

FROM NAVIER-STOKES TO EINSTEIN

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ARF

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$$\epsilon \quad G_{\mu\nu} = 0$$

$$\dot{V}_\mu = \eta \partial^\mu V_\mu + \partial_\mu P + V^\mu \partial_\mu V_\mu = 0$$

$$\partial_\mu V^\mu = 0$$

d. Amour, Price & Torne
Membrane Paradigm

RELATED?

Policastro Starinets
Kortun Son Battacharya
Minwalla Wadia AdS/CFT
Eling Fouxon OZ

HYDRODYNAMIC LIMIT

3

Scaling:

new solution:

$$v_i^\varepsilon(x, t) = \varepsilon v_i(\varepsilon x, \varepsilon^3 t)$$
$$P^\varepsilon(x, t) = \varepsilon^2 P(\varepsilon x, \varepsilon^3 t)$$

IF

$$v_i - \eta \partial^2 v_i + v^k \partial_k v_i - \partial_i P + v^k v^l \partial_k \partial_l v_i = 0$$

THEN

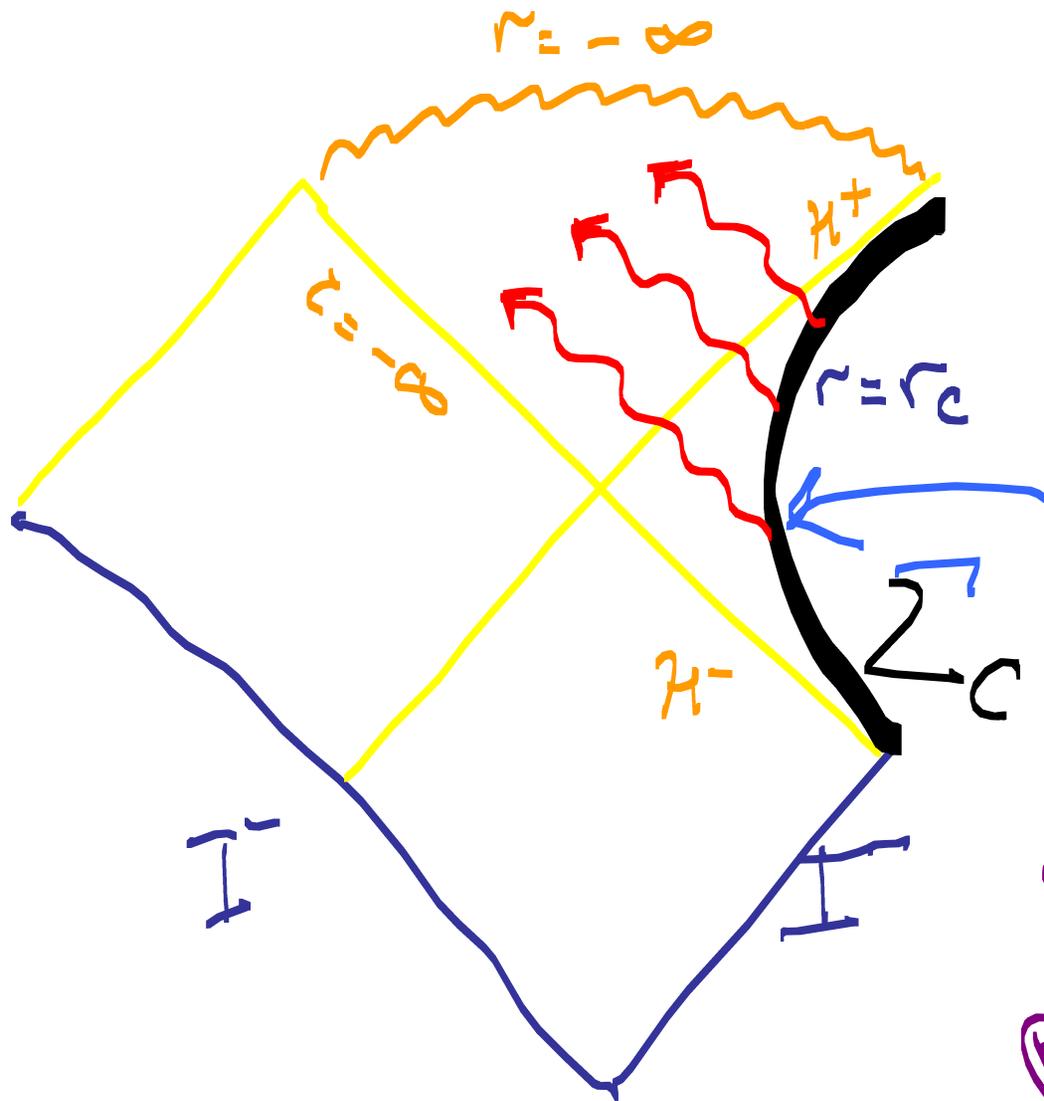
$$v_i^\varepsilon - \eta \partial^2 v_i^\varepsilon + v^\varepsilon{}^k \partial_k v_i^\varepsilon - \partial_i P^\varepsilon + \varepsilon^2 v^\varepsilon{}^k v^\varepsilon{}^l \partial_k \partial_l v_i^\varepsilon = 0$$

EXTRA TERMS "IRRELEVANT"

INCOMPRESSIBLE NS IS UNIVERSAL

OUR SETUP

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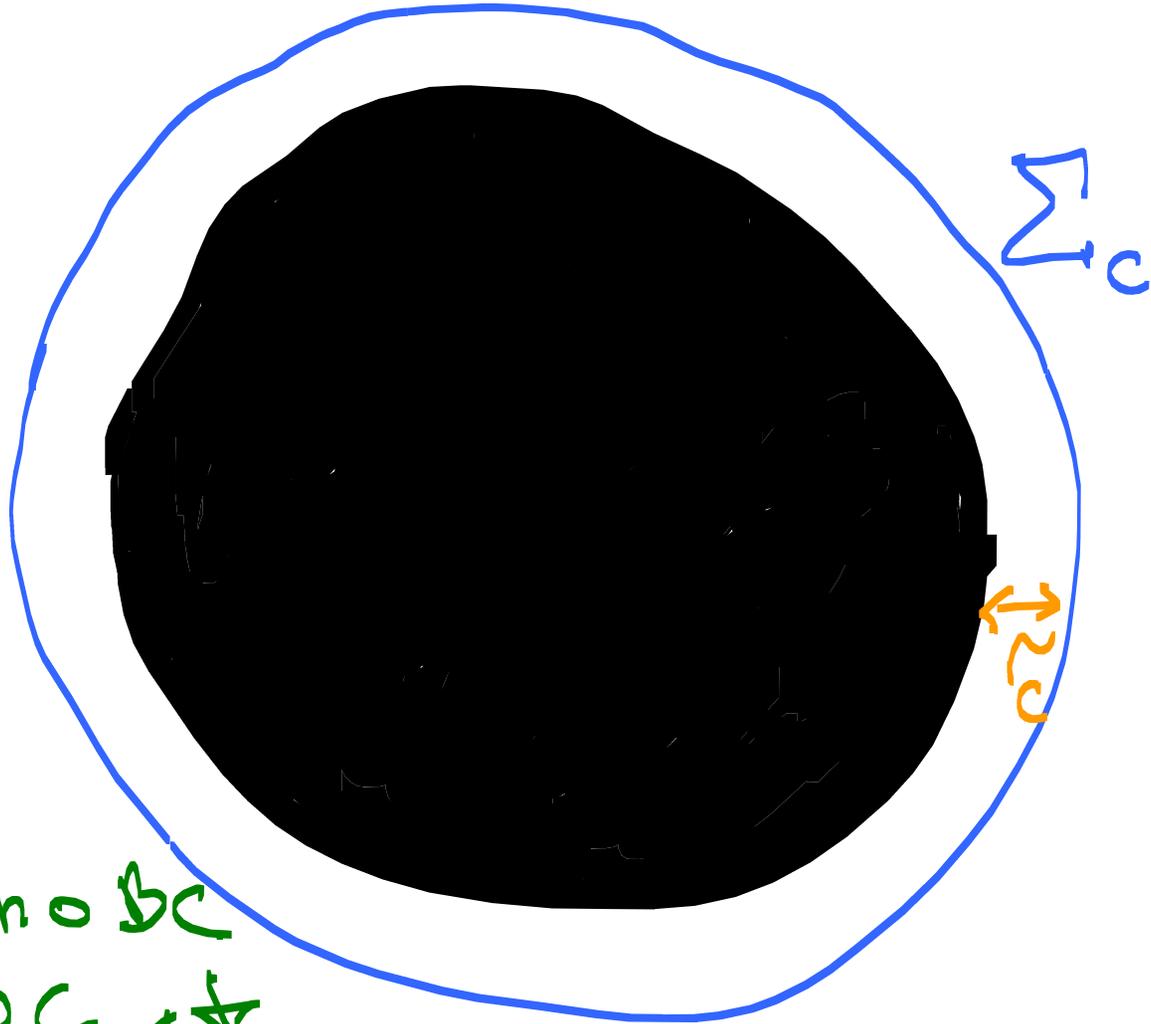


fluid
lives
here

Shooting
problem

ISOLATING B.C.S

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dAmour: no BC
BMW: BC at
 $r \rightarrow \infty$

THE SOLUTION

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$$ds^2_{p+2} = -r dz^2 + 2 dz dr + dx_i dx^i \quad \mathcal{O}(1)$$

$$- 2 \left(1 - \frac{r}{r_c}\right) v_i dx^i dz - 2 \frac{v_i}{r_c} dx^i dr \quad \mathcal{O}(\epsilon)$$

$$+ \left(1 - \frac{r}{r_c}\right) \left[(v^2 + 2p) dz^2 + \frac{v_i v_j}{r_c} dx^i dx^j \right] + \frac{r^2 + 2p}{\epsilon} dz dr \quad \mathcal{O}(\epsilon^2)$$

$$\sim \frac{r^2 - r_c^2}{r_c} \delta^2 v_\lambda dx^\lambda dz + \dots \quad \mathcal{O}(\epsilon^3)$$

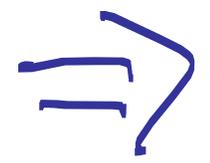
$$v_i \sim \mathcal{O}(\epsilon) \quad p \sim \mathcal{O}(\epsilon^2) \quad d_i \sim \mathcal{O}(\epsilon), \quad dz \sim \mathcal{O}(\epsilon^2)$$

Add Λ , take $r_c \rightarrow \infty$, $ds^2 \rightarrow \text{BMN}$

IRF $G_{\mu\nu} = O(\epsilon^4)$

$$\partial_\kappa v^\kappa = 0$$

$$\partial_\kappa v_i - \nu \partial^2 v_i + \partial_i P + v^\kappa \partial_\kappa v_i = 0$$



$$T_{\alpha\beta}$$

$$= \gamma^{\alpha\beta} \kappa - \kappa^{\alpha\beta}$$

incompressible fluid stress tensor

Compare McFadden

solution to all orders Skenderis Taylor

NEAR-HORIZON EXPANSION 8

Instead of expanding in ϵ , fix ϵ
expand in r_c (rescale $r \rightarrow \frac{r}{r_c}$ so
 Σ_c at $r=1$)

$$ds^2_{pt2} = -\frac{r}{r_c} dz^2 +$$

$$+ 2dz dr + dx_i dx^i + 2(r-1)v_i dx^i dt + \dots$$

Looks different, but after $z \rightarrow \frac{z}{\epsilon^2}$ $x^i \rightarrow \frac{x^i}{\epsilon^2}$
 $v \rightarrow \epsilon v$ + rescaling \rightarrow **JAME!!!**

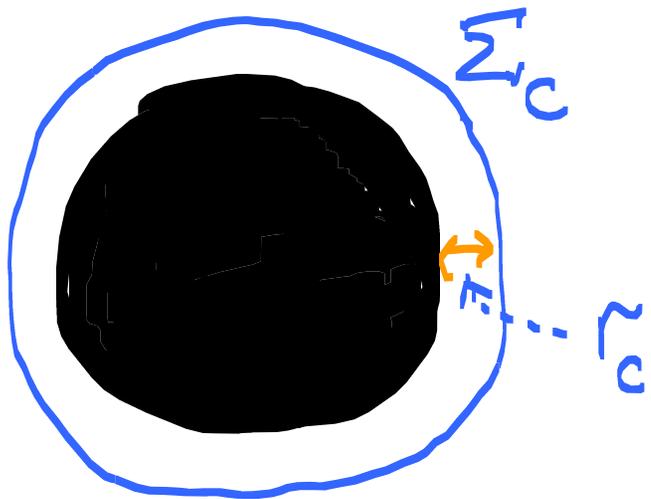
HYDRODYNAMIC
EXPANSION

=

NEAR-HORIZON
EXPANSION!!!

SPHERICAL HORIZONS ¹⁰

L. Bredberg EAS



Schwarzschild case similar upto one surprise. Obstruction at third order \Rightarrow

Neuman ~~Dirichlet~~ for

metric conformal factor.

$$ds^2_{\text{ptl}}(\Sigma_c) = \Omega^2(x,t) (-dt^2 + r_c^2 d\Omega_p^2)$$

$K(\Sigma_c) = K_0$ same at leading order
 \swarrow mathematically natural!

EINSTEIN-PETROV

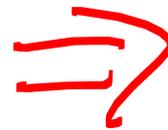
→ NAVIER-STOKES

V. Lysov E.A.S.

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Type I $\mathcal{L}(\xi^i, \xi^j) = 0$
+ constraints on ξ_c
+ $\kappa_0 \rightarrow \infty$



incompressible NS

The Petrov type I condition reduces
d.o.f. of bulk theory in ptZ to
boundary theory in $pt1, HO HO GRAPHIC$

FUTURE PROBLEMS

1. Find soluble examples.
2. How is bulk r -dependence related to N-S RG flow?
3. Map turbulence, global existence to GR problem.
4. What are the implications for the holographic nature of space time?