D-dimensional Log Gravity

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Critical Gravity

- Adding curvature square terms to the Einstein-Hilbert action renders it a renormalisable theory of gravity [K. S. Stelle:1977].

- In general this theory describes a massless spin-2 graviton, a massive spin-2 field and a massive scalar.

- The energies of excitations of the massive spin-2 field are negative. Thus although the theory is renormalisable, it suffers from having ghosts.

- It is possible to choose the parameters so that the massive spin- 2 field becomes massless [H. Lu,C. N. Pope:2011].

- The presence of the cosmological constant is essential for this step.

- To be more explicit, consider the generic 4-dimensional curvature squared gravity

$$I = \frac{1}{2\kappa^2} \int \sqrt{-g}\, d^4x \left(R - 2\Lambda + \alpha R^2 + \beta R^{\mu\nu} R_{\mu\nu}\right) . \tag{1}$$

- We have dropped the Riemann squared term since its combination with other curvature squared terms of the action can result in Gauss Bonnet action.

- For all values of α and β, AdS_4 is a solution of this theory.

- Linearisation of E.O.M around AdS_4 by making use of gauge condition $\nabla^\mu h_{\mu\nu} = \nabla_\nu h$, results in the equation for trace

$$\Lambda \left[h - 2(\beta + 3\alpha) \Box h \right] = 0 \,. \tag{2}$$

- According to this equation h is the massive scalar mode and it appears only for non-zero cosmological constant.

- We see that for the spatial case

$$\beta = -3\alpha\,, \tag{3}$$

the massive scalar mode h vanishes.

- Having imposed (3), we are left with the linearized equation

$$\frac{3\alpha}{2}\left(\Box-\frac{2\Lambda}{3}\right)\left(\Box-\frac{4\Lambda}{3}-\frac{1}{3\alpha}\right)h_{\mu\nu}=0\,. \tag{4}$$

- The fourth-order equation (4) describes a massless graviton, satisfying

$$\left(\Box-\frac{2\Lambda}{3}\right)h^{(m)}_{\mu\nu}=0\,, \tag{5}$$

and a massive spin-2 field, satisfying

$$\left(\Box-\frac{4\Lambda}{3}-\frac{1}{3\alpha}\right)h^{(M)}_{\mu\nu}=0\,. \tag{6}$$

- It is evident from equation (6) that for

$$\alpha = -\frac{1}{2\Lambda}, \tag{7}$$

the massive mode becomes massless.

- Hence we arrive at a 4-dimensional theory describing only massless gravitons. This theory is called critical gravity.

- For the values of α and β given by (3) and (7), the curvature-squared terms of action (1) can be written in the form

$$\frac{3}{4\Lambda} W_{\mu\nu\rho\sigma} W^{\mu\nu\rho\sigma} \tag{8}$$

where $W_{\mu\nu\rho\sigma}$ is the Weyl tensor.

- It was shown in [H. Lu,C. N. Pope:2011] that the energy of remaining massless modes in the critical point is zero. Moreover the mass and entropy of the black hole solutions of this theory are alsozero.

- Critical gravity may provide an interesting toy model for studying aspects of 4-dimensional quantum gravity.

Log Gravity

- Curvature-squared gravity at the critical point has new logarithmic solutions.

- To see them, we start from an AdS-Wave ansatz as

$$g_{\mu\nu} = \bar{g}_{\mu\nu} + F k_\mu k_\nu \tag{9}$$

where k_μ is a null vector field with respect to the metric $\bar{g}_{\mu\nu}$ with $\bar{g}_{\mu\nu}$ being the AdS$_4$ metric and F is an arbitrary function which is independent of the integral parameter along k_μ.

- In the coordinate which AdS_4 is given by

$$ds^2 = \frac{\ell^2}{r^2}\left(-2dx^+dx^- + dy^2 + dr^2\right) \ , \qquad (10)$$

The equations of motion reads as

$$\left[\frac{\beta r^3}{\ell^2}\frac{\partial^4}{\partial r^4} + \left(1 - \frac{8(3\alpha+\beta)}{\ell^2}\right)\left(r\frac{\partial^2}{\partial r^2} - 2\frac{\partial}{\partial r}\right)\right] F(x^+, r) = 0. \qquad (11)$$

- The most general solution of the above differential equation is in the form of r^x with constant x satisfying the following characteristic equation

$$x(x-3)\left[\beta(x-1)(x-2)-8(3\alpha+\beta)+\ell^2\right]=0. \tag{12}$$

- A generic solution of the equations of motion is

$$F(x^+,r)=f_4(x^+)+f_3(x^+)r^3+f_2(x^+)r^{\Delta_+}+f_1(x^+)r^{\Delta_-}. \tag{13}$$

where $\Delta_\pm = \frac{3}{2} \pm \sqrt{\frac{9}{4} - \frac{\ell^2 - 24\alpha - 6\beta}{\beta}}$, and f_i's are undetermined functions of x^+.

- we observe that at the critical point the characteristic equation degenerates leading to the new logarithmic solutions as follows

$$F(x^+, r) = f_4(x^+) + f_3(x^+) r^3 + \left[f_2(x^+) + f_1(x^+) r^3 \right] \log(r). \tag{14}$$

- This solution is not asymptotically AdS_4. More precisely, the equations of motion in the critical point admit asymptotic expansion

$$ds^2 = \frac{dr^2}{r^2} + \frac{1}{r^2} g_{ij}(x_i) dx^i dx^j, \tag{15}$$

where

$$g_{ij} = b_{(0)\ ij} \log(r) + g_{(0)\ ij} + g_{(3)\ ij} r^3 + b_{(3)\ ij} r^3 \log(r) + \cdots \tag{16}$$

- From the above expression for the asymptotic behavior of the metric one finds

$$g_{(0)\ ij} = \lim_{r \to 0} \left(g_{ij} - r \log(r) \frac{\partial g_{ij}}{\partial r} \right), \quad b_{(0)\ ij} = \lim_{r \to 0} r \frac{\partial g_{ij}}{\partial r} \tag{17}$$

- In the context of AdS/CFT correspondence,since in the presence of non-zero $b_{(0)\ ij}$ the geometry is not asymptotically locally AdS_4, the parameter $b_{(0)\ ij}$ should be considered sufficiently small[Skenderis, et. al:2009].

- In this way, $b_{(0)\,ij}$ is a source for an irrelevant operator in the dual CFT.

- Log modes also appear in the linearized equations. They are annihilated by the full fourth-order operator $(\Box + \frac{2}{3}\Lambda)^2$ but not by $(\Box + \frac{2}{3}\Lambda)$ alone [E. A. Bergshoeff, et. al:2011].

- These modes provide a logarithmic representation of $SO(2,3)$. This representation is neither unitary nor unitarizable [Porrati, M. Roberts:2011].

- Though in the lack of unitarity the critical gravity is not a viable theory of quantum gravity it may have a holographic description in terms of logarithmic conformal field theory.

Generalizing to D-dimensions

- The most general action for the quadratic gravity is

$$I = \frac{1}{\kappa}\int d^D x\,\sqrt{-g}\Big[R - 2\Lambda_0 + \alpha R^2 + \beta R^{\mu\nu}R_{\mu\nu} + \gamma\left(R^{\mu\nu\rho\sigma}R_{\mu\nu\rho\sigma} - 4R^{\mu\nu}R_{\mu\nu} + R^2\right)\Big]. \tag{18}$$

- The model has two distinct vacua such that $R_{\mu\nu} = \frac{2\Lambda}{D-2} g_{\mu\nu}$, where

$$\Lambda_0 - \Lambda = 2\Lambda^2 \Big[(D\alpha + \beta) \frac{D-4}{(D-2)^2} + \frac{(D-3)(D-4)}{(D-1)(D-2)} \gamma \Big]. \tag{19}$$

- It was shown in [S. Deser,et. al:2011] that for appropriate choice of the parameters there exists a critical point

$$
\begin{aligned}
\beta &= -\frac{4(D-1)}{D}\,\alpha, \\
\frac{(D-1)(D-2)}{4(-\Lambda)} &= (D-1)(D\alpha+\beta) \\
&\quad + (D-3)(D-4)\gamma,
\end{aligned} \tag{20}
$$

at which the model has only massless tensor gravitons.

- Similar to 4-dimensional case, we consider an ansatz in the form

$$ds^2 = \frac{\ell^2}{r^2}\Big(- F(x^+, r, x_i)du^2 - 2dx^+dx^- + dr^2 + (dx_i)^2\Big). \tag{21}$$

- Plugging this ansatz in the equations of motion, one finds a differential equation with generic solution r^x where constant x satisfies

$$\frac{\beta}{D-2}x(x-D+1)\left(x^2-(D-1)x+\frac{A}{\beta}\right)=0 \tag{22}$$

where
$A=\ell^2-2(D\alpha+\beta)(D-1)-2(D-3)(D-4)\gamma.$

- The most general solution is

$$F(x^+,r)=f_4(x^+)+f_3(x^+)r^{D-1}+f_2(x^+)r^{\Delta_+}+f_1(x^+)r^{\Delta_-}, \tag{23}$$

where $\Delta_\pm=\frac{D-1}{2}\pm\sqrt{(\frac{D-1}{2})^2-\frac{A}{\beta}}$.

- We note that at the critical point (20) where $A = 0$ the characteristic equation degenerates leading to new logarithmic solutions as follows

$$F(x^+, r) = f_4(x^+) + f_3(x^+) r^{D-1} + \left[f_2(x^+) + f_1(x^+) r^{D-1} \right] \log(r). \tag{24}$$

Conclusion

- We have studied curvature squared gravity at the critical point.

- It was shown that new logarithmic modes appear in the critical point which are not asymptotically AdS_4.

- Existence of logarithmic modes which have positive energy suggests that the critical gravity is not entirely trivial.

- Log modes provide a representation for the isometry group of AdS$_D$ which is not unitary. Hence critical gravity is not a unitary theory of quantum gravity.

- The logarithmic solutions may have a holographically dual description in terms of logarithmic conformal field theory.