Thermodynamics and Rotons in a Holographic Quantum Hall Model

Matti Järvinen

University of Crete

Milos – 21 June 2011

Outline

"Part II"

- Thermodynamics and phase structure of the D2-D8' model
- Fluctuation spectrum, e.g., magneto-roton
- Conclusions and open questions

Thermodynamics and phase structure (D2-D8')

Reminder of parameters

- u_T: "Temperature" ($\propto T^{2/3}$)
- h: Magnetic field ($\propto$ physical H)
- d: Charge density ($\propto D$) $\leftrightarrow$ μ: Chemical potential

We keep fixed

- $b = 1$: Bulk flux
- $m = 0.1$: Fermion mass

Thermodynamics and phase structure (D2-D8')

- Grand canonical ensemble:

$$\Omega(\mu, T, h) = \frac{1}{\mathcal{N}} S^E_{\mathrm{on-shell}} = \frac{1}{\mathcal{N}} \left(S^E_{DBI+CS} + S^E_{bdry} + S_{CT} \right)\Big|_{\mathrm{on-shell}}$$

with $\mu = a_0(u = \infty)$

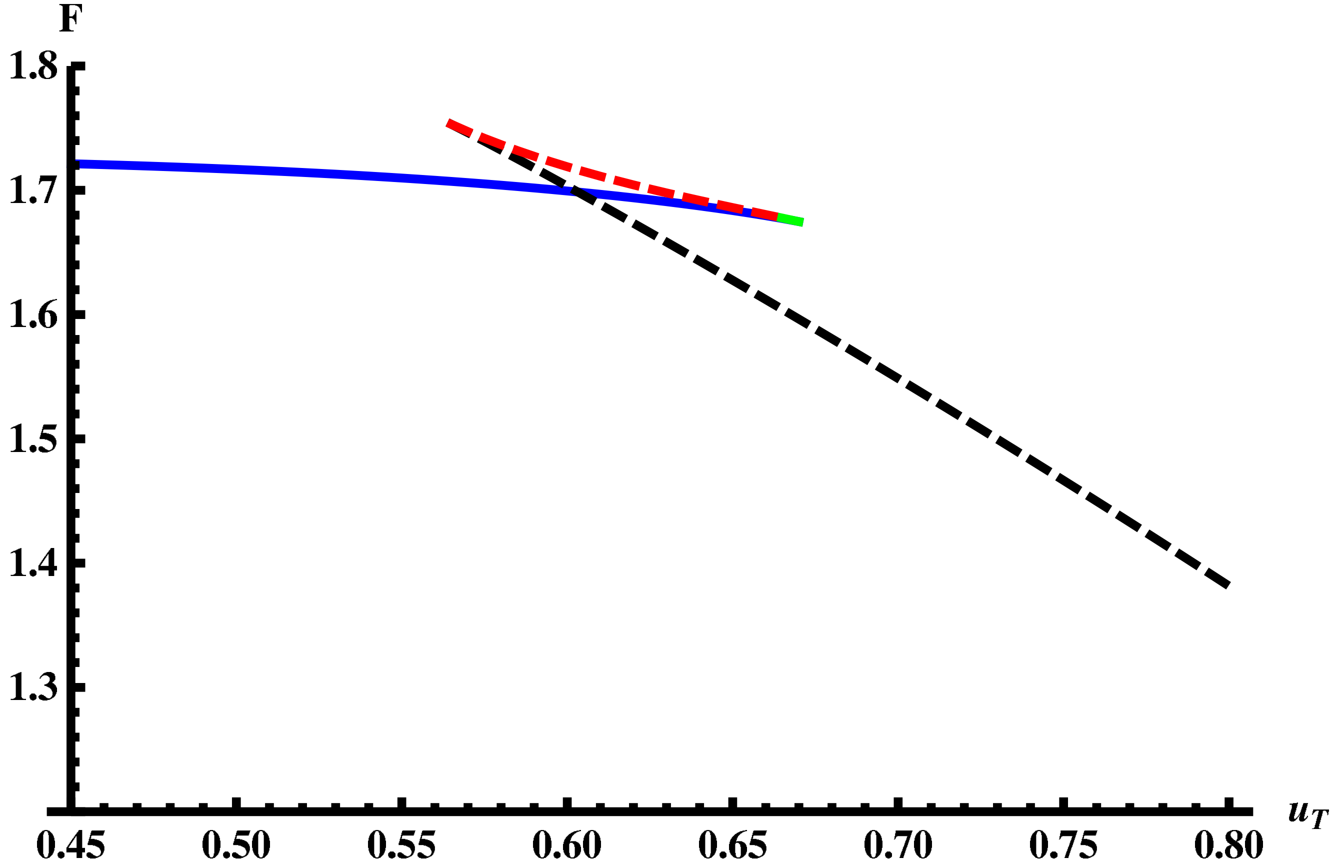
  - A boundary term S^E_{bdry} required for gauge invariance, and a counterterm S_{CT} to remove UV divergencies

- Canonical ensemble:

$$F(d, T, h) = \Omega(\mu(d), T, h) + d\mu(d)$$

Thermodynamics and phase structure (D2-D8')

High temperature phase transition at locking, $\frac{2}{3}h = d = 1$

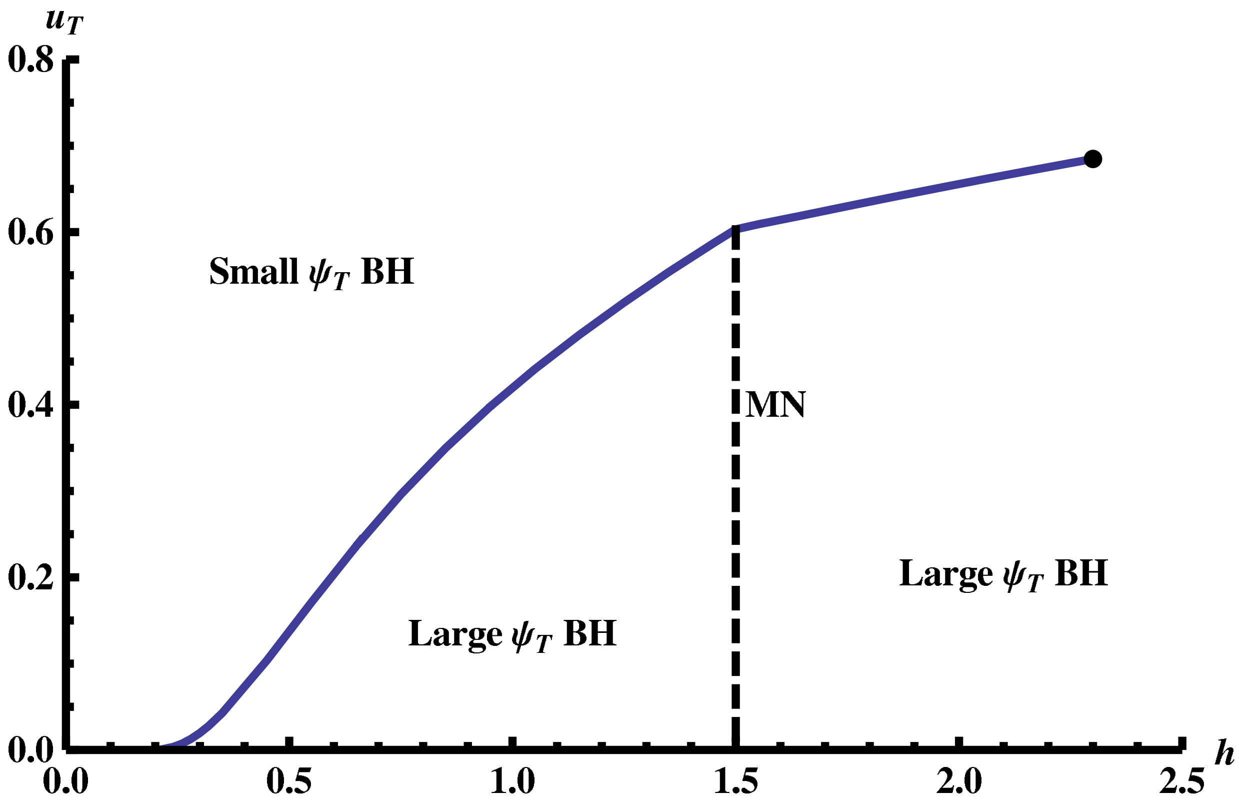


- With increasing u_T, first order transition MN $\longrightarrow$ BH
- Unstable MN and BH phases

Thermodynamics and phase structure (D2-D8')

(h, u_T) phase diagram for fixed charge density d

- BH $\longrightarrow$ BH transitions 1st order
- MN shrinks to a line – fixed d and h
- Spikey BH solutions in the vicinity of the MN line

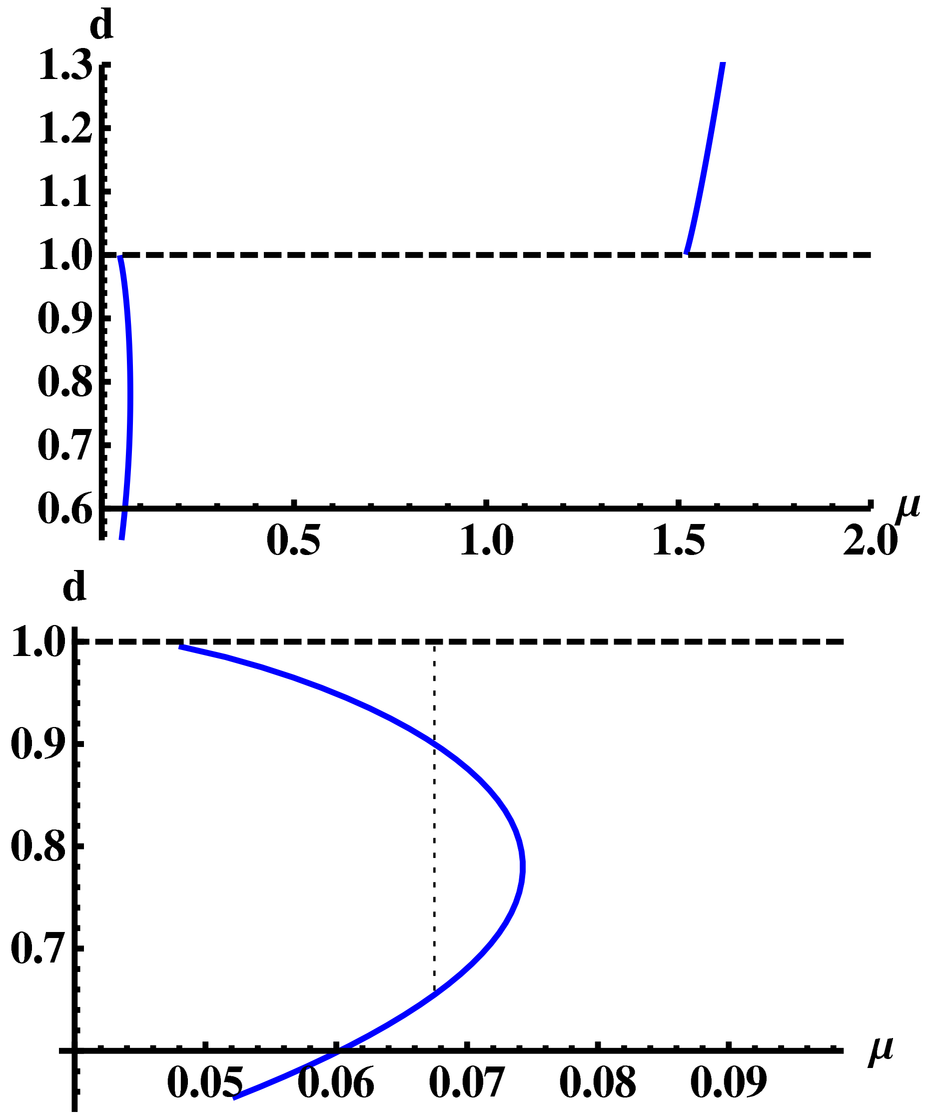


How about the MN $\longrightarrow$ large ψ_T BH transitions?

Thermodynamics and phase structure (D2-D8')

Transition from MN to BH (large ψ_T, fixed u_T):

- Better viewed in the grand canonical ensemble: $d \to \mu$; $F \to \Omega$
- First and second order transitions
- We plot $d = -\frac{\partial \Omega}{\partial \mu}$



Fluctuation analysis

- Earlier work: spectra for small fluctuations in the MN phase at zero temperature for D3-D7' [Jokela, Lifschytz, Lippert]
- This talk: finite temperature analysis of MN phase for D2-D8'
  - $\mathcal{L} = \mathcal{L}_0 + \mathcal{L}_1 + \mathcal{L}_2 + \cdots$
  - Use isotropic radial coordinate $\longrightarrow$ drastic simplifications

$$r^{5/2} = \left(\frac{u}{u_T}\right)^{5/2} + \sqrt{\left(\frac{u}{u_T}\right)^5 - 1}$$

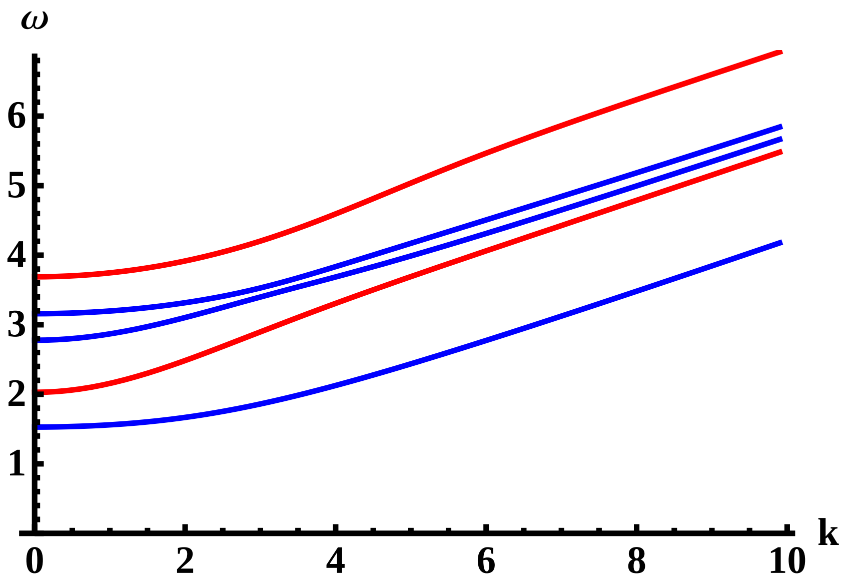
  - Still, lengthy EOMs . . .
  - Look for normalizable modes, Ansatz

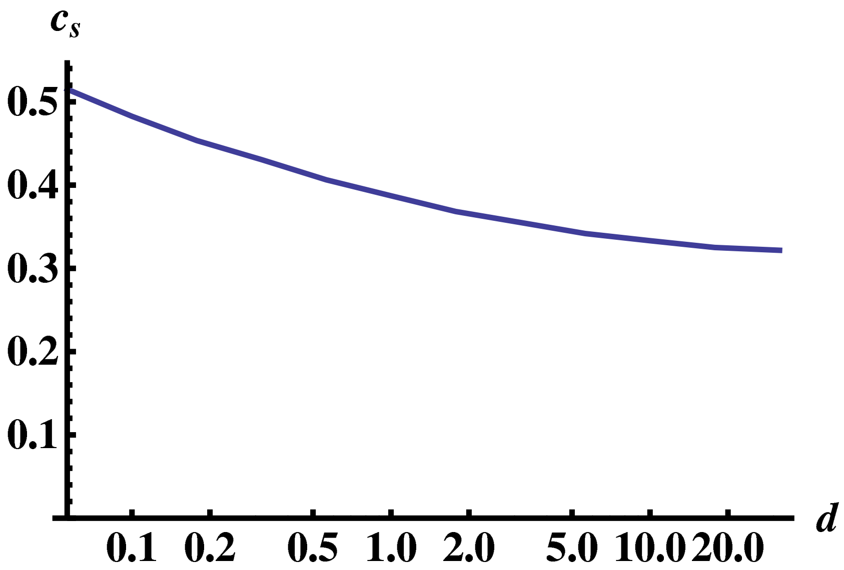
$\delta a_x(t, x, r) \sim \delta a_x(r) e^{-i\omega t + ikx}$ etc.

Fluctuation analysis

$$\omega \simeq \sqrt{\omega_0^2 + c_s^2 k^2}$$

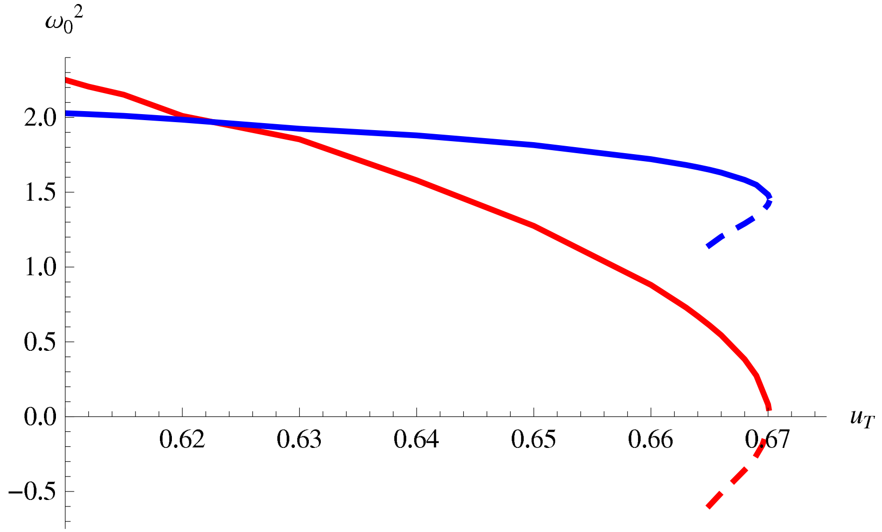
- Scalar (embedding) and vector (gauge field) fluctuations decouple at $k = 0$
- The system is stable and gapped
- Speed of sound c_s is independent of excitation level

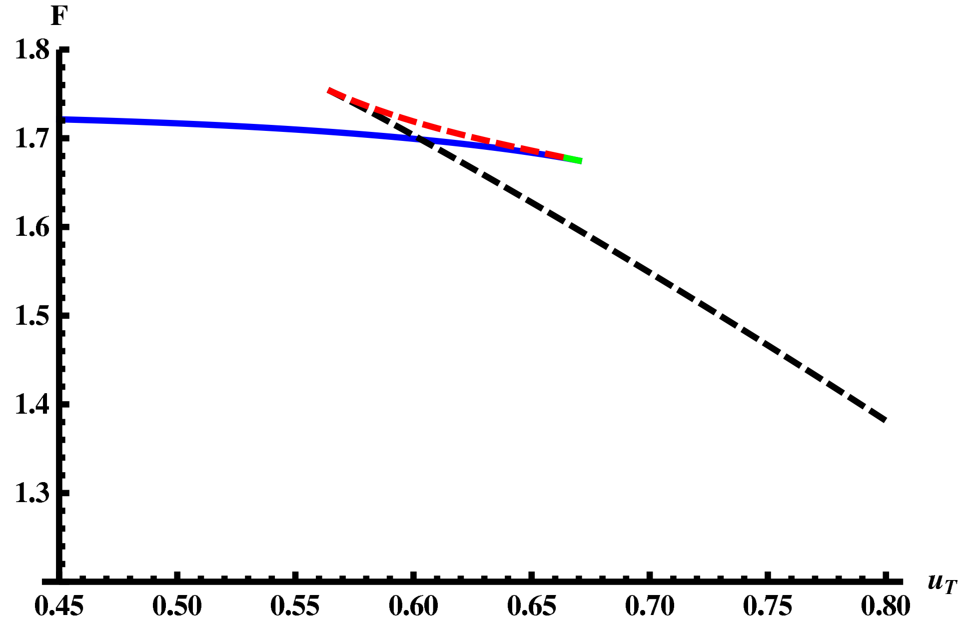




Fluctuation analysis

ω_0 vs. u_T

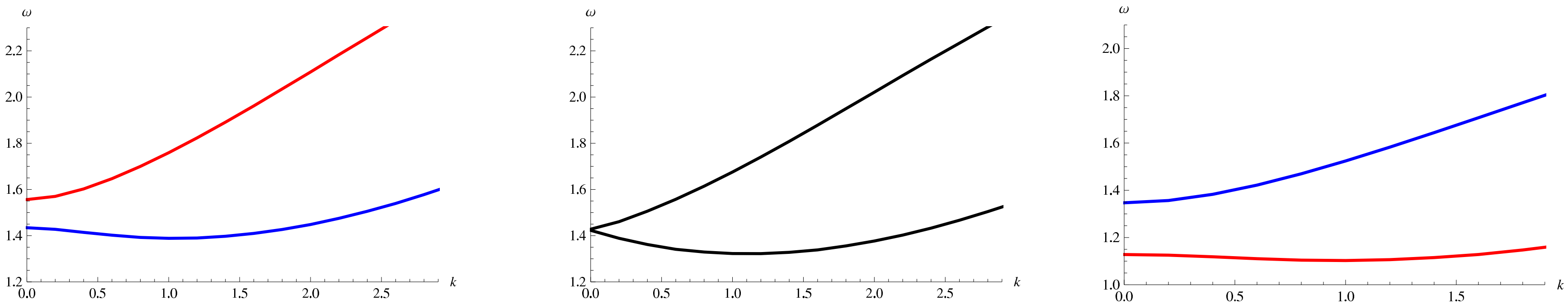




- A tachyonic mode only appears in the (perturbatively) unstable MN branch (green on the right)
- Level crossing of the two lowest modes seen

Magneto-roton excitation(s)

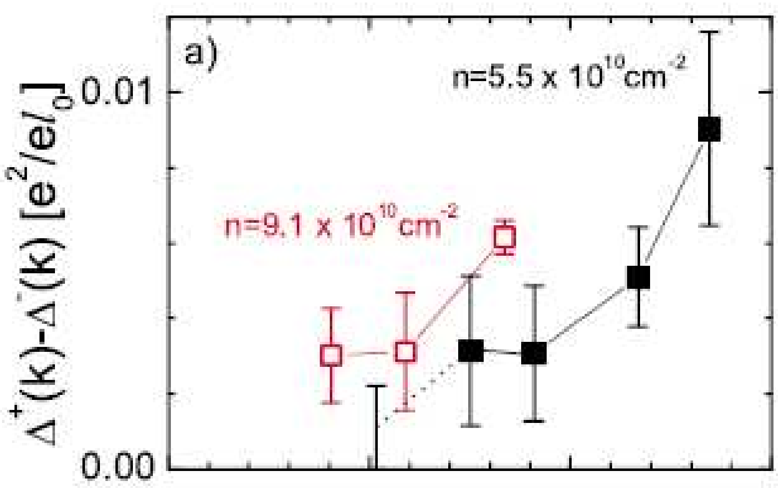
- Depending on the density (magnetic field) and temperature one finds a magneto-roton: $\omega^2 \simeq \sqrt{\omega_*^2 + c_s^2(k-k_*)^2}$



- Related to level crossing of scalar and vector modes

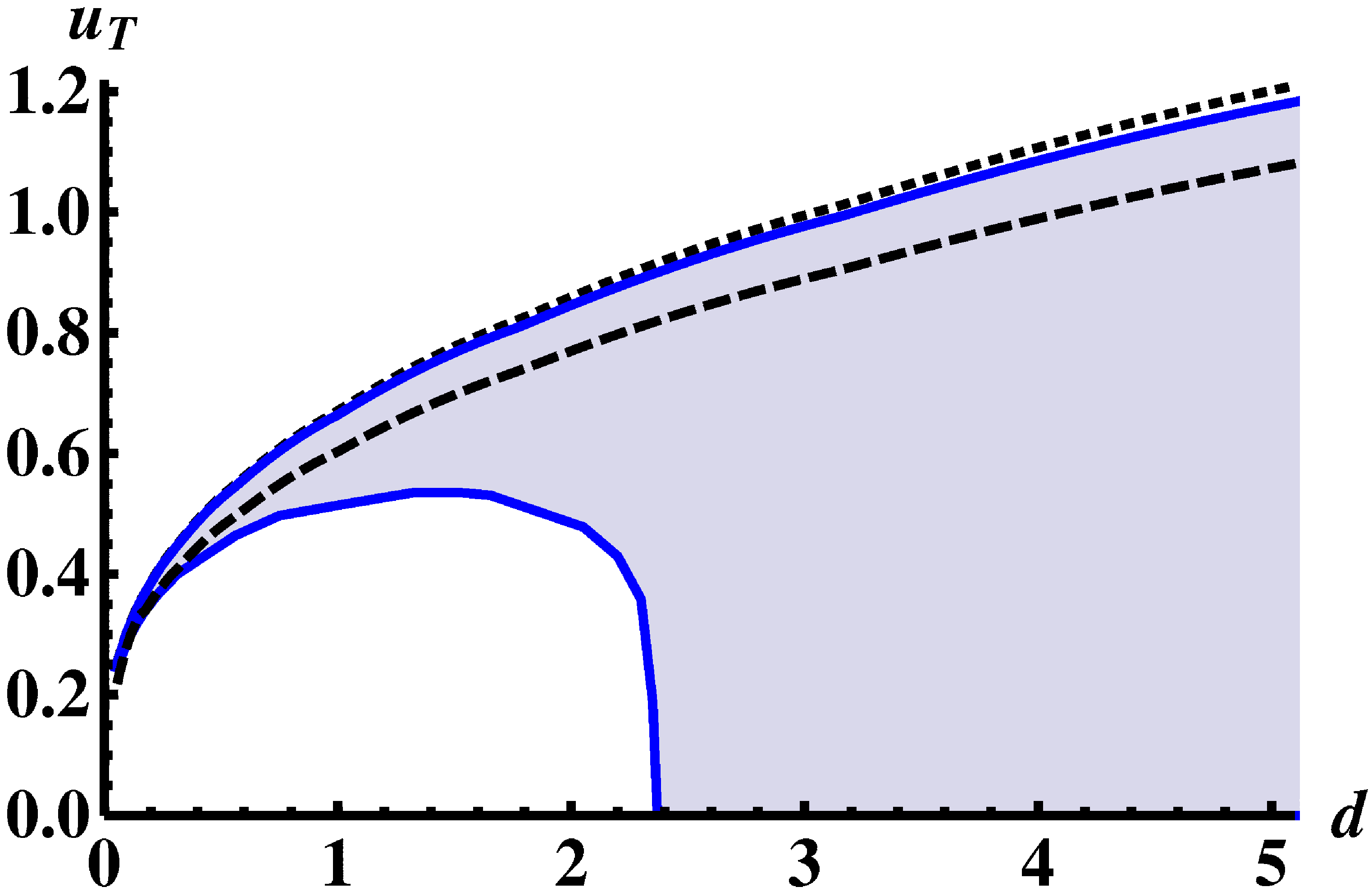
- Detected in many experiments, e.g., $\nu = 1/3$

[Hirjibehedin,Dujovne,Pinczuk,Dennis,Pfeiffer,West ’05]



Magneto-roton excitation(s)

Where's the roton? (First excitation)

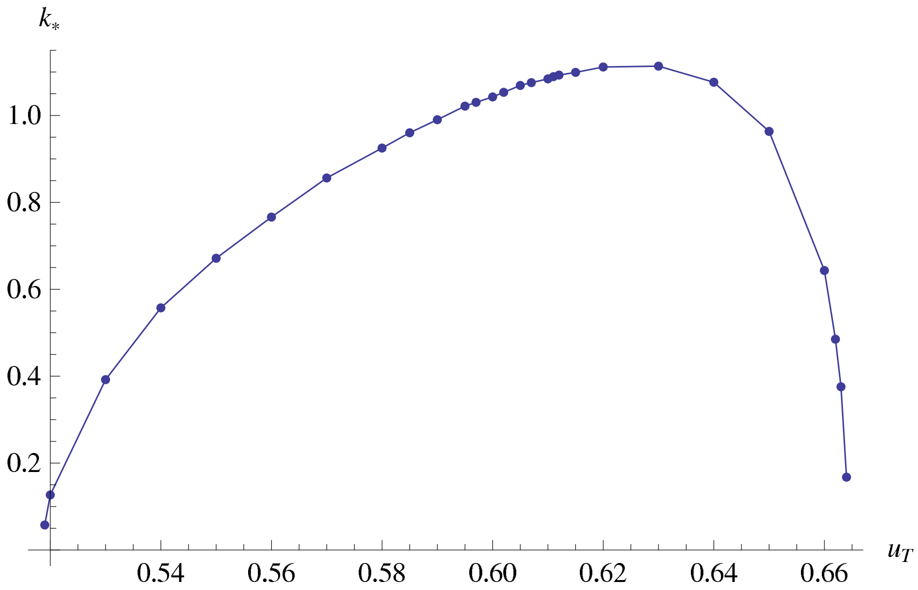


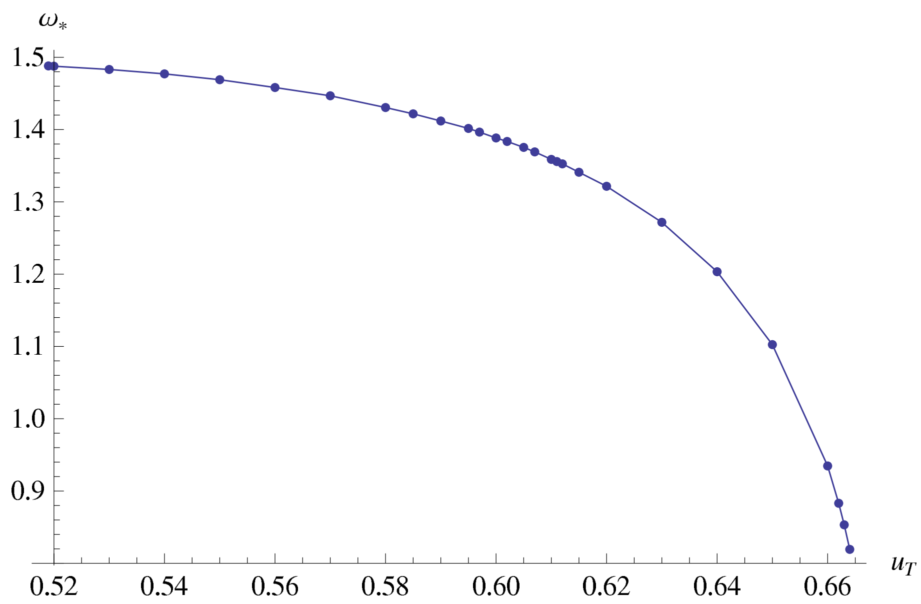
Dashed line: end of stable region – Dotted: end of MN solution

Magneto-roton excitation(s)

$$\omega^2 \simeq \sqrt{\omega_*^2 + c_s^2(k - k_*)^2}$$

k_* and ω_* of the roton with varying temperature ($d = 1$)





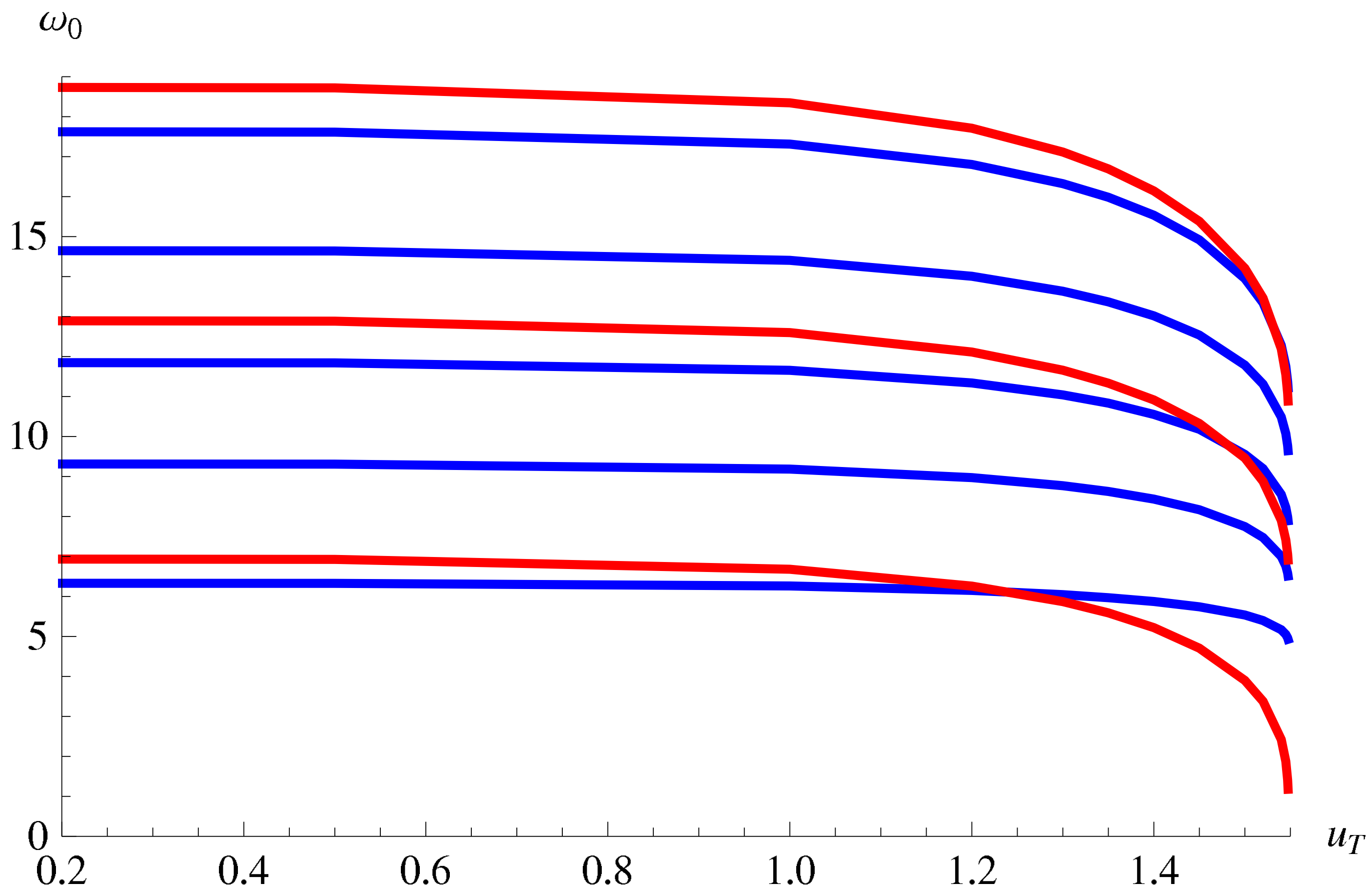
Magneto-roton excitation(s)

Level-crossings between scalar and vector modes in the high-excitation spectrum near the critical $T \Rightarrow$ more rotons

Regular pattern:

1st, 4th, 7th, . . .

modes rotonic



Conclusions (Summary of part II)

Holographic model of a QHF of strongly interacting charged fermions in 2+1 dimensions:

- Rich phase structure with first and second order transitions
- Excitation spectra: magneto-rotons $\longleftrightarrow$ level crossings

Conclusions (Open questions)

Things to do/understand better:

- ❏ Several filling fractions, and transitions between them?
- ❏ Quasiparticles/fractional charge?
- ❏ Edge states?
- ❏ Plateaux - impurities?
- ❏ Connection to bottom-up models?
- ❏ Universal properties? (Other Dp-Dq' systems are available via T-duality.)