An Intriguing Example of F-maximization in 3D SCFTs

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- Supersymmetric CFTs with 4 real SUSIES (N=2 in 3D, or N=1 in 4D) have a conserved U(1) R-symmetry that sits in the same supermultiplet as the stress-energy tensor.

  In a general interacting SCFT this symmetry receives quantum corrections and the quantum numbers associated with it become non-trivial functions of the parameters of the theory.

- The computation of the exact **non-perturbative** form of this symmetry is an important problem in field theory.

  Such exact knowledge can be used to determine the anomalous scaling dimensions of chiral operators, trace SUSY RG flows (hence a significant part of the topology/geometry of field theory space), test dualities, etc...

- In 4D this problem was solved by ***Intriligator and Wecht '05*** with the use of a-maximization:

  The exact U(1) R-symmetry in 4D N=1 SCFTs maximizes `a'

  (a = the coefficient of the Euler density in the conformal anomaly),

  or in terms of 't Hooft anomalies

$$a = \frac{3}{32}\left(3\mathrm{Tr}R^3 - \mathrm{Tr}R\right)$$

- Alternatives to a-maximization:

  (*a*) τ_{RR}-minimization (applies to any dimension, but hard to compute exactly)

  (*b*) Z-minimization: applies to AdS/CFT (***Martelli, Sparks and Yau '05***) the dual AdS space is AdS_{d+1} x Y_{2n-1}, Y_{2n-1} Sasaki-Einstein manifold

  the exact U(1) R-symmetry minimizes the Einstein-Hilbert action on Y_{2n-1}

  (equivalent to τ_{RR}-minimization, applies to general spacetime dimension, but requires a weakly curved AdS dual and specific regimes of parameters)

• In the last couple of years large classes of 3D N=2 SCFTs have been identified (constructed as Chern-Simons-Matter (CSM) theories).

Dynamics controlled by a set of discrete parameters (e.g. rank of gauge group, Chern-Simons level, etc...).

There are regimes where these theories are weakly coupled (tractable with perturbative methods) and regimes at strong coupling beyond perturbation theory.

Intriguing non-perturbative dynamics:

- ► the exact U(1) R-symmetry receives non-trivial corrections (some operators can become highly relevant and induce new RG flows/fixed points),
- ► dualities,
- ► drastic reduction of degrees of freedom (e.g. from N^2 to $N^{3/2}$ in ABJM)

F-maximization in 3D SCFTs
& recent developments

- The proposal of Jafferis (**1012.3210**)

the exact U(1) R-symmetry in 3D SCFTs maximizes the free energy F of the theory on S^3

$$Z_{S^3} = e^{-\mathcal{F}} \ , \quad F = \frac{1}{2}(\mathcal{F} + \overline{\mathcal{F}})$$

F is extensive in the dof of the system.
It can be computed exactly using localization techniques
(***Kapustin-Willett-Yaakov '09, Hama-Hosomichi-Lee '10, Jafferis '10***)

Very powerful technique:

► F seems to be a good measure of dof (c-function? ***Jafferis et al '11, Klebanov et al '11***, $N^{3/2}$ dof on M2-branes ***Drukker et al***), can be used to check dualities (holographic, Seiberg-like, mirror)

► The computation of more SUSY observables is possible via localization.

- Some care needs to be taken when coupling the SUSY theory with curvature. Doing things properly requires the introduction of extra couplings between the matter fields and curvature. These couplings are determined by the choice of R-symmetry.

- In this way F becomes a function of the trial R-charges.

$$\partial_{\Delta_j} |Z_{S^3}|^2 = 0$$

F-maximization

- Assuming that the R-symmetry does not mix with accidental flavor symmetries we can use the weak coupling formulation of the theory to compute F using localization techniques. For a CSM theory with gauge group G and chiral superfields in reps R_i one finds (after localization and appropriate regularization) a matrix integral:

$$Z_{S^3} = \int \prod_{\mathrm{Cartan}} du \; e^{i\pi \mathrm{Tr} u^2} \det_{Adj} (\sinh(\pi u)) \prod_{\mathrm{Chirals\ in\ rep\ R_i}} \det_{R_i} \left(e^{\ell(1-\Delta_i)+iu)} \right)$$

$$\ell(z) := -z \log\left(1 - e^{2\pi i z}\right) + \frac{i}{2}\left(\pi z^2 + \frac{1}{\pi}\mathrm{Li}_2\left(e^{2\pi i z}\right)\right) - \frac{i\pi}{12}$$

These arguments for F-maximization are plausible and the proposal has already passed a number of impressive non-trivial tests:

(1) Reproduces known perturbative results
(***Jafferis '10, Amariti '11, Amariti, Siani '11***)

(2) Reproduces Z-minimization
(***Herzog et al '10, Martelli-Sparks '11, Cheon-Kim^2 '11, Jafferis et al '11***)

(3) Verifies proposed Seiberg-like dualities
(***Kapustin '11, Willett-Yaakov '11***)

Besides a rigorous proof we would like to have more (qualitatively new) examples in order to:

(*i*) probe the validity and possible modifications of the principle in more `extreme' situations, e.g. when some fields decouple and identifiable accidental symmetries appear

(*ii*) obtain more intuition about the matrix integrals that appear in the localized expression of Z
(generally complicated integrals, in large-N limits many different saddle points, dualities imply complicated (new) mathematical identities)

(*iii*) is there always a unique extremum of F and is it always a maximum?

We will now discuss an example of a class of theories where we can probe most of these properties.

1-adjoint CSM theory

The theory of interest is:

$\hat{\mathbf{A}}$ CSM theory

N=2 Chern-Simons at level *k* with gauge group *G=U(N)* coupled to one chiral superfield X in the adjoint representation (NO superpotential)

- Important information about this theory can be obtained by studying the superpotential deformations

$\mathbf{A_{n+1}}$ CSM theory

$$\hat{\mathbf{A}}\ CSM\ theory\ \oplus\ W_{n+1} = \mathrm{Tr} X^{n+1}\ ,\quad n = 1, 2, \ldots$$

It is convenient to study these theories in the large-N ‘t Hooft limit

$$N, k \to \infty\ , \quad \lambda = \frac{N}{k} = \text{fixed}$$

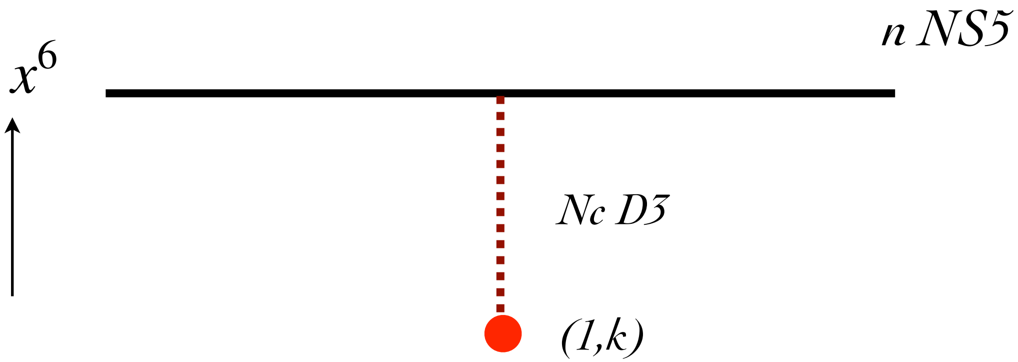
• It is believed (***Gaiotto, Yin '07***) that the $\hat{\mathbf{A}}$ theory is exactly superconformal at the quantum level at any value of the coupling λ.

At weak coupling the R-symmetry can be determined perturbatively and assigns R-charge

$$R(\lambda) \simeq \frac{1}{2} - 2\lambda^2 + \mathcal{O}(\lambda^4)\ , \quad \lambda \ll 1$$

to the chiral superfield X.

• No holographic description of this theory in supergravity is expected. Cannot appeal to AdS/CFT for any information about this theory.
We would like to know the full (non-perturbative) dependence of R on λ.



VN '08

NS5	:	0 1 2 3 4 5
(1,k)	:	0 1 2 [3,7] 8 9
D3	:	0 1 2 6

- There are regimes along the λ-line where the superpotential deformations

$$W_{n+1} = \frac{g_{n+1}}{n+1} \mathrm{Tr} X^{n+1}$$

are relevant and drive the theory to a new IR fixed point: the A_{n+1} theory.

- From a D-brane construction we learn that:

can be argued also directly in field theory

(i) the superpotential deformation W_{n+1} lifts the supersymmetric vacuum when

$$N > nk \quad (\text{equivalently in 't Hooft limit } \lambda > n)$$

(ii) the theory exhibits a Seiberg-like duality:

$$\boxed{\lambda \leftrightarrow n - \lambda}$$

$$U(N)_k \text{ with } W_{n+1} \;\sim\; U(nk-N)_k \text{ with } W_{n+1}$$

• This information has important implications for the undeformed $\hat{\mathbf{A}}$ theory.

(1) The fact that W_{n+1} can lift the supersymmetric vacuum at arbitrarily large integer values of λ implies that the R-charge decreases (with increasing λ) towards 0.

(2) More specifically, there has to be a sequence of critical couplings

$$0 = \lambda_2^* = \lambda_3^* = \lambda_4^* < \lambda_5^* < \cdots < \lambda_n^* < \lambda_{n+1}^* < \cdots$$

where each time one of the chiral operators $\mathrm{Tr}X^{n+1}$ becomes marginal. By definition

$$R(\lambda_{n+1}^*) = \frac{2}{n+1}$$

(3) The generic operator $\mathrm{Tr}X^{n+1}$ must become marginal ***before*** it becomes capable of lifting the SUSY vacuum at $\lambda=n$. This implies

$$\lambda^*_{n+1} < n \ , \quad R(n) < \frac{2}{n+1}$$

(4) The existence of a "conformal window" for Seiberg-like duality implies

$$\lambda^*_{n+1} < \frac{n}{2} \ , \quad R\left(\frac{n}{2}\right) < \frac{2}{n+1}$$

assuming $R(\lambda)$ is monotonic

(5) At $\lambda=\lambda^*_{4(n+1)}$ the operator $\mathrm{Tr}X^{n+1}$ hits the unitarity bound, becomes **free** and decouples from the rest of the theory. At that point we can no longer use it to deform the theory without destabilizing the SUSY vacuum (F-term SUSY breaking). Hence, spontaneous SUSY breaking must occur before this point:

$$n < \lambda^*_{4(n+1)} \ , \quad \frac{1}{2(n+1)} < R(n)$$

Summary

In the $\hat{\mathbf{A}}$ theory the following inequalities are expected to hold:

$$\left[\frac{n-3}{4}\right] \le \lambda^*_{n+1} < \frac{n}{2}$$

Seiberg-like duality imposes more constraints (see below)

$$\frac{1}{2(\lambda+1)} \le R(\lambda) < \frac{2}{\lambda+1} \ , \quad \lambda = 1, 2, \ldots$$

$$R(\lambda) < \frac{2}{2\lambda+1} \ , \quad \lambda = \frac{1}{2}, 1, \frac{3}{2}, \ldots$$

At weak coupling TrX is already free and decoupled. As we further increase the coupling more and more of the chiral ring operators TrX^{n+1} hit the unitarity bound and decouple. At strong coupling there is a sequential decommissioning of the bottom part of the chiral ring.

F-maximization answers

What does F-maximization have to say about all this?

- We are instructed to maximize the free energy

$$F = -\log \left| \int \left(\prod_{j=1}^{N} e^{\frac{i\pi N}{\lambda} t_j^2} dt_j \right) \prod_{i<j}^{N} \sinh^2\left(\pi(t_{ij})\right) \prod_{i,j=1}^{N} e^{\ell(1-R+it_{ij})} \right|$$

- We computed this function (and maximized) in the large-N limit using the saddle point approximation. This entails solving the algebraic equations

$$\mathcal{I}_i \equiv \frac{i}{\lambda} t_i + \frac{1}{N} \sum_{j\neq i} \left[\coth(\pi t_{ij}) - \frac{(1-R)\sinh(2\pi t_{ij}) + t_{ij}\sin(2\pi R)}{\cosh(2\pi t_{ij}) - \cos(2\pi R)} \right] = 0 \ , \quad i = 1, 2, \ldots, N$$

at a saddle point configuration

$$-\mathcal{F}(\lambda, N) = \sum_{i=1}^{N} \frac{i\pi N}{\lambda} t_i^2 + \sum_{i<j}^{N} \log\sinh^2(\pi t_{ij}) + \sum_{i,j=1}^{N} \ell(1 - R + it_{ij})$$

• In general, the t_i's that solve these equations are complex numbers.

• In lack of a better strategy we solved these equations numerically.

• Practically we introduce a ficticious time coordinate τ and solve the differential equations

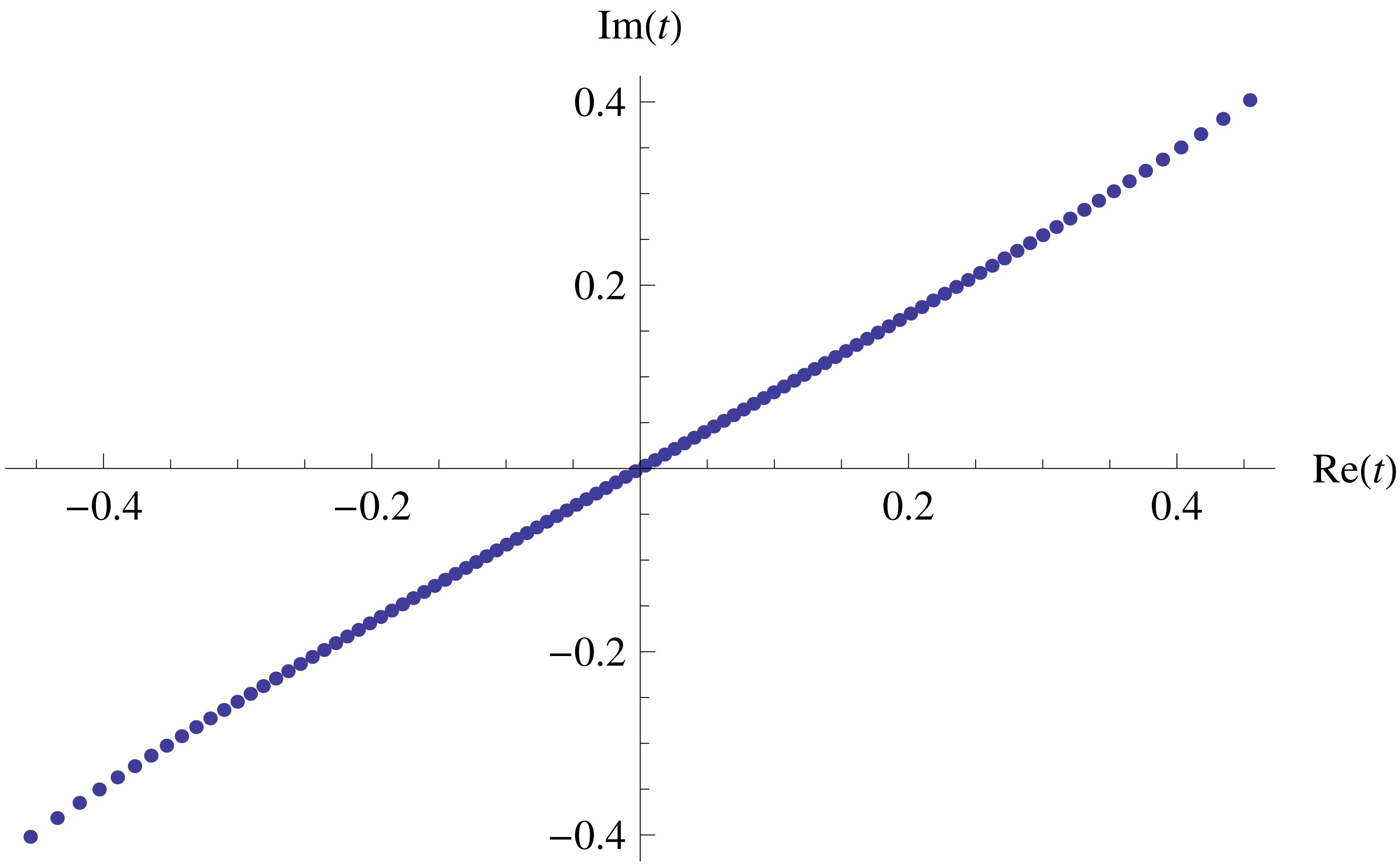
$$a\frac{dt_i(\tau)}{d\tau} = \mathcal{I}_i$$

With suitably chosen coefficient a the solution converges very quickly to the equilibrium configuration we are looking for.

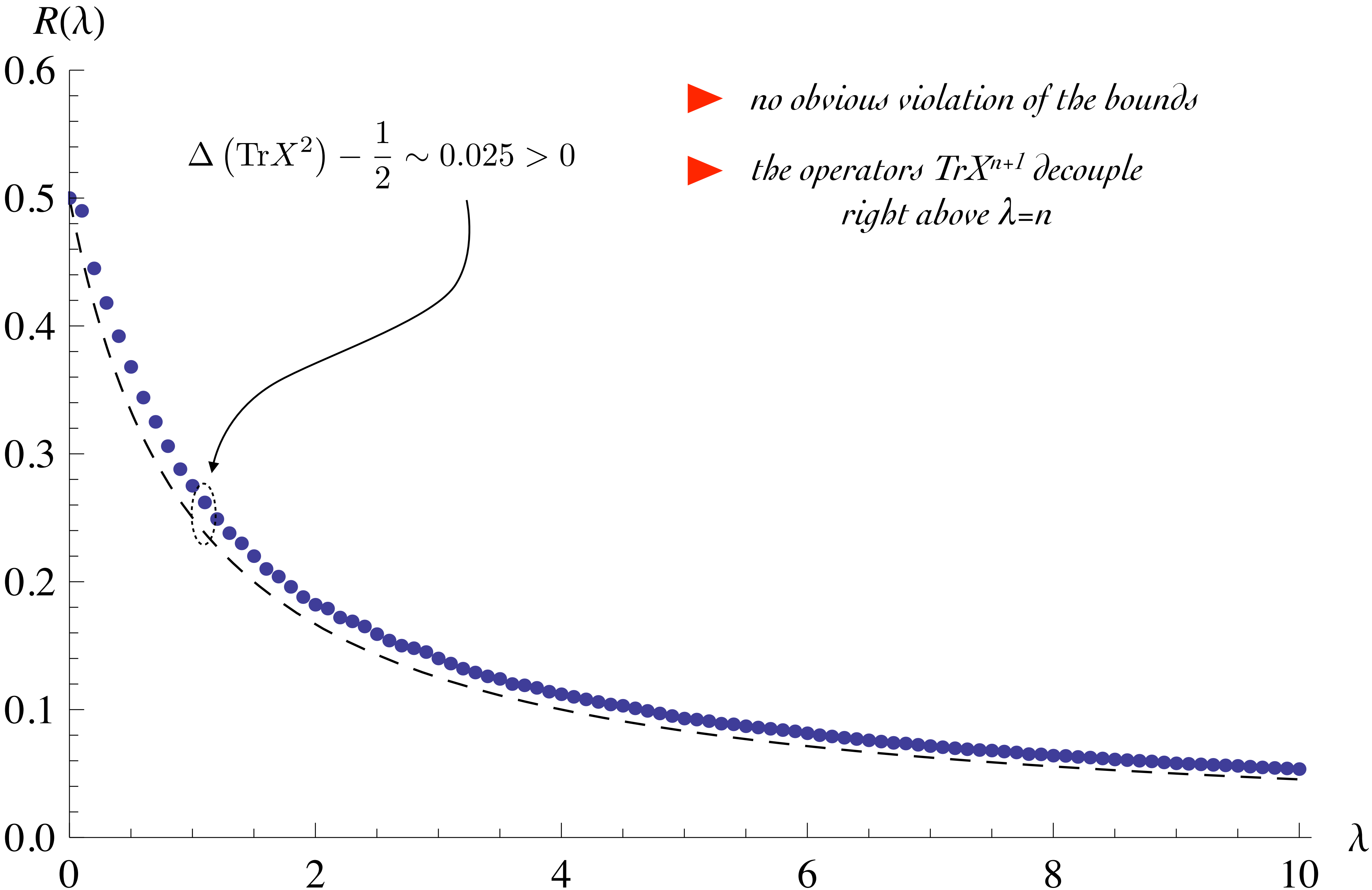
• Implemented this approach numerically for various values of N.
At N=100 the numerical result seems to approach the large-N asymptote within a few percent.

A typical distribution of the eigenvalues t_i in the complex plane.

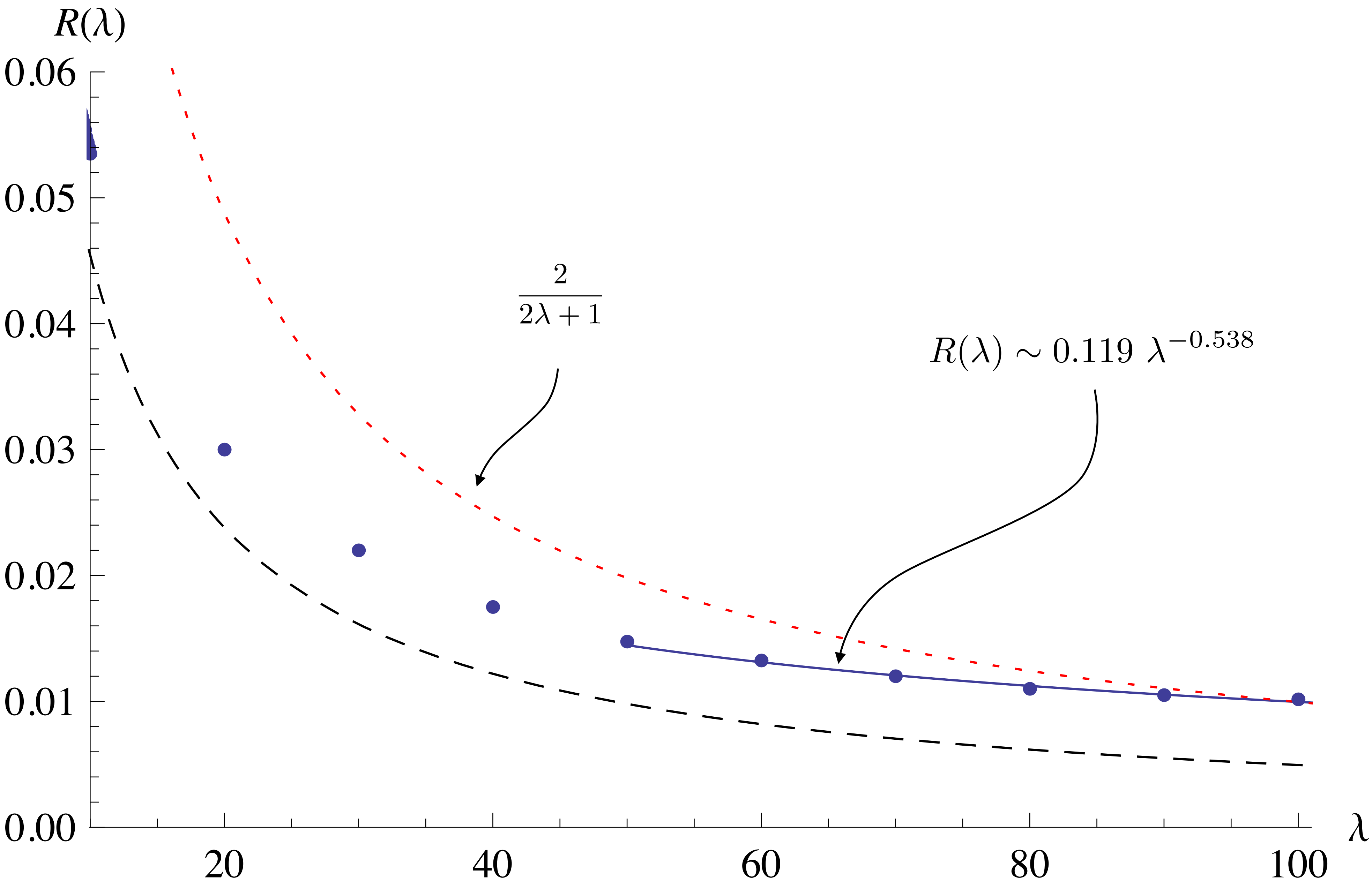
(this particular plot was obtained for N=100, λ=1, R=0.225)



R(λ) after F-maximization



- *no obvious violation of the bounds*
- *the operators TrX^{n+1} decouple right above $\lambda=n$*



$R(\lambda)$
0.06
0.05
0.04
0.03
0.02
0.01
0.00
20
40
60
80
100
λ
$\frac{2}{2\lambda+1}$
$R(\lambda) \sim 0.119\ \lambda^{-0.538}$

- **Checks**

► The numerical code reproduces very nicely the perturbative result. In fact, at leading order in λ the eigenvalue distribution is (***Minwalla et al '11***)

$$t = e^{\frac{\pi i}{4}}\sqrt{\lambda}y \ , \quad \rho(y) = \sqrt{\frac{2}{\pi} - y^2}$$

The numerical result verifies this behavior.

► We have written independently two different numerical codes (in Mathematica and Fortran) that reproduce the same result.

► We have explored a wide range of initial conditions for the τ-differential equations and parameters a.
We find many different multi-cut solutions (both numerically and analytically at weak coupling). The 1-cut solution appears to be the dominant one and is the one that reproduces the perturbative field theory result (saddle-point crosses at stronger coupling??? probably no).

• As we increase the coupling more and more operators hit the unitarity bound and decouple creating new accidental symmetries. The first operator that decouples non-perturbatively is TrX^2 at $\lambda\sim1$.
In principle, F-maximization in its current form can fail in such situations.

Recall what happens in 4D with a-maximization. In similar cases (e.g. in 4D 1-adjoint SQCD) when fields decouple one is instructed to subtract the anomalies of the decoupling fields from a and maximize the remaining contributions (***Kutasov, Parnachev, Sahakyan '05***).

In 4D 1-adjoint SQCD N^2 dof decouple (mesons). In our CSM example order 1 dof decouple at $\lambda \sim \mathcal{O}(1)$, hence their effects are not expected to have a sizable effect in the large-N limit in this regime.

Standard F-maximization should proceed unobstructed at large-N, finite λ.

- Accordingly, there are no obvious violations of the bounds for $\lambda \sim \mathcal{O}(1)$.

- Before changing behavior to cross the first upper bound curve (presumably an effect of the accumulating decoupling operators), the numerically determined R-charge curve appears to asymptote at large λ to the curve $\dfrac{1}{2\lambda}$.

- The fact that the curve remains in the vicinity of the lower-bound curve $\dfrac{1}{2(\lambda+1)}$ is a feature that has been observed also in 4D 1-adjoint SQCD and is natural to anticipate that it will persist for any λ.

Perspectives

We have identified a CSM theory with enough complex dynamics that can pose as a useful new testing ground for non-perturbative techniques in 3D QFT, like F-maximization.

A combination of field and string theory techniques can be used to pose (independent) constraints on the theory beyond the perturbative regime.

Open problems:

1) Is the 1-cut saddle point contribution always the dominant one?
Is the large-N limit amenable to analytic methods?

2) What happens at stronger coupling where a significant number of fields decouple? Is there a proper modification of F-maximization?
How does one implement such modifications?

3) In order to probe the effects of decoupling fields it will be interesting to consider the full CSM analog of 1-adjoint SQCD, namely

U(N_C) Chern-Simons theory at level k coupled to:
- 1 chiral superfield in the adjoint
- N_F chiral superfields in the fundamental
- N_F chiral superfields in the anti-fundamental.

To simplify things it is interesting to consider the Veneziano-like limit

$$k,\ N_C,\ N_F \to \infty\ ,\quad \lambda = \frac{N_C}{k}\ ,\quad x = \frac{N_C}{N_F}\quad \text{fixed}$$

Using information from string theory (***VN '08, '09***) one can set some constraints on the R-charge function R_X for the adjoint superfield X, e.g. now

$$\frac{\left[\frac{n-3}{4}\right] x}{x-\left[\frac{n-3}{4}\right]} < \lambda^*_{n+1} < \frac{nx}{x-n} \ , \quad n \leq \frac{[x]-3}{4}$$

$R_X(\lambda,x)$ is presumably a monotonically decreasing function of λ at fixed x that approaches at strong 't Hooft coupling a limiting lowest value

$$\frac{1}{2([x]+2)} < R_{X,\mathrm{lim}} < \frac{2}{[x]+1}$$

No corresponding information is currently available for R_Q, the R-charge functions for the quark multiplets.

4) CSM theories with 2 adjoint chiral superfields (+ additional matter) are also interesting.

In 4D a-maximization has led to an intriguing picture of 2-adjoint N=1 SCFTs that appear to admit a mysterious ADE classification. A web of RG flows connects different members of this classification.

In previous work ***VN '09*** we provided evidence for a similar structure in a subclass of 3D CSM SCFTs. F-maximization can help solidify and extend this picture.

It can also help find non-trivial evidence for another set of new Seiberg-like dualities proposed in ***VN '09***.

The web of RG flows can be used to further test the proposed F-theorem.