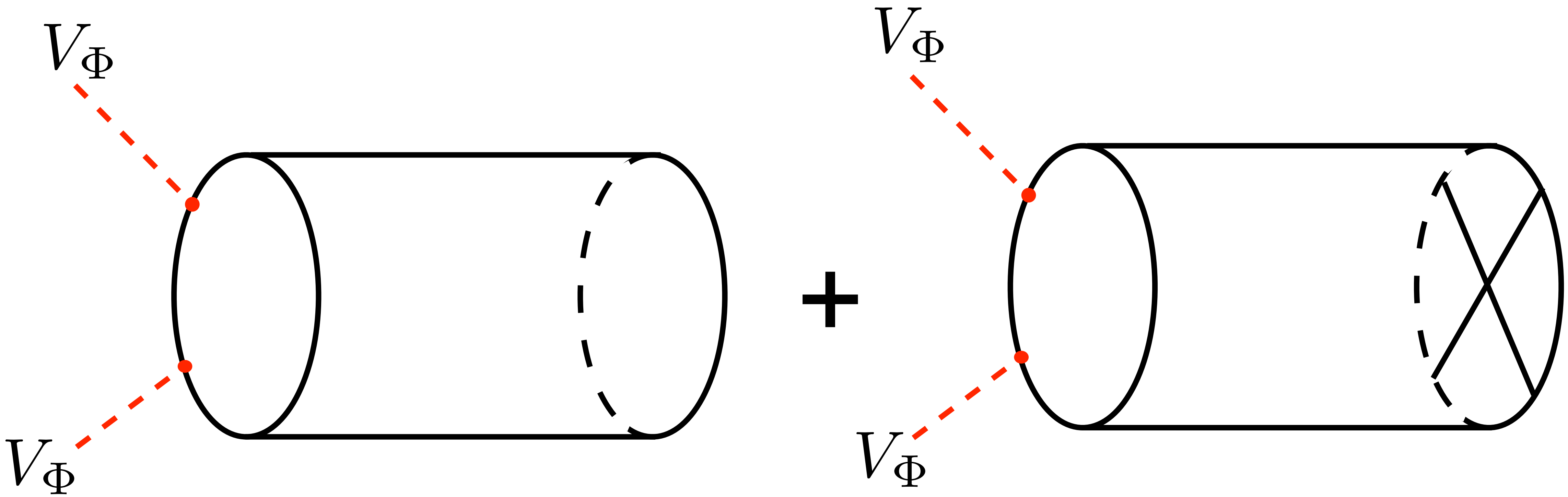



- Supersymmetry unbroken, if

$$\varphi^1_{ab} + \varphi^2_{ab} + \varphi^3_{ab} = 0$$

- $\mathcal{N} = 1$ contribution from strings stretching between branes $a$ and $b$ with non-vanishing angles along all 3 tori ← focus of this talk

# The amplitude



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- E.g.

$$\langle \Phi_3 \bar{\Phi}_3 \rangle_{\mathcal{A}} \sim \delta \int_0^\infty dt \int_0^{t/2} d\nu \sum_{\text{images}} \sum_{\substack{\alpha\beta \\ \text{even}}} \eta_{\alpha,\beta} \mathcal{Z}_{\mathcal{A}}^{\text{ext}}\left[\begin{smallmatrix}\alpha \\ \beta\end{smallmatrix}\right] \mathcal{Z}_{\mathcal{A}}^{\text{int}}\left[\begin{smallmatrix}\alpha \\ \beta\end{smallmatrix}\right]$$

$$e^{-\delta \langle X(i\nu) X(0) \rangle} \langle \Psi(i\nu) \bar{\Psi}(0) \rangle_{\mathcal{A}}^{\alpha,\beta} \langle \psi(i\nu) \psi(0) \rangle_{\mathcal{A}}^{\alpha,\beta}$$

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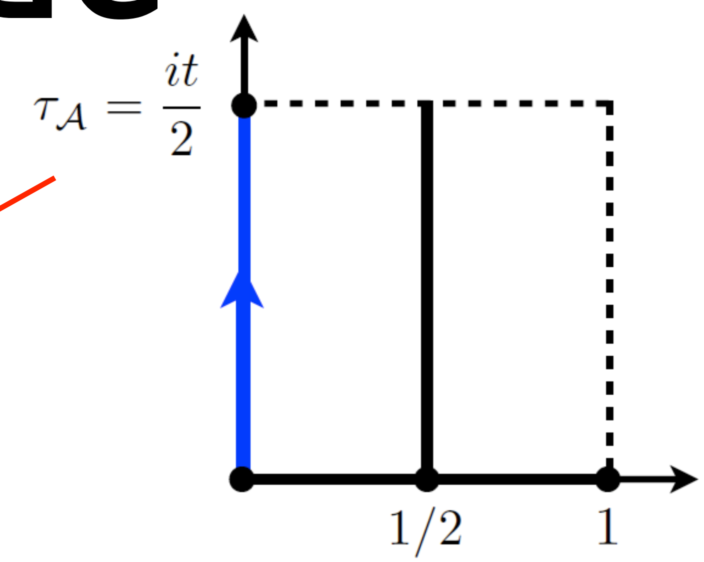


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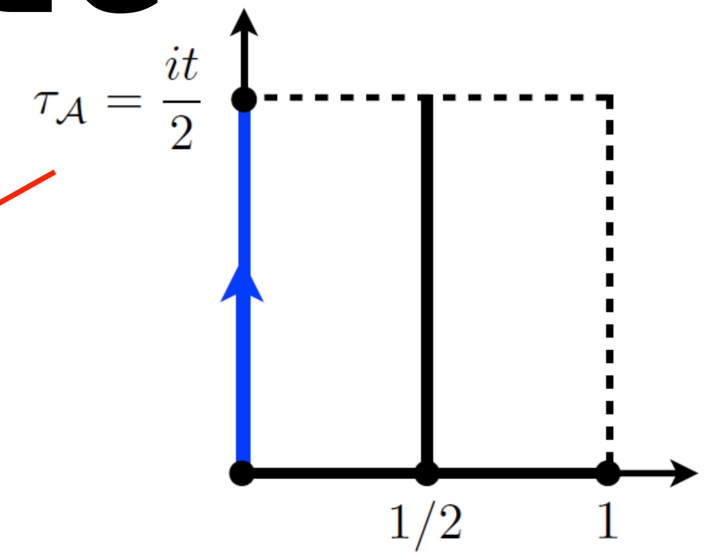


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- Spin structure sum & $\ell = 1/t$ & $\tilde{\nu} \equiv 2\nu\ell$

$$\langle \Phi_3 \bar{\Phi}_3 \rangle \sim \delta \sum_{\text{images}} \int_0^\infty d\ell \int_0^1 d\tilde{\nu}\, R_\delta(\tilde{\nu}, \ell) G_F[{}^{1/2}_{1/2+\varphi^3}](\tilde{\nu}, 2i\ell)$$

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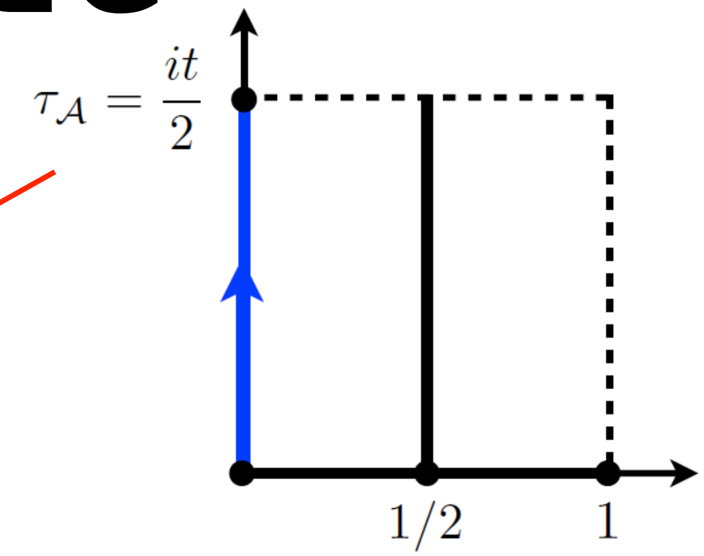


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$$\frac{\vartheta \left[ \begin{smallmatrix} 1/2 \\ 1/2+\varphi^3 \end{smallmatrix} \right] (\tilde{\nu}, 2i\ell) \vartheta_1'(0, 2i\ell)}{\vartheta \left[ \begin{smallmatrix} 1/2 \\ 1/2+\varphi^3 \end{smallmatrix} \right] (0, 2i\ell) \vartheta_1(\tilde{\nu}, 2i\ell)}$$

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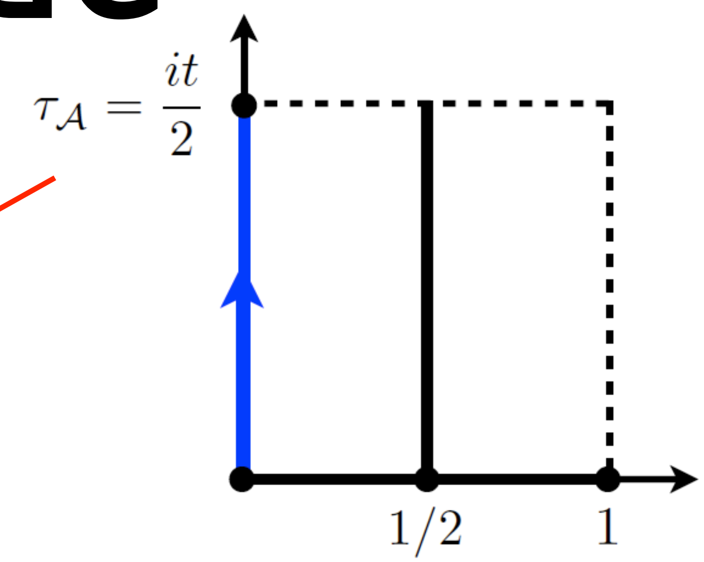


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$$\left| \frac{\vartheta_1(\tilde{\nu}, 2i\ell)}{2\ell\, \vartheta_1'(0, 2i\ell)} \right|^{\alpha'\delta}$$

$$\frac{\vartheta[\begin{smallmatrix}1/2 \\ 1/2+\varphi^3\end{smallmatrix}](\tilde{\nu}, 2i\ell)\vartheta_1'(0, 2i\ell)}{\vartheta[\begin{smallmatrix}1/2 \\ 1/2+\varphi^3\end{smallmatrix}](0, 2i\ell)\vartheta_1(\tilde{\nu}, 2i\ell)}$$

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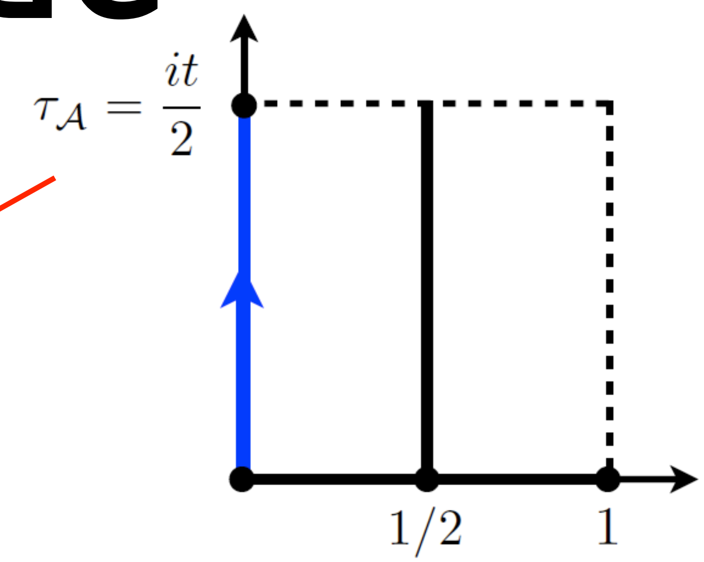


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$$\frac{\vartheta[{}^{1/2}_{1/2+\varphi^3}](\tilde{\nu}, 2i\ell) \vartheta_1'(0, 2i\ell)}{\vartheta[{}^{1/2}_{1/2+\varphi^3}](0, 2i\ell) \vartheta_1(\tilde{\nu}, 2i\ell)}$$

# Potential divergencies

- UV-divergence from $\ell \to \infty$ : cancels via vacuum tadpole cancellation condition
- Vertex collision divergence from $\tilde{\nu} \to 0$ or $1$
- Can be made explicit by [Whittaker, Watson: *A course of modern analysis*, 1927]

$$G_F[{}^{1/2}_{1/2+\varphi^3}](\tilde{\nu}, 2i\ell) = \pi \cot \pi\tilde{\nu} + \pi \cot \pi\varphi^3 + 4\pi \sum_{m,n=1}^{\infty} e^{-4\pi\ell mn} \sin(2\pi n\tilde{\nu} + 2\pi m\varphi^3)$$

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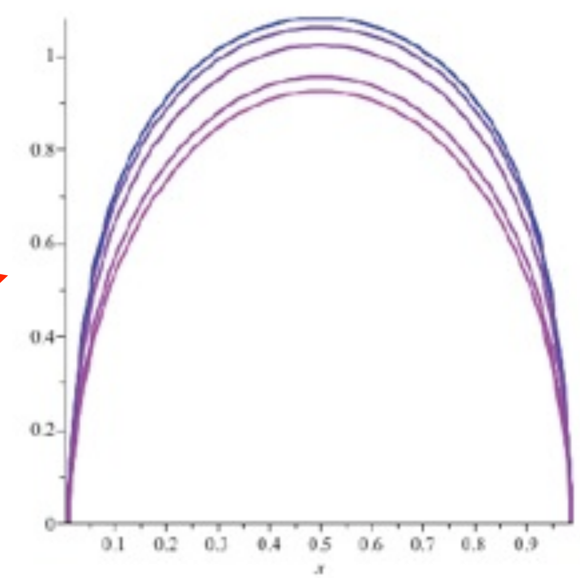
$$\int_0^x R_\delta(\tilde{\nu}) \cot(\pi\tilde{\nu}) d\tilde{\nu}$$



# Finite part

Given by

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$$= 0$$

Similar for Möbius

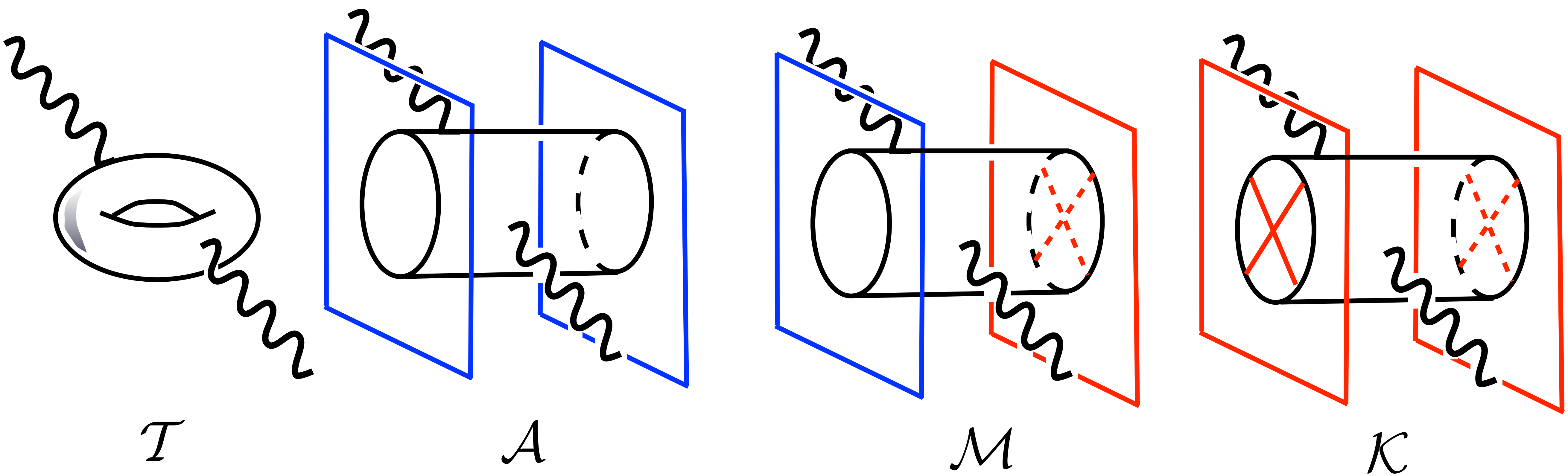
# Relation to other work

- Absence of mass renormalization due to $\mathcal{N} = 1$ contributions [Anastasopoulos, Antoniadis, Benakli, Goodsell, Vichi]

- Vanishing not to be expected
  - ★ from non-renormalization theorems
  - ★ for branes with world volume fluxes [Benakli, Goodsell; Bain, Berg; work in progress]
  - ★ for closed string scalars

# Closed string scalars in IIB

(No world-volume fluxes yet)



# What do we calculate?

- Compactification in type IIB: [Antoniadis, Ferrara, Minasian, Narain]

$$S_4 = \int d^4x\sqrt{-g}\left[e^{-2\phi_4} + \chi\left(\zeta(3)\frac{e^{-2\phi_4}}{\mathcal{V}} + 1\right)\right] R$$
$$+ \int d^4x\sqrt{-g}\left[e^{-2\phi_4} - \chi\left(\zeta(3)\frac{e^{-2\phi_4}}{\mathcal{V}} + 1\right)\right] G_{T\bar{T}}\,\partial_\mu T\partial^\mu\bar{T} + \ldots$$

In String frame!

Kähler moduli

- Weyl-rescaling to Einstein frame:

$$S_4^{(E)} = \int d^4x\sqrt{-g}\left(R + \left[1 - 2\chi\frac{\zeta(3)}{\mathcal{V}} - 2\chi e^{2\phi_4}\right] G_{T\bar{T}}\,\partial_\mu T\partial^\mu\bar{T}\right) + \ldots$$

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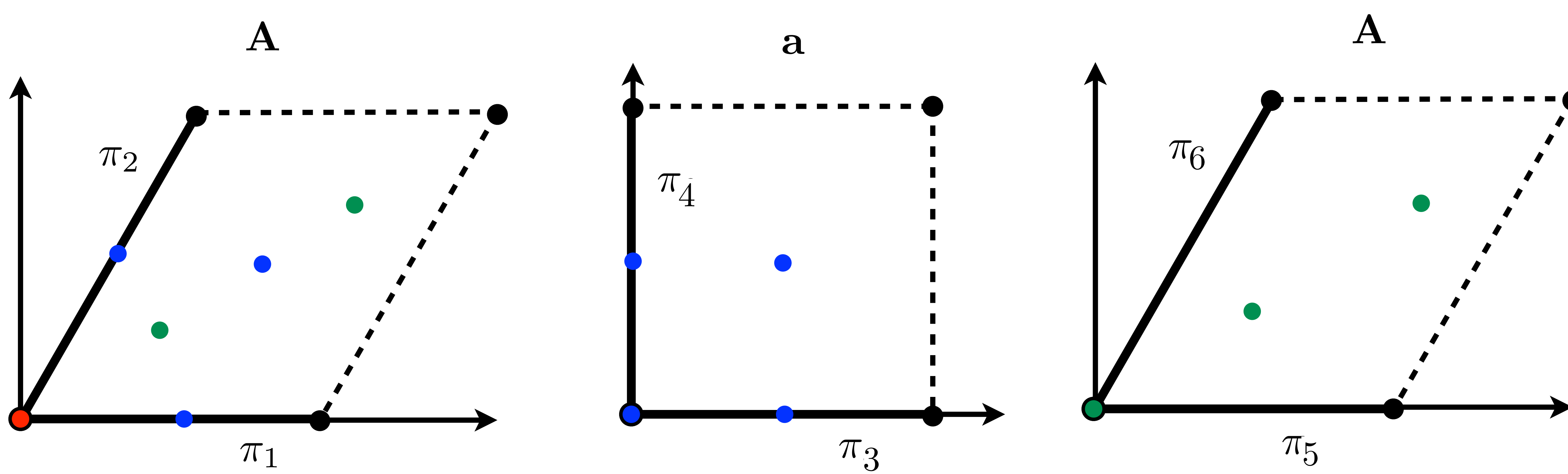
We are after corrections to this constant from $\mathcal{A}, \mathcal{M}, \mathcal{K}$

# $T^6/\mathbb{Z}'_6$

- $\Theta Z^1 = e^{2\pi i v_1} Z^1 \quad \Theta Z^2 = e^{2\pi i v_2} Z^2 \quad \Theta Z^3 = e^{2\pi i v_3} Z^3$

  $(v_1, v_2, v_3) = \left(\frac{1}{6}, -\frac{1}{2}, \frac{1}{3}\right)$

- 


- $\mathcal{N} = 1$ , if strings are twisted along all 3 tori

- We calculate 1-loop correction for volume modulus of the third torus, i.e. $T_3$

- In order to read off the Kähler metric for $T_3$ we need to know the correct definition of the moduli at 1-loop

- In type I usually corrections to definition of closed string moduli in presence of fluxes/angles [Blumenhagen, Schmidt-Sommerfeld; Camara, Condeescu, Dudas]

$\longrightarrow$ no issue here

- Vertex operator ( $t = \mathrm{Re}(T_3)$ ):

$$V_t \sim -\frac{2}{\alpha'}\int d^2\nu\, e^{ip\cdot X}\left[i\partial\bar{Z} + \frac{\alpha'}{2}(p\cdot\psi)\bar{\Psi}\right]\left[i\bar{\partial}Z + \frac{\alpha'}{2}(p\cdot\tilde{\psi})\tilde{\Psi}\right]$$
$$-\frac{2}{\alpha'}\int d^2\nu\, e^{ip\cdot X}\left[i\partial Z + \frac{\alpha'}{2}(p\cdot\psi)\Psi\right]\left[i\bar{\partial}\bar{Z} + \frac{\alpha'}{2}(p\cdot\tilde{\psi})\bar{\tilde{\Psi}}\right]$$

- $\langle V_t V_t\rangle$ =(4-fermion terms $\mathcal{O}(\delta)$ )+(8-fermion terms $\mathcal{O}(\delta^2)$)

8-fermion terms not negligible!

- Contact terms:

$$\int_{D_\epsilon} d^2\nu |\nu|^{-2+\delta} = \int_0^\epsilon d|\nu| \int_0^{2\pi} d\arg(\nu) |\nu|^{-1+\delta} = \underbrace{2\pi \frac{1}{\delta} |\nu|^\delta \Big|_0^\epsilon}_{\xrightarrow{\delta \to 0} \frac{1}{\delta}}$$

- 8-fermion terms necessary to reproduce torus contribution of [Antoniadis, Ferrara, Minasian, Narain]

# Some calculational steps

- Performing spin structure sum and $\nu$-integral leads to:

$$\int_0^\infty \frac{dt}{t^2} \frac{\vartheta_1'(\gamma, it)}{\vartheta_1(\gamma, it)}$$

orbifold twist

- Performing integral over torus parameter $t$ gives:

$$\int_0^\infty \frac{dt}{t^2} \frac{\vartheta_1'(\gamma, it)}{\vartheta_1(\gamma, it)} = \frac{\pi}{12}\left(2\psi'(\gamma) - \frac{\pi^2}{\sin^2(\pi\gamma)}\right)$$

(finite part)

- Separate contributions do not vanish (summing up in progress)

# Summary

- Quantum corrections important for string phenomenology
- Surprisingly, $\mathcal{N} = 1$ contributions to $K$ vanish for open string moduli of D-branes at angles
- Presented the steps for calculating the corrections to Kähler metric of closed string moduli

# Open questions

- Reason for vanishing of $\mathcal{N}=1$ contributions to $K$ for open string moduli of D-branes at angles?

- Extend the closed string calculation to branes at angles or with world volume fluxes ( $\longrightarrow$ additional moduli dependence)

- Phenomenology?

# Ευχαριστώ πολύ!

Thank You