Noncritical Holographic Models in External Electric Field

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arXiv:1102.4509

The generalization of AdS/CFT correspondence to more realistic gauge theories like QCD, provides new insights to understanding the dynamical non-perturbative effects in QCD, such as confinement, chiral symmetry breaking, color superconductivity and so on.

Dual gauge theories arising from brane constructions in ten-dimensional critical string theory are supersymmetric. In order to break supersymmetry, one may compactify the supersymmetric guage theory on a circle of radius R and impose anti-periodic boundary conditions for fermions around the circle. The resulting effective theory at low energy compared to the Kaluza-Klein mass scale, M_KK ∼ 1 R, is pure QCD without fundamental matter.

The main obstacle of this approach, however, is that resulting holographic QCD contains undesired Kaluza-Klein modes with the mass as the same order of hadrons and glueballs. One way to overcome this problem is to consider brane backgrounds in noncritical string theory . Since in this case holographic backgrounds live in lower dimensions, the problem of extra KK modes is more tractable .

Noncritical Model

S. Kuperstein and J. Sonnenschein, “Non-critical supergravity (d > 1) and holography,” [arXiv:hep-th/0403254], S. Kuperstein andJ. Sonnenschein, “Non-critical, near extremal AdS(6) background as a holographic laboratory of four dimensional YM theory,”[arXiv:hep-th/0411009]

The model is based on D4/D4-D4 brane system, where Nc D4-branes compactified on S1 with radius R and Nf D4-D4 flavor branes are transverse to the S1.

	t	x_1	x_2	x_3	x_4	x_5
D4	$\diamond$	$\diamond$	$\diamond$	$\diamond$	$\diamond$	
D4-$\overline{\text{D4}}$	$\diamond$	$\diamond$	$\diamond$	$\diamond$		$\diamond$

At zero temperature the 6-dimensional back-ground metric is given by

$$ds_6^2 = \left(\frac{u}{R_{AdS}}\right)^2 (-dt^2 + dx^i dx^i + f(u)\, dx_4^2) + \left(\frac{R_{AdS}}{u}\right)^2 \frac{du^2}{f(u)}$$

$$f(u) = 1 - \left(\frac{u_\Lambda}{u}\right)^5, \qquad R_{AdS} = \sqrt{\frac{15}{2}}\, l_s$$

$$\begin{aligned} F_{(6)} &= -Q_c \left(\frac{u}{R_{AdS}}\right)^4 dx_0 \wedge dx_1 \wedge dx_2 \wedge dx_3 \wedge dx_4 \wedge du \\ e^\phi &= \frac{2\sqrt{2}}{\sqrt{3}Q_c} \end{aligned}$$

At nonzero temperature there are two solutions with the same boundary condition

The one which dominates at low temperature is given by above metric where the periodicity of euclidian time is arbitrary, and periodicity of x4 is given by

$$x_4 \sim x_4 + 2\pi R = x_4 + \frac{4\pi R_{AdS}^2}{5u_\Lambda}$$

The flavor branes form a U embedding configuration, so this background corresponds to the confined phase with broken chiral symmetry.

By increasing temperature a confinement/deconfinement phase transition occurs at

$$T_c = \frac{1}{2\pi R}$$

the background for $T_c < T$ is represented by

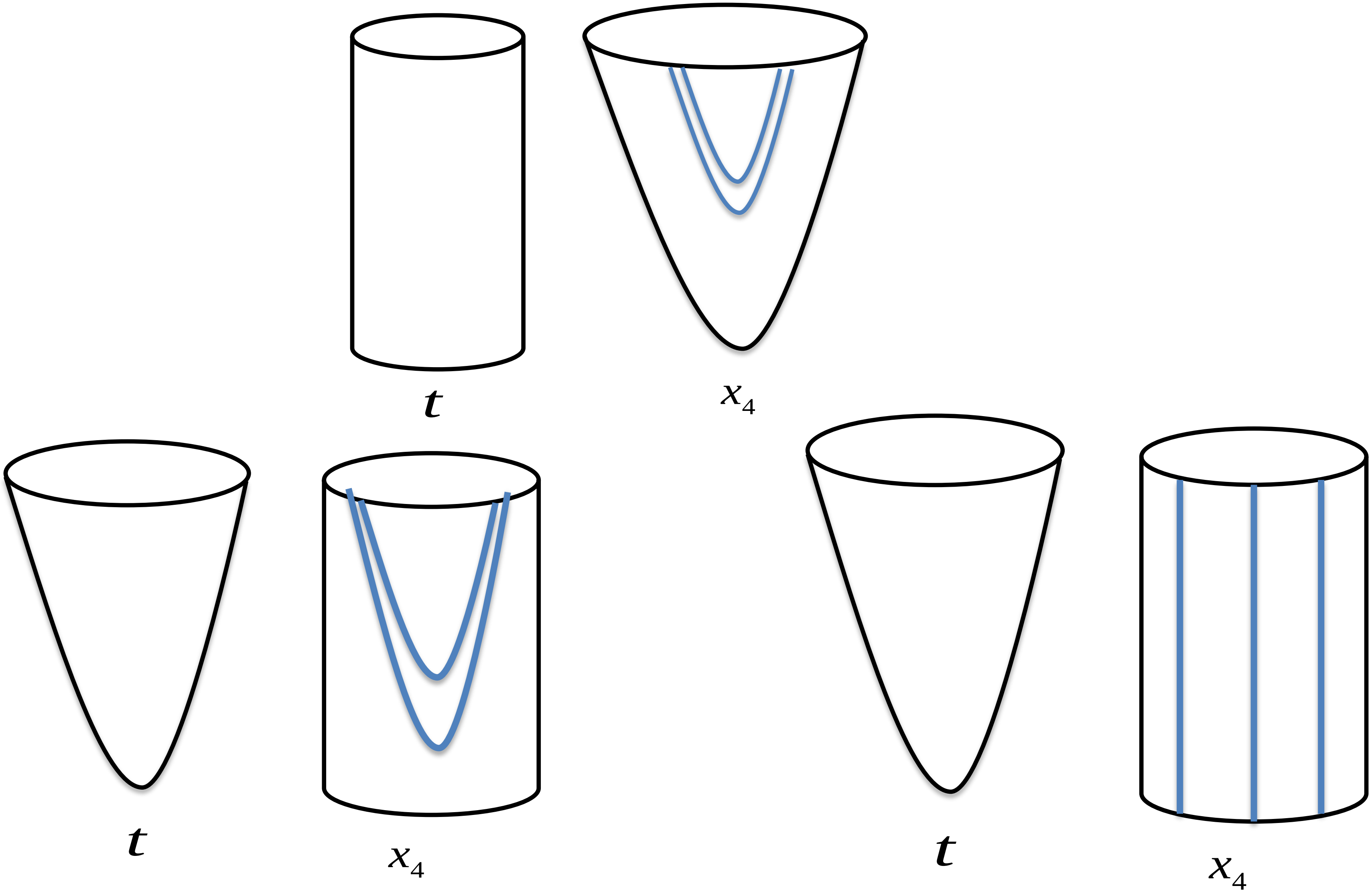
$$ds^2 = (\frac{u}{R_{AdS}})^2 (f(u) dt_E^2 + dx_i dx_i + dx_4^2) + (\frac{R_{AdS}}{u})^2 \frac{1}{f(u)} du^2$$

$$f(u) = 1 - (\frac{u_T}{u})^5, \qquad t_E \sim t_E + \frac{4\pi R_{AdS}^2}{5u_T}$$

$$T = \frac{5u_T}{4\pi R_{AdS}^2}$$

This background allows two embeddings for D4-D4 flavor branes: U embedding which is preferred configuration for $T_c < T < T_{\chi S}$ and parallel embedding which dominates for $T > T_{\chi S}$ where $T_{\chi S} = .169\, L$

Thus chiral symmetryrestores at temperatures above $T_{\chi S}$.



V. Mazu and J. Sonnenschein, “Non critical holographic models of the thermal phases of QCD,” JHEP 0806, 091 (2008) [arXiv:0711.4273 [hep-th]].

Adding U(1) gauge field

It is interesting to investigate the behavior of QCD matter in presence of other fundamental interactions. Indeed the physics of some astrophysical phenomena like quark stars is related to the behavior of thermal QCD in external electromagnetic field. Since QCD is strongly coupled at these circumstance, it is natural to use AdS/CFT and extracting the properties of QCD at these high densities and temperatures.

$$A_x(t_E, u) = -iE\, t_E + h(u)$$

time dependence part of gauge field describes a static electric field on the boundary and u dependence part encodes the response current.

We analyze various phases of noncritical holographic QCD in external electric field

A. Karch and A. O'Bannon, Metallic AdS/CFT," JHEP 0709, 024 (2007) [arXiv:0705.3870 [hep-th]]
O. Bergman, G. Lifschytz and M. Lippert, \Response of Holographic QCD to Electric and Magnetic Fields," JHEP 0805, 007 (2008) [arXiv:0802.3720 [hep-th]]

Deconfined Phase

The induced metric on flavor branes is given by

$$ds^2 = (\frac{u}{R_{AdS}})^2(f(u)dt_E^2 + dx_i dx_i) + \left((\frac{u}{R_{AdS}})^2 {x_4'}^2 + (\frac{R_{AdS}}{u})^2 \frac{1}{f(u)}\right) du^2$$

DBI action in presence of background gauge field takes the following form

$$S = \frac{\mathcal{N}}{R_{AdS}^5} \int du u^5 \sqrt{(f(u){x_4'}^2 + \frac{R_{AdS}^4}{u^4})(1 - \frac{e^2 R_{AdS}^4}{u^4 f(u)}) + \frac{f(u)R_{AdS}^4}{u^4} {a_x'}^2}$$

$$\mathcal{N} = 2N_f T_4 e^{-\phi}$$

By making use of the first integrals of motion one can find the asymptotic forms of gauge field and x' 4 at large u

$$a_x' \sim \frac{j}{u^3}, \qquad x_4' \sim \frac{c}{u^7}$$

$$S = \frac{\mathcal{N}}{R^5}\int du u^5 \sqrt{(f(u)x_4'^2 + \frac{R^4_{AdS}}{u^4})(f(u) - \frac{e^2 R^4_{AdS}}{u^4})(f(u) - \frac{j^2}{u^6})^{-1}}$$

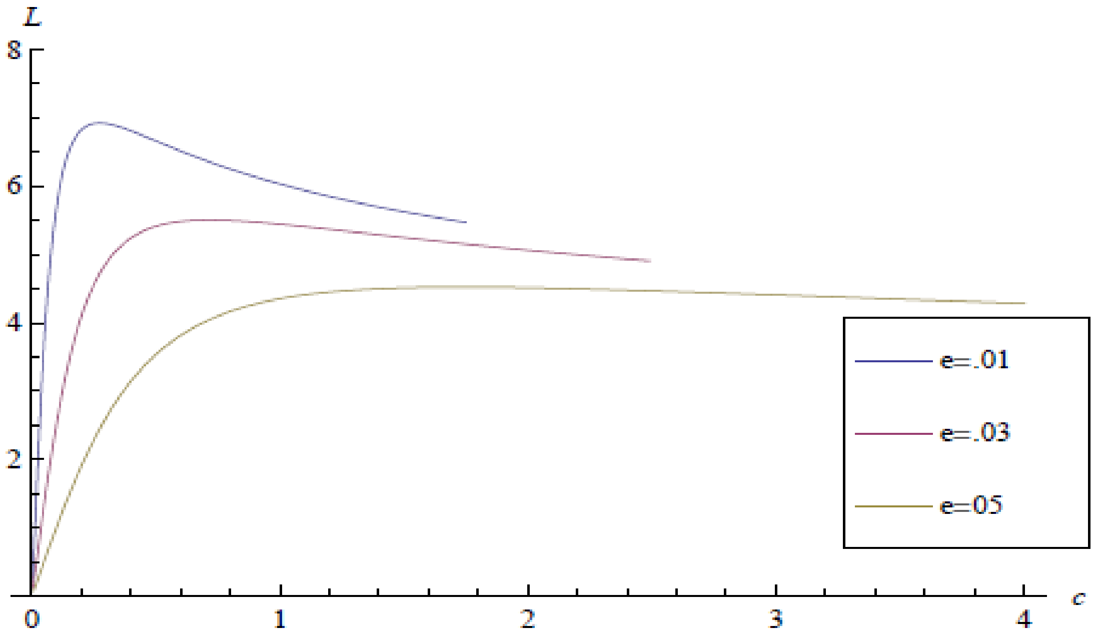
First consider the U embedding. In the case of vanishing response current, j = 0, the solution takes the following form

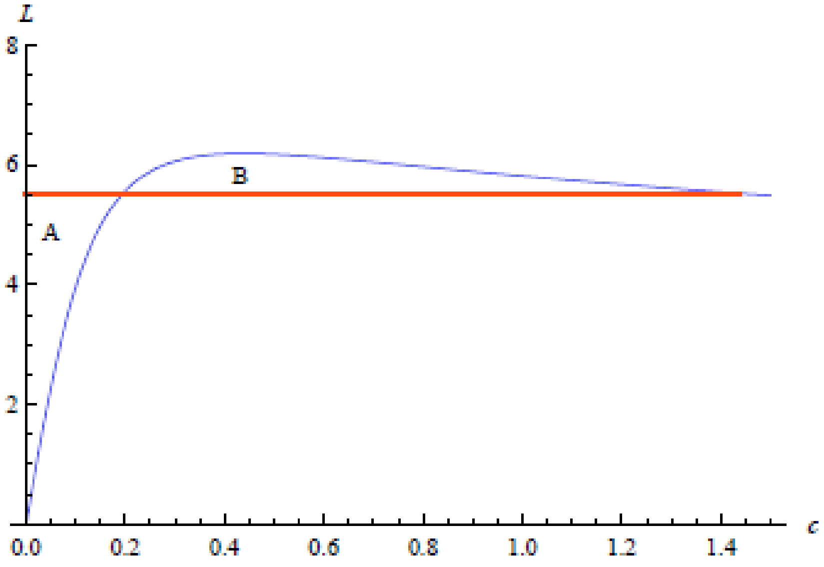
$$x_4'(u) = \frac{R^2_{AdS}}{u^2\sqrt{f}}\left[\frac{u^{10}(f(u) - \frac{e^2 R^4_{AdS}}{u^4})}{u_0^{10}(f(u_0) - \frac{e^2 R^4_{AdS}}{u_0^4})} - 1\right]^{\frac{-1}{2}}$$

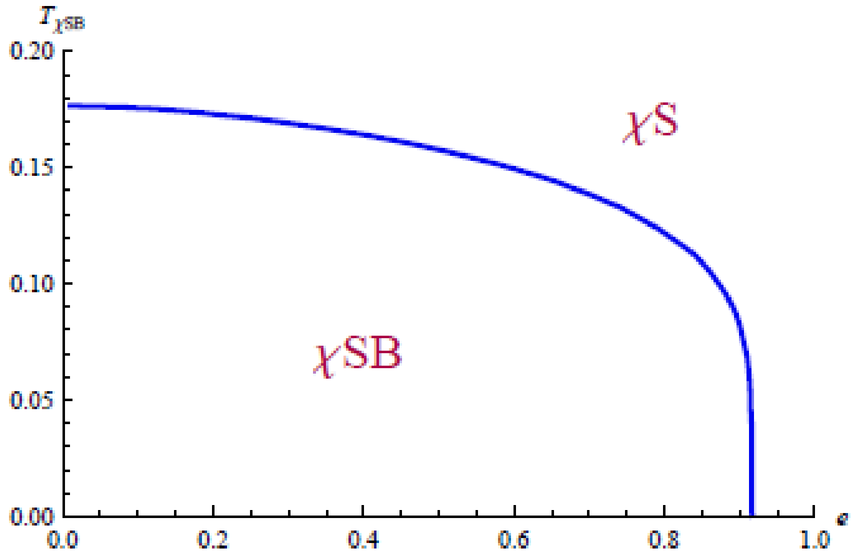
$$e_0^2 \leq \frac{1}{R^4_{AdS}} u_0^4 f(u_0)$$

Since by turning on the current, action increased, the favored configuration is a U embedding with vanishing current, j = 0 . In dual QCD this means that the deconfined chiral-symmetry breaking phase is an insulator.

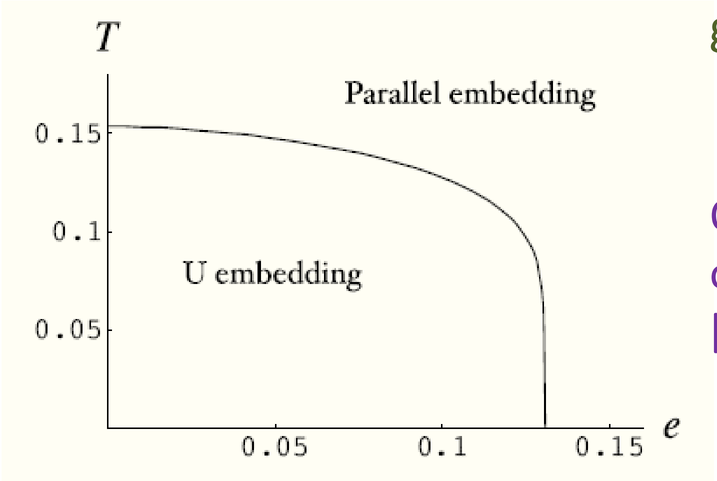
A natural question is that what happens for e > e0?. Figure 1 depicts L as a function of c for different values of electric field. As it is evident from this figure, L is a decreasing function of e, so there is a maximal value of e at fixed values of L and T, such that above which there are no U embedding solution. Thus we expect that the favorite solution in this regime becomes parallel embedding and there should be a phase transition from chiral-broken phase to chiral symmetric phase by increasing electric field.







According to this phase diagram, we observe that the critical temperature decreases with increasing external electric field, as we expected from the polarization effect of electric field.



general structure of phase diagram closely resembles phase diagram of Sakai-Sugimoto model

O. Bergman, G. Lifschytz and M. Lippert, “Response of Holographic QCD to Electric and Magnetic Fields [arXiv:0802.3720 [hep-th]].

In the parallel embedding the DBI action becomes

$$S_{||} = \frac{\mathcal{N}}{R_{AdS}^3} \int_{u_T}^{\infty} du\, u^3 \sqrt{\frac{f(u) - \frac{e^2 R_{AdS}^4}{u^4}}{f(u) - \frac{j^2}{u^6}}}$$

From this expression, it is clear that action becomes complex somewhere unless a nonzero current being turned on.

$$\sigma_{non-critical} = \frac{J^x}{E} = \frac{(2\pi\alpha')^2 \mathcal{N}}{R_{AdS}^3} \frac{j}{e} = \frac{(2\pi\alpha')^2 \mathcal{N}}{R_{AdS}} u_c(e, T)$$

$$f(u_c) - \frac{e^2 R_{AdS}^4}{u_c^4} = 0$$

There is no algebraic solution for above equation, however, we can study the weak filed and strong field behavior of conductivity

$$\sigma_{non-critical} = \begin{cases} \sqrt{\frac{12\pi}{5}}\,(2\pi\alpha')^{\frac{5}{2}}\mathcal{N}\ T & E \ll \frac{12\pi}{5}T^2 \\ (2\pi\alpha')^{\frac{5}{2}}\mathcal{N}\ E^{\frac{1}{2}} & E \gg \frac{12\pi}{5}T^2 \end{cases}$$

$$\sigma_{S-S} = \begin{cases} \frac{N_f N_c}{27\pi}\lambda_5\ T^2 & E \ll \frac{8\pi^2}{27}\lambda_5 T^3 \\ \frac{N_f N_c}{12\pi^{\frac{7}{3}}}\lambda_5^{\frac{1}{3}}\ E^{\frac{2}{3}} & E \gg \frac{8\pi^2}{27}\lambda_5 T^3 \end{cases}$$

$$\sigma_{D3-D7} = \begin{cases} \frac{N_f N_c}{4\pi}\ T & E \ll \frac{\pi}{2}\sqrt{\lambda_4}T^2 \\ \frac{N_f N_c}{(2\pi)^{\frac{3}{2}}\lambda_4^{\frac{1}{4}}}\ E^{\frac{1}{2}} & E \gg \frac{\pi}{2}\sqrt{\lambda_4}T^2 \end{cases}$$

finite density

we generalize our analysis in the parallel embedding for finite baryon density by turning on a nontrivial zero-component of the gauge field

$$S=\frac{\mathcal{N}}{R^5_{AdS}}\int du u^5\sqrt{(f(u)x_4'^2+\frac{R^4_{AdS}}{u^4})(1-\frac{e^2R^4_{AdS}}{u^4f(u)})+\frac{f(u)R^4_{AdS}}{u^4}a_x'^2-\frac{R^4_{AdS}}{u^4}a_0'^2}$$

$$a_0\simeq constant+\frac{d}{2u^2} \qquad\qquad D=\frac{(2\pi\alpha')\mathcal{N}}{R^3_{AdS}}d$$

$$\sigma=\frac{J}{E}=\frac{(2\pi\alpha')^2\mathcal{N}}{R_{AdS}}\sqrt{u_c^2+\frac{d^2}{u_c^4}}$$

$$\sigma_{noncritical} = \frac{25\alpha'^2\mathcal{N}}{4R_{AdS}^5}\frac{d}{T^2} = \frac{5}{12\pi}\frac{D}{T^2}$$

$$\sigma_{Sakai-Sugimoto} = \frac{27}{8\pi^2\lambda_5}\frac{D}{T^3}$$

$$\sigma_{D3-D7} = \frac{2}{\pi\sqrt{\lambda_4}}\frac{D}{T^2}$$

Again we observe that charge density dependence of conductivity in noncritical model and D3-D7 system is the same.

Kubo formula

The response of a thermodynamic system to an applied external field is described by transport coefficients of the system. For small deviation from equilibrium, Kubo formula relates transport coefficients to the equilibrium retarded green's functions of the system. In particular real-time correlator of two electromagnetism current determines electric conductivity of the medium via

$$\sigma = \frac{1}{2} \lim_{\omega \longrightarrow 0} \frac{1}{\omega} ImG_{\alpha}^{\alpha\ R}(\omega, \mathbf{k})$$

where G is retarded green's function of two transverse electromagnetic current.

By expanding the DBI action up to second order in the field strength around the background , we arrive at

$$\begin{aligned} ds^2|_{D4} &= \left(\frac{u}{R_{AdS}}\right)^2 (-f(u)dt^2 + dx^i dx^i) + \left(\frac{R_{AdS}}{u}\right)^2 \frac{du^2}{f(u)} \\ F_{tu} &= -A_t'(t, u) \end{aligned}$$

$$S=\frac{(2\pi\alpha')^2\mathcal{N}}{2R_{AdS}^3}\int\frac{du}{\sqrt{u^6+C^2}}[-\frac{(u^6+C^2)}{f(u)}(\frac{R_{AdS}}{u})^4F_{ti}^2-\frac{(u^6+C^2)^2}{u^6}F_{0u}^2$$
$$+R_{AdS}^4u^2\sum_{i<j}F_{ij}^2+f(u)(u^6+C^2)\sum F_{iu}^2]$$

equation of motion for transverse components

$$E''_\alpha+\frac{y}{f(y)\sqrt{1+C'^2y^6}}(y^{-1}f(y)\sqrt{1+C'^2y^6})'E'_\alpha+\frac{\omega^2-\frac{q^2f(u)}{(1+C'^2y^6)}}{f(y)^2}E_\alpha=0$$

From this equation the near horizon behavior of E can be read as

$$E_\alpha=(y_T-y)^{\pm iw/2}$$

± represent ingoing and outgoing wave into horizon, respectively. By imposing ingoing boundary condition at the horizon, the behavior of solution near y = 0 becomes

$$E_\alpha = \mathcal{A} + \mathcal{B}\, y^2$$

By applying AdS/CFT prescription for calculating real-time correlator, we have

$$G^R_{\alpha\alpha} = \frac{\delta^2 S}{\delta A_\alpha \delta A_\alpha} = \omega^2 \frac{\delta^2 S}{\delta E_\alpha \delta E_\alpha} = 2(2\pi\alpha')^2 \mathcal{N} R_{AdS} Im[\mathcal{A}/\mathcal{B}]$$

at low frequency, w –→ 0, the result is $\frac{\mathcal{A}}{\mathcal{B}} = 1 + \frac{5}{4} i \sqrt{1 + C' y_T^6}\, \frac{w}{y_T^2}$

$$\sigma = \frac{(2\pi\alpha')^2 \mathcal{N}}{R_{AdS}} u_T \sqrt{1 + \frac{d^2}{u_T^6}}$$

Comparison with Lattice simulations

Computation of transport coefficients by using lattice QCD requires an analytic continuation to real-time space, which leads to systematic errors in results. By using Maximum Entropy Method a lattice computation of conductivity has been performed in

S. Gupta, The electrical conductivity and soft photon emissivity of the QCD plasma," Phys. Lett. B 597, 57 (2004) [arXiv:hep-lat/0301006].

$$\frac{\sigma(T)}{T} = C_{EM} \begin{cases} 7.5 \pm 0.8, & T = 1.5T_c \\ 7.7 \pm 0.6, & T = 2T_c \\ 7.0 \pm 0.4, & T = 3T_c \end{cases}$$

where electromagnetic vertex factor is given by $C_{EM} = 4\pi\alpha \sum e_f{}^2$

α is fine structure constant and *ef* *is electric charge of a quark with* flavor f

For two flavors $C_{EM} \approx \frac{1}{20}$

G. Aarts, C. Allton, J. Foley, S. Hands and S. Kim, Phys. Rev. Lett. 99, 022002 (2007).
H.T.Ding, A.Francis, O.Kaczmarek, F.Karsch, E.Laermann, W.Soeldner, ``Thermal dilepton rate and electrical conductivity: An analysis of vector current correlation functions in quenched lattice QCD,'' Phys.Rev.D-83, 034504 (2011).

$$\frac{\sigma}{T} \simeq (0.4 \pm 0.1) C_{EM}$$

If we can trust these results, they show that, with good accuracy, conductivity is linear in temperature, in accordance with the predictions of noncritical model and D3-D7 system

Conclusion

- Non-criticla model predicts a linear dependence on temperature for conductivity of QGP in agreement with lattice simulations .

- The same behavior of conductivity as a function of temperature and chemical potential in noncritical model and D3-D7 system.

- The general structure of phase diagram closely resembles phase diagram of Sakai-Sugimoto model. This universal property may be related to the large-N limit.

M. Hanada and N. Yamamoto, Universality of Phases in QCD and QCD-like Theories," arXiv:1103.5480 [hep-ph].

Thank You!

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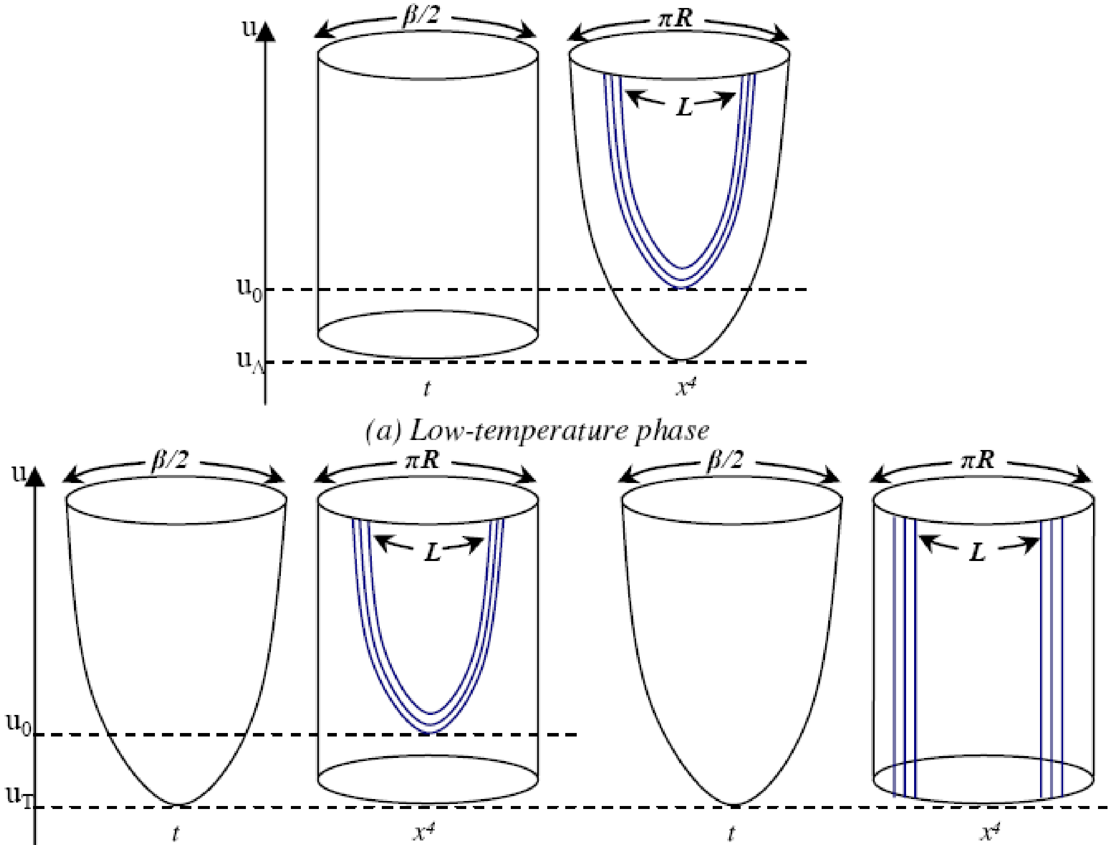
Noncritical Model

	t	x_1	x_2	x_3	x_4	x_5
D4	$\diamond$	$\diamond$	$\diamond$	$\diamond$	$\diamond$	
D4-$\overline{\text{D4}}$	$\diamond$	$\diamond$	$\diamond$	$\diamond$		$\diamond$

$$ds_6^2 = \left(\frac{u}{R_{AdS}}\right)^2 (-dt^2 + \delta_{ij}dx^i dx^j + f(u)dx_4^2) + \left(\frac{R_{AdS}}{u}\right)^2 \frac{du^2}{f(u)}$$

$$F_{(6)} = Q_c\left(\frac{u}{R_{AdS}}\right)^4 dx_0 \wedge dx_1 \wedge dx_2 \wedge dx_3 \wedge du \wedge dx_4$$

$$e^{\phi} = \frac{2\sqrt{2}}{\sqrt{3}Q_c} \qquad R_{AdS}^2 = \frac{15}{2} \qquad f(u) = 1 - \left(\frac{u_\Lambda}{u}\right)^5$$



(a) Low-temperature phase

(b) Intermediate-temperature phase

(c) High-temperature phase

$$T_{\chi SB} = \frac{0.169}{L}$$

$$T_d = \frac{1}{2\pi R} = \frac{0.159}{R}$$

Deconfined Phase

$$A_x(t_E, u) = -iEt_E + h(u)$$

$$S = \frac{\mathcal{N}}{R^5_{AdS}} \int du u^5 \sqrt{(f(u){x'_4}^2 + \frac{R^4_{AdS}}{u^4})(1 - \frac{e^2 R^4_{AdS}}{u^4 f(u)}) + \frac{f(u) R^4_{AdS}}{u^4} {a'_x}^2}$$

$$a'_x \sim \frac{j}{u^3}, \qquad x'_4 \sim \frac{c}{u^7}$$

$$J^x = \frac{2\pi\alpha' \mathcal{N}}{R^3_{AdS}} j$$

$$S = \frac{\mathcal{N}}{R^5} \int du u^5 \sqrt{(f(u){x'_4}^2 + \frac{R^4_{AdS}}{u^4})(f(u) - \frac{e^2 R^4_{AdS}}{u^4})(f(u) - \frac{j^2}{u^6})^{-1}}$$

$$S_{||} = \frac{\mathcal{N}}{R_{AdS}^3} \int_{u_T}^{\infty} du\, u^3 \sqrt{\frac{f(u) - \frac{e^2 R_{AdS}^4}{u^4}}{f(u) - \frac{j^2}{u^6}}}$$

$$\sigma_{non-critical} = \frac{J^x}{E} = \frac{(2\pi\alpha')^2 \mathcal{N}}{R_{AdS}^3} \frac{j}{e} = \frac{(2\pi\alpha')^2 \mathcal{N}}{R_{AdS}} u_c(e,T)$$

$$f(u_c) - \frac{e^2 R_{AdS}^4}{u_c^4} = 0$$

$$\sigma_{non-critical} = \begin{cases} \sqrt{\frac{12\pi}{5}}\,(2\pi\alpha')^{\frac{5}{2}}\mathcal{N}\ T & E \ll \frac{12\pi}{5}T^2 \\ (2\pi\alpha')^{\frac{5}{2}}\mathcal{N}\ E^{\frac{1}{2}} & E \gg \frac{12\pi}{5}T^2 \end{cases}$$

$$\sigma_{S-S} = \begin{cases} \frac{N_f N_c}{27\pi}\lambda_5\ T^2 & E \ll \frac{8\pi^2}{27}\lambda_5 T^3 \\ \frac{N_f N_c}{12\pi^{\frac{7}{3}}}\lambda_5^{\frac{1}{3}}\ E^{\frac{2}{3}} & E \gg \frac{8\pi^2}{27}\lambda_5 T^3 \end{cases}$$

$$\sigma_{D3-D7} = \begin{cases} \frac{N_f N_c}{4\pi}\ T & E \ll \frac{\pi}{2}\sqrt{\lambda_4}T^2 \\ \frac{N_f N_c}{(2\pi)^{\frac{3}{2}}\lambda_4^{\frac{1}{4}}}\ E^{\frac{1}{2}} & E \gg \frac{\pi}{2}\sqrt{\lambda_4}T^2 \end{cases}$$

$$\sigma_{D3-D7} = \frac{N_f N_c T}{4\pi}$$

$$\sigma_{non-critical} = \sqrt{\frac{12\pi}{5}}\,(2\pi\alpha')^{\frac{5}{2}} \mathcal{N}\ T$$

$$\sigma_{S-S} = \frac{N_f N_c}{27\pi}\lambda_5\ T^2$$

$$\frac{\sigma(T)}{T} = C_{EM}\begin{cases} 7.5 \pm 0.8, & T = 1.5T_c \\ 7.7 \pm 0.6, & T = 2T_c \\ 7.0 \pm 0.4, & T = 3T_c \end{cases} \qquad C_{EM} = 4\pi\alpha \sum e_f^2$$

S. Gupta, "The electrical conductivity and soft photon emissivity of the QCD plasma," Phys. Lett. B **597**, 57 (2004) [arXiv:hep-lat/0301006].

Material	σ [S/m] at 20 °C
Silver	6.30×10^{7}
Cooper	5.96×10^{7}
Water	4.8

1 ev of conductivity = 1.4×10^{4} S/m

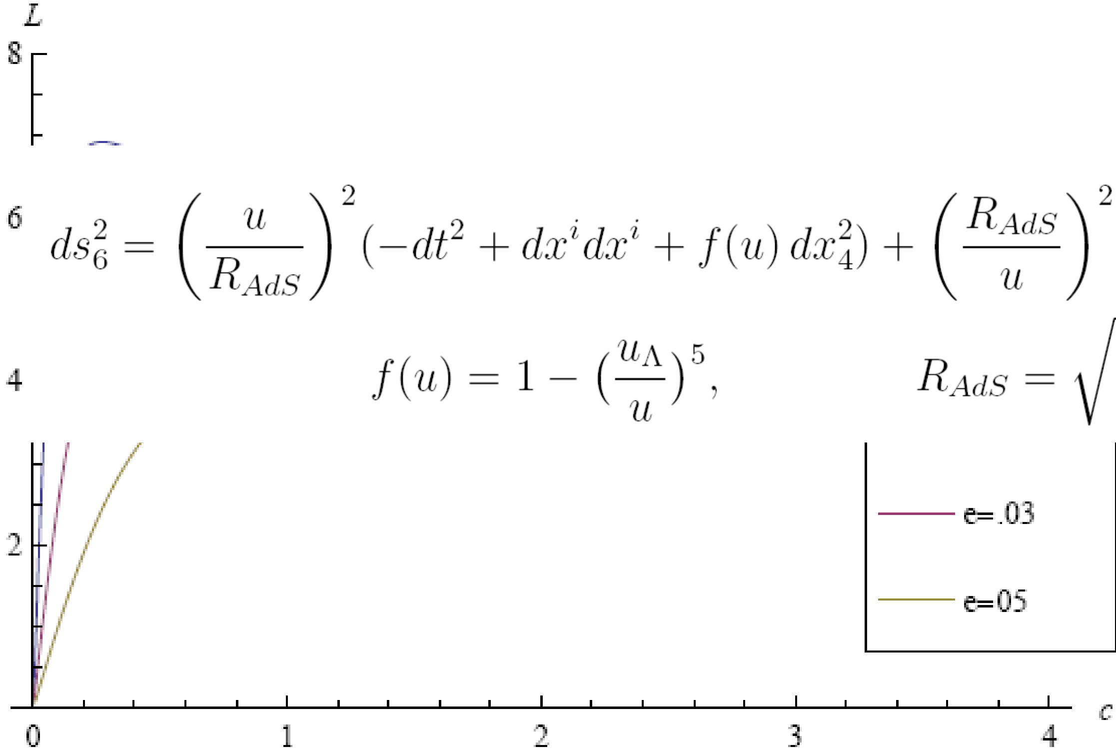
$$\sigma_{D3-D7} \approx 150\,\mathrm{Mev} \approx 10^{12}\,\mathrm{S/M}$$

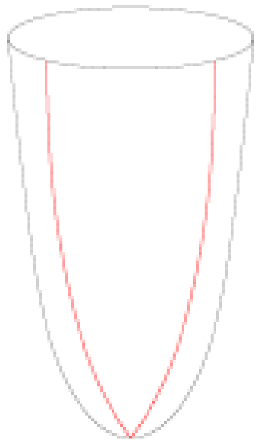
$$\sigma_{S-S} \approx \frac{1}{2\pi R} {T_c}^{2} \approx T_c \approx 10^{12}\,\mathrm{S/M}$$

$$\sigma_{lattice} \approx \frac{7}{20} T_c \approx 10^{11} S / M$$

$$ds_6^2 = \left(\frac{u}{R_{AdS}}\right)^2 (-dt^2 + dx^i dx^i + f(u)\, dx_4^2) + \left(\frac{R_{AdS}}{u}\right)^2 \frac{du^2}{f(u)}$$

$$f(u) = 1 - (\frac{u_\Lambda}{u})^5, \qquad R_{AdS} = \sqrt{\frac{15}{2}}\, l_s$$





$$S_V = \frac{\mathcal{N}}{R_{AdS}^3} \int_{u_T}^{\infty} du \, \frac{u^3}{\sqrt{f(u)}} \sqrt{\frac{1 - \frac{e^2 R_{AdS}^4}{u^4}}{1 - \frac{j^2}{u^6}}}$$

$$\sigma = (2\pi\alpha')^{\frac{5}{2}} \mathcal{N} \; E^{\frac{1}{2}}$$