Blackfolds:

a new approach to higher-dimensional black holes

5th Crete regional meeting on String Theory

Kolymbari, July 1, 2009

Niels Obers, Niels Bohr Institute

0902.0427 (**PRL)** (with R. Emparan, T. Harmark, V. Niarchos)

090y.xxxx:: To appear (with R. Emparan, T. Harmark, V. Niarchos)

0708.2181 (JHEP) (with R. Emparan, T. Harmark, V. Niarchos, M.J. Rodriguez)

0802.0519 (Springer Lectures Notes)

0701022 review CQQ (with V. Niarchos and T. Harmark)

Plan

- Introduction
- Separation of scales in higher-dimensional black holes
- Blackfold approach
- Examples of novel black hole families
- Lessons and outlook

Motivations to study higher-dimensional gravity

- Applications:
  - String/M theory
    - BH entropy, new brane solutions
  - AdS/CFT
    - new phases of thermal gauge theories, phase transitions
    - plasma balls/rings in AdS (fluid/gravity correspondence)
  - Large extra dimensions + TeV gravity
    - possible objects in universe/accelators
  - math: Lorentzian geometry
- Intrinsically interesting:

Can regard D as tunable parameter for gravity + black holes

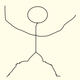
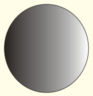
which BH properties are:

- intrinsic $\rightarrow$ Laws of BH mechanics
- D-dependent $\rightarrow$ uniqueness, topology, shape, stability

For various reviews see:

- Kol
- Harmark,Niarchos,NO
- Kleihaus,Kunz,Navarro-Larida
- Emparan,Reall
- NO

Progress in the last years

What do we know about black objects (i.e. with event horizon)
in higher dimensional Einstein gravity ?
$\rightarrow$ Dynamics of BHs in D $\geq$ 5 much richer than four dimensions

In this talk: restrict (mostly) to asymptotically flat solutions of pure gravity

$$R_{\mu\nu} = 0 \qquad \mathcal{M}^D$$

but:- interesting parallels with BHs in KK spaces
- techniques are readily generalized to AdS/dS space + adding charge

- D=4: black hole uniqueness

- D=5: MP black hole (S^3), ER black ring ($S^2 \times S^1$), black Saturn, …
  - 4D inspired techniques successful
  (assuming 2 axial Killing vector fields $\longrightarrow$ integrability
  full classification of BHs in terms of "rod-structure" + asympt. charges)

- D $\geq$ 6: MP black holes (S^{D-2}) are only known exact solutions
  - full dynamics too complex to be captured by conventional approaches

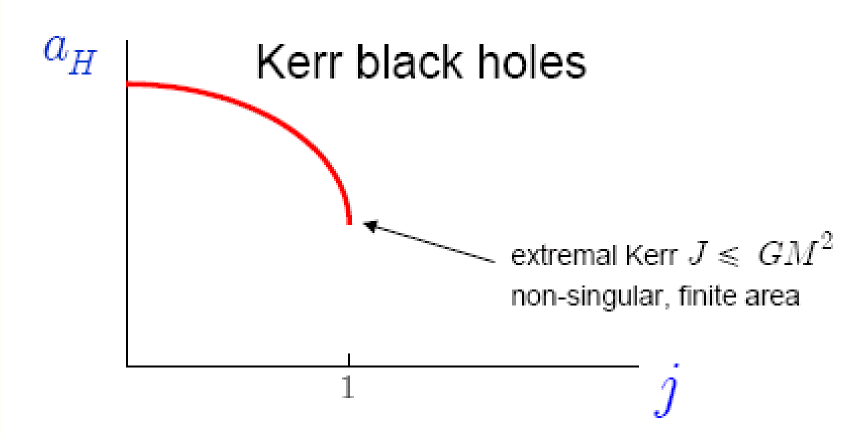
  $\Rightarrow$ but recent progress: thin black rings ($S^1 \times S^{D-3}$) in any dimension

Novel feature of higher D neutral BHs

- in some regimes horizons are characterized by (at least) two separate scales

$$r_0 \ll R$$

Cf. D=4



shape of Kerr BH is always approx. round with radius

$$r_0 \sim GM$$

D $\geq$ 5: no Kerr bound anymore

⇨ two classical length scales can be widely separated

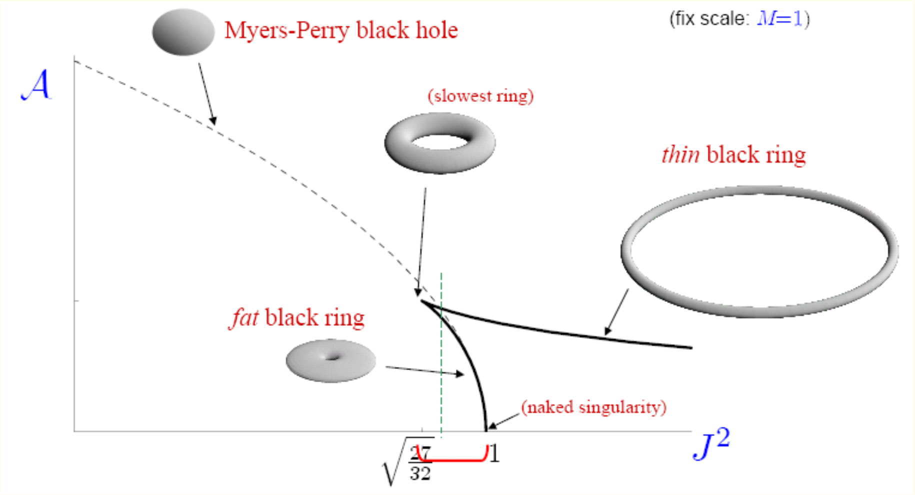
$$\ell_J \sim \frac{J}{M} \qquad \text{vs.} \qquad \ell_M \sim (GM)^{1/(D-3)}$$

Analogue for KK black holes: size of compact manifold vs. horizon radius

Separation of scales

► observe separation of scales in explicitly known solutions

D=5



- Kerr bound for MP

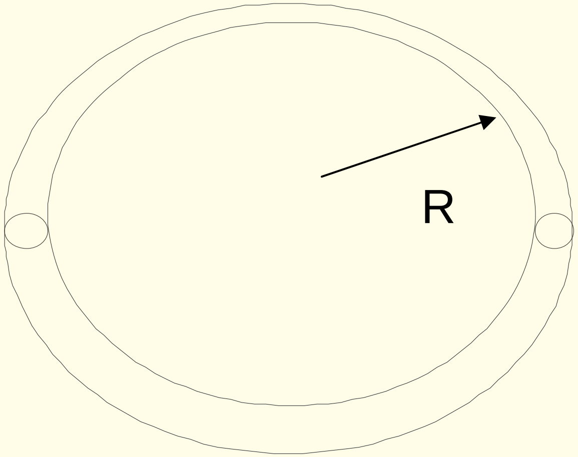
but: rotating black ring can have arbitrarily large angular momentum for given mass

Emparan,Reall

ultraspinning (small mass) limit

$$\frac{J^2}{GM^3} \to \infty$$

corresponds to: $R \gg r_0$



radius of ring $\gg$ thickness of ring

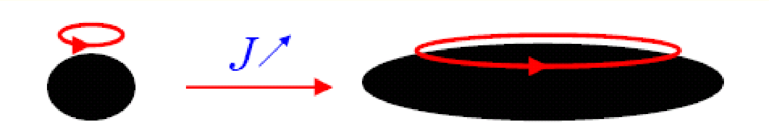
R = radius of S^1
r_0 = radius of $S^{D\text{-}3}$

Separation of scales (cont'd)

D $\geq$ 6: no Kerr bound for MP BHs:

$\longrightarrow$ ultraspinning regimes with pancaked horizons

Emparan,Myers



$$\frac{J^{D-3}}{GM^{D-2}} \to \infty$$

(approaches black membrane geometry $\mathbb{R}^2 \times S^{D-4}$ for large J)

radius of disc $\gg$ thickness of disc

Note: GL instability is also property of horizons in higher D depending on separation of two length scales along horizon

length vs. thickness (of black brane)

$\longrightarrow$ inhomogeneous black branes arise when the two begin to differ

Gregory,Laflamme

Higher D black holes organized according to scales

- dynamics of higher-dimensional black holes naturally organized in relative value of scales

$\ell_J \lesssim \ell_M$

• single length scale: Kerr BH behavior

$\ell_J \sim \ell_M$

• regime of mergers and connections between phases when two horizon scales meet $r_0 \sim R$
- not accessible to effective methods; requires extrapolation or numerics

$\ell_J \gg \ell_M$

• separation of scales allows effective description of long-wavelength description physics
$\rightarrow$ blackfold approach (subject of talk)

Based on idea that when $\ell_M/\ell_J \to 0$ black hole is locally a flat (possibly boosted) black brane (cf. known examples)

Effective theory describes how to bend black brane wv in background spacetime (similar to effective theories for other extended objects: cosmic strings, D-branes)

Blackfold approach

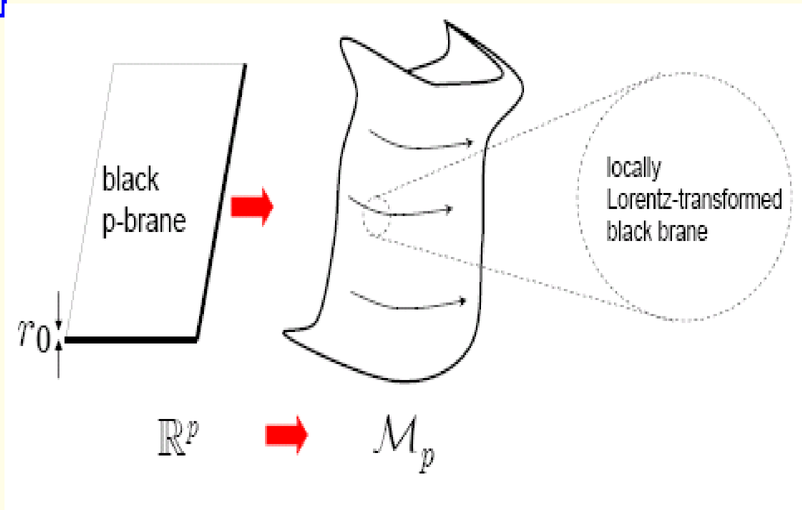
Emparan,Harmark,Niarchos,NO

Blackfold = **Black** p-brane whose worldvolume extends along a curved submani**fold** (of embedding space)

▶ start with flat p-brane: horizon $\mathbb{R}^p \times s^{n+1}$

size: r_0

bend spatial world-volume into submanifold $\mathcal{B}_p$ characterized by length scale: R



- consider regime of widely separated scales:

curvature radius of submanifold $\gg$ brane thickness $\qquad R \gg r_0$

$\longrightarrow$ can approximate the blackfold locally with flat black brane

Question: which $\mathcal{B}_p$ are possible ?

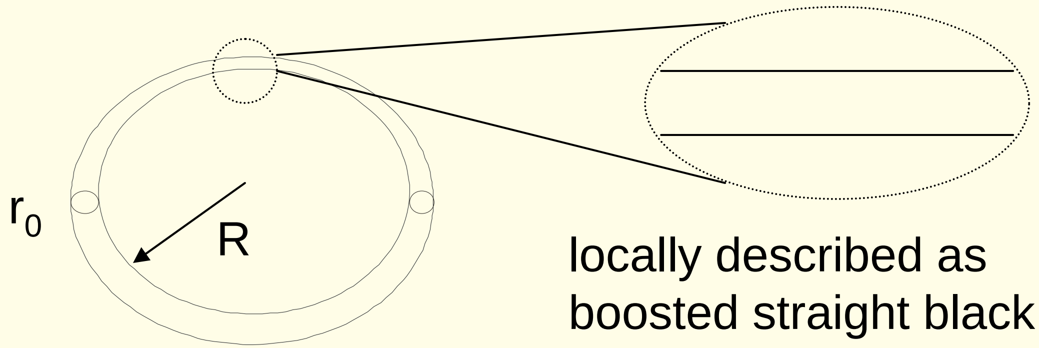
Long-wavelength effective theory

▶ two widely separated scales ⇨ integrate out short-distance dynamics

→ long-distance effective theory

• use to construct BHs perturbatively

e.g. 5D rotating black ring



locally described as
boosted straight black string

◀ by employing method of matched asymptotic expansion (MAE)
thin black ring solution for $D \geq 6$ has been constructed Emparan,Harmark,Niarchos,NO,Rodriguez

• MAE was first developed for localized BHs in KK space in limit: $L \gg r_0$

Harmark/Kol,Gorbonos/Karsik et.al
Dias,Harmark,Myers,NO

• other technique has been developed as well: classical effective field theory (ClEFT)

Chu,Goldberger,Rothstein/Kol

Goal: develop a leading order theory for the long-distance dynamics of high D BHs
start at the probe-brane or "test blackfold" level (i.e. ignore backreaction)

General idea

▶ similar to effective theories for other extended objects: cosmic strings, D-branes
difference: - short-distance d.o.f. = gravitational short-wavelength modes
- extended objects posses black hole horizon

$$g_{\mu\nu} = \{ g_{\mu\nu}^{(\text{long})}, g_{\mu\nu}^{(\text{short})} \}$$

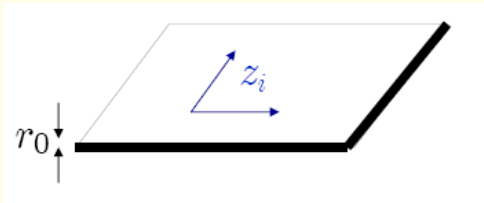
$$I_{EH} \approx \frac{1}{16\pi G} \int d^D x \sqrt{-g^{(\text{long})}} R^{(\text{long})} + I_{\text{eff}}[g_{\mu\nu}^{(\text{long})}, \phi]$$

- effective action from integrating out short-wavelength d.o.f.
- ϕ are `collective coordinates'

◀ main clue: known black holes in limit $\ell_M/\ell_J \to 0$ $\Rightarrow$ flat black branes
- need collective field dynamics black p-brane (define $D = p + n + 3$)

$$ds^2 = -f dt^2 + \sum_{i=1}^{p} dz_i^2 + f^{-1} dr^2 + r^2 d\Omega_{n+1}^2 \, , \quad f(r) = 1 - \frac{r_0^n}{r^n}$$

- coordinates (t,z_i) span brane worldvolume



Collective coordinates of black brane

► Goldstone modes of symmetries spontaneously broken by black brane
- positions in directions transverse to worldvolume $X^\perp$
- `horizon thickness' r_0
- boost parameters $\Lambda_i^{\ 0}$

(worldvolume is invariant under spatial rotations $SO(p) \in SO(1,p)$)

promote to collective field modes depending on wv coords σ^α

$$\phi(\sigma^\alpha) = \{X^\perp(\sigma^\alpha), r_0(\sigma^\alpha), \Lambda_i{}^0(\sigma^\alpha)\}$$

- total of (D-p-1) + 1 + p = D field variables

◄ validity of effective field approximation $1/R \sim |\partial_\alpha \phi| \ll r_0^{-1}$

► introduce embedding coordinates $X^\mu(\sigma)$ (gauge redundancy) of the blackfold $\mathcal{W}_{p+1}$ with spatial section $\mathcal{B}_p$:

determines induced metric: $\gamma_{\alpha\beta} = \partial_\alpha X^\mu \partial_\beta X^\nu g_{\mu\nu}$

Equations of motion and blackness condition

► instead of effective action:
more convenient to work directly with equations of motion

define effective stress tensor $T_{\mu\nu} = \frac{2}{\sqrt{-\gamma}} \frac{\delta I_{\text{eff}}}{\delta g^{\mu\nu}}$

- supported on the worldvolume of $\mathcal{W}_{\text{p+1}}$

◄ spacetime diffeomorphism invariance
i.e. consistent coupling of wv. theory to long-wavelength grav. field

$$\nabla_\mu T^{\mu\nu} = 0$$

transverse to worldvolume

- (D-p-1) field equations on D collective coords.

◄ need additional conditions: blackness (laws of black hole dynamics)

• 0^{th} law: surface gravity κ uniform over blackfold worldvolume
• rigidity: angular velocities on horizon $\Omega_{\text{H},i}$ uniform on blackfold worldvolume
+ directions along commuting spatial isometries of background

⇒ precisely enough to eliminate thickness + boosts in terms of $X^\perp$

Effective stress tensor

▶ effective stress tensor of blackfold results from

integrating out short-distance gravitational d.o.f.

\+ expressing coupling of long-wavelength metric to collective modes

solve classical EOM near blackfold ($r \ll R$)

\+ find effective stress tensor that reproduces effect of this soln

on gravitational field at distances $r \gg r_0$

\- blackfold $\sim$ black p-brane up to position dependent Lorentz boost

$\rightarrow$ determines equivalent distributional stress tensor (localized on blackfold wv.)

$$T_{\mu\nu}(\sigma^\alpha) = \tau_{\mu\nu}(\sigma^\alpha)\,\delta^{(D-p-1)}\,(x - X(\sigma^\alpha))$$

\- sources same field in matching region: $r_o \ll r \ll R$ (linearized gravity)

◀ static (flat) black p-brane has stress tensor (define $D = p + n + 3$)

$$\tau_{tt} = r_0^n(n+1)$$
$$\tau_{ii} = -r_0^n\ , \quad i = 1\dots p$$

Boosting the p-brane

- We now act with Lorentz transformation: $z \to \Lambda z$

$$\Lambda \in SO(1,m) \subset SO(1,p)$$

brane is invariant under spatial rotations: parameterize m boosts as:

$$\Lambda_0{}^0 = \cosh\alpha\,, \quad \Lambda_i{}^0 = \nu_i \sinh\alpha\,, \quad \sum_{i=1}^{m} \nu_i^2 = 1$$

boosted EM tensor $\tau_{ij} \to (\Lambda\tau\Lambda^{\mathrm{T}})_{\mathrm{ij}}\,(\sigma)$ is

$$\tau_{tt} = r_0^n[n\cosh^2\alpha + 1]$$
$$\tau_{ii} = r_0^n[n\nu_i^2\sinh^2\alpha - 1]\,, \quad i = 1\ldots m$$
$$\tau_{i\neq j} = r_0^n n\nu_i\nu_j \sinh^2\alpha\,, \quad i,j = 1\ldots m$$
$$\tau_{ti} = r_0^n n\nu_i \cosh\alpha\sinh\alpha\,, \quad i = 1\ldots m$$
$$\tau_{ii} = -r_0^n\,, \quad i = m+1\ldots p$$

m boost parameters $\alpha\,(\sigma)$, $\nu_i\,(\sigma)$ and brane thickness $r_0\,(\sigma)$ may depend on worldvolume coordinate !

- determined here EM tensor with flat indices since we are using local Lorentz frame to map with flat black brane

Blackness condition

- blackfold is now locally a boosted black brane
  but still need to impose blackness condition

⇨ surface gravity and angular velocities constant on the blackfold

can find these locally in terms of the embedding

$$\kappa = \frac{n}{2r_0(\sigma^\alpha)\cosh\alpha(\sigma^\alpha)}\,, \qquad \Omega_{Hi} = \frac{\nu_i(\sigma^\alpha)}{R_i(\sigma^\alpha)}\tanh\alpha(\sigma^\alpha)$$

- blackness determines the thickness and the boosts in terms of local velocity components and embedding coordinates $R_i(\sigma^\alpha)$

$$r_0(\sigma^\alpha) = \frac{n}{2\kappa}\sqrt{1-\mathcal{V}(\sigma^\alpha)^2}\,, \quad \tanh\alpha(\sigma^\alpha) = \mathcal{V}(\sigma^\alpha)\,, \quad \nu_i(\sigma^\alpha) = \frac{R_i(\sigma^\alpha)\Omega_{Hi}}{\mathcal{V}(\sigma^\alpha)}$$

with local velocity field defined by: $\mathcal{V}(\sigma^\alpha) = \left(\sum_{i=1}^m (r_i(\sigma^\alpha)\Omega_{Hi})^2\right)^{1/2}$

insert these in EM tensor $\rightarrow$ completely determined in terms of

$$\kappa, \Omega_{H,i}, R_i(\sigma)$$

Final form of boosted stress tensor

$$\tau_{00} = \left(\frac{n}{2\kappa}\right)^n (1-\mathcal{V}^2)^{\frac{n-2}{2}} \left(n+1-\mathcal{V}^2\right)$$

$$\tau_{0i} = \left(\frac{n}{2\kappa}\right)^n (1-\mathcal{V}^2)^{\frac{n-2}{2}} n r_i \Omega_i \,, \quad i = 1, \ldots, m$$

$$\tau_{ii} = \left(\frac{n}{2\kappa}\right)^n (1-\mathcal{V}^2)^{\frac{n}{2}} \left(\frac{n(r_i\Omega_i)^2}{1-\mathcal{V}^2} - 1\right) \,, \quad i = 1, \ldots, m$$

$$\tau_{ii} = -\left(\frac{n}{2\kappa}\right)^n (1-\mathcal{V}^2)^{\frac{n}{2}} \,, \quad i = m+1, \ldots, p$$

$$\tau_{i\neq j} = \left(\frac{n}{2\kappa}\right)^n (1-\mathcal{V}^2)^{\frac{n-2}{2}} r_i r_j \Omega_i \Omega_j \quad i, j = 1, \ldots, m$$

local velocity field: $\mathcal{V}(\sigma^\alpha) = \left(\sum_{i=1}^m (r_i(\sigma^\alpha)\Omega_{Hi})^2\right)^{1/2}$

EM tensor determined in terms of $\kappa, \Omega_i, r_i(\sigma)$

Blackfold equations as generalized geodesic eqs.

► EM conservation shown to be equivalent to Carter equation (brane probe approximation)

$$T^{\mu\nu} K_{\mu\nu}{}^{\rho} = 0 \qquad (\Leftarrow \quad \nabla_{\mu} T^{\mu\nu} = 0)$$

extrinsic curvature tensor (2nd fund. form)

energy momentum tensor on brane

- index ρ is orthogonal to worldvolume $\mathcal{W}_{p+1}$: D-p-1 equations

◄ Blackfold equations can be rewritten as generalized geodesic equation

$$\tau^{\alpha\beta}\left(\nabla_{\alpha}^{(\gamma)} \partial_{\beta} X^{\rho} + \Gamma^{\rho}_{\mu\nu} \partial_{\alpha} X^{\mu} \partial_{\beta} X^{\nu}\right) = 0$$

- generalizes geodesic eqn. in GR to extended objects

$\rightarrow$ Carter equation becomes a set of purely geometric equations for embedding of $\mathcal{B}$ and given temperature + angular velocities

⇨ (much stronger than topological restrictions) Geometric censorship for blackfolds

Thermodynamic quantities and horizon topology

◀ can compute mass and angular momentum by integrating appropriate EM tensor components over brane worldvolume

$$M = \int_{\mathcal{B}_p} \sqrt{-\gamma}\, \tau_{tt}\,, \qquad J_i = \int_{\mathcal{B}_p} \sqrt{-\gamma}\, r_i \tau_{ti}$$

◀ to compute total area: use that locally we have area of a boosted black brane

$$a_H(\sigma^\alpha) = \Omega_{n+1} r_0^{n+1}(\sigma^\alpha) \cosh\alpha(\sigma^\alpha)$$

- small s^{n+1}-sphere at each point of blackfold

$\rightarrow$ horizon is fibration of s^{n+1} over $\mathcal{B}_p$

• if fiber is regular, horizon topology: $(\text{topology of } \mathcal{B}_p) \times S^{n+1}$

• but $r_0(\sigma)$ can go go to zero at codimension-1 locus on $\mathcal{B}$ (where local boost is light-like)

- e.g. if $\mathcal{B}_p$ is p -ball with s^{n+1} shrinking at boundary: $S^{p+n+1} = S^{D-2}$

total area of horizon: $A_H = \int_{\mathcal{B}_p} \sqrt{-\gamma}\, a_H(\sigma^\alpha)$

Action principle for blackfolds and 1st law

► consider Gibbs free energy functional:

$$I_{\mathrm{G}}[x^\mu(\sigma^\alpha)] = M[x] - \Omega_i J_i[x] - 4\pi\kappa A_H[x]$$

varying I_G $\Rightarrow$ 1st law of thermodynamics

Claim:

$\Rightarrow$ $\boxed{\text{1st law of thermo} \Leftrightarrow \text{blackfold equations of motion}}$

explicit form of action in terms of embedding:

$$I_{\mathrm{G}} = \left(\frac{n}{2\kappa}\right)^n \int \sqrt{\gamma}\,[1 - \mathcal{V}(\sigma)^2]^{\frac{n}{2}}$$

• integrated version of local Smarr for black p-brane reads

Harmark,NO/Kastor,Traschen

$$(D-3)M = (D-2)\Big(\Omega_i J_i + TS\Big) + \mathcal{T}$$

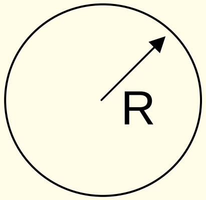
but asymptotically flat solutions should obey Smarr above with zero tension: $\mathcal{T} = 0$

$\longrightarrow$ total tension vanishes for blackfold (explicitly checked in examples)

◄ alternate ``NG'' type form of action $I_{\mathrm{G}} \propto I_{\mathrm{WV}} = \int \sqrt{-\gamma}\; \tau^{ab}\eta_{ab}$

Smarr

Example: Black ring



▶ wrap black string on a compact 1D space (topologically S^1)

specify embedding: S^1 in $\mathbb{R}^2$ (times point in $\mathbb{R}^{D-3}$)

$$\mathbb{R}^2 : \quad (r, \phi) \qquad r = R(\sigma) \; , \quad \phi = \sigma$$

action $\quad I_{\text{WV}} \propto \int \sqrt{-\gamma}(1-\Xi^2)^{\frac{n}{2}} = \int d\sigma \sqrt{(R')^2 + R^2}(1-\Omega^2 R^2)^{\frac{n}{2}}$

⟶ full EOM is:

$$(1-\Omega^2 R^2) R R'' + ((n+2)\Omega^2 R^2 - 2) R'^2 + ((n+1)\Omega^2 R^2 - 1) R^2 = 0$$

• highly non-linear DE; simple solution with constant R. $\quad R = \frac{1}{\sqrt{n+1}} \frac{1}{\Omega}$

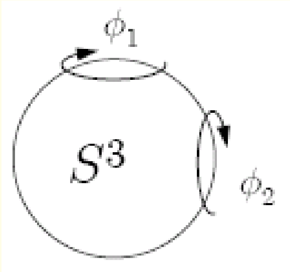
or directly from Carter equation: $\quad \frac{\tau_{11}}{R} = 0 \quad$ (total tension vanishes)

◀ zero tension condition is equivalent to balancing forces on ring

- centrifugal repulsion balances gravitational tension
- solution with horizon topology $S^1 \times S^{D-3}$

New solutions: odd-spheres

▶ black brane wrapped on $\mathcal{B}_{2k+1} = S^{2k+1}$



embed in $\mathbb{R}^{2k+2}$: $d\rho^2 + \rho^2 \left(\sum_{i=1}^{k+1} d\mu_i^2 + \mu_i^2 d\phi_i^2 \right) \,, \qquad \sum_{i=1}^{k+1} \mu_i^2 = 1$

sphere embedded as $\rho = R$ worldvolume coordinates: $\mu_i \ , \quad \phi_i$

- assume R = const. and take all Ω_i equal (simple solution ansatz)

▶ action is: $I_{\mathrm{WV}} \propto \int R^p (1 - \Omega^2 R^2)^{\frac{n}{2}}$

EOM solved by $$R = \sqrt{\frac{p}{n+p}} \frac{1}{\Omega}$$

(equivalent to $\sum_{i=1}^{p} \tau_{ii} = 0$ so total tension vanishes)

Novel family of blackfolds with horizon topology: $S^{2k+1} \times S^{n+1}$

- includes black rings for $k = 0$
- for $k \geq 1$: boosts depend on location on the S^{2k+1}
- uniform thickness

products of odd-spheres

► black brane wrapped on

$$\mathcal{B}_p = \prod_a S^{p_a} \ , \quad p_a = \mathsf{odd} \ , \quad \sum_a p_a = p$$

- number of spheres cannot be larger than n+2

•assume R = const. for each sphere
+ take all Ω_i equal for each sphere (simple solution ansatz)

► action is: $I_{\mathrm{WV}} \propto \prod_a \int R_a^p (1-(\Omega^{(a)})^2 R_a^2)^{\frac{n}{2}}$

EOM solved by $R_a = \sqrt{\frac{p_a}{n+p}} \frac{1}{\Omega^{(a)}}$

many new blackfolds with non-trivial horizon topology:

$$\mathbb{T}^p \times S^{n+1} \ , \quad (\mathbb{T}^{p-3} \times S^3) \times S^{n+1} \ ,$$
$$(S^3 \times S^3) \times S^{n+1} \ , \quad \ldots$$

Ultraspinning MP BHs as even-ball blackfolds

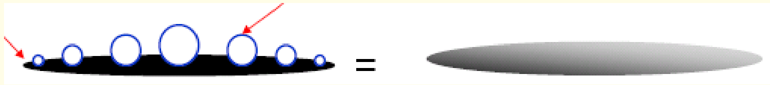
► blackfold eqs. do not admit even-sphere solutions for $\mathcal{B}_p$
- tension at fixed points of rotation group cannot be counterbalanced by centrifugal forces

→ instead solutions with $\mathcal{B}_p$ = ellipsoidal even-ball

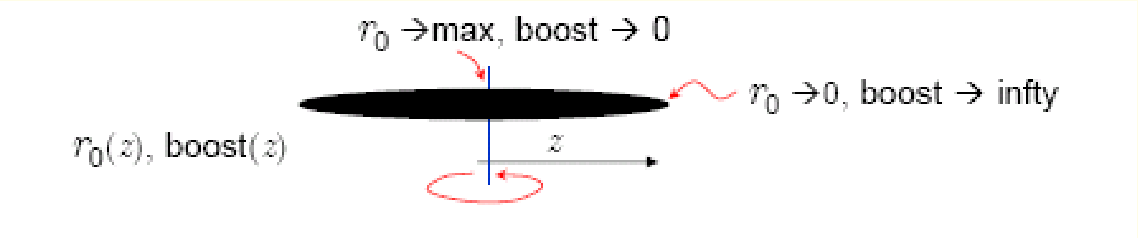
thickness r_0 shrinks to zero at boundary of ball so including the s^{n+1} fibers, horizon topology is S^{D-2}

• reproduce precisely all physical quantities of MP BH with $p/2$ ultra-spins
- highly non-trivial check on approach
(rotation has fixed points at center of ball, $r_0(\sigma)$ varying)

◄ simplest example: black disc: $D_2 \subset \mathbb{R}^2$



boost depends on radius:



• corresponds to MP BH with one angular momentum in ultraspinning limit

Blackfold Bestiary

► blackfold construction shows existence of new types of asymptotically flat stationary black holes in higher dimensions

$D=4$	$D=5$	$D=6$	$D=7$	$D=8$	$D=9$	
S^2	S^3	S^4	S^5	S^6	S^7	Kerr, MP BH
		$B_2 \otimes s^2$	$B_2 \otimes s^3$	$B_2 \otimes s^4$ $B_4 \otimes s^2$	$B_2 \otimes s^5$ $B_4 \otimes s^3$	ultraspinning MP BH
	$S^1 \times s^2$	$S^1 \times s^3$	$S^1 \times s^4$	$S^1 \times s^5$	$S^1 \times s^6$	black ring
		$\mathbb{T}^2 \times s^2$	$\mathbb{T}^2 \times s^3$	$\mathbb{T}^2 \times s^4$	$\mathbb{T}^2 \times s^5$	black torus
			$S^3 \times s^2$ $\mathbb{T}^3 \times s^2$	$S^3 \times s^3$ $\mathbb{T}^3 \times s^3$	$S^3 \times s^4$ $\mathbb{T}^3 \times s^4$	
				$S^1 \times S^3 \times s^2$	$S^1 \times S^3 \times s^3$ $\mathbb{T}^4 \times s^3$	

◄ for product odd-sphere and even-ball blackfolds with equal sizes and angular momenta (at fixed mass):

$$A(J) \sim J^{-p/n}$$

tori dominate entropically

Other cases

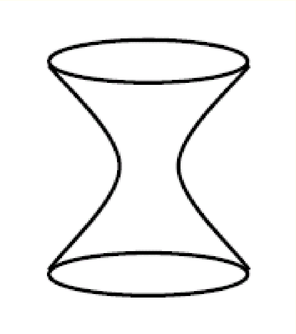
- static minimal blackfolds $\tau_{ij} = -P\eta_{ij}$

  (no boost)

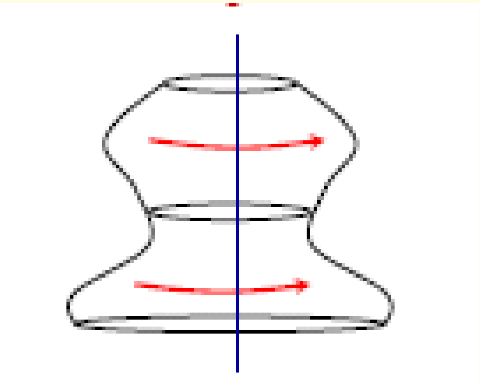
  $\longrightarrow \quad K^\rho = 0$ (mean curvature vector)

  minimal submanifold

  e,g. hyperboloid (static non-compact blackfold)



- axisymmetric blackfolds



use numerics or further perturbative approach ?

New blackfolds in 5D: helical rings and strings

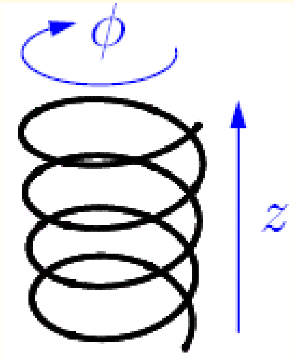
Emparan,Harmark,Niarchos,NO (in progress)

for black 1-folds we can take curves with tangent vector equal to
a linear combination of isometries

$$\zeta = \sum_i c_i \xi^{(i)}|_{x=X(\sigma)}$$

$\rightarrow$ for critical boost this satisfies Carter + blackness

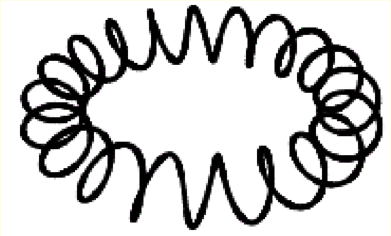
- helical black string: $\zeta \sim (k\partial_x + \partial_\phi)|_{r=R}$



  - helix with pitch k
  - boost along string gives momentum along x and angular momentum along ϕ

- helical black ring: $\zeta \sim (n\partial_\phi + m\partial_\psi)|_{r_1=R_1, r_2=R_2}$

  - helix of radius R_2 around circular trajectory of radius R_1 that closes on itself after m turns
  - boost is linear combo of two angular momenta



⇨ has only single spatial U(1) isometry:
first evidence of such a solution in 5D ! (as admited by rigidity theorem)

Hollands, Ishibashi, Wald

Charged blackfolds

Emparan,Harmark,Niarchos,NO (in progress)

Carter equation: $K_{\alpha\beta}^{\rho}\tau^{\alpha\beta} = F^{\rho}$, $\nabla_{\mu_1} J^{\mu_1 \cdots \mu_{p+1}} = 0$

worldvolume action $$I_{\mathrm{WV}} = \int \sqrt{\gamma}\,\tau^{ij}\gamma_{ij} + A \cdot J$$

• use branes in EMD-gravity (includes supergravities relevant for string theory)

- blackness condition involves in this case also constant chemical potential

⇒ seems to generate highly non-trivial blackfolds

- odd-sphere solutions

  S^1: dipole rings in any dimension (includes known dipole ring in 5D)

  - from boosting and bending a charged string

  Emparan

  higher spheres (in progress)
- even-ball solutions
  - charged rotating discs ? ……

could potentially be stable ! (under investigation)

Caveats

- regularity of black brane horizon after bending ?
  - shown for black 1-folds (i.e. black strings)
  - extension to p-folds (to appear)
    (use matched asymptotic expansion)

- backreaction of blackfold on background geometry is neglected
  (to leading order in r_0/R)
  - could make it impossible for leading-order solution to remain stationary
    (must be analyzed case-by-case)

- blackfolds may be (classically) unstable
- can use blackfold equations to analyze stability under long wavelength
  perturbations ($\lambda \gg r_0$)
- there are short wavelength ($\lambda \sim r_0$) instabilities (GL-type) outside approach

Further Outlook

• charged blackfolds
  - in progress Emparan,Harmark,Niarchos,NO

• method can also be applied to blackfolds in other backgrounds (AdS, dS)
  - black rings in (A)dS Caldarelli,Empran,Rodriguez

• SUSY blackfolds ?
  - extremal black holes and black rings Figueras,Kunduri,Lucetti,Rangamani

  cf. 5D supersymmetric black ring Elvang,Emparan,Mateos,Reall

• stability analysis

• relation with DBI

• higher-order analysis (via MAE/CIEFT) (in progress: horizons stay regular)

• blackfold motion + relation to fluid/gravity correspondence

• duality of higher D black holes to plasma balls + rings in AdS
  (cf. Lahiri,Minwalla) – many similar features

Lessons from blackfold approach

► dynamics of higher-dimensional black holes naturally organized in relative value of scales

$0 \leq J \lesssim M(GM)^{\frac{1}{D-3}}$ • single length scale: Kerr BH behavior

$J \gtrsim M(GM)^{\frac{1}{D-3}}$ • regime of mergers and connections between phases when two horizon scales meet $r_0 \sim R$
- not accesible to effective methods; requires extrapolation or numerics

$J \gg M(GM)^{\frac{1}{D-3}}$ • blackfolds
- extreme rich physics in this regime; study dynamics rather than exact solutions for all possible BHs

◄ blackfold horizon topologies $\mathcal{B}_p \times S^{n+1}$

supported by mechanical equilibrium

supported by internal structure of the BH

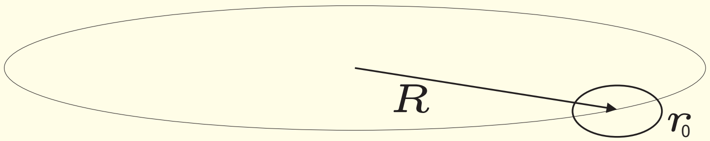
- purely topological analysis cannot distinguish between these two factors

The end

Matched asymptotic expansion

► MAE = systematic approach to iteratively construct solution given known solution in some limit + then correcting it in perturbative expansion

- applied e.g. to construct metric of small black holes on circle Harmark/Gorbonos,Kol

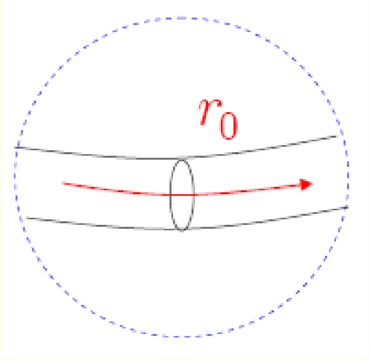


r_0: S^{n+1} radius

r is distance from ring

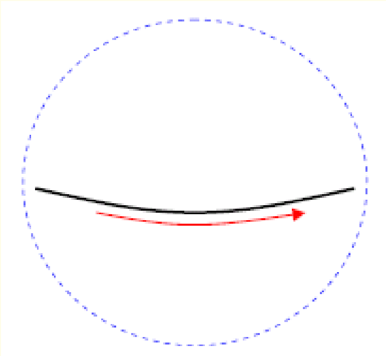
1. linearized solution around flat space

   - asymptotic zone: $r \gg r_0$



2. perturbations of boosted black string

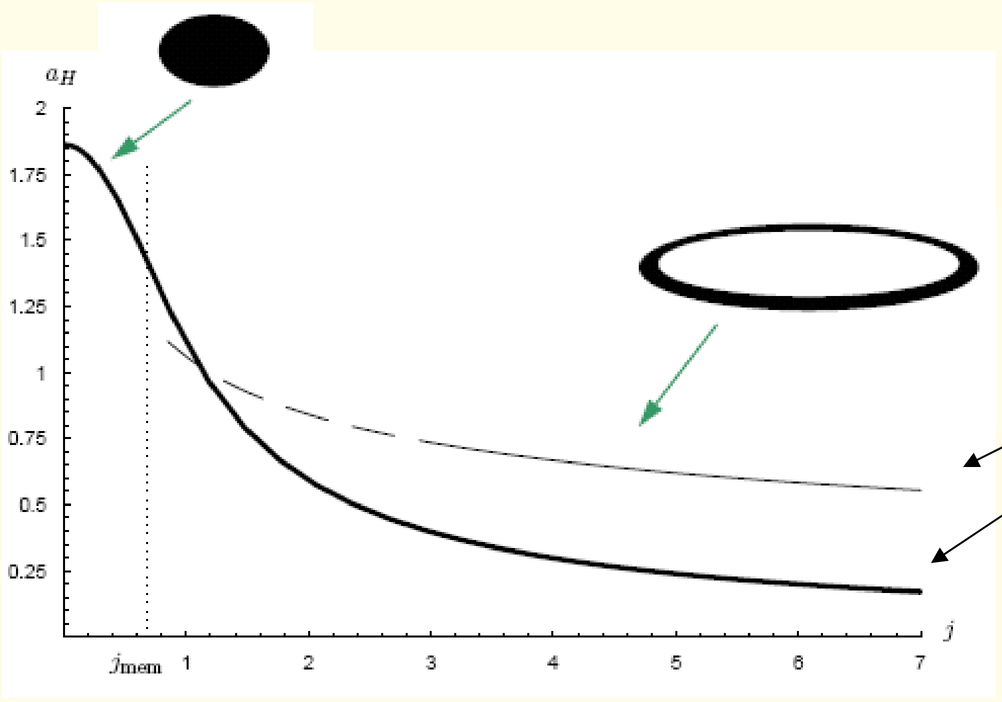
   - near-horizon zone: $r \ll R$



3. match in overlap zone

$$r_0 \ll r \ll R$$

Higher-dimensional black rings vs. MP black holes: $D \geq 6$



$a_H \sim j^{-\frac{1}{D-4}}$

$a_H \sim j^{-\frac{2}{D-5}}$

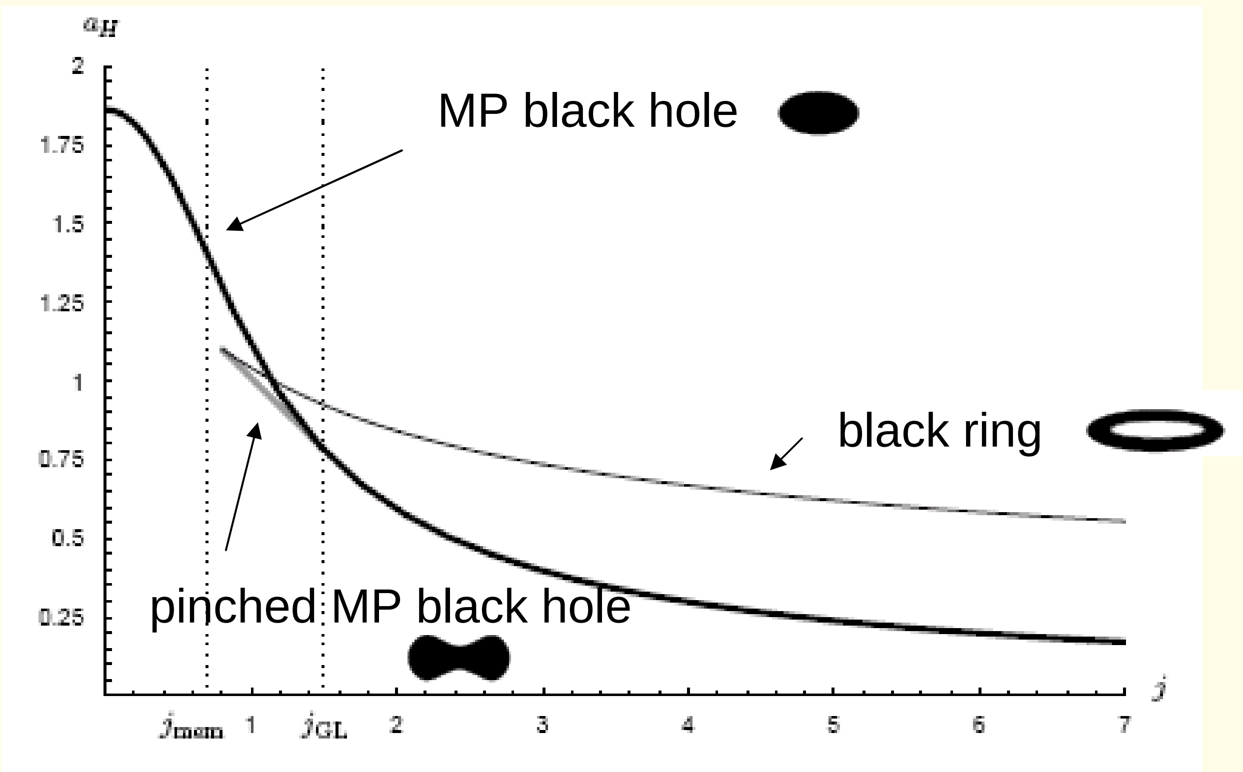
onset of membrane-like behavior of MP BH

black rings dominate entropically in ultraspinning regime

Towards completing the phase diagram

◀ based on analogy with phase diagram for KK BHs on torus:
extrapolate to $j = \mathcal{O}(1)$ regime

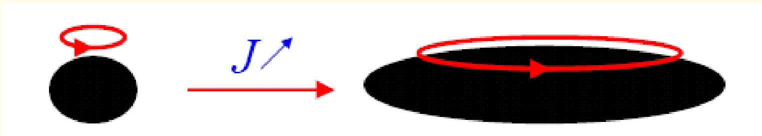
- proposal for phase diagram of stationary BHs (one angular momentum) in asymptotically flat space: main sequence = MP BH, pinched MP BH, black ring (uniform, non-uniform, localized)



Ultraspinning MP BHs and pinched (lumpy) BHs in $D \geq 6$

MP BH approaches black membrane geometry $\mathbb{R}^2 \times S^{D-4}$ for large J

Emparan,Myers



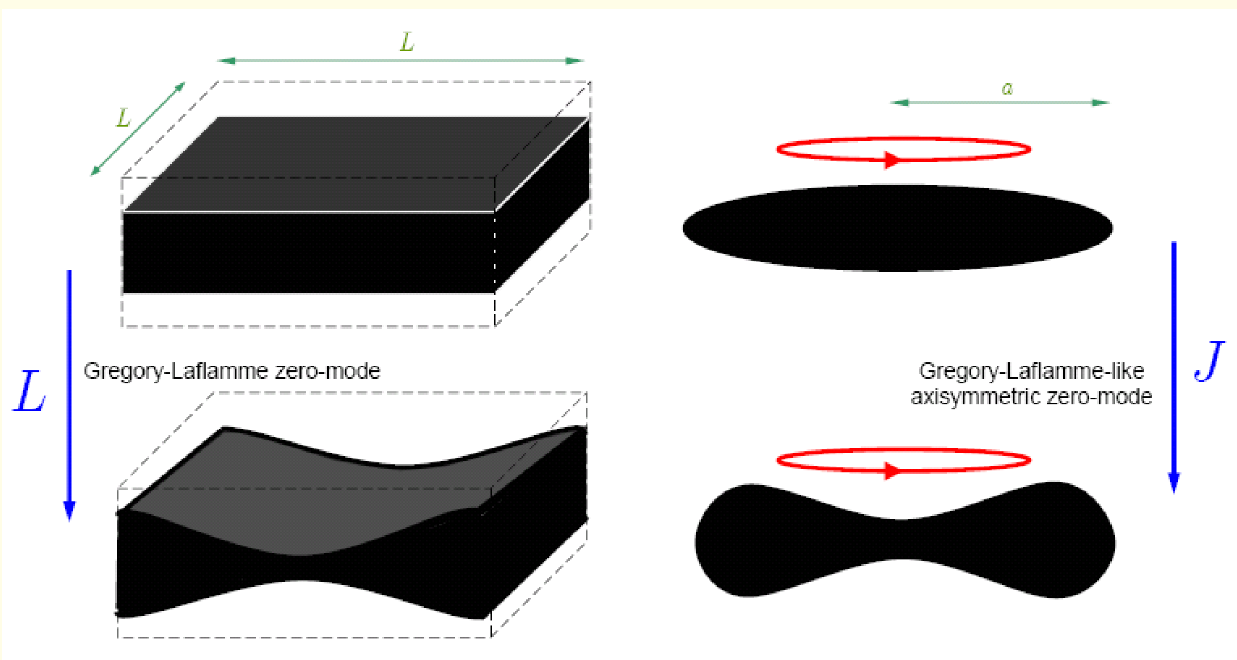
black membrane along rotation plane

black strings exhibit GL instablity:

Gregory,Laflamme/Gubser/Wiseman

in particular: new non-uniform black string at threshold (zero-) mode

- same holds for black branes, in particular black membrane



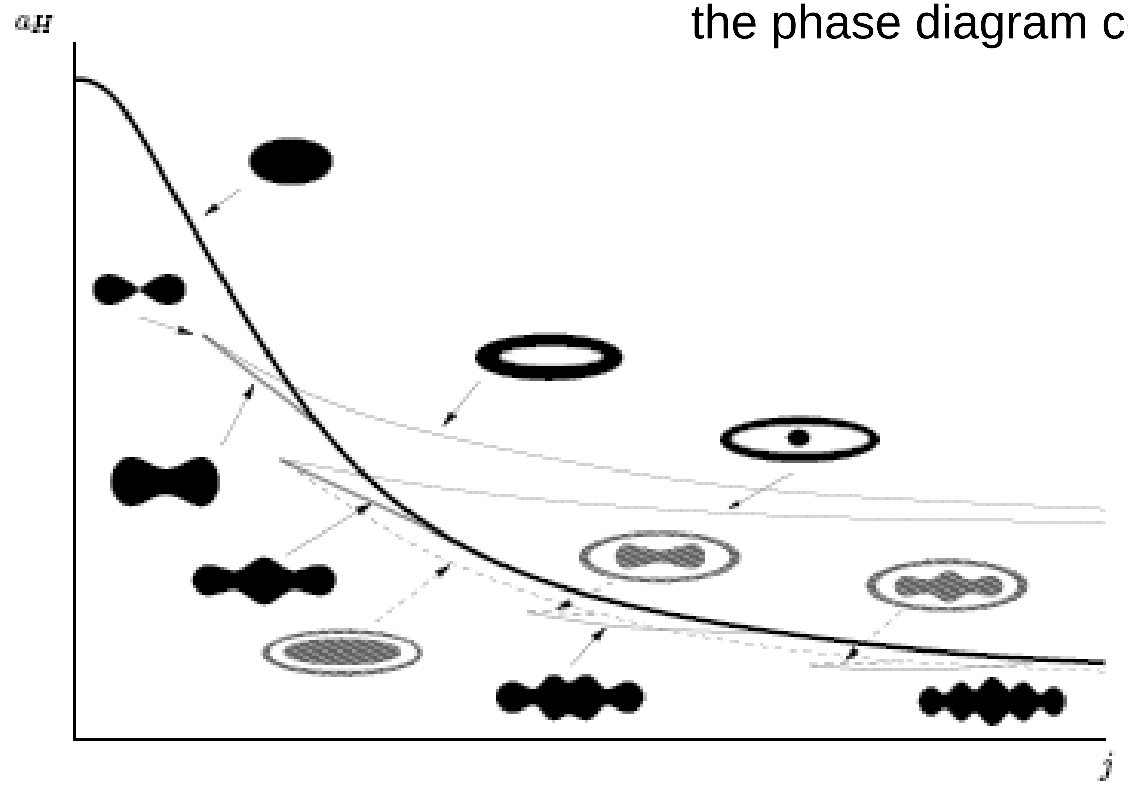
← pinched rotating BH

Black saturns and multi-pinches

most likely features

- main sequence: BH with pinch at rotation axis meets black ring phase
- infinite sequence of pinched BHs emanating from BH curve (from copies of the GL zero mode)
- upper black Saturn curve + merger to circular pinch

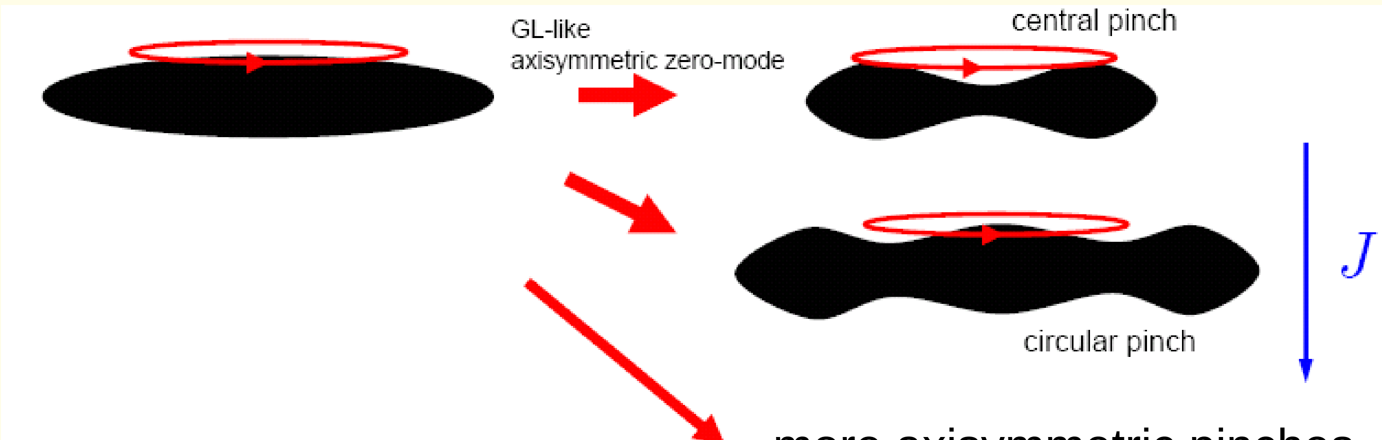
less compelling arguments for: pancaked + pinched black Saturns (but admit a simple and natural way for completing the phase diagram consistent with available info)



Multi-pinched BHs from axisymmetric zero-modes

for black string: integer multiples of the GL zero-mode also give rise to new non-uniform solutions (with repeated pattern of wiggles)

⇨ apply to pancaked (membrane-like) rotating BH



more axisymmetric pinches

- not yet found explicitly (need presumably numerics, cf. non-uniform string)
- necessary to complete phase diagram
  - will become black rings/black Saturns when pinched through
- pinched plasma balls in N=4 SYM recently found
  - dual to large pinched black holes in AdS

Lahiri,Minwalla