# Aspects of Higher Spin Gravity 

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## Topics

- Some background
- Structure and properties of Vasiliev's HS gravity
- Recent results on geometry and observables in HS gravity based on: E.S. and P. Sundell, arXiv:1103.2360


## Some Background

See, e.g., introduction in E.S. and P. Sundell, hep-th/0105001

- Fierz \& Pauli 1939: Free massive HS eqs in Minkowski ${ }_{4}$
- Weinberg 1964, Weinberg \& Witten 1980: No-go theorems for HS interactions
- Fronsdal 1978: Free massless limit in Minkowski ${ }_{4}$
- Fronsdal 1978: Free massless HS eqs in $A d S_{4}$
- Fradkin \& Vasiliev 1987: HS algebras, cubic gravitational interactions in $A d S_{4}$
- Vasiliev 1987-1992: Full HS gravity eqs of motion in $A d S_{4}$
- Connection with M-theory:
E. Bergshoeff, A. Salam, E.S. and Y. Tanii, 1988

| $\ell \backslash s$ | 0 | $\frac{1}{2}$ | 1 | $\frac{3}{2}$ | 2 | $\frac{5}{2}$ | 3 | $\frac{7}{2}$ | 4 | $\frac{9}{2}$ | 5 | $\frac{11}{2}$ | 6 | $\cdots$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 0 | 70 | 56 | 28 | 8 | 1 |  |  |  |  |  |  |  |  |  |
| 1 | $1+1$ | 8 | 28 | 56 | 70 | 56 | 28 | 8 | 1 |  |  |  |  |  |
| 2 |  |  |  |  | 1 | 8 | 28 | 56 | 70 | 56 | 28 | 8 | 1 |  |
| 3 |  |  |  |  |  |  |  |  | 1 | 8 | 28 | 56 | 70 | $\cdots$ |
| 4 |  |  |  |  |  |  |  |  |  |  |  |  | 1 | $\cdots$ |
| $\vdots$ |  |  |  |  |  |  |  |  |  |  |  |  |  |  |

The conjectured spectrum of massless states of M 2 brane on $A d S_{4} \times S^{7}$, given by symmetric product of $O S p(8 \mid 4)$ singletons.

- Connection with tensionless string on $A d S_{5} \times S^{5}$ and free Yang-Mills theory in the large N limit:

Haggi-Mani and Sundborg 2000, Sundborg 2001
In this limit the $\frac{\alpha^{\prime}}{R^{2}}=\frac{1}{g_{Y M}^{2} N} \rightarrow \infty$
Developed further by:
E.S. \& Sundell, 2002

Beisert, Bianchi, Morales, Samtleben, 2003 :
:
See also: E. Witten, talk given at J.H. Schwarz 60th Birthday Conference, 2001.

## $H S T_{4} / C F T_{3}$ correspondence

E.S. and Sundell conjecture, hep-th/0205131:

Vasilev's HS gravity (Type A with parity even bulk scalar) is dual to free $S U(N)$ valued scalar field theory in $1 / N^{2}$ expansion where

$$
\mathcal{L} \sim \operatorname{tr}(\partial \phi)^{2}, \quad S_{H S}=\frac{N^{2}}{R_{A d S}^{2}} \int d^{4} x \mathcal{L}_{H S}
$$

Klebanov and Polyakov conjecture, hep-th/0210114:
Vasiliev's HS gravity is dual to either free $O(N)$ vector model in $1 / N$ expansion
$\mathcal{L} \sim \frac{1}{2}(\partial \vec{\phi}) \cdot(\partial \vec{\phi}) \quad$ (UV fixed point) $\quad S_{H S}=\frac{N}{R_{A d S}^{2}} \int d^{4} x \mathcal{L}_{H S}$
or strongly coupled IR fixed point reached by double-trace deformation
$\Delta \mathcal{L} \frac{\lambda}{2 N}(\vec{\phi} \cdot \vec{\phi})^{2}$

Variant conjecture: E.S. and P. Sundell, 2003
Type B HS gravity which has with odd-parity bulk scalar is dual to free fermion field at IR fixed point, or a strongly coupled UV fixed point reached by double trace deformation
$\Delta \mathcal{L} \sim \frac{\lambda}{2 N}(\bar{\psi} \psi)^{2} \quad$ (Gross-Neveu model)
Evidence for the conjecture so far in theories in the case of both boundary conditions, and to leading order in $1 / N$ has been found.

Cubic scalar field couplings:
Petkou, 2003
E.S. and P. Sundell, 2003

Arbitrary three point functions:
X.Yin and S. Giombi, 2009 \& 2010

What about the matter coupled CS gauge theories in $3 D$ ? Are they dual to any HS gravity in 4D bulk?

Results to appear by:
Giombi, Minwalla, Prakash, Trivedi and Yin
Will come back to this later.
Visit also the web page for:
Simons Center Workshop on higher spin theories and holography, March 14-18, 2011

## Structure and Properties of HS Gravity

Free $O(N)$ vector model:
$S=\frac{1}{2} \int d^{3} x \sum_{a=1}^{N}\left(\partial_{\mu} \phi^{a}\right)^{2}$
$O(N)$ singlet conserved currents:
$J_{\left(\mu_{1} \cdots \mu_{s}\right)}=\phi^{a} \partial_{\left(\mu_{1}\right.} \cdots \partial_{\left.\mu_{s}\right)} \phi^{a}+\cdots \quad s=0^{+}, 2,4, \ldots$
This is the spectrum of massless HS fields in minimal bosonic HS gravity in 4D.

Type A Model: $\quad \phi^{a} \phi^{a} \quad \leftrightarrow \quad$ bulk scalar Type B Model: $\bar{\psi} \psi \quad \leftrightarrow$ bulk pseudo-scalar

Introduce commuting spinors $\left(y^{\alpha}, y_{\dot{\alpha}}\right)$, and package all the gauge fields into a master field $A_{\mu}(x, y, \bar{y})$ :

$$
\begin{aligned}
A_{\mu}(x, y, \bar{y})= & e_{\mu}^{\alpha \dot{\alpha}} y_{\alpha} \bar{y} \dot{\alpha}+\omega_{\mu}^{\alpha \beta} y_{\alpha} y_{\beta}+\omega_{\mu}^{\dot{\alpha} \dot{\beta}} \bar{y}_{\dot{\alpha}} \bar{y}_{\dot{\beta}}+\cdots \\
& +A_{\mu}^{\alpha_{1} \ldots \alpha_{s-1} \dot{\alpha}_{1} \cdots \alpha_{s-1}} y_{\alpha_{1}} \cdots y_{a_{s-1}} \bar{y}_{\dot{\alpha}_{1}} \cdots \bar{y}_{\dot{\alpha}_{s-1}}+\cdots
\end{aligned}
$$

Impose constraints on $A(x, y, \bar{y})$ such that only $s=2,4,6, \ldots$ are independent fields.

What about the scalar field $\phi$ ?
Combine $\phi$ and spin s Weyl tensors $\phi_{\alpha_{1} \cdots \alpha_{s+2}}$ into a master 0 -form:

$$
\begin{aligned}
\Phi(x, y, \bar{y})= & \phi(x)+\phi_{\alpha \dot{\alpha}}(x) y^{\alpha} \bar{y}^{\dot{\alpha}}+\cdots+\phi_{\alpha_{1} \cdots \alpha_{4}}(x) y^{\alpha_{1}} \cdots y^{\alpha_{4}} \\
& +\phi_{\alpha \dot{\alpha} \alpha_{1} \cdots \alpha_{4}}(x) y^{\alpha} \bar{y}^{\dot{\alpha}} y^{\alpha_{1}} \cdots y^{\alpha_{4}}+\cdots
\end{aligned}
$$

Impose constraint on $\Phi(x, y, y b)$ such that

$$
\phi_{\alpha \dot{\alpha}} \sim \partial_{\alpha \dot{\alpha}} \phi, \quad \phi_{\alpha \dot{\alpha} \alpha_{1} \cdots \alpha_{4}} \sim \partial_{\alpha \dot{\alpha}} \phi_{\alpha_{1} \cdots \alpha_{4}}, \quad \text { etc }
$$

To construct the HS algebra, call it hs(4), associated with the HS gauge fields, let $\left(y_{\dot{\alpha}}, \bar{y}^{\dot{\alpha}}\right)$ obey the oscillator algebra
$y_{\alpha} \star y_{\beta}=y_{\alpha} y_{\beta}+i \epsilon_{\alpha \beta}, \quad \bar{y}_{\dot{\alpha}} \star \bar{y}_{\dot{\beta}}=\bar{y}_{\dot{\alpha}} \bar{y}_{\dot{\beta}}+i \epsilon_{\dot{\alpha} \dot{\beta}}$
where the $y_{\alpha} \star y_{\beta}$ denotes the operator product and $y_{\alpha} y_{\beta}=y_{\beta} y_{\alpha}$ denotes the Weyl ordered product.
$S O(3,2)$ generators: $y_{\alpha} y_{\beta}, \quad \bar{y}_{\dot{\alpha}} \bar{y}_{\dot{\beta}}, \quad y_{\alpha} \bar{y}_{\dot{\alpha}}$
$h s(4)$ generators: $\quad y_{\alpha_{1}} y_{\alpha_{2}} \cdots y_{\alpha_{n}} \bar{y}_{\alpha_{1}} \bar{y}_{\alpha_{2}} \cdots \bar{y}_{\alpha_{m}}$ with

$$
n+m=4 \ell+2, \quad \ell=0,1,2, \ldots
$$

The $h s(4)$ algebra $\mathcal{A}$ has an anti-involution $\tau$ and involutions $\pi$ and $\bar{\pi}$ acting on Weyl ordered function $f \in h s(4)$ as

$$
\begin{aligned}
& \tau(f(y, \bar{y}))=f(i y, i \bar{y}) \\
& \pi(f(y, \bar{y}))=f(-y, \bar{y}), \quad \bar{\pi}(f(y, \bar{y}))=f(y,-\bar{y}) .
\end{aligned}
$$

The $h s(4)$ gauge theory is constructed by introducing a one-form gauge field $A=d x^{\mu} A_{\mu}(x, y, \bar{y})$ and a zero-form $\Phi(x, y, \bar{y})$ satisfying the conditions

$$
\tau(A)=-A, \quad A^{\dagger}=-A
$$

$$
\tau(\Phi)=\bar{\pi}(\Phi), \quad \Phi^{\dagger}=\pi(\Phi)
$$

The idea is to construct $h s(4)$ invariant gauge theory such that treating $\Phi$ and HS fields as weak fields, the y-expansion to lowest order in weak fields would yield:

$$
\text { Gravity }\left\{\begin{array}{l}
\mathcal{R}_{\alpha \beta, \gamma \delta}=\Phi_{\alpha \beta \gamma \delta} \\
\mathcal{R}_{\alpha \beta, \dot{\gamma} \dot{\delta}}=0 \\
\mathcal{R}_{\alpha \beta, \gamma \dot{\delta}}=0
\end{array}\right.
$$

(Einstein eq.)

$$
\begin{aligned}
& \text { Higher spins }\left\{\begin{array}{l}
F_{\alpha \beta, \gamma_{1} \ldots \gamma_{2 s-2}}^{(1)}=\Phi_{\alpha \beta \gamma_{1} \ldots \gamma_{2 s-2}} \\
F_{\alpha \beta, \gamma_{1} \ldots \gamma_{k} \dot{\gamma}_{k+1} \ldots \dot{\gamma}_{2 s-2}}^{(1)}=0
\end{array}\right. \\
& \text { Scalar }\left\{\begin{array}{l}
\nabla_{\alpha}^{\dot{\alpha}} \Phi_{\beta_{1} \ldots \beta_{m}} \dot{\beta}_{1} \ldots \dot{\beta}_{n}=i \Phi_{\alpha \beta_{1} \ldots \beta_{m}} \dot{\alpha} \dot{\beta}_{1} \ldots \dot{\beta}_{n} \\
\left.-i m n \epsilon_{\alpha\left(\beta_{1}\right.} \epsilon^{\dot{\alpha}\left(\dot{\beta}_{1}\right.} \Phi_{\left.\beta_{2} \ldots \beta_{m}\right)} \dot{\beta}_{2} \ldots \dot{\beta}_{n}\right)
\end{array}\right.
\end{aligned}
$$

The stated task turns out to be extremely difficult, as it requires an unmanageable Weyl curvature expansion, manageable just up to cubic order (Fradkin and Vasiliev). Breakthrough came with Vasiliev's work in 1992, which solved this problem. Introduce commuting internal spinorial coordinates $\left(z^{\alpha}, \bar{z}^{\dot{\alpha}}\right)$ and the associative product rules

$$
\begin{array}{ll}
y_{\alpha} \star y_{\beta}=y_{\alpha} y_{\beta}+i \epsilon_{\alpha \beta}, & z_{\alpha} \star z_{\beta}=z_{\alpha} z_{\beta}-i \epsilon_{\alpha \beta} \\
y_{\alpha} \star z_{\beta}=y_{\alpha} z_{\beta}-i \epsilon_{\alpha \beta}, & z_{\alpha} \star y_{\beta}=z_{\alpha} y_{\beta}+i \epsilon_{\alpha \beta}
\end{array}
$$

Next, introduce the Grassmann even master fields

$$
\widehat{A}=d x^{\mu} \widehat{A}_{\mu}+d z^{\alpha} \widehat{A}_{\alpha}+d \bar{z}^{\dot{\alpha}} \widehat{A}_{\dot{\alpha}} \quad, \quad \widehat{\Phi}
$$

obeying

$$
\tau(\widehat{A})=-\widehat{A}, \quad \widehat{A}^{\dagger}=-\widehat{A}
$$

$$
\tau(\widehat{\Phi})=\bar{\pi}(\widehat{\Phi}), \quad \widehat{\Phi}^{\dagger}=\pi(\widehat{\Phi})
$$

The $\tau, \pi$ and $\bar{\pi}$ maps are defined by

$$
\begin{aligned}
& \tau(y, \bar{y}, z, \bar{z})=(i y, i \bar{y},-i z,-i \bar{z}) \\
& \pi(y, \bar{y}, z, \bar{z})=(-y, \bar{y},-z, \bar{z}) \\
& \bar{\pi}(y, \bar{y}, z, \bar{z})=(y,-\bar{y}, z,-\bar{z})
\end{aligned}
$$

Next, define $\star$-product as

$$
\left.\left.\begin{array}{rl}
\widehat{f} * \widehat{g} & =\widehat{f} \exp \left[i\left(\frac{\overleftarrow{\partial}}{\partial z_{\alpha}}+\frac{\overleftarrow{\partial}}{\partial y_{\alpha}}\right)\left(\frac{\vec{\partial}}{\partial z^{\alpha}}-\frac{\vec{\partial}}{\partial y^{\alpha}}\right)\right. \\
& +i\left(\frac{\overleftarrow{\partial}}{\partial \bar{z}_{\dot{\alpha}}}-\frac{\overleftarrow{\partial}}{\partial \bar{y}_{\dot{\alpha}}}\right)\left(\frac{\vec{\partial}}{\partial \partial_{\bar{z}}}\right.
\end{array} \frac{\vec{\partial}}{\partial \bar{y}^{\dot{\alpha}}}\right)\right] \widehat{g} \quad \$
$$

Next, define curvatures as

$$
\begin{aligned}
\widehat{F} & =d \widehat{A}+\widehat{A} \star \widehat{A} \\
\widehat{D} \widehat{\Phi} & =d \widehat{\Phi}+\widehat{A} \star \widehat{\Phi}-\widehat{\Phi} \star \bar{\pi}(\widehat{A})
\end{aligned}
$$

Vasiliev HS field equations are then expressed as constraints on these curvatures:

$$
\begin{aligned}
\widehat{F} & =i c_{1} d z^{\alpha} \wedge d z_{\alpha} \widehat{\Phi} \star \kappa+i c_{2} d \bar{z}^{\dot{\alpha}} \wedge d \bar{z}_{\dot{\alpha}} \widehat{\Phi} \star \bar{\kappa} \\
\widehat{D} \widehat{\Phi} & =0, \quad \kappa=e^{i y^{\alpha} z_{\alpha}}, \quad \bar{\kappa}=e^{-i \bar{y}^{\dot{\alpha}} \bar{z}_{\dot{\alpha}}}
\end{aligned}
$$

Invariant under HS gauge transformations:
$\delta \widehat{A}=d \widehat{\epsilon}+[\widehat{A}, \widehat{\epsilon}]_{\star}, \quad \delta \widehat{\Phi}=\widehat{\Phi} \star \bar{\pi}(\widehat{\epsilon})-\widehat{\epsilon} \star \widehat{\Phi}$
Signature $(3,1)$ :

$$
c_{1}^{*}=c_{2}
$$

Signatures: $(4,0)$ and $(2,2): \quad c_{1}^{*}=c_{1}, \quad c_{2}^{*}=c_{2}$
Chiral Model:

$$
c_{2}=0 \quad \text { for }(4,0),(2,2)
$$

with suitable reality conditions on $y, \bar{y}, z, \bar{z}$ understood in $(4,0)$ and $(2,2)$ signatures.
C. Iazeolla, E. Sezgin and P. Sundell, arXiv:0706.2983

## Expansion in Curvatures

Start from the initial conditions
$\left.\widehat{A}_{\mu}\right|_{Z=0}=A_{\mu}(x, y, \bar{y}),\left.\quad \widehat{\Phi}\right|_{Z=0}=\Phi(x, y, \bar{y})$
Integrating the constraints

$$
\widehat{D}_{\alpha} \widehat{\Phi}=0, \quad \widehat{F}_{\alpha \beta}=i \epsilon_{\alpha \beta} \widehat{\Phi} \star \kappa, \quad \widehat{F}_{\alpha \mu}=0
$$

Assuming $\Phi$ weak, we obtain perturbative solution
$\widehat{\Phi}(\Phi), \quad \widehat{A}_{\mu}(\Phi, A), \quad \widehat{A}_{\alpha}(\Phi)$
Constraint $\widehat{F}_{\mu \nu}=0$ and $\widehat{D}_{\mu} \widehat{\Phi}=0$ can then be shown to be equivalent to:

$$
\left.\widehat{F}_{\mu \nu}\right|_{Z=0}=0,\left.\quad \widehat{D}_{\mu} \widehat{\Phi}\right|_{Z=0}=0
$$

Substituting the solutions for $(\widehat{A}, \widehat{\Phi})$, these are the full HS field equations.

## Lorentz Transformations

Vierbein and Lorentz connection in $E_{\mu}=e_{\mu}+\omega_{\mu}$ where
$e_{\mu}=\frac{1}{2 i} e_{\mu}^{\alpha \dot{\alpha}} y_{\alpha} \bar{y}_{\dot{\alpha}}, \quad \omega_{\mu}=\frac{1}{4 i} \omega_{\mu}^{\alpha \beta} y_{\alpha} y_{\beta}-$ h.c.
Writing $A_{\mu}=E_{\mu}+W_{\mu}^{\prime}$, one finds that $W_{\mu}^{\prime}$ do not transform as Lorentz tensors. The remedy is to define

$$
\begin{aligned}
A_{\mu} & =E_{\mu}+W_{\mu}+K_{\mu} \\
K_{\mu} & \equiv i \omega_{\mu}^{\alpha \beta}\left(\widehat{A}_{\alpha} \star \widehat{A}_{\beta}\right)_{Z=0}-\text { h.c. }
\end{aligned}
$$

The component field in $W_{\mu}$ defined in this way do transform correctly as Lorentz tensors.

## Working in $Z$-space

Very advantageous in finding exact solutions and in amplitude computations. In this approach, the constraints

$$
\widehat{F}_{\mu \nu}=0, \quad \widehat{F}_{\mu \alpha}=0, \quad \widehat{D}_{\mu} \widehat{\Phi}=0
$$

are integrated in simply connected spacetime regions, with

$$
\begin{aligned}
\widehat{A}_{\mu} & =\widehat{L}^{-1} \star \partial_{\mu} \widehat{L} \\
\widehat{A}_{\alpha} & =\widehat{L}^{-1} \star\left(\widehat{A}_{\alpha}^{\prime}+\partial_{\alpha}\right) \widehat{L} \\
\widehat{\Phi} & =\widehat{L}^{-1} \star \Phi^{\prime} \star \pi(\widehat{L})
\end{aligned}
$$

where $\widehat{L}=\widehat{L}(x ; y, \bar{y}, z, \bar{z})$ is a gauge function, and $\widehat{A}_{\alpha}^{\prime}$ and $\widehat{\Phi}^{\prime}$ are $x$-independent: $\partial_{\mu} \widehat{A}_{\alpha}^{\prime}=0, \partial_{\mu} \widehat{\Phi}^{\prime}=0$

Solve the remaining constraints in $Z$-space:

$$
\begin{aligned}
& 2 \partial_{[\alpha} \widehat{A}_{\beta]}^{\prime}+\left[\widehat{A}_{\alpha}^{\prime}, \widehat{A}_{\beta}^{\prime}\right]_{\star}=-\frac{i}{2} \epsilon_{\alpha \beta} \widehat{\Phi}^{\prime} \star \kappa \\
& \partial_{\alpha} \widehat{A}_{\dot{\beta}}^{\prime}-\partial_{\dot{\beta}} \widehat{A}_{\alpha}^{\prime}+\left[\widehat{A}_{\alpha}^{\prime}, \widehat{A}_{\dot{\beta}}^{\prime}\right]_{\star}=0 \\
& \partial_{\alpha} \widehat{\Phi}^{\prime}+\widehat{A}_{\alpha}^{\prime} \star \widehat{\Phi}^{\prime}+\widehat{\Phi}^{\prime} \star \pi\left(\widehat{A}_{\alpha}^{\prime}\right)=0
\end{aligned}
$$

with initial condition $C^{\prime}(y, \bar{y})=\left.\widehat{\Phi}^{\prime}\right|_{Z=0}$.

Let us restrict our attention to gauge functions of the form: $\widehat{L}=L(x ; y, \bar{y})$. The gauge fields can then be obtained from

$$
e_{\mu}+\omega_{\mu}+W_{\mu}=L^{-1} \partial_{\mu} L-K_{\mu}
$$

with

$$
K_{\mu}=\left.i \omega_{\mu}^{\alpha \beta}\left(L^{-1} \star \widehat{A}_{\alpha}^{\prime} \star \widehat{A}_{\beta}^{\prime} \star L\right)\right|_{Z=0}
$$

Gauge fields, including the metric, are thus obtained algebraically, without any other integration in spacetime!

Prokushkin and Vasiliev, 1999
lazeolla, E.S. and Sundell, 2007: $S O(3,1)$ invariant and other
exact solutions
Didenko and Vasiliev, 2009: Static BPS black holes
Giombi and Yin, 2010: Great simplifications in amplitude computations

## Vasiliev equations in terms of deformed oscillators:

$$
\widehat{S}_{\alpha}=z_{\alpha}-2 i \widehat{A}_{\alpha}, \quad \widehat{S}_{\dot{\alpha}}=\bar{z}_{\dot{\alpha}}-2 i \widehat{A}_{\dot{\alpha}}
$$

obeying $\pi\left(\widehat{S}_{\alpha}\right)=-\bar{\pi}\left(\widehat{S}_{\alpha}\right)$ and $\pi\left(\widehat{S}_{\dot{\alpha}}\right)=-\bar{\pi}\left(\widehat{S}_{\dot{\alpha}}\right)$. Then Vasiliev equations become:

$$
\begin{aligned}
& \widehat{S}_{[\alpha} \star \widehat{S}_{\beta]}=-i \epsilon_{\alpha \beta}(1-\widehat{\Phi} \star \kappa) \\
& \widehat{S}_{\alpha} \star \widehat{S}_{\dot{\alpha}}=\widehat{S}_{\dot{\alpha}} \star \widehat{S}_{\alpha}, \quad \widehat{S}_{\alpha} \star \widehat{\Phi}+\widehat{\Phi} \star \pi\left(\widehat{S}_{\alpha}\right)=0 \\
& d \widehat{W}^{\prime}+\widehat{W}^{\prime} \star \widehat{W}^{\prime}=0 \\
& d \widehat{\Phi}+\left[\widehat{W^{\prime}}, \widehat{\Phi}\right]_{\pi}=0, \quad d \widehat{S}+\left[W^{\prime}, \widehat{S}\right]_{\star}=0
\end{aligned}
$$

## Interaction Ambiguity

In the deformed oscillator algebra
$\widehat{S}_{[\alpha} \star \widehat{S}_{\beta]}=-i \epsilon_{\alpha \beta}(1-\widehat{\Phi} \star \kappa)$
we still have the freedom to have instead
$\widehat{S}_{[\alpha} \star \widehat{S}_{\beta]}=-i \epsilon_{\alpha \beta}\left(1-\widehat{\Phi} \star e^{i \Theta} \star \kappa\right)$
where $\Theta$ is an arbitrary function of $\widehat{\Phi} \star \pi(\widehat{\Phi})$. (Vasiliev, 1990).
Assigning the master scalar intrinsic parity $\pm 1$, and imposing parity symmetry reduces the interaction ambiguity to:

Type A model : $\Theta=0, \quad$ Type $B$ model : $\Theta=\pi / 2$.
(E.S. and Sundell, 2003). They contain the scalars in the $D(1,0)$ and $D(2,0)$ irreps of $A d S_{4}$ group, respectively. Relevant for holography.

## Further Generalization

Proposed by colorvioletE.S. nd P. Sundell, in arXiv:1103.2360.
Consider the system

$$
\begin{aligned}
& \widehat{F}+\mathcal{F}_{r}(\widehat{\Phi}) \star \widehat{J}^{r}=0, \quad \widehat{D} \widehat{\Phi}=0, \quad \widehat{d} \widehat{J}^{r}=0 \\
& {\left[\widehat{f}, \widehat{J}^{r}\right]_{\pi}=0, \quad \forall \widehat{f} \in \widehat{\mathcal{A}}, \quad \pi\left(\widehat{J}^{r}\right)=\widehat{J}^{r}, \quad r=1, \ldots, n}
\end{aligned}
$$

for some $n$, and where

$$
\begin{aligned}
& \mathcal{F}_{r}(\widehat{\Phi})=\widehat{\Phi} \star \sum_{n=0} f_{2 n+1, r}(\widehat{\Phi}) \star(\pi(\widehat{\Phi}) \star \widehat{\Phi})^{* n} \\
& \widehat{d} f_{2 n+1, r}=0, \quad\left[f_{2 n+1, r}, \widehat{f}\right]_{\star}=0
\end{aligned}
$$

Interaction ambiguity is possibly relevant to the study of matter coupled CS theories in 3D as holographic duals of bulk HS gravity. This remains to be seen.
(Giombi and Yin, Simons Workshop, 2011).
S. Giombi, S. Minwalla, S. Prakash, S. Trivedi, X. Yin, to appear

Aharony et al, private communication
Wadia et al: Talk in this conference on $W_{\infty}$ symmetry of CS gauge theory coupled to fermions.

## Global Formulation and Structure Groups

Recall that we have gauge fields on $\mathcal{M}$ (the $x$-space), valued in higher spin Lie algebra

$$
\begin{gathered}
\widehat{\mathfrak{h}}(4)=\left\{\widehat{P}(Y, Z): \tau(\widehat{P})=(\widehat{P})^{\dagger}=-\widehat{P}\right\} \\
\operatorname{ad}_{\widehat{P}_{1}}\left(\widehat{P}_{2}\right)=\left[\widehat{P}_{1}, \widehat{P}_{2}\right]_{\star}
\end{gathered}
$$

The structure group $\widehat{\mathfrak{t}} \subseteq \widehat{\mathfrak{h}}(4) \oplus \mathfrak{s l}(2, \mathbb{C})$
Decompose $\mathcal{M}$ into coordinate charts $\mathcal{M}_{I}$ labelled by I, i.e. $\mathcal{M}=\bigcup_{I} \mathcal{M}_{I}$.

The classical moduli space then consists of gauge orbits of locally defined field configurations

$$
\left\{\widehat{W}_{I}, \widehat{\omega}_{I}, \widehat{\Phi}_{I}, \widehat{S}_{\alpha, I}\right\}
$$

glued together by transition functions $\widehat{G}_{I}^{I^{\prime}}=\exp _{\star}\left(\widehat{t}_{I}^{I^{\prime}}\right)$ with $\widehat{t}_{I}^{I^{\prime}} \in \widehat{\mathfrak{t}}$ defined on $\mathcal{M}_{I} \cap \mathcal{M}_{I^{\prime}}$. The classical moduli space thus encodes a principal $\widehat{\mathfrak{t}}$-bundle with connection

$$
\widehat{\Gamma}=\Pi_{\mathfrak{t}}(\widehat{W} \oplus \omega)
$$

where $\Pi_{\mathfrak{t}}$ denotes the projection to $\widehat{\mathfrak{t}}$, and an associated $\widehat{\mathfrak{t}}$-bundle with section $\left(\widehat{E}, \widehat{\Phi}, \widehat{S}_{\alpha}\right)$ where

$$
\widehat{E}=\left(1-\Pi_{\hat{\mathfrak{t}}}\right)(\widehat{W} \oplus \omega)
$$

Observables: $\mathcal{O}\left[\widehat{W}_{I}, \widehat{\omega}_{I}, \widehat{\Phi}_{I}, \widehat{S}_{\alpha, I}\right]$ have the invariance properties:

- $\delta_{\widehat{\Lambda}_{I}} \mathcal{O} \equiv 0$ off-shell for locally defined $\widehat{\Lambda}_{I}=\Pi_{\mathfrak{t}}\left(\widehat{\epsilon}_{I} \oplus \Lambda_{I}\right)$
- $\delta_{\widehat{\xi}_{I}} \mathcal{O}=0$ on-shell for parameters $\widehat{\xi}_{I}=\left(1-\Pi_{\mathfrak{t}}\right)\left(\widehat{\epsilon}_{I} \oplus \widehat{\Lambda}\right)$ that form sections of a $\widehat{\mathfrak{t}}$-bundle associated to the principal bundle.

Thus, the observables are left invariant on-shell by the diffeomorphisms of $\mathcal{M}$.

Four natural phases (choices of structure group) in minimal bosonic higher spin gravity:

- an unbroken phase with structure algebra $\widehat{\mathfrak{h}}(4) \oplus \mathfrak{s l}(2, \mathbb{C})$;
- a broken phase with $\pi$-even higher-spin structure algebra $\mathfrak{h} \mathfrak{S}_{+}(4) \oplus \mathfrak{s l}(2, \mathbb{C})$ where

$$
\widehat{\mathfrak{h}}_{+}(4)=\frac{1}{2}(1+\pi) \widehat{\mathfrak{h}} \mathfrak{s}(4)
$$

(proposed by E.S. and P. Sundell).

- a broken phase with chiral higher-spin structure algebra consisting of all polynomials in $\widehat{\mathfrak{h}}{ }_{+}(4)$ that are purely holomorphic or anti-holomorphic (proposed by Vasiliev).
- the broken Lorentz invariant phase with structure algebra given by the canonical $\mathfrak{s l}(2, \mathbb{C})$ which is typically assumed in the literature on HS gravity.

Using the geometric formulation $\pi$-even structure group, we have:

- proposed a formulae minimal areas of higher spin metrics
- constructed on-shell closed abelian forms of positive even degrees.
- we introduced tensorial coset coordinates and demonstrate how single derivatives with respect coordinates of higher ranks factorize into multiple derivatives with respect to lower ranks.

