Towards non-SUSY Duality

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Introduction

Interesting to find the low energy degrees of freedom of strongly interacting systems. Canonical Example: massless QCD

$$[SU(N_c)] \times SU(N_f)_L \times SU(N_f)_R \times U(1)_B$$

UV: quarks and gluons

IR: pions

Introduction

Correlation functions of gauge invariant operators $\psi\tilde{\psi}$ $F^2_{\mu\nu}$ at very large distances are expressible by correlation functions of pions fields.

WHAT IS THE MAPPING?

$$\int [dA][d\psi][d\tilde{\psi}] < F^2 ... > e^{-S(A,\psi,\tilde{\psi})} = \int [dU] <?.. > e^{-S(U)}$$

We do not know the answer in general.

Introduction

But some operators can be mapped:

- **Currents corresponding to the global symmetry** $SU(N_f)_L \times SU(N_f)_R$

$$j_\mu^{UV} = \psi\gamma_\mu T_L \bar{\psi} + \tilde{\psi}\gamma_\mu T_R \bar{\tilde{\psi}}$$

$$\longrightarrow j_\mu^{IR} = (T_L - T_R)\partial\pi + (T_L + T_R)\pi\partial\pi + \cdots$$

- **A similar formula for** $U(1)_B$ **but slightly more complicated to derive (WZ and also Witten)**

Introduction

- **No rigorous method for the $U(1)$ axial current**

$$j_A = \psi\gamma_\mu\bar{\psi} - \tilde{\psi}\gamma_\mu\bar{\tilde{\psi}}$$

because of the anomaly

$$\partial j_A \sim F\tilde{F}$$

Mapping Operators in SUSY

- Many strongly interacting theories have a known low energy description (duality).
- E.g. SUSY QCD (in the appropriate regime) is described at low energies by a weakly coupled dual gauge theory.
- The mapping of operators extends to the CHIRAL-RING, so we can do much more than in nonSUSY theories.

Mapping Operators in SUSY

- For instance we can map mesons and baryons

  $Q\tilde{Q} \longrightarrow M$ and $Q^{N_c} \longrightarrow q^{N_f - N_c}$

- As in QCD we can also map global currents, e.g.

$$QQ^\dagger - \tilde{Q}\tilde{Q}^\dagger = \frac{N_f - N_c}{N_c}\left(qq^\dagger - \tilde{q}\tilde{q}^\dagger\right)$$

Mapping Operators in SUSY

Can we go beyond the chiral ring and conserved currents?

We will derive an EXACT result for the mapping of the axial current

$$J_A = QQ^\dagger + \tilde{Q}\tilde{Q}^\dagger$$

and will discuss several applications of the general method.

Beyond the Chiral Ring

Consider the anomaly equation

$$\bar{D}^2 \left(Q Q^\dagger + \tilde{Q} \tilde{Q}^\dagger \right) \sim W_\alpha^2,$$

but W_α^2 is related to trace anomaly via

$$T_\mu^\mu \sim F^2$$

and the trace anomaly is related to the R-current by SUSY

$$\{Q, [\bar{Q}, R_\mu]\} \sim T_{\mu\nu}$$

Beyond the Chiral Ring

Formally: a theory with an R-symmetry has a multiplet

$$\mathcal{R}_\mu = R_\mu + \theta S_\mu + c.c. + \theta\bar{\theta}(T_{\mu\nu} + \ldots) + \ldots$$

The conservation equations are subsumed in

$$\bar{D}\mathcal{R}_\mu = \bar{D}^2 D_\alpha U$$

with real U. For conformal theories U is a conserved and

$$R_\mu + U|_{\theta\bar{\theta}}$$

is the superconformal R-symmetry.

Beyond the Chiral Ring

Mathematically

$$\bar{D}\mathcal{R}_\mu = \bar{D}^2 D_\alpha U$$

implies that R_μ is reducible (U is a sub-multiplet) but not decomposable.

If we can follow the R-symmetry, we can follow U!

Beyond the Chiral Ring

General Algorithm: Pick an R-symmetry along the flow.

Determine the superconformal R-symmetry in the UV

and IR and then we find

$$U^{UV} = 3J^{UV}/2 \longrightarrow U^{IR} = 3J^{IR}/2$$

where $J^{UV,IR} = R^{UV,IR}_{SCFT} - R_\mu$.

In many examples U turns out to be the Konishi current – so we provide an exact mapping for it under duality.

Beyond the Chiral Ring

The equation

$$\bar{D}\mathcal{R}_\mu = \bar{D}^2 D_\alpha U$$

does not fix U uniquely. We can always redefine

$$U \longrightarrow U + f(\Phi) + \bar{f}(\bar{\Phi})$$

and this is annihilated by $\bar{D}^2 D_\alpha$.

This is an ambiguity in the procedure that arises rarely, not in the examples we discuss here. In most cases the holomorphic pieces can be forbidden by symmetries.

Beyond the Chiral Ring

In summary the procedure can be fully carried out in the following cases

- There is a duality
- A-maximization or other methods tell us what is the IR superconformal symmetry.

Example: SQCD

SQCD in the range $N_c < N_f < 3N_c/2$ flows from a free UV fixed point to a free IR fixed point.

There is one exact R-symmetry along the flow $U(1)_R(Q,\tilde{Q}) = 1 - N_c/N_f$. From the algorithm above we get an exact result

$$QQ^\dagger + \tilde{Q}\tilde{Q}^\dagger \longrightarrow \frac{2N_f - 3N_c}{3N_c - N_f}\left(qq^\dagger + \tilde{q}\tilde{q}^\dagger - 2MM^\dagger\right)$$

Example: SQCD

The divergence of this identity gives

$$\bar{D}^2\left(QQ^\dagger + \tilde{Q}\tilde{Q}^\dagger\right) \longrightarrow \frac{2N_f - 3N_c}{3N_c - N_f}\bar{D}^2\left(qq^\dagger + \tilde{q}\tilde{q}^\dagger - 2MM^\dagger\right)$$

$$\Downarrow$$

$$W_{ele}^2 \longrightarrow \frac{2N_f - 3N_c}{3N_c - N_f} W_{mag}^2$$

The coefficient is negative in the free-magnetic phase, as appropriate. This result disagrees with many previous proposals by various people.

Example: Adjoint SQCD

Adjoint SQCD has been solved using a-maximization.

As a function of $x = N_f/N_c < 2$ there are free fields accompanied by nontrivial SCFTs. For some x where there are N free fields we find

$$QQ^\dagger + \tilde{Q}\tilde{Q}^\dagger \longrightarrow \left(-\frac{1}{2} + \frac{N_c}{N_f}\right)^{-1} \sum_{j=0}^{N} \left(2\frac{N_c}{N_f} - j - 2\right) M_j M_j^\dagger$$

which again maps the Konishi current to something nontrivial in the IR.

Applications: Soft Breaking

Consider SQCD in the free-magnetic phase

- $K = Qe^V Q^\dagger + \tilde{Q} e^{-V} \tilde{Q}^\dagger, W = 0$
- In the IR this flow to the dual theory
  $K = qe^{\tilde{V}} q^\dagger + \tilde{q} e^{-\tilde{V}} \tilde{q}^\dagger + MM^\dagger, W = qM\tilde{q}$
- One natural question is what happens to the dynamics once we deform in the UV
  $\delta\mathcal{L} = m_\lambda \lambda_{ele} \lambda_{ele} - m^2 (QQ^\dagger + \tilde{Q}\tilde{Q}^\dagger)$

Applications: Soft Breaking

- This is a soft deformation, like the ones people consider in the MSSM
- However, following it to the IR is nontrivial, as we go through strong coupling – we are dealing with “non-SUSY” duality.

Applications: Soft Breaking

- **Our mapping gives the answer:**

$$\delta\mathcal{L} = m_\lambda \frac{2N_f - 3N_c}{3N_c - N_f} \lambda_{mag}\lambda_{mag}$$

$$-m^2 \frac{3N_c - 2N_f}{3N_c - N_f} \left(2MM^\dagger - qq^\dagger - \tilde{q}\tilde{q}^\dagger\right)$$

- **Hence, the meson field is massive at the origin, but the magnetic quarks are tachyonic.**

Applications: Soft Breaking

Particles in the IR can be tachyonic even if all masses in the UV are positive (violating persistent mass condition!).

One can verify that the F-terms

$$q M \tilde{q}$$

and the D-terms of the magnetic $SU(N_f - N_c)$ group do not stabilize the runaway, so we do not know where is the vacuum

Applications: Soft Breaking

This runaway can be stabilized by gauging baryon number symmetry. Then one finds a vacuum located at

$$q \sim \tilde{q} \sim m 1_{(N_f - N_c)\times(N_f - N_c)} \ , \qquad M = 0$$

- The magnetic gauge symmetry is completely higgsed so vacuum well defined
- The global symmetry is color-flavor locked: it mixes with the magnetic gauge symmetry and survives.
- The scale of the breaking is set by the soft scale.

Alternative to MSSM EWSB

The usual mechanism for electroweak symmetry breaking in SUSY extensions of the SM is through stop squark loops

$$\mathcal{L}_{MSSM} \supset H H^\dagger t t^\dagger$$

which drive the mass of the higgs negative and trigger EWSB.

Alternative to MSSM EWSB

Our mechanism given an alternative: The soft mass turns negative due to strong coupling effects, which are *calculable* and yield unsuppressed answers.

We do not need loop-suppressed effects from stop squarks.

Alternative to MSSM EWSB

The little hierarchy problem in the MSSM exists because the quartic coupling for the Higgs field is

$$g^2|H|^4/8$$

This is too small and necessitates large one-loop corrections.

Such large corrections only arise with a heavy stop squark, which leads to some appreciable tuning.

Alternative to MSSM EWSB

Our framework avoids this problem because the Higgs is identified with a magentic quark and thus also participates in interactions of the form

$$q M \tilde{q}$$

which give additional quartic couplings.

This is reminiscent of the so called NMSSM, so here we see that our “nonSUSY” duality provides a sorely needed completion of this...

Conclusions

- We have shown that the R-current multiplet contains a real operator U, and this coincides with the Konishi current in many examples
- This object is a long operator and cannot be followed via the usual tools
- We have used the indecomposable structure of the multiplet to follow U
- In this way we obtained some exact mappings of various nonSUSY operators

Conclusions

- Deforming by these operators looks like adding soft terms and we thus enter the regime of “nonSUSY duality.”
- We have found interesting low energy weakly coupled vacua with higgsed gauge symmetries and Yukawa interactions
- Such vacua possess several phenomenologically desirable properties that one can utilize

Open Questions

- Brane constructions for softly deformed theories? hints about the vacuum before $U(1)_B$ gauging?
- Non-SUSY tests of duality
- More concrete model building. Top quark mass can also arise from the $\mathcal{O}(1)$ Yukawa couplings, unlike what common lore in technicolor suggests. (Higgs is composite and yet easy to get top mass – no need for walking etc.)