Holograms of Conformal Chern-Simons Gravity

Hamid Afshar

IPM-Tehran, ITP-Vienna

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Milos

Branislav Cvetković, Sabine Ertl, Daniel Grumiller and Niklas Johansson
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Introduction

- (2+1)- dimensional Einstein gravity has no local bulk d.o.f.

$$S = \frac{1}{16\pi G}\int d^3x\sqrt{-g}\,\left(R + \tfrac{2}{\ell^2}\right) \qquad \text{Diffeomorphism}$$

- One can remove all bulk d.o.f. using diffeo's. Boundary d.o.f. emerge by the nonvanishing diffeo's acting on the boundary.

- They can generate asymptotic symmetries whose generators $\tilde{G}_\xi$ satisfy two copies of Virasoro algebra with central charge $c = \frac{3\ell}{2G}$, for AdS boundary conditions.

$$[L_n, L_m] = (n-m)L_{n+m} + \tfrac{c}{12}(n^3 - n)\delta_{n+m,0}$$

where $L_n = \tilde{G}_\xi[\varepsilon^+ = e^{inx^+}]$ Brown- Henneaux 1986

The purpose of this talk

- Conformal Chern- Simons gravity has no local bulk d.o.f.

$$S = \frac{k}{4\pi}\int d^3x\, \epsilon^{\mu\nu\lambda}\, \Gamma^{\rho}{}_{\mu\sigma}\left(\partial_\nu \Gamma^{\sigma}{}_{\lambda\rho} + \tfrac{2}{3}\Gamma^{\sigma}{}_{\nu\tau}\Gamma^{\tau}{}_{\lambda\rho}\right)$$

$$C_{\mu\nu} \equiv \varepsilon_{\mu}{}^{\lambda\sigma}\nabla_{\lambda}\left(R_{\nu\sigma} - \tfrac{1}{4}g_{\nu\sigma}R\right) = 0$$

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$\nabla_\mu C^{\mu\nu} = 0, \quad C^\mu{}_\mu = 0$ Diffeomorphism $\times$ Weyl

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$$\nabla_\mu C^{\mu\nu}=0,\quad C^{\mu}{}_{\mu}=0\qquad\qquad \text{Diffeomorphism}\times\text{Weyl}$$

- Boundary d.o.f. emerge by the nonvanishing transformations generated by, $\tilde{G}_\xi$ and $\tilde{G}_W$ acting on the boundary:
  - Diffeomorphism only $\tilde{G}_\xi$.
  - Diffeomorphism with compensating Weyl $\tilde{G}_\xi+\tilde{G}_{W[\xi]}$.
  - Diffeomorphism $\tilde{G}_\xi$ and an independent Weyl $\tilde{G}_W$.

Setup

- We explore boundary d.o.f. by introducing a cylindrical conformal boundary and a metric that asymptotes to,

$$g_{\mu\nu}dx^\mu dx^\nu = e^{2\phi(x^\pm,y)}\left(\frac{dx^+dx^- + dy^2}{y^2} + h_{\mu\nu}dx^\mu dx^\nu\right)$$

  near the boundary $y = 0$. This defines our boundary condition with the subleading part $h_{\mu\nu}$,

$$\begin{pmatrix} h_{++} = \mathcal{O}(1/y) & h_{+-} = \mathcal{O}(1) & h_{+y} = \mathcal{O}(1) \\ & h_{--} = \mathcal{O}(1) & h_{-y} = \mathcal{O}(1) \\ & & h_{yy} = \mathcal{O}(1) \end{pmatrix}$$

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- Boundary conditions imposed on the Weyl factor $\Rightarrow$ Three different cases.

Case 1: Trivial Weyl factor/ Trivial Weyl rescaling

- This is the BH case, $\phi = 0$. The boundary conditions are preserved, $\mathcal{L}_\xi g_{\mu\nu} = h_{\mu\nu}$, by asymptotic Killing vectors:

$$\begin{aligned} \xi^\pm &= \epsilon^\pm(x^\pm) - \tfrac{1}{2} y^2 \partial_\mp^2 \epsilon^\mp(x^\mp) + \mathcal{O}(y^3), \\ \xi^y &= \frac{y}{2}\left(\partial_+\varepsilon^+ + \partial_-\varepsilon^-\right) + \mathcal{O}(y^3), \end{aligned}$$

  and also a trivial Weyl rescaling,

$$g_{\mu\nu} \to e^{2\Omega} g_{\mu\nu} \quad \text{where} \quad \Omega = \mathcal{O}(y^2).$$

- The corresponding generators satisfy two copies of Virasoro algebra with central charges,

$$c_{R/L} = \pm 12k \quad \text{with} \quad L_n = \tilde{G}_\xi[\varepsilon^+ = e^{inx^+}, \varepsilon^- = 0].$$

- All results in this case are reproduced as a limiting case of TMG: $\mu \to 0$, $\kappa^2\mu \to$ finite. $S_{TMG} = S_{EH} + \frac{1}{\mu} S_{CS}$

- Calculating the response functions using AdS/CFT dictionary. First variation of the on-shell action $\delta S = \delta S|_{\mathrm{EOM}} + \delta S_b|_{\mathrm{EOM}}$

$$\delta S = \frac{1}{2}\int_{\partial M} d^2x\sqrt{-\gamma^{(0)}}\left(T^{\alpha\beta}\delta\gamma^{(0)}_{\alpha\beta} + J^{\alpha\beta}\delta\gamma^{(1)}_{\alpha\beta}\right)$$

Gaussian normal coordinates, asymptotic expansion, $e^\rho \propto 1/y$

$$ds^2 = d\rho^2 + \left(\gamma^{(0)}_{\alpha\beta}\, e^{2\rho} + \gamma^{(1)}_{\alpha\beta}\, e^{\rho} + \gamma^{(2)}_{\alpha\beta} + \dots\right) dx^\alpha dx^\beta$$

where $\gamma^{(0)}$ is the boundary metric, $\gamma^{(1)}$ describes Weyl graviton mode, and $\gamma^{(2)}$ contains information about the left- and right-moving massless boundary gravitons. The response functions $T^{\alpha\beta}$ and $J^{\alpha\beta}$ are correlated as,

$$\langle J(z,\bar z)J(0,0)\rangle = \frac{2k\bar z}{z^3}, \qquad \langle T^R(z)T^R(0)\rangle = \frac{6k}{z^4}$$

Li *et al*, Skenderis *et al.*

Case 2: Fixed Weyl factor/ Fixed Weyl rescaling

- An arbitrary but fixed Weyl factor of the form,

$$\phi = \alpha \log y + f(x^+, x^-) + \mathcal{O}(y)$$

- The boundary conditions are preserved, $\mathcal{L}_\xi g_{\mu\nu} = \Omega g_{\mu\nu}$, by asymptotic conformal Killing vectors:

$$\begin{aligned}
\xi^\pm &= \epsilon^\pm(x^\pm) - \tfrac{1}{2} y^2 \partial_\mp^2 \epsilon^\mp(x^\mp) + \mathcal{O}(y^3), \\
\xi^y &= \frac{y}{2}\left(\partial_+ \varepsilon^+ + \partial_- \varepsilon^-\right) + \mathcal{O}(y^3), \\
\Omega &= -\frac{\alpha}{2}\left(\partial_+ \varepsilon^+ + \partial_- \varepsilon^-\right) - \left(\varepsilon^+ \partial_+ + \varepsilon^- \partial_-\right) f + \mathcal{O}(y^2).
\end{aligned}$$

- To remove gravitational anomaly $f(x^\pm) = f_+(x^+) + f_-(x^-)$.

- The generators of these asymptotic CKV's satisfy two copies of Virasoro algebra with central charges,

$$c_{R/L} = \pm 12k \quad \text{with} \quad L_n = \tilde{G}_\xi[\varepsilon^+ = e^{inx^+}, \varepsilon^- = 0] + \tilde{G}_W[\Omega(\varepsilon^+)].$$

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Case 3: Free Weyl factor/ Free Weyl rescaling

- An arbitrary and free Weyl factor of the form $(\alpha = 0)$,

$$\phi = f_+(x^+) + f_-(x^-) + \mathcal{O}(y)$$

- The boundary conditions are preserved, $\delta g_{\mu\nu} = \Omega g_{\mu\nu}$, by asymptotic Weyl rescalings,

$$\Omega \;=\; \Omega_+(x^+) + \Omega_-(x^-) + \mathcal{O}(y^2).$$

  and by an independent asymptotic diffeomorphism as before.

- The generators of the asymptotic Weyl rescalings satisfy a chiral algebra,

$$[\mathcal{J}_n, \mathcal{J}_m] = 2k\, n\, \delta_{n+m,0} \qquad with \qquad \mathcal{J}_n = \tilde{G}_W[\Omega = -e^{inx^+}]$$

- The generators of asymptotic CKV's satisfy two copies of Virasoro algebra with central charges,

$$c_{R/L} = \pm 12k \quad \text{with} \quad L_n = \tilde{G}_\xi[\varepsilon^+ = e^{inx^+}, \varepsilon^- = 0] + \tilde{G}_W[\Omega(\varepsilon^+)].$$

CFT interpretation

- The generators of the asymptotic Weyl rescaling and asymptotic diffeomorphism cummute unless, $\alpha \neq 0$.

- We construct the generators of asymptotic Killing vectors by a Sugawara- shifting mechanism,

$$\mathcal{L}_n = L_n + \frac{1}{4k} \sum_{m \in Z} : \mathcal{J}_m \mathcal{J}_{n-m} :$$

- The new algebra contains a U(1) current algebra with $c_R = 12k + 1$ and $c_L = -12k$,

$$\begin{aligned}
[\mathcal{L}_n, \mathcal{L}_m] &= (n-m)\mathcal{L}_{n+m} + \tfrac{c_R}{12}(n^3 - n)\delta_{n+m,0} \\
[\bar{L}_n, \bar{L}_m] &= (n-m)\bar{L}_{n+m} + \tfrac{c_L}{12}(n^3 - n)\delta_{n+m,0} \\
[\mathcal{J}_n, \mathcal{J}_m] &= 2k\, n\, \delta_{n+m,0} \\
[\mathcal{J}_n, \mathcal{L}_m] &= n\, \mathcal{J}_{n+m}
\end{aligned}$$

Discussion

- In case 3 the holomorphic Weyl factor constitutes an additional free chiral boson in the theory shifting $c_R \longrightarrow c_R + 1$.

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- Requiring conservation of the stress-energy tensor is equivalent to the flatness of boundary and demanding the scalar field to obey the massless Klein-Gordon equation.
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- Allowing for a curved boundary metric with Ricci scalar R we expect the scalar field to obey the Liouville equation $\Box f \propto R$.
- Allowing $\alpha \neq 0$ violates the asymptotic AdS boundary condition and leads to another central charge in the commutator $[L_n, \mathcal{J}_m]$ proportional to $\alpha k m^2 \delta_{n+m,0}$.
- This new anomaly can be removed by a topological twist,

$$\mathcal{L}_n \to \hat{\mathcal{L}}_n = \mathcal{L}_n - \gamma\,(n+1)\,\mathcal{J}_n\,.$$

which also changes the value of c_R (work in progress).

Thank you!

Questions...