P-wave Holographic Superconductor

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Plan of the talk

- Few comments on superconductivity and (non) Fermi liquid
- Holographic superconductor
- P-wave insulator/conductor/superconductor phase diagram(A. Akhavan, M. A., 1011.6158)
- Fermions on asymptotically AdS background

Few comments on superconductivity

Superconductivity is an electrical resistance of exactly zero which occurs in certain materials below a characteristic temperature known as critical temperature T_c. These material are called superconductor.

It is important to note that superconductivity is a quantum mechanical phenomenon. It is also characterized by a phenomenon called the Meissner effect.

The ejection of any sufficiently weak magnetic field from the interior of the superconductor as it transitions into the superconducting state.

A phenomenological description of superconductivity was first given by London brothers (Fritz and Heinz London in 1935) with simple equation

$$\vec{J} = -\frac{ne^2}{mc}\vec{A}$$

In terms of the electric and magnetic fields E and B one has

$$\frac{\partial \vec{J}}{\partial t} = -\frac{ne^2}{mc}\vec{E}, \qquad \nabla \times \vec{J} = -\frac{ne^2}{mc}\vec{B}$$

Known as London's equations.

If the second of London's equations is manipulated by applying Ampere's law one finds

$$\nabla^2 \vec{B} = \frac{1}{\lambda^2}\vec{B}, \qquad \lambda^2 = \frac{mc^2}{4\pi ne^2}$$

Where λ is London penetration depth which is a a characteristic length over which external magnetic fields are exponentially suppressed.

A more complete theory of superconductivity was given by Bardeen, Cooper and Schrieffer in 1957 and is known as BCS theory.

They showed that interactions with phonons can cause pairs of elections with opposite spin to bind and form a charged boson called a Cooper pair.

Below a critical temperature T_c, there is a second order phase transition and the Cooper pair, being bosons, condenses.

The DC conductivity becomes infinite producing a superconductor.

Depending on the orbital angular momentum of the wave function of Cooper pairs one has

- s-wave superconductor; $L = 0$

- p-wave superconductor; $L = 1$

- d-wave superconductor; $L = 2$

It was thought that the highest T_c for a BCS superconductor was around 30 K.

The highest T_c known today (at atmospheric pressure) is $T_c = 134$ K.

There is evidence that electron pairs still form in these high T_c materials, but the pairing mechanism is not well understood. It is believed that high T_c superconductor is d-wave.

At normal phase the system cannot be described by standard Fermi liquid.

Fermionic systems

- Free electron gas

N non-interacting Fermi particles of spin $s = \frac{1}{2}$. The single particle eigen-states are plane wave states with momentum K , and energy $E_k = \frac{h^2 k^2}{8\pi^2 m}$.

The ground state $\longrightarrow$ Fermi sea : All single particles stats are filled up to a limiting wave vector $k_f = (3\pi^2 N)^{1/3}$.

$$E_f = \frac{p_f^2}{2m}$$

- Fermi liquid

Electrons in a metal may be thought of a free fermion gas with some effective mass m_*.

Effectively we have quasiparticles and in the ground state the quasiparticle fill the Fermi see up to the Fermi momentum

$$E_f = \frac{p_f^2}{2m}$$

One may also add the interaction of the quasiparticles (Landau Fermi liquid)

The predictions for temperature dependence of specific heat and electrical resistivity in the Fermi liquid scenario are

$$C \sim T \qquad \rho \sim T^2$$

- Non-Fermi liquid

In 1991 Seaman et al, Phys. Rev. Lett 67 presented measurements of specific heat and electric resistivity of some material which strongly disagreed with the Fermi liquid model .

The temperature dependence of specific heat and electrical resistivity of (Non-Fermi liquid) are

$$C \sim T \ln T, \qquad \rho \sim T$$

There is no long lived quasiparticles

The information can be read from poles structure of the fermion Green's function near Fermi surface.

Moreover unlike BCS theory, it is believed that in high T_C superconductor it may involve strong coupling system.

Therefore AdS/CFT may proved a framework to study it.

Holographic superconductor

How to construct a holographic model for superconductor. This means we want to have gravity dual which exhibits certain features of superconductivity.

• We want to have a holographic model $\longrightarrow$ gravity with negative cosmological constant (AdS solution).

• We want finite density $\longrightarrow U(1)$ gauge field in the bulk.

• Finite temperature CFT $\longrightarrow$ Embedding the Schwarzschild black hole solution into AdS.

• To describe superconductor one needs a black hole solution which has hair at low temperature, though has no hair at high temperature. Gubser 0801.2977

The simplest model may be given by the following action

$$S = \int d^4x \sqrt{-g} \left[R + \frac{6}{L^2} - \frac{1}{4} F^2 + |\nabla \psi|^2 - m^2 |\psi|^2 \right]$$

This is charged scalar coupled to a $U(1)$ gauge field.

For sufficiently low temperature the black hole solution is unstable to develop hair where the $U(1)$ gauge symmetry is also broken.

P-wave holographic superconductor

Consider a five dimensional $SU(2)$ Einstein-Yang-Mills theory with a negative

$$S = \int d^5x\sqrt{-g}\left[\frac{1}{2}\left(R + \frac{12}{L^2}\right) - \frac{1}{4}F^a_{\mu\nu}F^{a\,\mu\nu}\right],$$

The equations of motion are

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R - 6g_{\mu\nu} = T_{\mu\nu},$$

$$\frac{1}{\sqrt{-g}}\partial_\mu(\sqrt{-g}F^{a\,\mu\nu}) + qf^{abc}A^b_\mu F^{c\,\mu\nu} = 0,$$

where

$$T_{\mu\nu} = F^a_{\mu\rho}F^{a\ \rho}_{\nu} - \frac{1}{4}g_{\mu\nu}F^a_{\mu\nu}F^{a\,\mu\nu},$$

$$F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + qf^{abc}A^b_\mu A^c_\nu.$$

The equations of motion support an AdS solitonic solution with zero gauge field

$$ds^2 = \frac{1}{r^2 g(r)} dr^2 + r^2(-dt^2 + dx^2 + dy^2) + r^2 g(r) d\chi^2, \qquad g = 1 - \frac{r_0^4}{r^4}.$$

- This provides a gravity description of a three dimensional field theory with a mass gap.

- It is still a solution with a constant non-zero gauge potential $A_t = \mu$.

The action admits another analytic solution with non-zero gauge field

$$ds^2 = \frac{dr^2}{r^2 g} - r^2 g dt^2 + r^2(dx^2 + dy^2 + dz^2), \qquad A = \rho\left(1 - \frac{1}{r^2}\right)\sigma^3 dt,$$

$$g = 1 - \frac{1 + \rho^3/3}{r^4} + \frac{\rho^2}{3r^3},$$

σ^3 is the generator of $U(1)$ subgroup. The horizon is located at $r = 1$. In this notation the Hawking temperature of the black hole is $T = \frac{2-\rho^2/3}{2\pi}$.

This is, indeed, an AdS Reissner-Nordström black hole which carries the charge of the $U(1)$ abelian subgroup of the $SU(2)$ gauge group.

Probe limit

For canonical normalized field

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R - 6 g_{\mu\nu} = \frac{1}{q} T_{\mu\nu},$$

For large q keeping $T_{\mu\nu}$ finite, the back reactions will be negligible

To study the insulator/superconductor phase transition, following Gubser, 0803.3483 , we will consider the following ansatz for the gauge field

$$A = \phi(r)\,\sigma^3 dt + \psi(r)\,\sigma^1 dx.$$

t-component of the gauge field represents the $U(1)$ gauge field, while the second term plays the role of the charged field whose condensation breaks the $U(1)$ gauge symmetry.

Plugging this ansatz into the equations of motion, for the AdS soliton solution, one arrives at

$$\phi'' + \left(\frac{3}{r} + \frac{g'}{g}\right)\phi' - \frac{\psi^2}{r^4 g}\phi = 0, \qquad \psi'' + \left(\frac{3}{r} + \frac{g'}{g}\right)\psi' + \frac{\phi^2}{r^4 g}\psi = 0.$$

See also Gubser and Pufu 0805.2960, Roberts and Hartnoll 0805.3898

The equations are invariant under the rescaling $r \to r_0 r, \phi \to r_0\phi$ and $\psi \to r_0\psi$, so that r_0 may be dropped from the equations.

- Near the boundary

$$\phi = \mu - \frac{\rho}{r^2}, \qquad \psi = \psi_0 + \frac{\psi_1}{r^2}.$$

- Near the tip

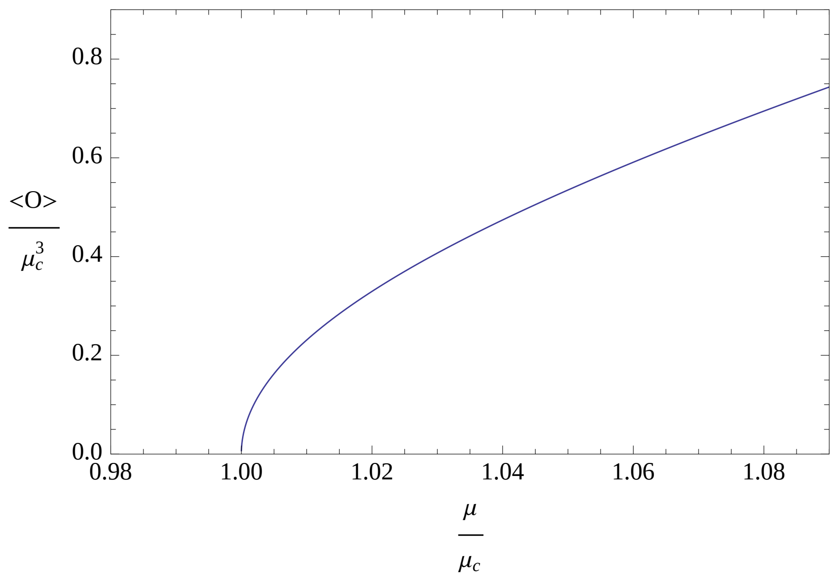
$$\psi = \alpha_0 + \alpha_1(1-\frac{1}{r}) + \alpha_2(1-\frac{1}{r})^2 \cdots ,$$

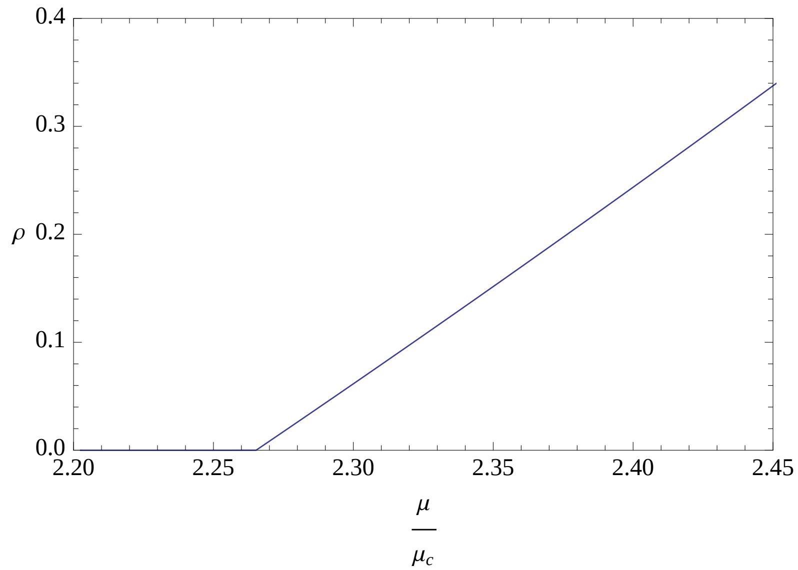
$$\phi = \beta_0 + \beta_1(1-\frac{1}{r}) + \beta_2(1-\frac{1}{r})^2 \cdots .$$

We set $\psi_0 = 0$ and Also up to a normalization one has $\langle O \rangle \sim \psi_1$.

With these boundary conditions one may solve the equations to find the expectation value of the dual operator as a function of the chemical potential.

One finds that the solution is unstable to develop a hair for the chemical potential bigger than a critical value, $\mu > \mu_c$.





The phase transition is second order ($\rho = \partial F / \partial \mu$).

The phase transition may be interpreted as an insulator/superconductor phase transition.

See Nishioka, Ryu and Takayanagi 0911.0962. for s-wave case.

To study the conductivity of the theory we will consider an extra magnetic field along y direction.

From gravity description point of view this can be done by turning on a non-zero gauge field in y direction.

$$A_y = A(r)e^{i\omega t}\sigma^3.$$

The corresponding equation of motion for A_y is given by

$$A'' + \left(\frac{3}{r} + \frac{g'}{g}\right) A' + \frac{\phi^2}{r^4 g}\left(\omega^2 - \psi^2\right) A = 0.$$

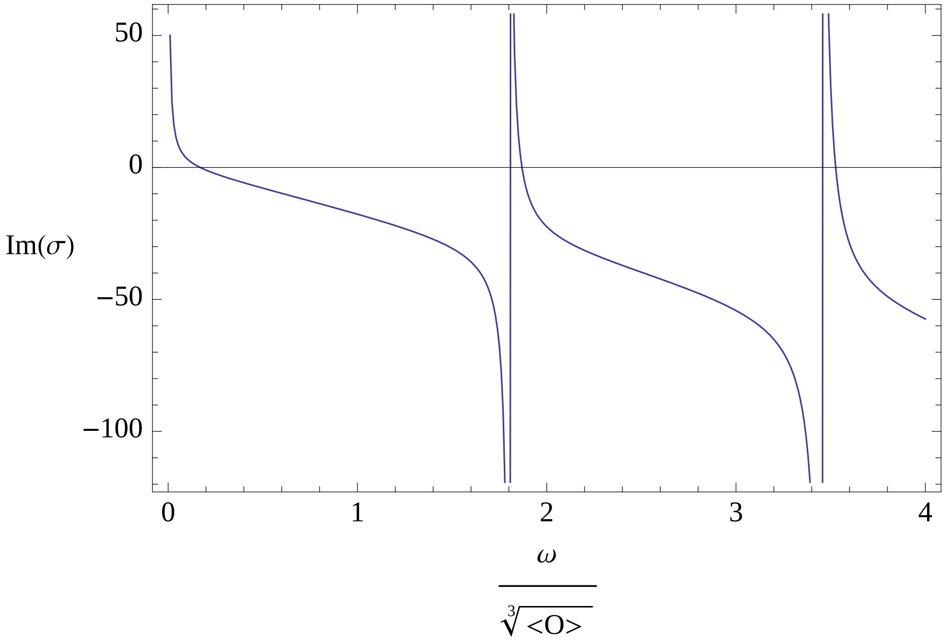
For large r

$$A = A_0 + \frac{A_1}{r^2}.$$

The conductivity in y direction is (see for example Hartnoll, Herzog and Horowitz 0803.3295)

$$\sigma_{yy} = \frac{-iA_1}{\omega A_0}.$$

Using the numerical solution we had found for ψ and ϕ in the previous subsection we can find the behavior of the AC conductivity in terms of the energy ω.



At $\omega \to 0$ we find a pole showing that we have an infinite conductivity as expected for the superconductor phase.

$$\mathrm{Im}(\sigma) = \int d\omega' \frac{\mathrm{Re}(\sigma)}{\omega - \omega'}$$

One can redo the same computations on the RN AdS black hole.

Again consider the following ansatz for the gauge field

$$A = \phi(r)\, \sigma^3 dt + \psi(r)\, \sigma^1 dx.$$

• The solution is unstable to develop a vector hair for sufficiently low temperature where the $U(1)$ gauge symmetry is also broken.

• This corresponds to a second order conductor/superconductor phase transition from field theory point of view.

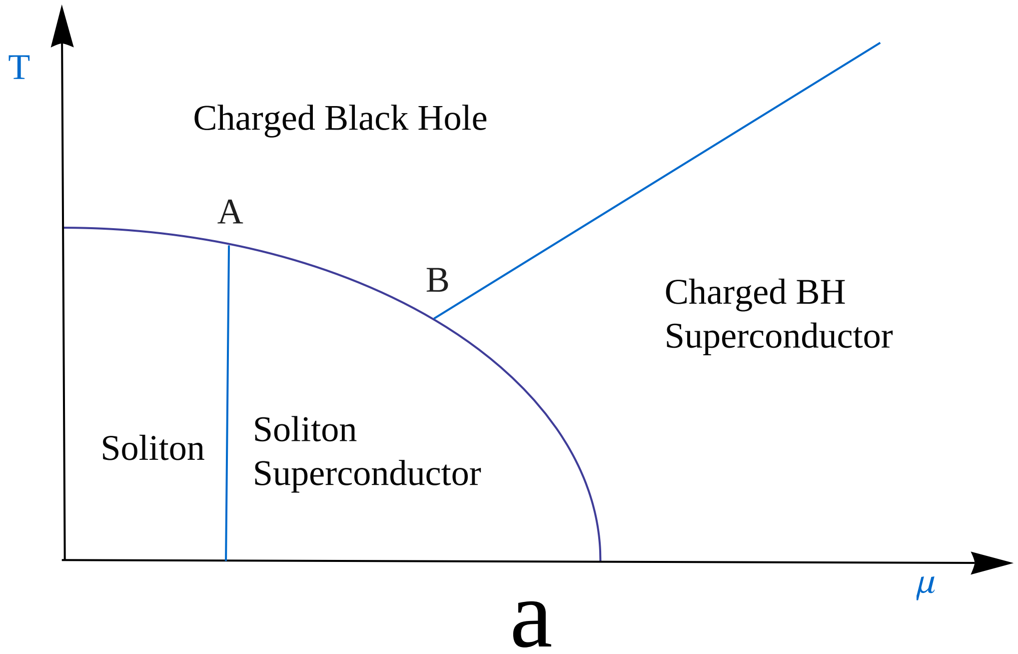
Phase structure in probe limit

- The equations of motion support two distinctive solutions; AdS soliton and AdS charged black hole. [Insulator, conductor]

- In each case the solution becomes unstable to develop a vector hair as we change the parameters of the model. [insulator/superconductor phase transition, conductor/superconductor phase transition]

- There is a first order phase transition from AdS soliton to AdS charged black hole (Witten hep-th/9803131) [insulator/conductor phase transition]

So altogether we get four different phases as follows.



Beyond probe limit

The aim is to study the effects of the gauge field on the background metric. (see also Ammon, Erdmenger, Grass, Kerner and O'Bannon 0912.3515, Manvelyan, Radu and Tchrakian 0812.3531, Basu, He, Mukherjee and Shieh 0911.4999)

- AdS soliton

$$ds^2 = \frac{dr^2}{g(r)} + r^2\left(-f(r)dt^2 + h(r)dx^2 + dy^2\right) + g(r)e^{-\chi(r)}d\eta^2,$$

$$A = \phi(r)\sigma^3 dt + \psi(r)\sigma^1 dx.$$

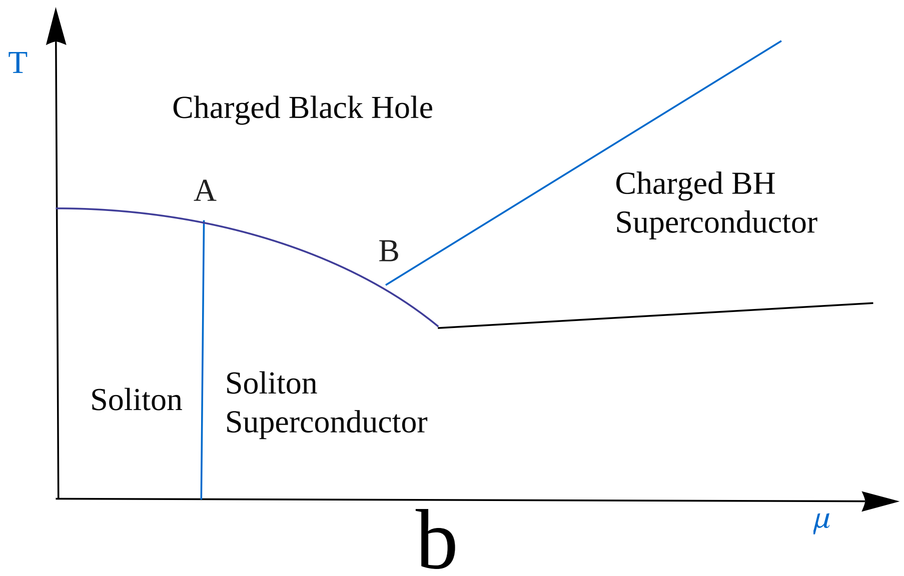
- AdS charged black hole

$$ds^2 = \frac{dr^2}{g(r)} + r^2\left(h(r)dx^2 + dy^2 + dz^2\right) - g(r)e^{-\chi(r)}dt^2,$$

$$A = \phi(r)\sigma^3 dt + \psi(r)\sigma^1 dx.$$

Taking into account the back reactions we observe two new features in the phase diagram of the model.

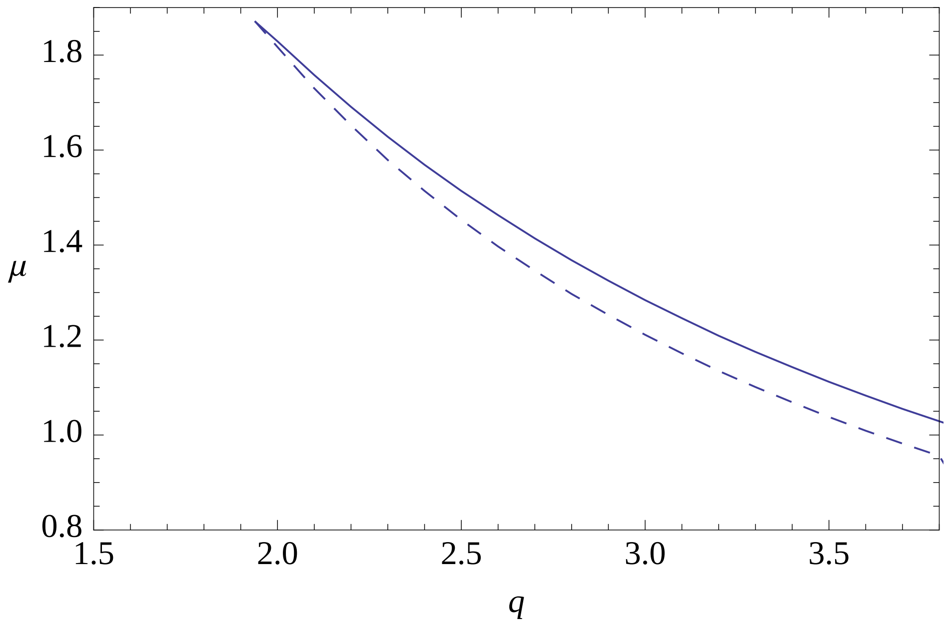
The first observation is that for large chemical potential as we decrease temperature the favored phase is soliton superconductor.

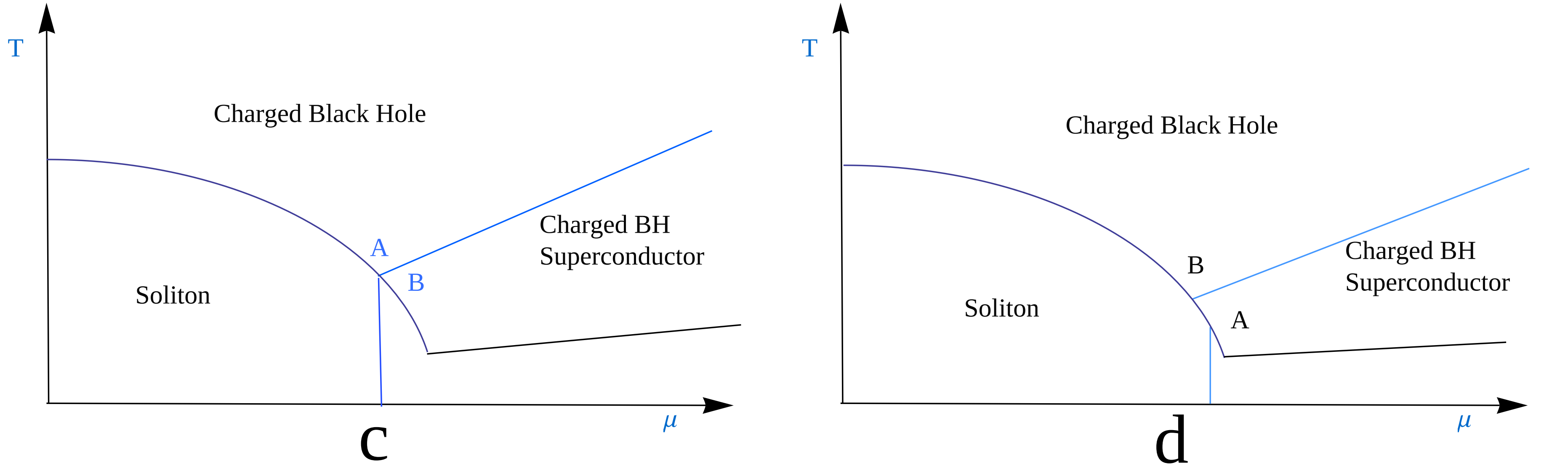


In particular we cannot have a phase describing hairy charged black hole at zero temperature which could have been the case if the phase diagram had been given by probe limit.

On the other hand there are two special points, named by A and B in figure which in order to understand their physical significant, it requires to study the system beyond the probe limit.

Actually our numerical computations show that the positions of these two points labeled by μ_A and μ_B change as we are changing q.





As we further decrease q we observe that while both μ_A and μ_B increase, the point A passes though the point B .

We encounter a new phase transition. Actually as we decrease temperature there is a range of μ between which the superconductor becomes an insulator via a first order phase transition.

see Horowitz and Way 1007.3714 for the case of s-wave.

Going further it seems that the phase where we have soliton superconductor becomes smaller and smaller and eventually disappears from the phase diagram, though due to the uncertainty of our numerical results, we have not been able to explore the situation exactly.

In particular for low temperature (for small enough q, i.e. $q \approx 0.86$) the numerical solution develops a singularity and one has to study $T \to 0$ limit of hairy charged black hole more carefully.

Zero temperature limit

• It is then natural to pose the equation what happens when we send the temperature of the AdS charged black hole to zero?

• Sending temperature to zero we will end up with an extremal black hole whose near horizon geometry develops an AdS_2 throat with non-zero entropy.

• To study holographic superconductors at zero temperature the extremal black hole cannot provide the gravity dual descriptions.

• One must have a geometry with zero size horizon ensuring that the ground state is a single state (entropy is zero).

See Horowitz and Roberts 0908.3677 for the case of s-wave.

See also Basu, He, Mukherjee and Shieh 0911.4999 for 3D p-wave superconductors

One needs to solve the equations of motion with a particular boundary condition ensuring that the resultant geometry would have zero size horizon.

Consider the ansatz

$$ds^2 = \frac{dr^2}{g(r)} + r^2\left(h(r)dx^2 + dy^2 + dz^2\right) - g(r)e^{-\chi(r)}dt^2,$$

$$A = \phi(r)\sigma^3 dt + \psi(r)\sigma^1 dx.$$

with the following behaviors in $r \to 0^+$ limit.

$$\phi \sim \phi_0(r), \quad \psi \sim \psi_0 - \psi_1(r), \quad \chi \sim \chi_0 - \chi_1(r), \quad g \sim r^2 + g_1(r), \quad h \sim h_0 + h_1(r)$$

with the assumption that $\phi_0, \psi_1, \chi_1, g_1$ and h_1 go to zero sufficiently fast in the limit of $r \to 0^+$.

Plugging these behaviors into the corresponding equations of motion, at leading order, one finds

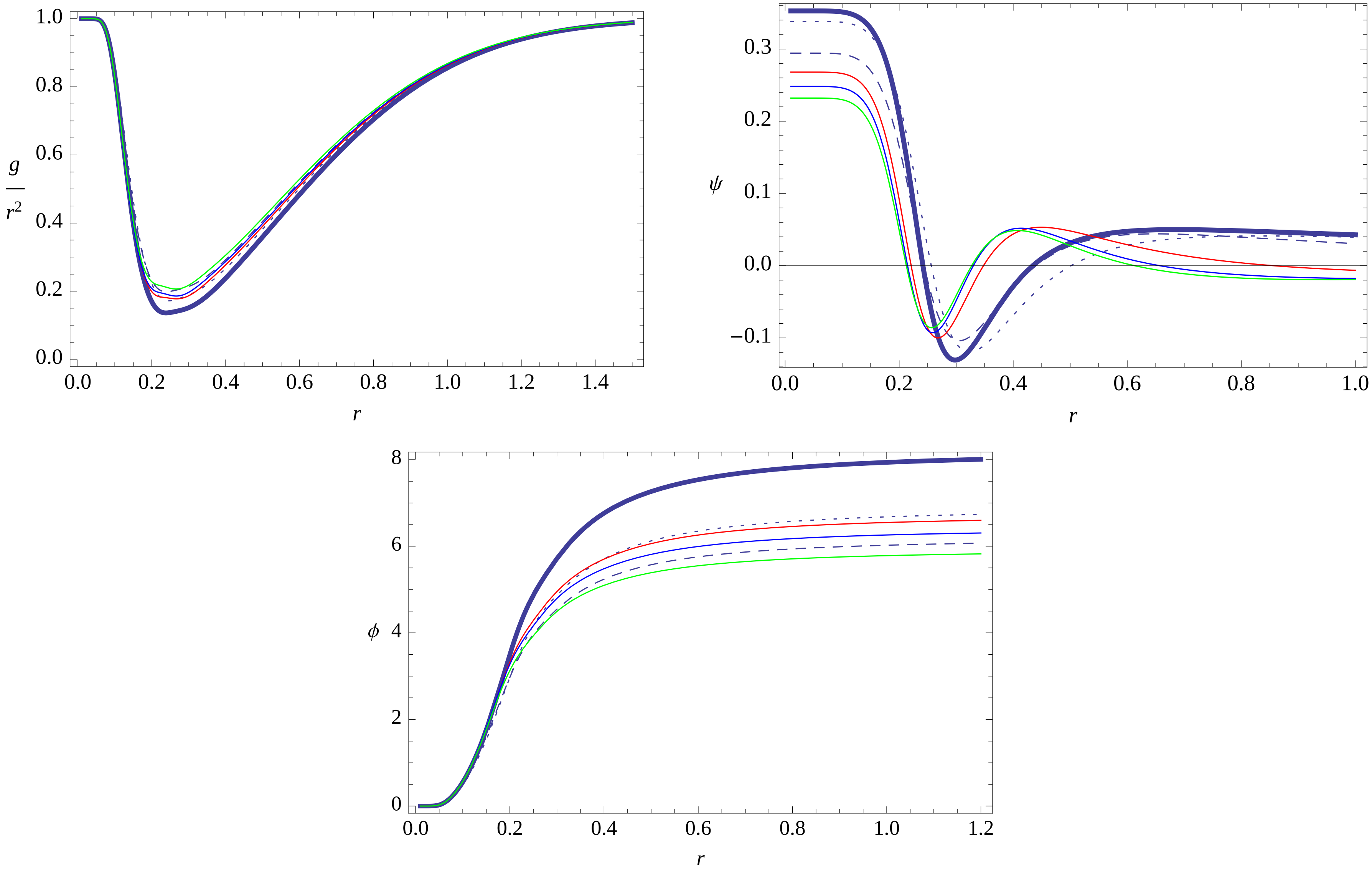
$$\phi = \phi_0 \frac{e^{-\frac{\alpha}{r}}}{\sqrt{r}}, \quad \chi = \chi_0 - \frac{e^{\chi_0} \alpha \phi_0^2}{6r^2} e^{-\frac{2\alpha}{r}}, \quad g = r^2 - \frac{e^{\chi_0} \alpha \phi_0^2}{6r^2} e^{-\frac{2\alpha}{r}},$$

$$\psi = \psi_0 \left(1 - \frac{e^{\chi_0} q^2 \phi_0^2}{4r\alpha^2} e^{-\frac{2\alpha}{r}}\right), \qquad h = h_0 \left(1 + \frac{e^{\chi_0} \phi_0^2}{8r} e^{-\frac{2\alpha}{r}}\right).$$

where $\alpha = q\psi_0/h_0$.

On the other hand for large r one impose the following asymptotic conditions for the gauge field components

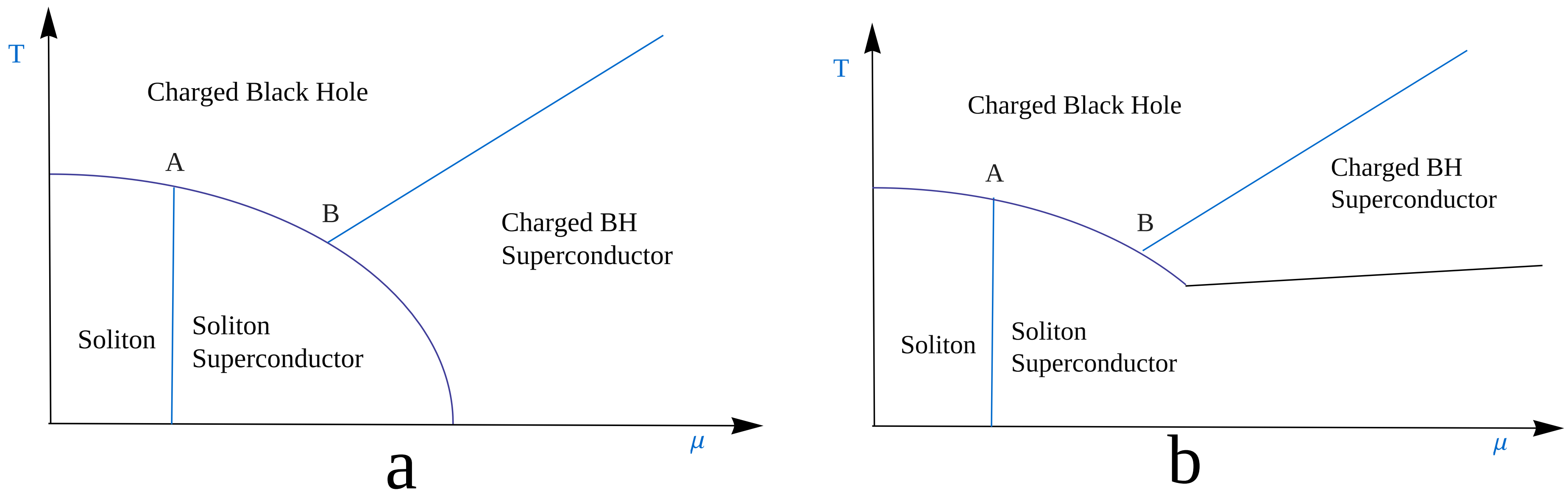
$$\phi = \mu - \frac{\rho}{r^2}, \qquad \psi = \frac{\tilde{\psi}}{r^2}.$$



1.0
0.8
0.6
0.4
0.2
0.0
$\frac{g}{r^2}$
0.0
0.2
0.4
0.6
0.8
1.0
1.2
1.4
r
0.3
0.2
0.1
0.0
−0.1
ψ
0.0
0.2
0.4
0.6
0.8
1.0
r
8
6
4
2
0
ϕ
0.0
0.2
0.4
0.6
0.8
1.0
1.2
r

Fermion on asymptotically AdS solutions

Typically we have following phase structure for the holographic superconductor.



One may probe different regions by a fermionic field.

Some references

S.S. Lee 0809.3402

Liu, McGreevy and Vegh 0903.2477

Faulkner, Liu, MacGreevy and Vegh 0907.2694

Faulkner and Polchinski 1001.3402

Faulkner, Iqbal, Liu, MacGreevy and Vegh 1101.0597

....

Mostly for s-wave superconductor.

For p-wave see, however, Gubser, Rocha and Yarom 1002.4416; Ammon, Erdmenger, Kaminski and O'Bannon, 1003.1134; M. A , work in progress

When the system is in normal conductor phase the corresponding geometry is given by RN AdS black hole

$$ds^2 = \frac{dr^2}{r^2 g} - r^2 g dt^2 + r^2(dx^2 + dy^2 + dz^2), \qquad A = \rho \left(1 - \frac{1}{r^2}\right) \sigma^3 dt,$$

$$g = 1 - \frac{1 + \rho^3/3}{r^4} + \frac{\rho^2}{3r^3},$$

s-wave: A is $U(1)$. p-wave A is $U(1) \subset SU(2)$.

Fermions in this background may provide a framework to study non Fermi liquid.

We consider following action in the bulk

$$S = \int d^4x \sqrt{g}\bar{\psi}(i\Gamma^{\mu} D_{\mu} - m)\psi$$

where $D_{\mu} = \partial_{\mu} + \frac{1}{4}\omega_{\mu}^{ab}\Gamma_{ab} - iqA_{\mu}$. (s-wave)

• For $T = 0$ the near horizon metric is $AdS_2 \times R^2$.

• Therefore the low energy physics of the boundary theory can be described by the AdS_2 geometry: IR CFT

• It is easy to find the retarded two point function of the emergent IR CFT (Liu, McGreevy and Vegh 0903.2477)

$$\mathcal{G}_k(\omega) = c(k)\omega^{2\nu_k}$$

$c(k)$ is a complex function and $\delta_k = \frac{1}{2} + \nu_k$ is the conformal dimension in 2D CFT

This not the retarded Green function of the whole theory. It can be found in low energy around Fermi surface as follows (Faulkner, Liu, MacGreevy and Vegh 0907.2694)

$$G_R(k,\omega) = \frac{h_1}{k_\perp - \frac{1}{v_f}\omega - h_2 c(k)\omega^{2\nu_k}}$$

The pole structure of the Green function depends on ν_k.

- $\nu_k > \frac{1}{2}$: Stable quasiparticle (nor Fermi liquid)
- $\nu_k < \frac{1}{2}$: No quasiparticle
- $\nu_k = \frac{1}{2}$: there is log solution; strange metal

For $\nu_k = 1$ it is very similar to Fermi liquid.

Other parts of the phase structure can also be considered both in s and p wave models

Summary

- AdS/CFT is found useful to find different models for superconductor
- To see more interesting physics one needs to go beyond probe limit
- One may probe different region of the phase structure by fermions
- In the conductor phase the physics around the Fermi surface is governed by emergent IR CFT