Minimal Model Holography (From Large N to Large $\mathcal{N}$)

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Plan of the Talk

- Quick Overview of Vector Coset Holography

  The things we have learnt. Novel features and the need for a string embedding.

- The large $\mathcal{N} = 4$ SUSY case

  Motivation - why focus on this case? The HS theory and the dual coset.

- Relation to String Theory on $AdS_3 \times S^3 \times S^3 \times S^1$

  Described by a non-abelian Vasiliev theory? Evidence.

Duals to 2d Coset CFTs: A Good Laboratory for AdS/CFT

Boundary 2d CFT

- $\frac{G}{H}$ WZW theories are solvable. There is an intricate spectrum (for any N) and correlators computable. As a bonus, in many cases, genuinely non-supersymmetric.
- Presence of a large unbroken higher spin symmetry ($\mathcal{W}_N$ or often much larger) in an interacting theory.
- A 'tHooft like large N limit appears to be sensible.

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Bulk AdS_3

- Higher spin gauge fields ($s = 2, 3 \ldots$) in AdS_3 are non-propagating (though can have propagating scalars/fermions).
- Classical Vasiliev theories describing them are relatively tractable - "string-field-theory-lite". Many non-trivial classical solutions.
- Can already see novel string-like generalizations of geometry.
- Perhaps these Vasiliev theories are related to conventional AdS_3 string theories (in a tensionless limit).

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Two kinds of Coset Dualities

Vector-Like Cosets: Central charge $c \propto N$. For example $\mathcal{W}_N$ minimal models

$$\frac{SU(N)_k \times SU(N)_1}{SU(N)_{k+1}}.$$

Take $k, N \to \infty$ with ratio fixed. (Gaberdiel and R. G.)

Two kinds of Coset Dualities

Matrix-Like Cosets: Central charge $c \propto N^2$. For example, extended $\mathcal{W}_N$ models

$$\frac{SU(N)_k \times SU(N)_\ell}{SU(N)_{k+\ell}}.$$

Take $k, \ell, N \to \infty$ with ratios fixed. E.g. when $k = \ell = N$ (R. G., Hashimoto, Klebanov, Schoutens, Sachdev). Also other examples Niarchos, Kiritsis; Creutzig-Hikida-Ronne.

Presumably dual to a full fledged AdS_3 string theory. Background not yet identified.

Vector Coset Holography

The CFT: Minimal $\mathcal{W}_N$ series (generalising the Virasoro unitary series for $N = 2$)

$$\frac{SU(N)_k \times SU(N)_1}{SU(N)_{k+1}}.$$

A line of fixed points when $k, N \to \infty$ (labelled by 'tHooft coupling $0 \leq \lambda = \frac{N}{N+k} \leq 1$) with $c_N(\lambda) = N(1 - \lambda^2)$.

The Bulk: Vasiliev higher spin theory in AdS_3 with (infinite dimensional) gauge group $hs[\lambda]$ coupled to one complex scalar of mass

$$M^2 = -1 + \lambda^2.$$

Central charge $c = \frac{3R_{AdS}}{2G_N}$ (Campoleoni et.al., Henneaux-Rey) i.e. $G_N \propto \frac{1}{N}$.

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What we know about the Duality

Matching symmetries

- Underlying $\mathcal{W}_\infty[\lambda]$ symmetry.
- At finite N, (i.e. finite c) both sides have $\mathcal{W}_N$ symmetry - due to a nontrivial equivalence of the quantum $\mathcal{W}_\infty[\lambda]$ symmetry algebra (Gaberdiel-R.G.).
- Powerful non-linear symmetry which largely constrains the structure of the theory.

What we know about the Duality

Matching Spectrum

- Perturbative excitations. (Gaberdiel-R.G.-Hartman-Raju) quanta of gravitons, scalars and higher spin fields.
- Non-perturbative excitations ("conical defects") (Castro-R.G.-Gutperle-Raeymakers; Perlmutter, Prochazka, Raeymakers). Related by analytic continuation to a class of "light states" (bound to perturbative quanta).
- Black Hole States. (Kraus-Perlmutter, Gaberdiel-Hartman-Jin) Growth of CFT states matches the entropy at asymptotically high temperatures.

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Generalisations

- Can be generalized to other 2d vector cosets with $SO(N)$ (Ahn, Gaberdiel-Vollenweider)
- $\mathcal{N} = 1, 2$ Kazama-Suzuki models (Creutzig-Hikida-Ronne; Beccaria et.al.).
- Large $\mathcal{N} = 4$ Cosets and relation to String theory on $AdS_3 \times S^3 \times S^3 \times S^1$ (Gaberdiel-R.G.).

Analogue of 3d/4d proposal relating vector models (in general with Chern-Simons interactions) to higher spin/string theories on AdS_4. (Klebanov-Polyakov; Sezgin-Sundell; Giombi-Yin ; GMPTWY; Aharony et.al.; Chang et.al.)
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What Can We Learn from These Dualities?

- How does AdS/CFT work in a non-SUSY context?
- Exploit higher spin symmetry as a substitute for SUSY to constrain dynamics and extract information.
- Opens up the possibility to study holographic duals of solved large N 2d QFTs (generically massive) e.g. 't Hooft model of QCD_2.
- Integrate solvable 2d theories into the framework of gauge-string dualities and gain a general understanding of the domain of applicability of these ideas.
- Can try to address the tension between integrability and thermalisation (formation of black holes) in a simple situation. (cf. Gautam's talk)
- Toy model for understanding features of stringy geometry in a gauge invariant way. How does one talk about a horizon and its thermal entropy? Also entanglement entropy - de Boer-Jottar; Ammon et.al.).

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HS AdS/CFT and Gauge-String Duality

Can we fit in (HS AdS)/(vector CFT) examples into the general framework of Gauge-String dualities?

- Need to do so to make sense of some novel features.
- Dual CFTs on $S^1 \times \mathbb{R}$ (or on $\Sigma_{g\geq 1} \times \mathbb{R}$) has a continuum of light states for $N \to \infty$.
- No independent bulk prescription for computing quantum corrections ($\frac{1}{N}$ corrections). An embedding into string theory would give a definition.

Are there tensionless non-SUSY AdS_3 string theories for non-SUSY cosets?
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Large $\mathcal{N} = 4$ Holography

Motivation

- An AdS_3 background with a large amount of supersymmetry is much more constrained.
- Many backgrounds with $\mathcal{N} = 2$ SUSY but much fewer with $\mathcal{N} = 4$ (i.e. (4,4) SUSY).
- Analogue of ABJ theories in 3d dual to non-abelian Vasiliev theory and string theory on $AdS_4 \times \mathbb{C}P^4$ (Chang-Minwalla-Sharma-Yin).

- A general $\mathcal{N} = 4$ SCFT has the so-called large $\mathcal{N} = 4$ SUSY algebra.
- Contains *two* $SU(2)$ Kac-Moody algebras (unlike a single $SU(2)$ in the usual/small $\mathcal{N} = 4$ case). Labelled by a single parameter - the ratio of the two levels.
- CFT dual to string theory on $AdS_3 \times S^3 \times S^3 \times S^1$ should have this large $\mathcal{N} = 4$ symmetry (Boonstra et.al., Elitzur et.al.).

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Large $\mathcal{N} = 4$: A Crash Course

- The global part of the large $\mathcal{N} = 4$ algebra is the $D(2,1|\alpha)$ Lie superalgebra.
- Generated by $L_{0,\pm 1}, G^a_{\pm\frac{1}{2}}, A^{\pm,i}_0$.

$$
\begin{aligned}
[L_m, L_n] &= (m-n)\, L_{m+n}; \qquad [L_m, G^a_r] = (\tfrac{m}{2} - r) G^a_{m+r} \\
[A^{\pm,i}_0, G^a_r] &= \mathrm{i}\, \alpha^{\pm\, i}_{ab} G^b_r; \qquad [A^{\pm,i}_0, A^{\pm,j}_0] = \mathrm{i}\, \epsilon^{ijl} A^{\pm,l}_0 \\
\{G^a_r, G^b_s\} &= 2\delta^{ab}\, L_{r+s} + 4\,(r-s) \left(\gamma\, \mathrm{i}\, \alpha^{+\, i}_{ab}\, A^{+,i}_{r+s} + (1-\gamma)\, \mathrm{i}\, \alpha^{-\, i}_{ab}\, A^{-,i}_{r+s} \right) ,
\end{aligned}
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Here $\alpha = \frac{\gamma}{1-\gamma}$ is a free parameter of the algebra.

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Here $\alpha = \frac{\gamma}{1-\gamma}$ is a free parameter of the algebra.

Large $\mathcal{N} = 4$: A Crash Course

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- Generated by $L_{0,\pm 1}, G^a_{\pm\frac{1}{2}}, A_0^{\pm,i}$.

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- String theory on $AdS_3 \times S^3 \times S^3 \times S^1$ preserves $(4,4)$ SUSY.
- Characterised by two D5 brane charges (fluxes) $Q_5^{\pm}$ and a D1 charge Q_1.
- Dual CFT should have two $SU(2)$'s with levels $k^{\pm} = Q_5^{\pm} Q_1$.
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- However, no candidate CFT dual for the general case.
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Large $\mathcal{N}=4$ Vector Coset Holography

Strategy: Start with a vector coset duality with large $\mathcal{N}=4$. Then generalize to $U(M)$ Vasiliev theory and identify a candidate M-extended coset for the string background (in the tensionless limit).

The CFT

$$\frac{\mathfrak{su}(N+2)^{(1)}_{\kappa}}{\mathfrak{su}(N)^{(1)}_{\kappa}} \cong \frac{\mathfrak{su}(N+2)_k \oplus \mathfrak{so}(4N+4)_1}{\mathfrak{su}(N)_{k+2}} .$$

Here $c=\frac{6(k+1)(N+1)}{k+N+2}$. Take $k, N \to \infty$ keeping $\lambda = \frac{N+1}{N+k+2} = \gamma$ fixed.

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The Vasiliev theory

- Bulk theory based on a higher spin algebra $shs_2[\gamma]$ which has global superalgebra $D(2,1|\alpha)$.
  Just as $hs[\lambda]$ has $sl(2)$ as its global subalgebra.
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Spectrum

- There are 8 fields of each spin $s>1$ (seven with $s=1$). They are organized into multiplets $R^{(s)}$ of $D(2,1|\alpha)$ as follows:

$$R^{(s)}: \begin{array}{ll} s: & (\mathbf{1},\mathbf{1}) \\ s+\frac{1}{2}: & (\mathbf{2},\mathbf{2}) \\ s+1: & (\mathbf{3},\mathbf{1})\oplus(\mathbf{1},\mathbf{3}) \\ s+\frac{3}{2}: & (\mathbf{2},\mathbf{2}) \\ s+2: & (\mathbf{1},\mathbf{1})\ . \end{array}$$

- Also two massive BPS multiplets of fields

$$\phi_+ : (\mathbf{2},\mathbf{1})_{s=0}\oplus(\mathbf{1},\mathbf{2})_{s=\frac{1}{2}}\ , \qquad \phi_- : (\mathbf{1},\mathbf{2})_{s=0}\oplus(\mathbf{2},\mathbf{1})_{s=\frac{1}{2}}\ ,$$

ϕ_- has two complex scalars $M_-^2=-1+\gamma^2$ and two dirac fermions with $m^2=(\gamma-\frac{1}{2})^2$. Similarly for ϕ_+ with $M_+^2=-1+(1-\gamma)^2$.

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Comparison

- Higher spin symmetry currents of coset match the gauge fields of the Vasiliev theory.
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- Can we generalize this coset duality for the Vasiliev theory to one for the string theory?
- In analogy with the case for ABJ theories (Chang et.al.), generalize the Vasiliev theory to one with additional $U(M)$ Chan-Paton indices.
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- Important to identify a viable CFT candidate to complete the triality between cosets, Vasiliev theory and string theories.

To Wrap Up

- Learnt many things about vector coset holography from explicit checks and construction of classical solutions.
- Good launch pad for understanding non-SUSY AdS/CFT.
- But also time to integrate these examples into string theory.
- Made a start with the large $\mathcal{N} = 4$ SUSY example - a nice example of vector coset holography.
- Non-abelian Vasiliev theory seems to capture stringy (BPS) spectrum.
- But CFT for stringy limit yet to be identified.