Democratic superstring field theory and its gauge fixing arXiv:0911.2962, arXiv:1010.1662

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A short summary: arXiv:1101.1314

General review (with E. Fuchs): arXiv:0807.4722

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Outline

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- How should we treat the superstring? SFT of what?
- RNS world-sheet formalism
- Do we have a reliable RNS string field theory?

RNS string field theories

- ~~Witten's open RNS theory~~ (Witten 86), (Wendt 89)
- Open ~~R~~NS "modified" theory (Arefeva et. al. 89, Preitschopf et. al. 89), (M. K 09)
- Heterotic NS (closed NS-NS) theory (Saroja and Sen 92)
- Open NS nonpolynomial theory (Berkovits 95)
- Heterotic NS theory (Berkovits, Okawa and Zwiebach 04)
- Open NS cubic "non-minimal" theory (Berkovits and Siegel 09, M. K 09)

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- Open RNS "democratic" theory (M. K 09)

Open string field theory – Ingredients

- The (classical) string field lives in the space including ghosts, the "Hilbert space": $\Psi \in \mathcal{H}$
- Ghost number one (classically)
- In flat space:
  $\Psi = \big(T(x_0)c_1 + A_\mu(x_0)\alpha^\mu_{-1}c_1 + C(x_0)c_0 + \ldots\big)\,|0\rangle$
- String fields interact by half-string gluing
- Perform gluing by "integration" and "star product"
  $$\int : \mathcal{H} \to \mathbb{C} \qquad \star : \mathcal{H} \otimes \mathcal{H} \to \mathcal{H}$$
- The BRST operator Q is a derivation
- The string mid-point is invariant under the star product

The large Hilbert space and picture number (Friedan et. al. 85)

"Bosonisation" of the superghosts: $\beta = e^{-\phi}\partial\xi \quad \gamma = \eta e^{\phi}$

ξ zero mode not included $\Rightarrow$ Physical space=small Hilbert space

The large Hilbert space: $H_L = H_S \oplus (\xi_0 H_S)$

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The large space is to the small one like $\mathbb{C}$ is to $\mathbb{R}$

- It is a natural (CFT) extension
- Manifests the symmetries: $\mathbb{Z}_2$ for Q and η_0 (Berkovits and Vafa 94)
- Trivializes Q (η_0): $QV = 0 \Rightarrow V = Q(PV) \quad P = -c\xi\xi' e^{-2\phi}$

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Vertex operators carry "picture number"

$pic(\xi) = 1 \qquad pic(\eta = -1) \qquad pic(e^{n\phi}) = n$

Only picture number -2 (-1 in H_L) leads to non-zero vev

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Vertex operators in the large space at arbitrary picture (Berkovits 01):

$(Q - \eta_0)\Psi = 0 \qquad \Psi \sim \Psi + (Q - \eta_0)\Lambda$

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Promoting to string field theory

Action: $S = -\oint \mathcal{O}\Big(\frac{1}{2}\Psi(Q-\eta_0)\Psi + \frac{1}{3}\Psi^3\Big)$

Gauge symmetry: $\delta\Psi = (Q-\eta_0)\Lambda + [\Psi, \Lambda]$

The Ramond sector is "trivially" included: $\Psi = A + \alpha$

The action includes a mid-point insertion:

$$\mathcal{O} \approx \xi \sum_{p\in\mathbb{Z}} X_p \qquad \Longleftrightarrow \qquad Q\mathcal{O} = \eta_0\mathcal{O} = \sum_{p\in\mathbb{Z}} X_p$$

X_p are multi-picture changing operator: $X_0 = 1, X_1 = X, X_{-1} = Y$
They are zero-weight primaries and so is $\mathcal{O}$

Successes

- Includes the Ramond sector: $\Psi = A + \alpha$
- Elegant and natural (Ramond sector, $\mathcal{O}$ insertion)
- BV master action is easily derived
- Generalization to general D-brane system is straightforward
- The mid-point insertion probably has no kernel
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  How about other gauge fixings?

Fixing the picture number (partial gauge fixing)

Could we gauge fix the theory to an arbitrary picture number?

- The picture-number gauge symmetry becomes non-linear
- E.O.M components at different pictures hold independently
- The (off-shell) string field is picture-dependent

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"Special" picture numbers (NS sector): $A \to e^{-\Lambda}(Q - \eta_0 + A)e^{\Lambda}$

- One "special" picture (0) for the gauge string field
- Two "special" pictures (0 and -1) for the physical string field

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Gauge fixing to $pic = 0$

Let $pic(A) = 0$ and consider first the linearized theory:

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Gauge symmetry – equivalent representations

- $\delta A = (Q-\eta_0)\Lambda$
- $\delta A = Q\Lambda_0 - \eta_0\Lambda_1 \qquad \eta_0\Lambda_0 = Q\Lambda_1 = 0$
- $\delta A = Q\eta_0\tilde{\Lambda}_1$

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Two possible resolutions of the gauge symmetry

- $\delta A = Q\Lambda_0 \quad \eta_0\Lambda_0 = 0 \quad \Rightarrow \quad \eta_0 A = 0 \quad \Rightarrow \quad A = \eta_0\Phi$
- $\delta A = \eta_0\Lambda_1 \quad Q\Lambda_1 = 0 \quad \Rightarrow \quad QA = 0 \quad \Rightarrow \quad A = Q\Phi$

Gauge fixing to $pic = 0$ – nonlinear level

$\eta_0 A = 0 \quad \Leftrightarrow \quad A = \eta_0 \Phi$

This condition is unchanged at the non-linear level. The residual gauge symmetry takes the form: $\delta A = Q\Lambda + [A, \Lambda] \qquad \eta_0 \Lambda = 0$, which coincides with the modified theory, as does the action

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$QA = 0 \quad \Leftrightarrow \quad A = Q\Phi$

This condition has to be modified at the non-linear level

A natural generalization: $A = e^{-\Phi} Q e^{\Phi}$

This is not a restriction to gauge-solutions, or to solutions at all:

A obeys $QA + A^2 = 0$, while the E.O.M is: $(Q - \eta_0)A + A^2 = 0$

A is a solution iff $\eta_0 A = 0$, which is exactly the E.O.M of the non-polynomial theory

Nonpolynomial open string field theory (Berkovits 95)

The nonpolynomial theory is a generalization of WZW

SFT	WZW
Φ (string field, $gh(\Phi) = pic(\Phi) = 0$)	algebra element
e^Φ	g (group element)
Q, η_0 (odd derivations)	∂, $\bar\partial$ (even)
t (auxiliary coordinate)	t (auxiliary coordinate)
$\oint$ (CFT vev in the large space)	$\int d^2 z Tr$
g_o (open string coupling const.)	quant. for compact groups

Table: NP-SFT - WZW dictionary

Action: $S = \dfrac{1}{2g_o^2} \oint \left(\eta_0 e^\Phi Q e^{-\Phi} - \int_0^1 dt\, \Phi \left[e^{-t\Phi} \eta_0 e^{t\Phi}, e^{-t\Phi} Q e^{t\Phi} \right] \right)$

Variation: $\delta S = \dfrac{1}{g_o^2} \oint \left(\eta_0 \left(e^{-\Phi} Q e^\Phi \right) e^{-\Phi} \delta e^\Phi \right)$

"Two-dim." representation of NP-SFT (WZW) (M. K 10)

Define J_Q, J_η: $QX = [J_Q, X]$, $\eta_0 X = [J_\eta, X]$ (Horowitz et. al. 86)

Define L: $LX = [\Phi, X]$

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$$\eta_0 \Phi = -LJ_\eta, \quad \oint XLY = -\oint LXY, \quad \left(e^{-L} - 1\right) J_Q = e^{-\Phi} Q e^{\Phi}$$

Action ($g_o = 1$):

$$S = -\oint \eta_0 \Phi \frac{e^{-L} - 1 + L}{L^2} Q\Phi = \oint J_\eta e^{-L} J_Q = \oint J_\eta e^{-\Phi} J_Q e^{\Phi}$$

Define: $\Phi(\alpha) = \alpha\Phi \quad (0 \le \alpha \le 1)$

Initial condition: $S(\alpha = 0) = 0$

$$\frac{\partial S(\alpha)}{\partial \alpha} = -\oint J_\eta L e^{-\alpha L} J_Q = -\oint \eta_0 \Phi e^{-\alpha L} J_Q = \oint \Phi \eta_0 \left(e^{-\alpha\Phi} Q e^{\alpha\Phi}\right)$$

Gauge fixing to $pic = 0$ – gauge symmetry

The gauge fixing $A = e^{-\Phi} Q e^{\Phi}$ leaves a residual gauge symmetry:

$$\delta e^{\Phi} = e^{\Phi} \eta_0 \hat{\Lambda}_1$$

It also introduces a new gauge symmetry:

$$\delta e^{\Phi} = Q \hat{\Lambda}_0 e^{\Phi}$$

Together, these two gauge symmetries are exactly those of the non-polynomial theory

Gauge fixing to $pic = 0$ – action

The action takes the form: $S = -\oint \mathcal{O}\Big(\frac{1}{6}AQA - \frac{1}{2}A\eta_0 A\Big)$

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Then, $\dfrac{dS(\alpha)}{d\alpha} = \oint (Q\mathcal{O})A\eta_0\Phi$

In agreement with the expressions of the non-polynomial theory

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String field theory version of CS $\leftrightarrow$ WZW (Elitzur et. al. 89)

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The picture-fixed action can be generalized to include a Ramond string field at picture number $\frac{1}{2}$

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- Off-shell supersymmetry?
- Relation to pure-spinor SFT and "extended universality"?
- Generalize to heterotic and closed theories

Thank You